16. Probability

Question 1.
Events A and B are independent if
(a) P (A ∩ B) = P (A/B) P (B)
(b) P (A ∩ B) = P (B/A) P (A)
(c) P (A ∩ B) = P (A) + P (B)
(d) P (A ∩ B) = P (A) × P (B)

Answer: (d) P (A ∩ B) = P (A) × P (B)
Events are said to be independent if the occurrence or non-occurrence of one event does not affect the probability of the occurrence or non-occurrence of the other.
Now, by the multiplication theorem,
P(A ∩ B) = P(A) × P(B/A) ……….. 1
Since A and B are independent events,
So, P(B/A) = P(B)
From equation 1, we get
P(A ∩ B) = P(A) × P(B)

Question 2.
A single letter is selected at random from the word PROBABILITY. The probability that it is a vowel is
(a) 2/11
(b) 3/11
(c) 4/11
(d) 5/11

Answer: (b) 3/11
There are 11 letters in the word PROBABILITY out of which 1 can be selected in $^{11}C_1$ ways.
So, exhaustive number of cases = 11
There are 3 vowels i.e. A, I, O
So, the favorable number of cases = 3
Hence, the required probability = 3/11

Question 3.
A die is rolled, find the probability that an even prime number is obtained
(a) 1/2
(b) 1/3

(c) 1/4
(d) 1/6

Answer: (d) 1/6
When a die is rolled, total number of outcomes = 6 (1, 2, 3, 4, 5, 6)
Total even number = 3 (2, 4, 6)
Number of even prime number = 1 (2)
So, the probability that an even prime number is obtained = 1/6

Question 4.
When a coin is tossed 8 times getting a head is a success. Then the probability that at least 2 heads will occur is
(a) 247/265
(b) 73/256
(c) 247/256
(d) 27/256

Answer: (c) 247/256
Let x be number a discrete random variable which denotes the number of heads obtained in n (in this question n = 8)
The general form for probability of random variable x is
P(X = x) = nCx × px × qn-x
Now, in the question, we want at least two heads
Now, p =q = 1/2
So, $P(X \geq 2) = {}^8C_2 \times (1/2)^2 \times (1/2)^{8-2}$
$\Rightarrow P(X \geq 2) = {}^8C_2 \times (1/2)^2 \times (1/2)^6$
$\Rightarrow 1 - P(X < 2) = {}^8C_0 \times (1/2)^0 \times (1/2)^8 + {}^8C_1 \times (1/2)^1 \times (1/2)^{8-1}$
$\Rightarrow 1 - P(X < 2) = (1/2)^8 + 8 \times (1/2)^1 \times (1/2)^7$
$\Rightarrow 1 - P(X < 2) = 1/256 + 8 \times (1/2)^8$
$\Rightarrow 1 - P(X < 2) = 1/256 + 8/256$
$\Rightarrow 1 - P(X < 2) = 9/256$
$\Rightarrow P(X < 2) = 1 - 9/256$
$\Rightarrow P(X < 2) = (256 - 9)/256$
$\Rightarrow P(X < 2) = 247/256$

Question 5.
The probability that the leap year will have 53 sundays and 53 monday is
(a) 2/3
(b) 1/2

(c) 2/7
(d) 1/7

Answer: (d) 1/7
In a leap year, total number of days = 366 days.
In 366 days, there are 52 weeks and 2 days.
Now two days may be
(i) Sunday and Monday
(ii) Monday and Tuesday
(iii) Tuesday and Wednesday
(iv) Wednesday and Thursday
(v) Thursday and Friday
(vi) Friday and Saturday
(vii) Saturday and Sunday
Now in total 7 possibilities, Sunday and Monday both come together is 1 time.
So probabilities of 53 Sunday and Monday in a leap year = 1/7

Question 6.
Let A and B are two mutually exclusive events and if P(A) = 0.5 and $P(\bar{B})$ =0.6 then P(AUB) is
(a) 0
(b) 1
(c) 0.6
(d) 0.9

Answer: (d) 0.9
Given, A and B are two mutually exclusive events.
So, $P(A \cap B) = 0$
Again given P(A) = 0.5 and $P(\bar{B}) = 0.6$
$P(B) = 1 - P(\bar{B}) = 1 - 0.6 = 0.4$
Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$\Rightarrow P(A \cup B) = P(A) + P(B)$
$\Rightarrow P(A \cup B) = 0.5 + 0.4 = 0.9$

Question 7.
Seven white balls and three black balls are randomly placed in a row. The probability that no two black balls are placed adjacently equals
(a) 1/2
(b) 7/15
(c) 2/15
(d) 1/3

Answer: (b) 7/15
While placing 7 while balls in a row, total gaps = 8
3 black balls can be placed in 8 gaps = C = (8 × 7 × 6)/(3 × 2 × 1) = 8 × 7 = 56
So, the total number of ways of arranging white and black balls such that no two black balls are adjacent = 56 × 3! × 7!
Actual number of arrangement possible with 7 white and 3 black balls = (7 + 3)! = 10!
So, the required Probability = (56 × 3! × 7!)/10!
= (56 × 3! × 7!)/(10 × 9 × 8 × 7!)
= (56 × 3!)/(10 × 9 × 8)
= (56 × 3 × 2 × 1)/(10 × 9 × 8)
= (7 × 3 × 2 × 1)/(10 × 9)
= (7 × 2)/(10 × 3)
= 7/(5 × 3)
= 7/15

Question 8.
The events A, B, C are mutually exclusive events such that P (A) = (3x + 1)/3, P (B) = (x – 1)/4 and P (C) = (1 – 2x)/4. The set of possible values of x are in the interval
(a) [1/3, 1/2]
(b) [1/3, 2/3]
(c) [1/3, 13/3]
(d) [0, 1]

Answer: (a) [1/3, 1/2]
P(A) = (3x + 1)/3
P(B) = (x – 1)/4
P(C) = (1 – 2x)/4
These are mutually exclusive events.
⇒ -1 ≤ 3x ≤ 2, -3 ≤ x ≤ 1, -1 ≤ 2x ≤ 1
⇒ -1/3 ≤ x ≤ 2/3, -2 ≤ x ≤ 1, -1/2 ≤ x ≤ 1/2
Also, 0 ≤ (3x + 1)/3 + (x – 1)/4 + (1 – 2x)/4 ≤ 1
⇒ 1/3 ≤ x ≤ 13/3
⇒ max {-1/3, -3, -1/2, 1/3} ≤ x ≤ min {2/3, 1/2, 1, 13/3}
⇒ 1/3 ≤ x ≤ 1/2
⇒ x ∈ [1/3, 1/2]

Question 9.
A bag contains 2 red, 3 green and 2 blue balls. Two balls are drawn at random. The probability that none of the balls drawn is blue is
(a) 10/21
(b) 11/21

(c) 2/7
(d) 5/7

Answer: (a) 10/21
Total number of balls = 2 + 3 + 2 = 7
Two balls are drawn.
Now, P(none of them is blue) = ${}^5C_2 / {}^7C_2$
= {(5 × 4)/(2 × 1)}/{(7 × 6)/(2 × 1)}
= (5 × 4)/(7 × 6)
= (5 × 2)/(7 × 3)
= 10/21

Question 10.
If 4-digit numbers greater than 5000 are randomly formed from the digits 0, 1, 3, 5 and 7, then the probability of forming a number divisible by 5 when the digits are repeated is
(a) 1/5
(b) 2/5
(c) 3/5
(d) 4/5

Answer: (b) 2/5
Given digits are 0, 1, 3, 5, 7
Now we have to form 4 digit numbers greater than 5000.
So leftmost digit is either 5 or 7.
When digits are repeated
Number of ways for filling left most digit = 2
Now remaining 3 digits can be filled = 5 × 5 × 5
So total number of ways of 4 digits greater than 5000 = 2 × 5 × 5 × 5 = 250
Again a number is divisible by 5 if the unit digit is either 0 or 5. So there are 2 ways to fill the unit place.
So total number of ways of 4 digits greater than 5000 and divisible by 5 = 2 × 5 × 5 × 2 = 100
Now probability of 4 digit numbers greater than 5000 and divisible by 5
= 100/250
= 2/5

Question 11.
Events A and B are said to be mutually exclusive iff
(a) P (A U B) = P (A) + P (B)
(b) P (A ∩ B) = P (A) × P (B)
(c) P(A U B) = 0
(d) None of these

Answer: (a) P (A U B) = P (A) + P (B)
If A and B are mutually exclusive events,
Then $P(A \cap B) = 0$
Now, by the addition theorem,
$P(A \cup B) = P(A) + P(B) – P(A \cap B)$
$\Rightarrow P(A \cup B) = P(A) + P(B)$

Question 12.
Two numbers are chosen from {1, 2, 3, 4, 5, 6} one after another without replacement. Find the probability that the smaller of the two is less than 4.
(a) 4/5
(b) 1/15
(c) 1/5
(d) 14/15

Answer: (a) 4/5
Total number of ways of choosing two numbers out of six = $^6C_2$ = (6 × 5)/2 = 3 × 5 = 15
If smaller number is chosen as 3 then greater has choice are 4, 5, 6
So, total choices = 3
If smaller number is chosen as 2 then greater has choice are 3, 4, 5, 6
So, total choices = 4
If smaller number is chosen as 1 then greater has choice are 2, 3, 4, 5, 6
So, total choices = 5
Total favourable case = 3 + 4 + 5 = 12
Now, required probability = 12/15 = 4/5

Question 13.
If the integers m and n are chosen at random between 1 and 100, then the probability that the number of the from $7^m + 7^n$ is divisible by 5 equals
(a) 1/4
(b) 1/7
(c) 1/8
(d) 1/49

Answer: (a) 1/4
Since m and n are selected between 1 and 100,
Hence total sample space = 100 × 100
Again, 71 = 7, 72 = 49, 73 = 343, 74 = 2401, 75 = 16807, etc
Hence 1, 3, 7 and 9 will be the last digit in the power of 7.
Now, favourable number of case are
→ 1,1 1,2 1,3 ………… 1,100
2,1 2,2 2,3 ………… 2,100

3,1 3,2 3,3 ………… 3,100
……………….
……………….
100,1 100,2 100,3 ………… 100,100
Now, for m = 1, n = 3, 7, 11, ………, 97
So, favourable cases = 25
Again for m = 2, n = 4, 8, 12, ………, 100
So, favourable cases = 25
Hence for every m, favourable cases = 25
So, total favourable cases = 100 × 25
Required Probability = (100 × 25)/(100 × 100)
= 25/100
= 1/4

Question 14.
A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn. Then the probability that they both are diamonds is
(a) 84/452
(b) 48/452
(c) 84/425
(d) 48/425

Answer: (d) 48/425
Total number of cards = 52 and one card is lost.
Case 1: if lost card is a diamond card
Total number of cards = 51
Number of diamond cards = 12
Now two cards are drawn.
P(both cards are diamonds) = ${}^{12}C_2 / {}^{51}C_2$
Total number of cards = 52 and one card is lost.
Case 2: If lost card is not a diamond card
Total number of cards = 51
Number of diamond cards = 13
Now two cards are drawn.
P(both cards are diamonds) = ${}^{13}C_2 / {}^{51}C_2$
Now probability that both cards are diamond = ${}^{12}C_2 / {}^{51}C_2 + {}^{13}C_2 / {}^{51}C_2$
= $({}^{12}C_2 + {}^{13}C_2) / {}^{51}C_2$
= {(12 × 11)/(2 × 1) + (13 × 12)/(2 × 1)}/{(51 × 50)/(2 × 1)}
= (12 × 11 + 13 × 12)/(51 × 50)
= (132 + 156)/2550
= 288/2550
= 96/850 (288 and 2550 divided by 3)

= 48/425 (96 and 850 divided by 2)
So probability that both cards are diamond is 48/425

Question 15.
The probability that when a hand of 7 cards is drawn from a well-shuffled deck of 52 cards, it contains 3 Kings is
(a) 1/221
(b) 5/716
(c) 9/1547
(d) None of these

Answer: (c) 9/1547
Total number of cards = 52
Number of king card = 4
Now, 7 cards are drawn from 52 cards.
P (3 cards are king) = $\{^{4}C_{3} \times {}^{48}C_{4}\}/\, {}^{52}C_{7}$
= {4 × (48 × 47 × 46 × 45)/(4 × 3 × 2 × 1)}/{(52 × 51 × 50 × 49 × 48 × 47 × 46)/(7 × 6 × 5 × 4 × 3 × 2 × 1)}
= {4 × (48 × 47 × 46 × 45) × (7 × 6 × 5 × 4 × 3 × 2 × 1)}/{(4 × 3 × 2 × 1) × {(52 × 51 × 50 × 49 × 48 × 47 × 46)}
= (7 × 6 × 5 × 4 × 45)/(52 × 51 × 50 × 49)
= (6 × 5 × 4 × 45)/(52 × 51 × 50 × 7)
= (6 × 4 × 45)/(7 × 52 × 51 × 10)
= (6 × 45)/(7 × 13 × 51 × 10)
= (6 × 3)/(7 × 13 × 17 × 2)
= (3 × 3)/(7 × 13 × 17)
= 9/1547

Question 16.
A die is rolled, then the probability that an even number is obtained is
(a) 1/2
(b) 2/3
(c) 1/4
(d) 3/4

Answer: (a) 1/2
When a die is rolled, total number of outcomes = 6 (1, 2, 3, 4, 5, 6)
Total even number = 3 (2, 4, 6)
So, the probability that an even number is obtained = 3/6 = 1/2

Question 17.
Six boys and six girls sit in a row at random. The probability that the boys and girls sit

alternatively is
(a) 1/462
(b) 11/462
(c) 5/121
(d) 7/123

Answer: (a) 1/462
Given, 6 boys and 6 girls sit in a row at random.
Then, the total number of arrangement of 6 boys and 6 girls = arrangement of 12 persons = 12!
Now, boys and girls sit alternatively.
So, the total number of arrangement = $2 \times 6! \times 6!$
Now, P(boys and girls sit alternatively) = $(2 \times 6! \times 6!)/12!$
$= (2\times 6 \times 5! \times 6!)/(12 \times 11!)$
$= (5! \times 6!)/11!$
$= (5 \times 4 \times 3 \times 2 \times 1 \times 6!)/(11 \times 10 \times 9 \times 8 \times 7 \times 6!)$
$= (5 \times 4 \times 3 \times 2)/(11 \times 10 \times 9 \times 8 \times 7)$
$= (4 \times 3)/(11 \times 9 \times 8 \times 7)$
$= 3/(11 \times 9 \times 2 \times 7)$
$= 1/(11 \times 3 \times 2 \times 7)$
$= 1/462$

Question 18.
Two dice are thrown the events A, B, C are as follows A: Getting an odd number on the first die. B: Getting a total of 7 on the two dice. C: Getting a total of greater than or equal to 8 on the two dice. Then AUB is equal to
(a) 15
(b) 17
(c) 19
(d) 21

Answer: (d) 21
When two dice are thrown, then total outcome = $6 \times 6 = 36$
A: Getting an odd number on the first die.
A = {(1, 1), (1, 2), (1, 3), (1, 4),(1, 5), (1, 6), (3, 1), (3, 2), (3, 3), (3, 4),(3, 5), (3, 6), (5, 1), (5, 2), (5, 3), (5, 4),(5, 5), (5, 6)}
Total outcome = 18
B: Getting a total of 7 on the two dice.
B = {(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)}
Total outcome = 6
C: Getting a total of greater than or equal to 8 on the two dice.
C = {(2, 6), (3, 5), (3, 6), (4, 4),(4, 5), (4, 6), (5, 3), (5, 4), (5, 5), (5, 6),(6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}
Total outcome = 15

Now $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$\Rightarrow n(A \cup B) = 18 + 6 - 3$
$\Rightarrow n(A \cup B) = 21$

Question 19.
Let A and B are two mutually exclusive events and if $P(A) = 0.5$ and $P(\overline{B}) = 0.6$ then $P(AUB)$ is
(a) 0
(b) 1
(c) 0.6
(d) 0.9

Answer: (d) 0.9
Given, A and B are two mutually exclusive events.
So, $P(A \cap B) = 0$
Again given $P(A) = 0.5$ and $P(\overline{B}) = 0.6$
$P(B) = 1 - P(\overline{B}) = 1 - 0.6 = 0.4$
Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$\Rightarrow P(A \cup B) = P(A) + P(B)$
$\Rightarrow P(A \cup B) = 0.5 + 0.4 = 0.9$

Question 20.
A certain company sells tractors which fail at a rate of 1 out of 1000. If 500 tractors are purchased from this company, what is the probability of 2 of them failing within first year
(a) $e^{-1/2}/2$
(b) $e^{-1/2}/4$
(c) $e^{-1/2}/8$
(d) none of these

Answer: (c) $e^{-1/2}/8$
This question is based on Poisson distribution.
Now, $\lambda = np = 500 \times (1/1000) = 500/1000 = 1/2$
Now, $P(x = 2) = \{e^{-1/2} \times (1/2)^2\}/2! = e^{-1/2}/(4 \times 2) = e^{-1/2}/8$
