CUET (UG)

Mathematics Sample Paper - 06

Solved

Time Allowed: 50 minutes **Maximum Marks: 200**

General Instructions:

1. There are 50 questions in this paper.
2. Section A has 15 questions. Attempt all of them.
3. Attempt any 25 questions out of 35 from section B.
4. Marking Scheme of the test:

a. Correct answer or the most appropriate answer: Five marks (+5).

b. Any incorrectly marked option will be given minus one mark (-1).

c. Unanswered/Marked for Review will be given zero mark (0).

Section A

1. If A and B are symmetric matrices of order n ($A \neq B$), then **[5]**

a) A + B is skew symmetric

b) A + B is a diagonal matrix

c) A + B is a zero matrix

d) A + B is symmetric

2. If A is any square matrix then which of the following is not symmetric? **[5]**

a) $A+A^t$

b) $A - A^t$

c) A^tA

d) AA^t

3. The trace of the matrix $A = \begin{bmatrix} 1 & -5 & 7 \\ 0 & 7 & 9 \\ 11 & 8 & 9 \end{bmatrix}$ is **[5]**

a) 12

b) 17

c) 3

d) 25

4. The slope of the normal to the curve $y = 2x^2 + 3 \sin x$ at $x = 0$ is **[5]**

a) 3

b) $\frac{1}{3}$

c) $-\frac{1}{3}$

d) -3

5. If the function $f(x) = 2x^2 - kx + 5$ is increasing on (1, 2), then k lies in the interval **[5]**

a) $(4, \infty)$

b) $(-\infty, 8)$

c) $(8, \infty)$ d) $(-\infty, 4)$

6. The values of a for which $y = x^2 + ax + 25$ touches the x-axis are **[5]**

a) 0 b) ± 10

c) ± 5 d) 4, - 6

7. $\int \frac{(ax+b)}{(cx+d)} dx = ?$ **[5]**

a) $\frac{ax}{c} + \log |cx + d| + C$ b) $\frac{a}{c} + \log |cx + d| + C$

c) $\frac{ax}{c} + \frac{(bc-ad)}{c^2} \log |cx + d| + C$ d) None of these

8. $\int \cos 3x \sin 2x \, dx = ?$ **[5]**

a) $-\frac{1}{2}\cos x + \frac{1}{10}\cos 5x + C$ b) $-\frac{1}{2}\sin x + \frac{1}{10}\sin 5x + C$

c) $\frac{1}{2}\cos x - \frac{1}{10}\cos 5x + C$ d) None of these

9. $\int \frac{10x^9 + 10^x \log_e 10 dx}{x^{10} + 10^x}$, equals **[5]**

a) $(10^x - x^{10})^{-1} + C$ b) $10^x + x^{10} + C$

c) $10^x - x^{10} + C$ d) $\log (10^x + x^{10}) + C$

10. The area of the region bounded by the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$is **[5]**

a) $20\pi^2$ sq. units b) 25π sq. units

c) 20π sq. units d) $16\pi^2$ sq. units

11. The general solution of the differential equation $(x^2 + x + 1)\, dy + (y^2 + y + 1)\, dx = 0$ is $(x + y + 1) = A(1 + Bx + Cy + Dxy)$, where B, C and D are constants and A is parameter. What is B equal to? **[5]**

a) 2 b) -1

c) 1 d) -2

12. The general solution of the DE $x^2 \frac{dy}{dx} = x^2 + xy + y^2$ is **[5]**

a) $\tan^{-1} \frac{y}{x} = \log x + C$ b) $\tan^{-1} \frac{y}{x} = \log y + C$

c) $\tan^{-1}\frac{x}{y}=\log x+C$

d) none of these

13. A feasible region of a system of linear inequalities is said to be ..., if it can be enclosed within a circle. **[5]**

a) unbounded

b) In squared form

c) bounded

d) in circled form

14. If A and B are two independent events with $\mathrm{P(A)}=\frac{3}{5}$ and $P(B)=\frac{4}{9}$, then $\mathrm{P}\left(\mathrm{A}'\cap\mathrm{B}'\right)$ equals **[5]**

a) $\frac{2}{9}$

b) $\frac{8}{45}$

c) $\frac{4}{15}$

d) $\frac{1}{3}$

15. If P(A) $=\frac{6}{11}$, P(B) $=\frac{5}{11}$ and P(A ∪ B) $=\frac{7}{11}$.find P(A∩B) **[5]**

a) $\frac{4}{13}$

b) $\frac{5}{17}$

c) $\frac{5}{11}$

d) $\frac{4}{11}$

Section B

Attempt any 25 questions

16. The relation **congruence modulo m** on the set Z of all integers is a relation of type **[5]**

a) Transitive only

b) Symmetric only

c) Reflexive only

d) Equivalence

17. The number of solutions of the equation $\sin^{-1}x-\cos^{-1}x=\sin^{-1}\left(\frac{1}{2}\right)$is **[5]**

a) 2

b) 1

c) 3

d) Infinite.

18. The matrix $\begin{bmatrix}0 & 5 & -7\\ -5 & 0 & 11\\ 7 & -11 & 0\end{bmatrix}$ is **[5]**

a) a diagonal matrix

b) a skew-symmetric matrix

c) a symmetric matrix

d) an upper triangular matrix

19. If A and B are any 2 $\times$ 2 matrices, then det. (A+B) = 0 implies **[5]**

a) detA + det B = 0

b) det A = 0 or det B = 0

c) None of these

d) det A = 0 and det B = 0

20. The equations x + 2y + 2z = 1 and 2x + 4y + 4z = 9 have **[5]**

a) no solution

b) only one solution

c) only two solutions

d) infinitely many solutions

21. The greatest value of c ∈ R for which the system of linear equations x - cy - cz = 0, cx - y + cz = 0, cx + cy - z = 0 has a non-trivial solution, is **[5]**

a) -1

b) 2

c) $\frac{1}{2}$

d) 0

22. If y = log $\left(\frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}-x}\right)$ then $\frac{dy}{dx}$ =? **[5]**

a) $\frac{-2}{\sqrt{1+x^2}}$

b) $\frac{2\sqrt{1+x^2}}{x^2}$

c) none of these

d) $\frac{2}{\sqrt{1+x^2}}$

23. If the function f(x) $= \begin{cases} \frac{k\cos x}{(\pi-2x)}, & \text{when } x \neq \frac{\pi}{2} \\ 3, & \text{when } x = \frac{\pi}{2} \end{cases}$ be continuous at x $= \frac{\pi}{2}$, then the value of k is **[5]**

a) 6

b) 3

c) -3

d) -5

24. Let f(x) = a + b |x| + c $|x|^4$, where a, b, and c are real constants. **[5]**

a) none of these

b) $4+[x]^2 \neq 0$

c) c = 0

d) a = 0

25. If y = $\sqrt{\frac{\sec x-1}{\sec x+1}}$ then $\frac{dy}{dx}$ = ? **[5]**

a) $\frac{1}{2}\sec^2\frac{x}{2}$

b) $\frac{-1}{2}cosec^2\frac{x}{2}$

c) $\sec^2 x$

d) none of these

26. If y = a sin mx + b cos m x, then $\frac{d^2y}{dx^2}$ is equal to **[5]**

a) my_1

b) None of these

c) $-m^2y$

d) m^2y

27. The point on the curve $y^2 = x$, where the tangent makes an angle of $\frac{\pi}{4}$ with x-axis is **[5]**

a) (4, 2)

b) $\left(\frac{1}{4}, \frac{1}{2}\right)$

c) (1, 1)

d) $\left(\frac{1}{2}, \frac{1}{4}\right)$

28. The sum of two non-zero numbers is 8, the minimum value of the sum of their reciprocals is **[5]**

a) $\frac{1}{2}$

b) $\frac{1}{8}$

c) $\frac{1}{4}$

d) none of these

29. Function f(x) = $\log_a$ x is increasing on R , if **[5]**

a) a < 1

b) 0 < a < 1

c) a > 1

d) a > 0

30. The function $f(x) = 4 - 3x + 3x^2 - x^3$ is decreasing **[5]**

a) Strictly decreasing on R

b) Strictly increasing on R

c) Decreasing on R

d) Increasing on R

31. $\int \frac{(x^2+1)}{(x^4+1)} dx = ?$ **[5]**

a) $\frac{1}{\sqrt{2}} \tan^{-1}\left\{\frac{1}{\sqrt{2}}\left(\mathrm{x} - \frac{1}{\mathrm{x}}\right)\right\} + \mathrm{C}$

b) $\frac{1}{\sqrt{2}} \tan^{-1}\left(x - \frac{1}{x}\right) + C$

c) $\frac{1}{\sqrt{2}} \cot^{-1}\left\{\left(x - \frac{1}{x}\right)\right\} + C$

d) None of these

32. $\int x^2 e^{x^3} dx$ equals **[5]**

a) $\frac{1}{2} e^{x^3} + C$

b) $\frac{1}{2} e^{x^2} + C$

c) $\frac{1}{3} e^{x^3} + C$

d) $\frac{1}{3} e^{x^2} + C$

33. $\int_0^{\pi/2} \cos^2 x dx = ?$ **[5]**

a) π

b) $\frac{\pi}{2}$

c) 1

d) $\frac{\pi}{4}$

34. The value of the integral $\int_{\frac{1}{3}}^{1} \frac{(x-x^3)^{\frac{1}{3}}}{x^4} dx$ is **[5]**

a) 4

b) 6

c) 0

d) 3

35. The area of the region (in square units) bounded by the curve $x^2 = 4y$, line x = 2 and x-axis is **[5]**

a) $\frac{8}{3}$

b) 1

c) $\frac{2}{3}$

d) $\frac{4}{3}$

36. The general solution of the DE $\frac{dy}{dx} = (\sqrt{1-x^2})(\sqrt{1-y^2})$ is **[5]**

a) none of these

b) $2\sin^{-1}y - \sin^{-1}x = C$

c) $2\sin^{-1} y - \sin^{-1} x = x\sqrt{1-x^2} + C$

d) $\sin^{-1} y - \sin^{-1} x = x\sqrt{1-x^2} + C$

37. The general solution of the differential equation $(x^2 + x + 1)\,dy + (y^2 + y + 1)\,dx = 0$ is $(x + y + 1) = A(1 + Bx + Cy + Dxy)$, where B, C and D are constants and A is parameter. What is D equal to? **[5]**

a) -1

b) 1

c) 2

d) -2

38. The solution of the differential equation $2x \cdot \frac{dy}{dx} - y = 3$ represents a family of **[5]**

a) circles

b) parabolas

c) straight lines

d) ellipses

39. Show that the points A(1, – 2, – 8), B (5, 0, – 2) and C (11, 3, 7) are collinear, and find the ratio in which B divides AC. **[5]**

a) 3 :2

b) 2 :4

c) 2 : 3	d) 2 :1

40. The projection of the vector $\hat{i} + \hat{j} + \hat{k}$ along the vector of $\hat{j}$ is **[5]**

a) -1
b) 2
c) 0
d) 1

41. Let the vectors $\vec{a}$ and $\vec{b}$ be such that $|\vec{a}| = 3$ and $|b| = \frac{\sqrt{2}}{3}$, then $\vec{a} \times \vec{b}$ is a unit vector if the angle between $\vec{a}$ and $\vec{b}$ is **[5]**

a) $\frac{\pi}{4}$
b) $\frac{\pi}{3}$
c) $\frac{\pi}{6}$
d) $\frac{\pi}{2}$

42. The position vector of the point which divides the join of points with position vectors $\vec{a} + \vec{b}$ and $2\vec{a} - \vec{b}$ in the ratio 1 : 2 is **[5]**

a) $\frac{3\vec{a}+2\vec{b}}{3}$
b) $\frac{4\vec{a}+\vec{b}}{3}$
c) $\vec{a}$
d) $\frac{5\vec{a}-\vec{b}}{3}$

43. If $\vec{a} = (\hat{i} + 2\hat{j} - 3\hat{k})$ and $\vec{b} = (3\hat{i} - \hat{j} + 2\hat{k})$ then the angle between $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$ is **[5]**

a) $\frac{\pi}{2}$
b) $\frac{2\pi}{3}$
c) $\frac{\pi}{4}$
d) $\frac{\pi}{3}$

44. If O is the origin and P(1 , 2, -3) is a given point, then the equation of the plane through P and perpendicular to OP is **[5]**

a) x - 2y + 3z = 12
b) x - 2y - 3z = 14
c) None of these
d) x + 2y - 3z = 14

45. If a plane meets the coordinate axes in A, B and C such that the centroid of $\triangle$ABC is (1, 2, 4), then the equation of the plane is **[5]**

a) x + 2y + 4z = 7
b) 4x + 2y + z = 12
c) x + 2y + 4z = 6
d) 4x + 2y + z = 7

46. The distance between the point (3, 4, 5) and the point where the line $\frac{x-3}{1} = \frac{y-4}{2} = \frac{z-5}{2}$ meets the plane x + y + z = 17, is **[5]**

a) 1

b) None of these

c) 2

d) 3

47. A speaks truth in 75% cases and B speaks truth in 80% cases. Probability that they contradict each other in a statement, is **[5]**

a) $\frac{2}{5}$

b) $\frac{13}{20}$

c) $\frac{7}{20}$

d) $\frac{3}{5}$

48. Assume that in a family, each child is equally likely to be a boy or a girl. A family with three children is chosen at random. The probability that the eldest child is a girl given that the family has at least one girl is **[5]**

a) $\frac{1}{3}$

b) $\frac{4}{7}$

c) $\frac{2}{3}$

d) $\frac{1}{2}$

49. The probability that a person is not a swimmer is 0.3. The probability that out of 5 persons 4 are swimmers is **[5]**

a) ${}^{5}C_{1}(0.7)(0.3)^{4}$

b) $(0.7)^{4}(0.3)$

c) ${}^{5}C_{4}(0.7)(0.3)^{4}$

d) ${}^{5}C_{4}(0.7)^{4}(0.3)$

50. If A and B are events such that $P(A)=\frac{1}{2}, P(B)=\frac{7}{12}$ and $P\left(A' \cup B'\right)=\frac{1}{4}$, then A and B are **[5]**

a) Independent and mutually exclusive

b) Independent

c) None of these

d) Mutually exclusive

Solutions

Section A

1.

(d) A + B is symmetric

Explanation: Sum of two symmetric matrices is also symmetric.

2.

(b) A - A^t

Explanation: For every square matrix (A – A') is always skew – symmetric.

3.

(b) 17

Explanation: As the trace of a matrix is the sum of all diagonal elements,

Therefore, 1 + 7 + 9 = 17

Trace = 17.

Which is the required solution.

4.

(c) $-\frac{1}{3}$

Explanation: Equation of the curve is given as, $y = 2x^2 + 3sinx$

Clearly, slope of the tangent at x = 0 is

$$\left.\frac{dy}{dx}\right]_{x=0} = 4x + 3\cos x\Big]_{x=0} = 0 + 3\cos 0 = 3$$

Therefore, the slope of the normal to the curve at x = 0 is

$$= \frac{-1}{\text{Slope of the tangent at } x=0} = \frac{-1}{3}$$

Therefore, the slope of the normal to the curve y = $2x^2$ + 3 sin x at x = 0 is $-\frac{1}{3}$

5.

(d) (-∞, 4)

Explanation: $f(x) = 2x^2 - kx + 5$

$f'(x) = 4x - k$

for f(x) to be increasing, we must have

f(x) > 0

4x - k > 0

K < 4 x

since $x \in [1, 2], 4x \in [4, 8]$

so, the minimum value of 4 x is 4.

since K < 4x, K < 4.

k $\in (-\infty, 4)$

6.

(b) $\pm$ 10

Explanation: Given, $y = x^2 + ax + 25 \Rightarrow \frac{dy}{dx} = 2x + a$ ...(i)

The curve (i) touches the x-axis implies that x-axis is tangent to curve at meeting point.

$\Rightarrow \frac{dy}{dx} = 0 \Rightarrow 2x + a = 0$

$\Rightarrow x = -\frac{a}{2}$

$\Rightarrow$ The co-ordinate of meeting point are $\left(-\frac{a}{2}, 0\right)$, therefore it satisfies the curve (i)

$\Rightarrow \left(-\frac{a}{2}\right)^2 + a\left(-\frac{a}{2}\right) + 25 = 0$

$\Rightarrow \frac{a^2}{4} - \frac{a^2}{2} + 25 = 0 \Rightarrow -a^2 + 100 = 0$

$\Rightarrow a = \pm 10$

7.

(c) $\frac{ax}{c} + \frac{(bc-ad)}{c^2}\log|cx + d| + C$

Explanation: Given :

$\int \frac{(ax+b)}{(cx+d)}dx = \int \frac{ax}{cx+d} + \frac{b}{cx+d}dx$

$= a\int \frac{x}{cx+d} \times \frac{c}{c}dx + b\int \frac{1}{cx+d}dx$

$= \frac{a}{c}\left(\int \frac{cx+d}{cx+d}dx - \frac{d}{cx+d}\right) + b\log|cx + d| + c$

$= \frac{a}{c}\left(x - \frac{d}{c}\log|cx + d|\right) + \frac{b}{c}\log|cx + d| + c$

$= \frac{a}{c}x + \frac{(bc-ad)}{c^2}\log|cx + d| + c.$

Which is the required solution.

8.

(d) None of these

Explanation: Given:

$\int \cos 3x \sin 2x\, dx = \frac{1}{2}\int 2\cos 3x \sin 2x\, dx$

$= \frac{1}{2}\int \sin 5x + \cos x\, dx$

$= \frac{1}{2}\left\{\frac{-\cos 5x}{5} + \frac{\sin x}{1}\right\} + c$

$= -\frac{\cos 5x}{10} + \frac{\sin x}{2} + c.$

Which is the required solution.

9.

(d) $\log(10^x + x^{10}) + C$

Explanation: Let $x^{10} + 10^x = t$

$\Rightarrow (10x^9 + 10^x \log_e 10)dx = dt$

$\Rightarrow \int \frac{10x^9 + 10^x \log_e 10}{x^{10} + 10^x} dx = \int \frac{dt}{t}$

$= \log t + C$

$= \log(x^{10} + 10^x) + C$

10.

(c) 20π sq. units

Explanation: The area of the standard ellipse is given by ; πab. Here, a = 5 and b = 4
Therefore, the area of curve is $\pi(5)(4) = 20\pi$.

11.

(b) -1

Explanation: Given differential equation is

$(x^2 + x + 1)dy + (y^2 + y + 1)dx = 0$

$\Rightarrow (x^2 + x + 1)dy = -(y^2 + y + 1)dx$

$\Rightarrow \frac{dy}{(1 + y + y^2)} = -\frac{dx}{(1 + x + x^2)}$

$\Rightarrow \frac{dx}{(1 + x + x^2)} + \frac{dy}{(1 + y + y^2)} = 0$

$\Rightarrow \int \frac{dx}{\left(x + \frac{1}{2}\right)^2 + \frac{3^2}{4}} + \int \frac{dy}{\left(y + \frac{1}{2}\right)^2 + \frac{3}{4}} = 0$

$$\Rightarrow \int \frac{dx}{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} + \frac{dy}{\left(y+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} = 0 \text{ [on integrating]}$$

$$\Rightarrow \frac{1}{\left(\frac{\sqrt{3}}{2}\right)}\tan^{-1}\left\{\frac{\left(x+\frac{1}{2}\right)}{\frac{\sqrt{3}}{2}}\right\} + \frac{1}{\frac{\sqrt{3}}{2}}\tan^{-1}\left\{\frac{y+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right\} = \frac{2}{\sqrt{3}}\tan^{-1}C_1$$

$$\left[\because \int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}\right]$$

$$\Rightarrow \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2y+1}{\sqrt{3}}\right) = \frac{2}{\sqrt{3}}\tan^{-1}C_1$$

$$\Rightarrow \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + \tan^{-1}\left(\frac{2y+1}{\sqrt{3}}\right) = \tan^{-1}C_1$$

$$\Rightarrow \tan^{-1}\left\{\frac{\left(\frac{2x+1}{\sqrt{3}}\right)+\left(\frac{2y+1}{\sqrt{3}}\right)}{1-\left(\frac{2x+1}{\sqrt{3}}\right)\left(\frac{2y+1}{\sqrt{3}}\right)}\right\} = \tan^{-1}C1 \left[\because \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$$

$$\Rightarrow \frac{\sqrt{3}[(2x+1)+(2y+1)]}{3-(2x+1)\cdot(2y+1)} = C_1$$

$$\Rightarrow 2\sqrt{3}(x+y+1) = 2C(1-x-y-2xy)$$

$$\Rightarrow (x+y+1) = \frac{1}{\sqrt{3}}(1-x-y-2xy)$$

On comparing with $(x + y + 1) = A(1 + Bx + Cy + Dxy)$
Here, A is parameter and B, C and D are constants.
The value of B = -1

12. **(a)** $\tan^{-1}\frac{y}{x} = \log x + C$

Explanation: We have, $x^2\frac{dy}{dx} = x^2 + xy + y^2$

$\frac{dy}{dx} = 1 + \frac{y}{x} + \frac{y^2}{x^2}$

Let y = vx

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$1 + v + v^2 = v + x\frac{dv}{dx}$

$1 + v^2 = x\frac{dv}{dx}$

$\frac{dx}{x} = \frac{dv}{v^2+1}$

On integrating on both sides, we obtain

log x = $\tan^{-1}$v + C

$\tan^{-1}\frac{y}{x} = \log x + c$

13.

(c) bounded

Explanation: A feasible region of a system of linear inequalities is said to be bounded, if it can be enclosed within a circle.

14. **(a)** $\frac{2}{9}$

Explanation: $P\left(A' \cap B'\right) = 1 - P(A \cup B)$

$= 1 - [P(A) + P(B) - P(A \cap B)]$

$= 1 - \left[\frac{3}{5} + \frac{4}{9} - \frac{3}{5} \times \frac{4}{9}\right][\because P(A \cap B) = P(A) \cdot P(B)]$

$= 1 - \left[\frac{27+20-12}{45}\right] = 1 - \frac{35}{45} = \frac{10}{45} = \frac{2}{9}$

15.

(d) $\frac{4}{11}$

Explanation: If P(A) = $\frac{6}{11}$, P(B) = $\frac{5}{11}$ and P(A ∪ B) = $\frac{7}{11}$.

P(A ∪ B) = P(A) + P(B) - P(A ∩ B)

$$\Rightarrow \frac{7}{11} = \frac{6}{11} + \frac{5}{11} - P(A \cap B)$$

$$\Rightarrow P(A \cap B) = \frac{4}{11}$$

Section B

16.

(d) Equivalence

Explanation: Equivalence

17.

(b) 1

Explanation: $\sin^{-1}x - \cos^{-1}x = \sin^{-1}\frac{1}{2}$

$$\Rightarrow \sin^{-1}x - \cos^{-1}x = \frac{\pi}{6}$$

$$\Rightarrow \sin^{-1}x - \left(\frac{\pi}{2} - \sin^{-1}x\right) = \frac{\pi}{6}$$

$$\Rightarrow 2\sin^{-1}x = \frac{\pi}{6} + \frac{\pi}{2} = \frac{2\pi}{3}$$

$$\Rightarrow \sin^{-1}x = \frac{\pi}{3} \Rightarrow x = \frac{\sqrt{3}}{2}$$

Hence, there is only one solution

18.

(b) a skew-symmetric matrix

Explanation: A = $\begin{bmatrix} 0 & 5 & -7 \\ -5 & 0 & 11 \\ 7 & -11 & 0 \end{bmatrix}$

$$A^T = \begin{bmatrix} 0 & -5 & 7 \\ 5 & 0 & -11 \\ -7 & 11 & 0 \end{bmatrix}$$

$$-A = \begin{bmatrix} 0 & 5 & -7 \\ -5 & 0 & 11 \\ 7 & -11 & 0 \end{bmatrix}$$

$\because\ A^T = -A$

Then, the given matrix is a skew–symmetric matrix.

19.

(c) None of these

Explanation: If det (A+B) = 0 implies that A+B a Singular matrix.

20. **(a)** no solution

Explanation: The given system of equations does not have solution if $\begin{vmatrix} 1 & 2 & 2 \\ 2 & 4 & 4 \end{vmatrix} = 0$

21.

(c) $\frac{1}{2}$

Explanation: $\frac{1}{2}$

22.

(d) $\frac{2}{\sqrt{1+x^2}}$

Explanation: Given that y $= \log_e\left(\frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}-x}\right)$

Differentiating with respect to x, we obtain

$$\Rightarrow \frac{dy}{dx} = \frac{\sqrt{1+x^2}-x}{\sqrt{1+x^2}+x}$$

$$\times \frac{(\sqrt{1+x^2}-x) \times \left(\frac{1}{2\sqrt{1+x^2}} \times 2x+1\right) - (\sqrt{1+x^2}+x) \times \left(\frac{1}{2\sqrt{1+x^2}} \times 2x-1\right)}{(\sqrt{1+x^2}-x)^2}$$

Hence, $\frac{dy}{dx} = \frac{2}{\sqrt{1+x^2}}$

23.

(b) 3

Explanation: Here,it is given that the function f(x) is continuous at x = $\frac{\pi}{2}$.

$\therefore$ L. H. L = $\lim_{x \to \frac{\pi}{2}} f(x)$

$= \lim_{x \to \frac{\pi}{2}} \frac{k\cos x}{\pi - 2x}$

Substituting, x = $\frac{\pi}{2} - h$;

As $x \to \frac{\pi}{2}^{-}$ then $h \to 0$

$$\therefore \lim_{x \to \frac{\pi}{2}} \frac{k\cos\left(\frac{\pi}{2} - h\right)}{\pi - 2\left(\frac{\pi}{2} - h\right)} = k \cdot \lim_{h \to 0} \frac{\sin h}{h}$$

$\therefore$ L.H.L = k

As it is continuous which implies right hand limit equals left hand limit equals the value at that point.

$\therefore$ k = 3

24.

(b) $4+[x]^2 \neq 0$

Explanation: Given that f(x) = a + b |x| + c $|x|^4$, where a, b, and c are real constants and f(x) is differentiable at x = 0.

$$f(x) = \begin{cases} a + bx + cx^4, x \geq 0 \\ a - bx + cx^4, x < 0 \end{cases}$$

$\because$ f(x) is differentiable at x =0

$\therefore$ LHD = RHD

$$\Rightarrow \lim_{x \to 0^-} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0^+} \frac{f(x) - f(0)}{x - 0}$$

$$\Rightarrow \lim_{x \to 0^-} \frac{a - bx + cx^4 - a}{x} = \lim_{x \to 0^+} \frac{a + bx + cx^4 - a}{x}$$

$$\Rightarrow \lim_{h\to 0} \frac{a-b(-h)+c(-h)^4-a}{-h} = \lim_{h\to 0} \frac{a+bh+ch^4-a}{h}$$

$$\Rightarrow \lim_{h\to 0} \frac{a-b(-h)+c(-h)^4-a}{-h} = \lim_{h\to 0} \frac{a+bh+ch^4-a}{h}$$

$$\Rightarrow \lim_{h\to 0} -b-\text{ch}^3 = \lim_{h\to 0} b+ch^3$$

$\Rightarrow$ - b = b

$\Rightarrow$ 2b = 0

$\Rightarrow$ b = 0

25. **(a)** $\frac{1}{2}\sec^2\frac{x}{2}$

Explanation: Given that $y = \sqrt{\frac{\sec x-1}{\sec x+1}}$

Multiplying by cos x in numerator and denominator, we obtain

$$y = \sqrt{\frac{1-\cos x}{1+\cos x}}$$

Using 1 - cos x = $2\sin^2\frac{x}{2}$ and 1 + cos x = $2\cos^2\frac{x}{2}$, we obtain

$$y = \sqrt{\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}}$$

$$= \tan\left(\frac{x}{2}\right)$$

Differentiating with respect to x, we obtain

$$y = \sec^2\frac{x}{2} \times \frac{1}{2}$$

$$= \frac{1}{2}\sec^2\frac{x}{2}$$

26.

(c) $-m^2y$

Explanation: $y = a\sin mx + b\cos mx \Rightarrow y_1 = am\cos mx - bm\sin mx$

$\Rightarrow y_2 = -am^2\sin mx - bm^2\cos mx$

$\Rightarrow y_2 = -m^2(a\sin mx + b\cos mx) = -m^2y$

27.

(b) $\left(\frac{1}{4}, \frac{1}{2}\right)$

Explanation: $\frac{dy}{dx} = \frac{1}{2y} = \tan\frac{\pi}{4} = 1$

$\Rightarrow y = \frac{1}{2} \Rightarrow x = \frac{1}{4}$

28. **(a)** $\frac{1}{2}$

Explanation: Let, the numbers whose sum is 8 are 8, 8 - x.

Given f(x) = $\frac{1}{x} + \frac{1}{8-x}$

$\Rightarrow$ f'(x) = $\frac{-1}{x^2} + \frac{1}{(8-x)^2}$

to find minima or maxima

f'(x) = 0

$\Rightarrow \frac{-1}{x^2} + \frac{1}{(8-x)^2} = 0$

$\Rightarrow$ x = 4

f''(x) = $\frac{2}{x^3} - \frac{2}{(8-x)^3}$

$\Rightarrow$ f''(4) = $\frac{2}{4^3} - \frac{2}{(8-4)^3} = 0$

Minimum value of the sum of their reciprocals = $\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

29.

(c) a > 1

Explanation: a > 1

30.

(c) Decreasing on R

Explanation: Given, f'(x) = $-x^3 + 3x^2 - 3x + 4$

f'(x) = $-3x^2 + 6x - 3$

f'(x) = $-3(x^2 - 2x + 1)$

f'(x) = $-3(x - 1)^2$

As f'(x) has -ve sign before 3

$\Rightarrow$ f'(x) is decreasing over R.

31. **(a)** $\frac{1}{\sqrt{2}}\tan^{-1}\left\{\frac{1}{\sqrt{2}}\left(x-\frac{1}{x}\right)\right\}+C$

Explanation: Formula:- $\int x^n dx = \frac{x^{n+1}}{n+1}+c; \int\frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a}+c$

Therefore ,

$$\Rightarrow \int\frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}}dx = \int\frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}-2+2}dx$$

$$=\int\frac{1+\frac{1}{x^2}}{\left(x-\frac{1}{x}\right)^2+2}dx$$

Put $x-\frac{1}{x}=t \Rightarrow \left(1+\frac{1}{x^2}\right)dx = dt$

$$\Rightarrow \int\frac{1}{t^2+2}dt = \frac{1}{\sqrt{2}}\tan^{-1}\frac{t}{\sqrt{2}}+c$$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\left[\frac{1}{\sqrt{2}}\left(x-\frac{1}{x}\right)\right]+c$$

32.

(c) $\frac{1}{3}e^{x^3}+C$

Explanation: Let $I = \int x^2 e^{x^3}dx$

Also, let $x^3 = t$, $\Rightarrow$ $3x^2dx = dt$

Thus,

$$\Rightarrow I = \frac{1}{3}\int e^t dt$$

$= \frac{1}{3}\left(e^{t}\right) + C$

$= \frac{1}{3}\left(e^{x^3}\right) + C$

33.

(d) $\frac{\pi}{4}$

Explanation: $I = \int_0^{\frac{\pi}{2}} \frac{1+\cos 2x}{2} dx$

$= \left(\frac{x}{2} + \frac{\sin 2x}{4}\right)_0^{\frac{\pi}{2}}$

$= \left(\frac{\frac{\pi}{2}}{2} + \frac{\sin \pi}{4}\right) - \left(\frac{0}{2} + \frac{\sin 0}{4}\right)$

$= \frac{\pi}{4}$

34.

(b) 6

Explanation: Given: $\int_{\frac{1}{3}}^{1} \left(\frac{\left(x-x^3\right)^{\frac{1}{3}}}{x^4}\right) dx$

Let $I = \int_{\frac{1}{3}}^{1} \left(\frac{\left(x-x^3\right)^{\frac{1}{3}}}{x^4}\right) dx$

Now, let x = sin θ $\Rightarrow$ dx = cos θ dθ

Now, when x $= \frac{1}{3}, \theta = \sin^{-1}\left(\frac{1}{3}\right)$ and when $x = 1, \theta = \frac{\pi}{2}$

$$\Rightarrow I = \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}} \left(\frac{\left(\sin\theta - \sin^3\theta\right)^{\frac{1}{3}}}{\sin^4\theta} \right) \cos\theta d\theta$$

$$= \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}} \left(\frac{(\sin\theta)^{\frac{1}{3}}) \left(1 - \sin^2\theta\right)^{\frac{1}{3}}}{\sin^4\theta} \right) \cos\theta d\theta$$

$$= \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}} \left(\frac{(\sin\theta)^{\frac{1}{3}} \left(\cos^2\theta\right)^{\frac{1}{3}}}{\sin^4\theta} \right) \cos\theta d\theta$$

$$= \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}} \left(\frac{(\sin\theta)^{\frac{1}{3}} (\cos\theta)^{\frac{2}{3}}}{\sin^2\theta \cdot \sin^2\theta} \right) \cos\theta d\theta$$

$$= \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}} \left(\frac{(\cos\theta)^{\frac{2}{3}+1}}{(\sin\theta)^{2-\frac{1}{3}}} \right) \cdot \frac{1}{\sin^2\theta} d\theta$$

$$= \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}} \left(\frac{(\cos\theta)^{\frac{5}{3}}}{(\sin\theta)^{\frac{5}{3}}} \right) \cdot cosec^2\theta d\theta$$

$$= \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}} \left((\cot\theta)^{\frac{5}{3}} \right) \cdot cosec^2\theta d\theta$$

$$= \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}} \left((\cot\theta)^{\frac{5}{3}} \right) \cdot cosec^2\theta d\theta$$

put, $cot\theta = t$, $then\ - cosec^2\theta d\theta = dt$

When, $\theta = \sin^{-1}\left(\frac{1}{3}\right)$, $t = 2\sqrt{2}$ and when $\theta = \frac{\pi}{2}$, t = 0

$\therefore$ I $= \int_{2\sqrt{2}}^{0} -(t)^{\frac{5}{3}} \cdot dt$

$$= -\left[\frac{(t)^{\frac{5}{3}+1}}{\frac{5}{3}+1}\right]_{2\sqrt{2}}^{0}$$

$$= -\left[\frac{(t)^{\frac{8}{3}}}{\frac{8}{3}}\right]_{2\sqrt{2}}^{0}$$

$$= -\frac{3}{8}\left[(0)^{\frac{8}{3}} - (2\sqrt{2})^{\frac{8}{3}}\right]$$

$$= -\frac{3}{8}\left[-(\sqrt{8})^{\frac{8}{3}}\right]$$

$$= \frac{3}{8}\left[(8)^{\frac{4}{3}}\right]$$

$$= \frac{3}{8}[16]$$

$= 6$

35.

(c) $\frac{2}{3}$

Explanation: The area of the region bounded by the curve $x^2 = 4y$ and line x = 2 and x-axis

$\Rightarrow \int_0^2 y dx = \int_0^2 \frac{x^2}{4} dx$

$$\Rightarrow \int_0^2 y dx = \left[\frac{x^3}{12}\right]_0^2$$

$$\Rightarrow \int_0^2 y dx = \frac{8}{12} = \frac{2}{3}$$

36.

(c) $2\sin^{-1}y - \sin^{-1}x = x\sqrt{1-x^2} + C$

Explanation: Here, $\frac{dy}{dx} = (\sqrt{1-x^2})(\sqrt{1-y^2})$

$$\frac{dy}{\sqrt{1-y^2}} = \sqrt{1-x^2}dx$$

Let x = sin t

dx = cot t dt

We know that $\cos x = \sqrt{1-x^2}$

On integrating on both sides,we obtain

$$\sin^{-1}y = \frac{t}{2} + \frac{\sin 2t}{4} + C$$

Sin 2t = 2 sin t cost

$$= 2 \times \sqrt{1-x^2}$$

$$\sin^{-1}y = \frac{\sin^{-1}x}{2} + \frac{x\sqrt{1-x^2}}{2} + C$$

$$2\sin^{-1}y - \sin^{-1}x = x\sqrt{1-x^2} + C$$

37.

(d) -2

Explanation: Given differential equation is

$(x^2 + x + 1)dy + (y^2 + y + 1)dx = 0$

$\Rightarrow (x^2 + x + 1)dy = -(y^2 + y + 1)dx$

$$\Rightarrow \frac{dy}{\left(1+y+y^2\right)} = -\frac{dx}{\left(1+x+x^2\right)}$$

$$\Rightarrow \frac{dx}{\left(1+x+x^2\right)} + \frac{dy}{\left(1+y+y^2\right)} = 0$$

$$\Rightarrow \int \frac{dx}{\left(x+\frac{1}{2}\right)^2+\frac{3^2}{4}}+\int \frac{dy}{\left(y+\frac{1}{2}\right)^2+\frac{3}{4}}=0$$

$$\Rightarrow \int \frac{dx}{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}+\frac{dy}{\left(y+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}=0 \text{ [on integrating]}$$

$$\Rightarrow \frac{1}{\left(\frac{\sqrt{3}}{2}\right)}\tan^{-1}\left\{\frac{\left(x+\frac{1}{2}\right)}{\frac{\sqrt{3}}{2}}\right\}+\frac{1}{\frac{\sqrt{3}}{2}}\tan^{-1}\left\{\frac{y+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right\}=\frac{2}{\sqrt{3}}\tan^{-1}C_1$$

$$\left[\because \int \frac{dx}{a^2+x^2}=\frac{1}{a}\tan^{-1}\frac{x}{a}\right]$$

$$\Rightarrow \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2y+1}{\sqrt{3}}\right)=\frac{2}{\sqrt{3}}\tan^{-1}C_1$$

$$\Rightarrow \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+\tan^{-1}\left(\frac{2y+1}{\sqrt{3}}\right)=\tan^{-1}C_1$$

$$\Rightarrow \tan^{-1}\left\{\frac{\left(\frac{2x+1}{\sqrt{3}}\right)+\left(\frac{2y+1}{\sqrt{3}}\right)}{1-\left(\frac{2x+1}{\sqrt{3}}\right)\left(\frac{2y+1}{\sqrt{3}}\right)}\right\}=\tan^{-1}C1 \left[\because \tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$$

$$\Rightarrow \frac{\sqrt{3}\,[\,(2x+1)+(2y+1)\,]}{3-(2x+1)\cdot(2y+1)}=C_1$$

$$\Rightarrow 2\sqrt{3}(x+y+1)=2C(1-x-y-2xy)$$

$$\Rightarrow (x+y+1)=\frac{1}{\sqrt{3}}(1-x-y-2xy)$$

On comparing with $(x + y + 1) = A(1 + Bx + Cy + Dxy)$

Here, A is parameter and B, C and D are constants.
The value of D = -2

38.

(b) parabolas

Explanation: Given equation can be written as

$$\frac{2dy}{y+3} = \frac{dx}{x}$$

$\Rightarrow$ 2log (y + 3) = log x + log c

$\Rightarrow$ $(y + 3)^2$ = cx which represents the family of parabolas

39.

(c) 2 : 3

Explanation: $\overrightarrow{AB} = 4\hat{i} + 2\hat{j} + 6\hat{k} = 2\left(2\hat{i} + \hat{j} + 3\hat{k}\right)$

$\overrightarrow{BC} = 6\hat{i} + 3\hat{j} + 9\hat{k} = 3(2\hat{i} + \hat{j} + 3\hat{k})$

$\therefore \overrightarrow{AB} = 2 \times \frac{\overrightarrow{BC}}{3}$

Therefore , AB and BC are parallel, but point B is common , so points , A,B,C are collinear.

As $\frac{\overrightarrow{AB}}{\overrightarrow{BC}} = \frac{2}{3}$, thus , point B divides AC in the ratio 2 : 3.

40.

(d) 1

Explanation: projection of $\hat{i} + \hat{j} + \hat{k}$ on $\hat{j}$ is $\frac{(\hat{i}+\hat{j}+\hat{k})\,.\hat{j}}{|\hat{j}|}$=1

41. **(a)** $\frac{\pi}{4}$

Explanation: It is given that $\vec{a} \times \vec{b}$ is a unit vector, then:

$\Rightarrow |\vec{a} \times \vec{b}| = 1 \Rightarrow |\vec{a}|\,|\vec{b}|\,sin\theta = 1$

$\Rightarrow 3.\frac{\sqrt{2}}{3} sin\theta = 1 \Rightarrow sin\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{\pi}{4}$

42.

(b) $\frac{4\vec{a}+\vec{b}}{3}$

Explanation: $\frac{4\vec{a}+\vec{b}}{3}$ is the correct answer. Applying section formula the position vector of the required point is

$$\frac{2(\vec{a}+\vec{b})+1(2\vec{a}-\vec{b})}{2+1} = \frac{4\vec{a}+\vec{b}}{3}.$$

43. **(a)** $\frac{\pi}{2}$

Explanation: Given vectors $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ *and* $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$

Now, $\vec{a} + \vec{b} = 4\hat{i} + \hat{j} - \hat{k}$ and $\vec{a} - \vec{b} = -2\hat{i} + 3\hat{j} - 5\hat{k}$

let θ be the angle between the vectors $\vec{a} + \vec{b}$ *and* $\vec{a} - \vec{b}$

$$\Rightarrow cos\theta = \frac{-8+3+5}{\sqrt{16+1++} \times \sqrt{4+9+25}} = 0 = \frac{\pi}{2}$$

44.

(d) x + 2y - 3z = 14

Explanation: Let the required equation of the plane through P(1, 2, -3) be
a(x - 1) + b (y - 2) + c(z + 3) = 0.
D.r.'s of OP are (1 - 0), (2 - 0), (-3, 0), i.e., 1, 2, -3.
∴ a = 1, b = 2, c = -3.
Hence, the required equation of the plane is
1(x - 1) + 2(y - 2) - 3(z + 3) = 0
⇒ x + 2y - 3z = 14.

45.

(b) 4x + 2y + z = 12

Explanation: Let the required equation of the plane be $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$.

Then, it meets the coordinate axes in A(a, 0, 0), B(0, b, 0), C(0, 0, c).

∴ centroid of △ABC is $G\left(\frac{a+0+0}{3}, \frac{0+b+0}{3}, \frac{0+0+c}{3}\right)$

i.e., $G\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$.

∴ $\left(\frac{a}{3} = 1, \frac{b}{3} = 2, \frac{c}{3} = 4\right)$

⇒ a = 3, b = 6, c = 12.

Hence, the required equation of the plane is

$$\frac{x}{3} + \frac{y}{6} + \frac{z}{12} = 1$$

⇒ 4x + 2y + z = 12

46.

(d) 3

Explanation: The coordinates of any point on the given line are of the form

$\frac{x-3}{1} = \frac{y-4}{2} = \frac{z-5}{2} = \lambda$

$\Rightarrow x = \lambda + 3; y = 2\lambda + 4; z = 2\lambda + 5$

So, the coordinates of the point on the given line are (λ + 3, 2λ + 4, 2λ + 5). This point lies on the plane,

x + y + z = 17

$\Rightarrow \lambda + 3 + 2\lambda + 4 + 2\lambda + 5 = 17$

$\Rightarrow 5\lambda = 5$

$\Rightarrow \lambda = 1$

So, the coordinates of the point are

(λ + 3, 2λ + 4, 2λ + 5)

= (1 + 3, 2(1) + 4, 2(1) + 5)

= (4, 6, 7)

Now, the distance between the points (4, 6, 7) and (3, 4, 5) is

$\sqrt{(3-4)^2 + (4-6)^2 + (5-7)^2}$

$= \sqrt{1 + 4 + 4}$

= 3 units

47.

(c) $\frac{7}{20}$

Explanation: P(A speaks truth) = 0.75

P(A lies) = 1-0.75 = 0.25

P(B speaks truth) = 0.8

P(B lies) = 1 - 0.8 = 0.2

P(contradicting each other in a statement) = P (A speaks truth and B lies) + P (B speaks truth and A lies)

= 0.75 × 0.2 + 0.8 × 0.25

= 0.15 + 0.2

= 0.35

$= \frac{35}{100} = \frac{7}{20}$

48.

(b) $\frac{4}{7}$

Explanation: Here, S={(B, B, B), (G, G, G), (B, G, G), (G, B, G), (G, G, B), (G, B, B), (B, G, B), (B, B, G)}

E_1 = Event that a family has at least one girl, then

E_1 = {(G, B, B), (B, G, B), (B, B, G), (G, G, B), (B, G, G), (G, B, G), (G, G, G)}

E_2 = Event that the eldest child is a girl, then

E_2= {(G, B, B), (G, G, B), (G, B, G), (G, G, G)}

$\therefore E_1 \cap E_2 = \{(G, B, B), (G, G, B), (G, B, G), (G, G, G)\}$

$$\therefore P\left(E_2/E_1\right) = \frac{P\left(E_1 \cap E_2\right)}{P\left(E_1\right)} = \frac{4/8}{7/8} = \frac{4}{7}$$

49.

(d) ${}^5C_4(0.7)^4(0.3)$

Explanation: Here, $\bar{p} = 0.3 \Rightarrow p = 0.7$ and q = 0.3, n = 5 and r = 4

$\therefore$ Required probability $= {}^5C_4(0.7)^4(0.3)$

50.

(c) None of these

Explanation: We are having two events A and B such that

$P(A) = \frac{1}{2}, P(B) = \frac{7}{12}$ and $\left(A' \cup B'\right) = \frac{1}{4}$,

$P\left(A' \cup B'\right) = P(A \cap B)' = 1 - P(A \cap B) = \frac{1}{4}$

$\Rightarrow P(A \cap B) = \frac{3}{4}$

$\Rightarrow$ As $P(A \cap B) \neq P(A).P(B)$ … thus, they are not independent,

$\Rightarrow$ And as $P(A \cup B) \neq P(A) + P(B)$ … thus, they are not mutually exclusive.