Sets

Exercise – 1.1

Solution 1:

A well-defined collection of objects is called a set:
The collection of prime numbers is a well-defined collection of objects and is hence a set.
The collection of easy subtopics in this chapter is not a well-defined collection of objects and is hence not a set.
The collection of good teachers in your school is not a well-defined collection of objects and is hence not a set.
The collection of girls in your class is a well-defined collection of objects and is hence a set.
The collection of odd natural numbers is a well-defined collection of objects and is hence a set.

Solution 2:

i. A = {January, February, March, May, July, August, October, December}
ii. B = {violet, indigo blue, green, yellow, orange, red}.
iii. C = {-3,-2,-1, 0, 1, 2, 3}.
iv. D = {-2,-1, 0, 1, 2, 3}.
v. Given that, $(n - 1)^3$, $n < 3$, $n \in W$

Here $n < 3$ and $n \in W$ means n = 0, 1, 2

Hence, for n = 0, $(n - 1)^3 = (0 - 1)^3 = (-1)^3 = (-1)$

For n = 1, $(n - 1)^3 = (1 - 1)^3 (0)^3 = 0$

For n = 2, $(n - 1)^3 = (2 - 1)^3 = (1)^3 = 1$

Hence, E = {,-1,0,1}

Solution 3

i. We observe that there are 4 elements in set F and the elements are the multiples of natural numbers from 1 to 4.

$\therefore F = \{x|x = 5n, n \in N, n \leq 4\}$

ii. We observe that the elements in set G are the squares of natural numbers from 3 and 10 and whose perfect square is less than or equal to 81.

$G = \{x|x = n^2, n \in N, 3 \leq n < 10\}$

iii. We observe that the elements in set H are powers of 5.

$\therefore H = \{x|x = 5^n, n \in N, n \leq 4\}$

iv. We observe that the elements in set X are square roots of 64.

$\therefore X = \{x|x = \text{Square root of } 64\}$

v. We observe that the elements in set Y are reciprocal of the cubes of natural numbers less than or equal to 5.

$\therefore Y = \left\{x \mid x = \frac{1}{n^3}, n \in N, n \leq 5\right\}$

Solution 4:

Let 'A' be the set of first five integers whose square is odd.

A = {1,3,5,7,9}

Exercise – 1.2

Solution 1:

Singleton sets have only one element:

i. $A = \{x|\sqrt{x} = 16\}$

$\sqrt{x} = 16$

Squaring on both the sides, we have,

$\therefore (\sqrt{x})^2 = 16^2$

$\therefore x = 256$

$\therefore A = \{256\}$

$\therefore A = \{x|\sqrt{x} = 16\}$ is a singleton set

ii. $B = \{y|y^2 = 36\}$

$\therefore B = \{-6, 6\}$

$\therefore B = \{y|y^2 = 36\}$ is not a singleton set

iii. $C = \{p|p \in I, p^3 = -8\}$

$\therefore C = \{-2\}$

$\therefore C = \{p|p \in I, p^3 = -8\}$ is a singleton set

iv. $D = \{q \mid (q-4)^2 = 0\}$

$(q-4)^2 = 0$

$\therefore\ q - 4 = 0$

$\therefore\ q = 4$

$\therefore\ D = \{4\}$

$\therefore\ D = \{q \mid (q-4)^2 = 0\}$ is a singleton set

v. $E = \{x \mid 1 + 2x = 3x, x \in W\}$

$E = \{1\}$

$\therefore\ 1 + 2x = 3x,$

$\therefore\ 1 = 3x - 2x$

$\therefore\ 1 = x, x = 1$

$\therefore\ E = \{x \mid 1 + 2x = 3x, x \in W\}$ is a singleton set

Solution 2:

An empty set has no elements.
i. Let ‘A’ be the set of all even prime numbers
A = {2}; Set A has only one element.
Thus, A is a singleton set.
Hence, A is not an empty set
ii. B = {x|x is a capital of India}
∴B = {Delhi}; Set B has only one element.
Thus, B is a singleton set.
Hence, B is not an empty set
iii. F = {y|y is a point of intersection of two parallel lines}
The two parallel lines do not intersect each other.
∴There are 0 elements in set F
∴Hence, F is an empty set.
iv. G = {z|z ∈ N, 3 < Z < 4}
There is no natural number between 3 and 4
∴G = ϕand hence G is an empty set.
v. H = {t|triangle having four sides}
A triangle always has three sides and it cannot have four sides.
Hence, H is an empty set.

Solution 3:

Finite sets:
Finite sets are those in which the counting of elements terminates at a certain stage.
Infinite sets:
Infinite sets are those in which the counting of elements does not terminate at a certain stage.

i. A= {1,3,5,7,......}

Here the counting of elements does not terminate.

Hence, A= {1,3,5,7,......} is an infinite set

ii. B = {101,102,103,.........1000}.

Here the counting of elements terminates at the number 1000.

Hence, B = {101, 102, 103,.........1000} is a finite set.

iii. $C = \{ x | x \in Q, 3 < x < 5\}$

Q is the set of all rational numbers.

Since $x \in Q$, there can be infinite rational numbersbetween 3 and 5

Hence, $C = \{ x | x \in Q, 3 < x < 5\}$ is an infinite set.

iv. $D = \{y | y = 3^n \; n \in N\}$

$D = \{3^1, 3^2, 3^3, 3^4 .....\}$

Here the counting of elements does not terminate

$\therefore D = \{y | y = 3^n \; n \in N\}$ is an infinite set.

Solution 4:

Since the number of boys and girls in a class terminates at a certain stage, G and H are finite sets.

Exercise – 1.3

Solution 1:

i. The subset relation between the set A and set C is:
A = The set of all residents in Mumbai

C = The set of all residents in Maharashtra
Since Mumbai is in Maharashtra, every element of Set A is an element of Set C, hence, set A is a subset of set C.
But there are some elements in set C which are not in set A, therefore A is the proper subset of set C.
Hence, $A \subset C$
ii.The subset relation between the sets E and D is:
$E \subset D$
E = The set of all residents in Madhya-Pradesh.
D = The set of all residents in India
Since Madhya-Pradesh is inIndia, every element of Set E is an element of Set D, hence, set E is a subset of set D.
But there are some elements in set D which are not in set E, therefore D is the proper subset of set E.
Hence, $E \subset D$
iii. Mumbai, Bhopal, Maharashtra, and Madhya-Pradesh all are in India. Therefore, all the sets under consideration are the subsets of set D.
Hence, $A \subset D$, $B \subset D$, $C \subset D$, $E \subset D$
Therefore Set D can be chosen as the universal set.

Solution 2:

i. B = {a} and C = {a, b} are the proper subsets of set A ={a, b, c}.

Here, $a \in A$ and $a \in C$

And, $a \in B$, $b \in B$ and $a \in C$, $b \in C$

Every element of Set B and Set C is an element of Set A, hence, set B and set C are subsets of set A.

Therefore, B = {a} and C = {a, b} are the proper subsets of the set A ={a, b, c}.

But there are some elements in set A which are not in set B.

That is, $b \in A$, $c \in A$.

And $b \notin B$ and $c \notin B$.

Hence, set B is the proper subset of set A, or $B \subset A$

Also there is one element in set A which is not in set C.

That is $c \in A$ and $c \notin C$.

Hence, set C is the proper subset of set A, or $C \subset A$

ii. Here, $a \in C$, $b \in C$ and $a \in A$, $b \in A$.

Thus, every element of Set C is an element of Set A.

And $c \in A$ and $c \notin C$.

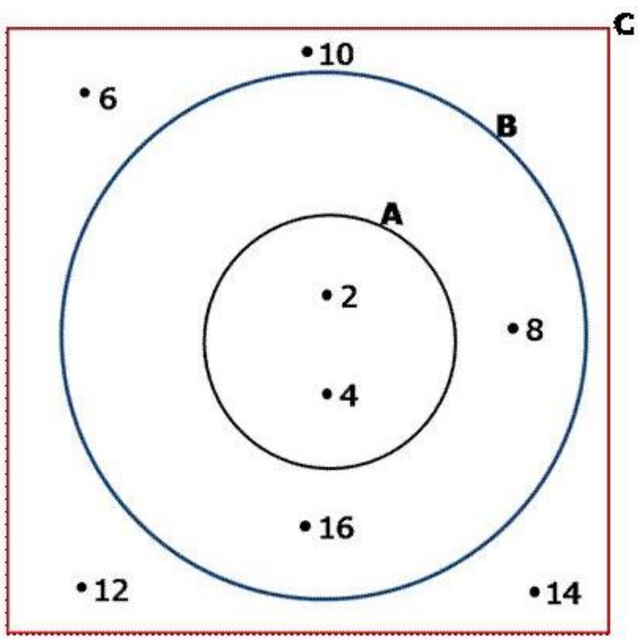
Thus, there is one element in set A which is not in set C.

Hence, set C is the proper subset of set A.

And set A is the Superset of set C or $A \supset C$.

Solution 3:

$A = \{2, 4\}$
$B = \{x | x = 2^n, n < 5, n \in N\}$
$\therefore B = \{2, 4, 8, 16\}$
$C = \{x | x$ is an even natural number $\leq 16\}$
$\therefore C = \{2,4,6,8,10,12,14,16\}$
The Venn diagram will be as given below:



solution 4:

Let x be an element of set A.
Thus, ∴ $x \in A$.
Given that $A \subset B$.
∴ Every element of set A is an element of B.
∴$x \in B$
Also given that $B \subset C$
∴ Every element of set B is an element of C.
∴$x \in C$
∴If $x \in A$, then $x \in C$ for all $x \in A$.
∴ Every element of the set A is an element of the set C.
Hence, $A \subset C$.

Solution 5:

All possible subsets of X = {1, 2, 3} are:
{1},{2},{3},{1, 2},{1, 3},{2, 3}, {1, 2, 3},{} or φ

Exercise – 1.4

Solution 1:

P = {x|x is a letter in the word 'CATARACT'}
Let us avoid the repetition of letters and write the distinct letters in curly brackets, with comma as a separator.
∴ P = {C, A, T, R}
Q = {y|y is a letter in the word 'TRAC'}
∴ Q = {T, R, A, C}
The order in which the letters are listed is immaterial.
∴Every element of set P is an element of set Q.
∴P Í Q.
Also, every element of set Q is an element of set P
∴Q Í P.
All the elements in set P and all the elements in set Q are equal, hence, P = Q.

Solution 2:

The union of pairs of sets A and B is the set of all elements which are in set A or in set B. It is denoted by $A \cup B$.
i. A = {2, 3, 5, 6, 7}, B ={4, 5, 7, 8}
$A \cup B$ = {2, 3, 4, 5, 6, 7, 8}
ii. C = {a, e, i, o, u}, D ={a, b, c, d}

C ∪ D = { a, b, c, d, e, i, o, u }
iii. E= {x|x ∈ N and x is a divisor of 12}
∴E= {1, 2, 3, 4, 6, 12}
F= {y|y ∈ N and y is a divisor of 18}
∴F = {1, 2, 3, 6, 9, 18}
∴E ∪ F = {1, 2, 3, 4, 6, 9, 12, 18}

Solution 3:

The set of all common elements of A and B is called the intersection of A and B, and it is denoted by A ∩ B.

i. A = {1, 2, 4, 5, 7}, B ={2, 3, 4, 8}

∴A ∩ B = {2, 4}

ii. C = {6, 7, 8, 9, 10}

D = {5, 6, 7, 8, 9}

∴C ∩ D = {6, 7, 8, 9}

iii. E= {-1, -2 ...}

F= {1, 2, 3 ...}

E ∩ F = {} or ϕ

Solution 4:

U = {x|x = 2^n n ∈ W, n < 8}
∴U = {2^0, 2^1, 2^2, 2^3, 2^4 2^5, 2^6, 2^7}
∴U = {1, 2, 4, 8, 16, 32, 64, 128}

A = {y|y = 4^n n ∈ W, n < 4}
∴A = {4^0, 4^1, 4^2, 4^3}
∴A = {1, 4, 16, 64}

B = {z|z = 8^n n ∈ W, n ≤ 2}
∴B = {8^0, 8^1, 8^2}
∴B = {1, 8, 64}
Hence, U = {1, 2, 4, 8, 16, 32, 64, 128}
A = {1, 4, 16, 64}
B = {1, 8, 64}

i. A'

A' is called the complement of set A.

A' is the set of all the elements in U which are not in A.

A = {1, 4, 16, 64} and U = {1, 2, 4, 8, 16, 32, 64, 128}

∴A' = {2, 8, 32,128}

ii. B'

B' is called the complement of set B.

B' is the set of all the elements in U which are not in B

B = {1, 8, 64} and U = {1, 2, 4, 8, 16, 32, 64, 128}

∴B' = {2, 4, 16, 32,128}

iii. (A ∪ B)'

A = {1, 4, 16, 64}

B = {1, 8, 64}

A ∪ B = {1, 4, 8, 16, 64} and U = {1, 2, 4, 8, 16, 32, 64, 128}

(A ∪ B)' is the set of all the elements in U which are not in A ∪ B.

∴(A ∪ B)' = {2, 32, 128}

iv. (A ∩B)'

A = {1, 4, 16, 64}

B = {1, 8, 64}

A∩ B is the set of all common elements of A and B.

(A ∩ B) = {1, 64} and

U = {1, 2, 4, 8, 16, 32, 64, 128}

(A∩ B)' is the set of all the elements in U which are not in A ∩ B.

(A ∩ B)' = {2, 4, 8, 16, 32, 128}

Solution 5:

i. If A and B are two sets, then

$(A \cup B)' = A' \cap B'$

$A = \{a|a$ is a letter in the word 'college'$\}$

$A = \{c, o, l, e, g\}$

$B = \{b|b$ is a letter in the word 'luggage'$\}$

$\therefore B = \{l, u, g, a, e\}$

$U = \{a, b, c, d, e, f, g, l, o, u\}$,

$A \cup B = \{a, c, e, g, l, o, u\}$

$(A \cup B)' = \{b, d, f\}$ ...(1)

$A' = \{a, b, d, f, u\}$,

$B' = \{b, c, d, e, f, o\}$

$A' \cap B' = \{b, d, f\}$ ...(2)

From equations (1) and (2),

$(A \cup B)' = A' \cap B'$ is proved.

ii. If A and B are the two sets, then

$(A \cap B)' = A' \cup B'$

$A = \{c, o, l, e, g\}$

$B = \{l, u, g, a, e\}$

$A \cap B = \{l, e, g\}$

$(A \cap B)' = \{a, b, c, d, f, o, u\}$...(3)

$A' = \{a, b, d, f, u\}$,

$B' = \{b, c, d, f, o\}$

$A' \cup B' = \{a, b, c, d, f, o, u\}$...(4)

From equations (3) and (4),

$(A \cap B)' = A' \cup B'$ is proved.

Exercise – 1.5

Solution 1:

A = {1, 3, 5, 6, 7}, B ={4, 6, 7, 9}
∴A ∪ B = {1, 3, 4, 5, 6, 7, 9}
The number of elements of the set A ∪ B is denoted by n(A ∪ B).
∴n(A ∪ B) = 7
∴L.H.S. = 7
Now consider the R.H.S, n (A) + n (B) – n (A ∩ B).
n (A) = 5, n (B) = 4, n (A ∩ B) = 2
∴n (A) + n (B) = 5 + 4 = 9
∴n (A) + n (B) – n (A ∩ B) = 5 + 4 – 2 = 7
∴L.H.S = R.H.S
∴n(A ∪ B) = n (A) + n (B) – n (A ∩ B)
Hence, verified.

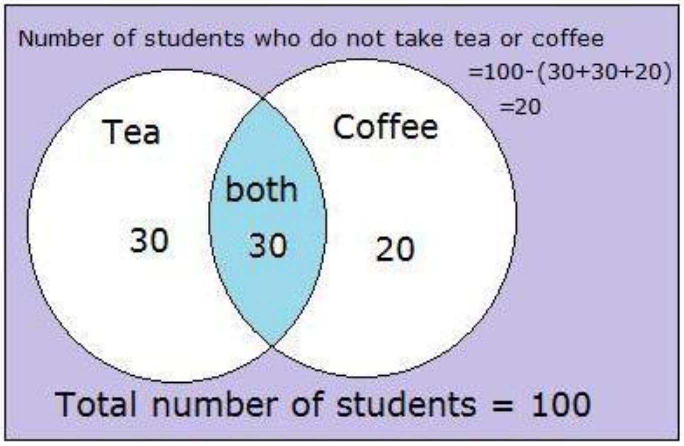
Solution 2:

We know the identity,
n(A ∪ B) = n(A) + n(B) – n(A ∩ B)
Substituting the values n(A) = 5, n(A ∪ B) = 9 and n(A ∩ B) = 2 in the above identity, we have,
9= 5 + n(B) – 2
∴ n(B) = 9 + 2 – 5
∴ n(B) = 11 – 5
∴ n(B) = 6

Solution 3:

Let A be the set of students who drink tea.
∴ n (A) = 60
Let B be the set of students who drink coffee.
∴n (A) = 60
30 students drink both coffee and tea.
∴n (A ∩ B) = 30
(A ∪ B) will be the set of students who take at least one drink (tea or coffee or both)
n (A ∪ B) = n (A) + n (B) - n (A ∩ B)
∴ n (A ∪ B) = 60 + 50 - 30
∴ n (A ∪ B) = 80
Hence, there are 80 students who drink either tea or coffee or both. Consider the

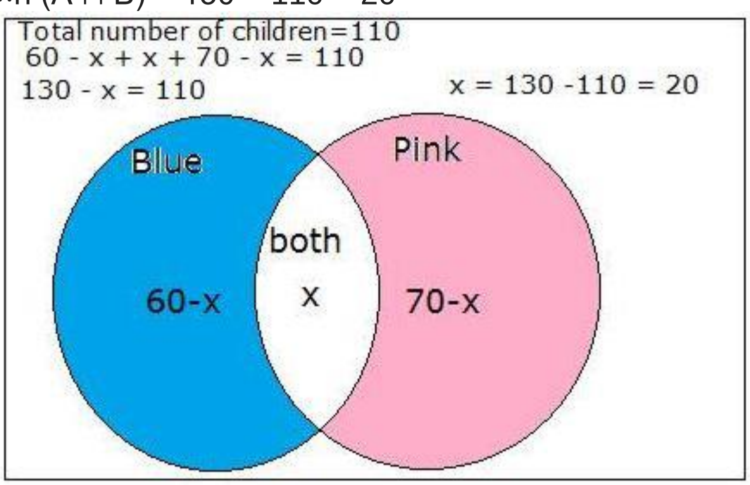
following Venn diagram.



∴ Number of students who neither drink tea nor coffee
= Total number of students in the class – number of students who drink either tea or coffee or both.
= 100 – 80 = 20
∴ Number of students who do neither drink tea nor coffee are 20

Solution 4:

Let A be the set of students who choose blue.
∴ n (A) = 60
Let B be the set of students who choose pink.
∴ n (A) = 70
(A ∩ B) will be the students who choose both the colours.
Every student has to choose at least one of the colour Therefore, n (A ∩ B) = n (A) + n (B) – total number of students
∴n (A ∩ B) = 60 + 70 – 110
∴n (A ∩ B) = 130 – 110 = 20



Hence, there are 20 students who choose both the colours as their favourite colour.

Solution 5:

Consider the given equation:
$n (A \cup B \cup C) = n (A) + n (B) + n (C) – n (A \cap B) – n (B \cap C) – n (C \cap A) + n (A \cap B \cap C)$
$n (A) = 5, n (B) = 5, n (C) = 4$
$n (A \cap B) = 2$
$n (B \cap C) = 2$
$n (C \cap A) = 2$
$n (A \cap B \cap C) = 1$
Substituting the values in the given equation,
$n (A \cup B \cup C) = n (A) + n (B) + n (C) – n (A \cap B) – n (B \cap C) – n (C \cap A) + n (A \cap B \cap C)$
$\therefore n (A \cup B \cup C) = 5 + 5 + 4 – 2 – 2 – 2 + 1$
$\therefore n (A \cup B \cup C) = 9$
From the given figure it can be seen that there are 9 elements in all.
Hence, the equation is verified.