4. Direct Current

4.1. Two conductors, *1-3-5* and *2-4-6*, connected points with equal potentials on the resistors R_a and R_b, so that no current flows through either of them. Will there be currents flowing through them and through the *3-4* section if the key K is closed? Will this lead to a change in the reading of the ammeter?

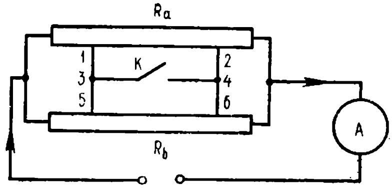


Fig. 4.1

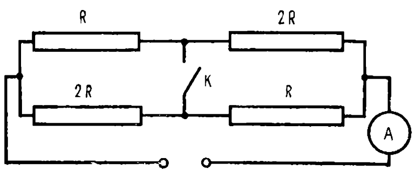


Fig. 4.2

4.2. How will the reading of the ammeter change if the key K is closed?

4.3. A voltage U_0 is applied to a potentiometer whose sliding contact is exactly in the middle. A voltmeter V is connected between the sliding contact and one fixed end of the potentiometer. It is assumed that the resistance of the voltmeter is not very high if compared with the resistance of the potentiometer. What voltage will the voltmeter show: higher than, less than, or equal to $U_0/2$?

4.4. A "black box" is an electric unit with four terminals, *1*, *2*, *3*, and *4*, and an unknown internal structure. The box shown in Figure (a) and (b) possesses the following properties: if a constant voltage of 220 V is applied to terminals *1* and *2*, a voltage of 127 V appears across terminals *3* and *4* (Figure (a)), while if a voltage of 127 V is applied to terminals *3* and *4*, the same voltage of 127 V appears across terminals *1* and *2* (Figure (b)). What is inside the "black box"? The formulation of the problem is quite meaningful if the voltages are measured by electrostatic voltmeters, which do not consume electric current. If voltmeters of the magnetoelectric, thermal, or electromagnetic type are employed, the voltages across the "out" terminals of the "black box" may somewhat differ from the ones indicated in Figures (a) and (b).

4.5. Two potentiometers are connected in series, and their sliding contacts are connected electrically, too. In one potentiometer the sliding contact remains fixed at

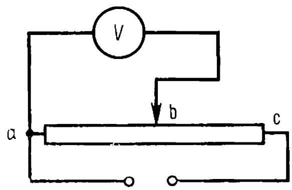


Fig. 4.3

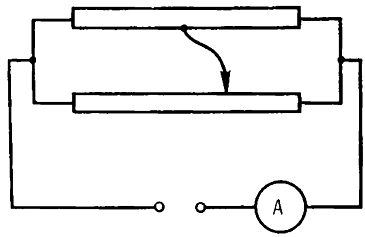
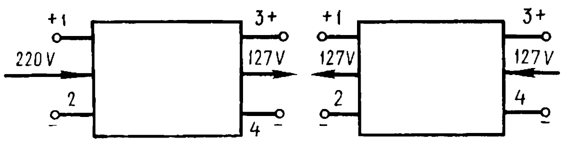


Fig. 4.5



(a) (b)

Fig. 4.4

the midpoint. How will the reading of the ammeter vary as the sliding contact of the second potentiometer is moved from one end of the potentiometer to the other?

4.6. A constant voltage U_0 is applied to a potentiometer of resistance R connected to an ammeter. A constant

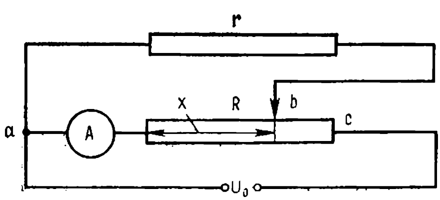


Fig. 4.6

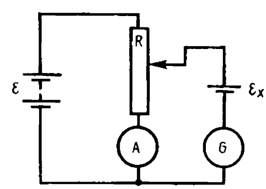


Fig. 4.7

resistor r is connected to the sliding contact of the potentiometer and the fixed end of the potentiometer (after an ammeter). How will the reading of the ammeter vary as the sliding contact is moved from one end of the potentiometer to the other? The resistance of the ammeter is assumed to be negligible.

4.7. To measure a small emf (of, say, a galvanic cell or a thermocouple) the so-called balancing method is employed. The circuit diagram of this method is shown in the figure. Here $\mathscr{E}_x$ is the sought emf, $\mathscr{E}$ is the source of current whose emf is much higher than $\mathscr{E}_x$, G is a galvanometer with the zero in the middle of the scale, A is an ammeter, and R is the resistance box. How should one operate this circuit so as to ensure an accuracy in measuring $\mathscr{E}_x$ that is determined by the precision of the measuring devices?

4.8. Two resistors with resistances R_1 and R_2 are connected in series, and so are two capacitors with capaci-

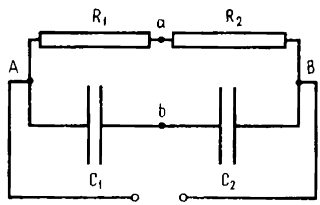


Fig. 4.8

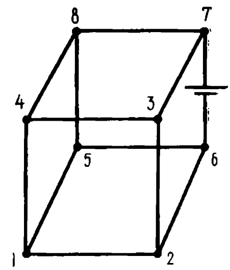


Fig. 4.10

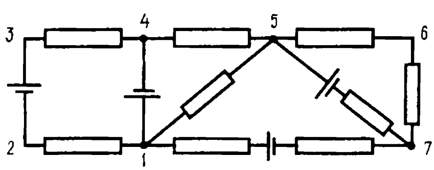


Fig. 4.9

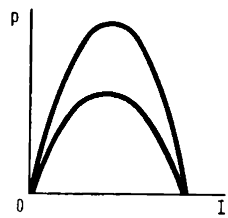


Fig. 4.12

tances C_1 and C_2. The two systems are connected in parallel and an external voltage is applied to the new system (see the figure accompanying the problem). What must be the relationship between R_1, R_2, C_1, and C_2 for the potential difference between the points a and b to be zero?

4.9. All the resistances and emf's shown in the figure accompanying the problem are assumed known. How many values of current can exist for such a circuit? How many equations for finding these values must we construct on the basis of Kirchhoff's first law and how many must we construct on the basis of Kirchhoff's second law?

4.10. Twelve conductors are connected in such a way that they form a cube, and an emf source is connected into an edge of the cube. All the resistances and the emf's are known. There are eight junctions (eight vertices of the cube) and six loops (six faces of the cube) in the circuit. Construct the equations for determining all the currents in the circuit.

4.11. A source of electric current with an emf $\mathscr{E}_0$ and an internal resistance r is connected to an external circuit with a resistance R. What must be the relationship between r and R for the power output in the external circuit to be maximal? What is the efficiency of the current source in this case, provided that the power output in the external circuit is assumed to be the useful output?

4.12. In two circuits, each of which contains a DC source and an external resistance, the maximal currents are the same, while the maximum power output in the external resistance of one circuit is twice that in the other. In what parameters do these circuits differ?

4.13. ADC source is connected to a rheostat. When the sliding contact is x distant from either end of the rheo-

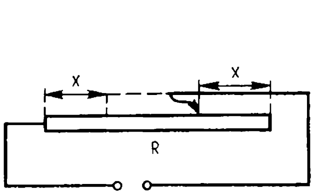


Fig. 4.13

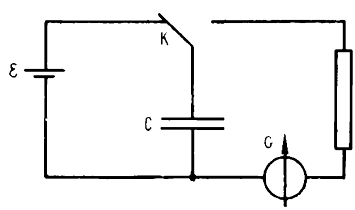


Fig. 4.16

stat (the length of the rheostat is set at unity), the power output in the rheostat is the same in both cases. Determine the internal resistance of the DC source if the resistance of the theostat is R.

4.14. How must a large number of galvanic cells, each having the same emf $\mathscr{E}$ and the same internal resistance r, be connected so that in an external circuit whose resistance is R the power output is maximal?

4.15. Can a circuit be constructed in which the displacement current in the capacitor remains practically constant over a definite time interval?

4.16. A DC source with known emf $\mathscr{E}$ is charging a capacitor C. After the charging process has been completed, the capacitor is disconnected, via a key K, from the DC source and is connected to a resistor R, through which the capacitor discharges. The capacitance of the capacitor and the resistance of the resistor are selected in such a way that the charging process takes several minutes, so that the discharge current can be registered by a measuring device, G. The results of measurements are used to draw a rough curve on a diagram in which the time of discharge is laid off on the horizontal axis and the logarithm of the current, on the vertical axis. Determine the law by which the current varies and the curve representing the dependence of the logarithm of the current on the time of discharge. How can the curve help in determining the parameters of the discharge circuit, R and C?

4.17. A capacitor of capacitance C is charged to a potential difference U_0 and is then discharged through a re-

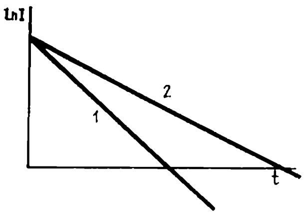


Fig. 4.17

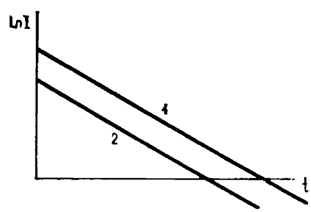


Fig. 4.18

sistance R. The discharge current gradually decreases, with a straight line *1* corresponding to this process (see the figure accompanying the problem, where time is laid off on the horizontal axis and the logarithm of the current, on the vertical axis). Then one of the three parameters, U_0, R, or C, is changed in such a manner that the $\ln I$ vs. t dependence is represented by the straight line *2*. Which of the three parameters was changed and in what direction?

4.18. A charged capacitor is discharged through a resistor two times. The time dependence of the logarithm of the discharge current obtained in the two experiments is represented by the two straight lines, *1* and *2*, in the figure accompanying the problem. The experimental conditions differed only in one of the three parameters: the initial voltage of the capacitor U, the capacitance C, or the resistance R. Determine the parameter that was varied in these experiments and in which case this parameter is greater.

4.19. Prove that when a capacitor of capacitance C that has been charged to a potential difference U_0 is discharged through a resistance R, the amount of heat liberated in the conductors is equal to the initial energy stored in the capacitor.

4.20. Prove that when a capacitor is charged through a resistor R from a DC source with an emf equal to $\mathscr{E}$ half of the energy supplied by the source goes to the capacitor and half, to heating the resistor.

4.21. A charged capacitor is connected to an uncharged capacitor with the same capacitance. Determine the changes in the energies stored by the two capacitors and explain the origin of these changes from the viewpoint of energy conservation.

4.22. A conducting disk is rotating with an angular velocity ω. Allowing for the fact that electrons are the cur-

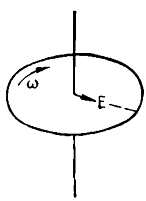


Fig. 4.22

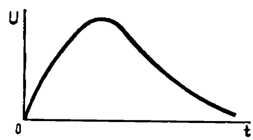


Fig. 4.23

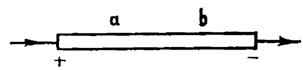


Fig. 4.24

rent carriers in a conductor, determine the potential difference between the center of the disk and the edge.

4.23. In the Tolman-Stewart experiment, a cylinder is mounted on a shaft and is rotated very rapidly. The surface of the cylinder is wound with many turns of wire of length l in a single layer. After the cylinder has been set spinning at a large angular velocity, it is braked to a stop as quickly as possible. In the circuit consisting of the wire and a measuring device, this braking manifests itself in a pulse of current caused by the potential difference that appears between the ends of the wire. If the potential difference is registered by an oscillograph, we obtain a curve similar to the one shown in the figure accompanying the problem, where time is laid off on the horizontal axis.* How, knowing the initial linear velocity of the winding, the length of the wire, and the voltage oscillogram, can one determine the electron charge-to-mass ratio?

* In the Tolman-Stewart experiment, the quantity measured was not the potential difference but the amount of electricity passing through the circuit. This was done using a device called the ballistic galvanometer.

4.24. The section of a conductor between the points a and b is being heated. Does this lead to a redistribution of potential along the conductor (the arrow indicates the direction in which the current is flowing)? Will the passage of current change the temperature distribution in the conductor?

4.25. A constant voltage is applied to a metal wire. The current passing through the wire heats the wire to a certain temperature. Then half of the wire is cooled by a stream of air from a fan. How will the temperature of the other half of the wire change in the process?

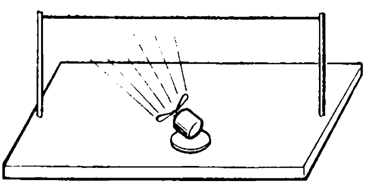


Fig. 4.25

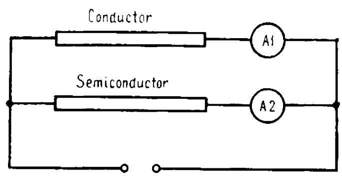


Fig. 4.27

4.26. Two electric bulbs whose rated voltage is 127 V and whose rated wattages are 25 and 150 W are connected in series to a DC source of 220 V. Which of the two bulbs will burn out?

4.27. A conductor and a semiconductor are connected in parallel. At a certain voltage both ammeters register the same current. Will this condition remain as such if the voltage of the DC source is increased?

4.28. A conductor and a semiconductor are connected in series. The voltage applied to this system is selected in such a way that the readings of the voltmeters $V1$ and $V2$ coincide. Will this condition remain unchanged if the voltage of the DC source is increased?

4.29. A thermionic valve, or diode, has a heated filament and a plate near it. The dependence of the current flowing between filament and plate on the voltage applied to valve (the current-voltage characteristic) is as follows. First the current grows with voltage, but then goes into a plateau at a sufficiently high voltage. Why, notwithstanding the fact that the filament may emit the number of

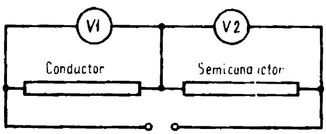


Fig. 4.28

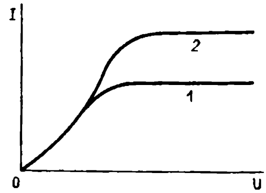


Fig. 4.29

electrons required for the saturation current to set in, the latter does not manifest itself at an arbitrarily small voltage between the electrodes? In which respect does curve *1* differ from curve *2* from the standpoint of the experimental conditions if the two are obtained using the same device?

4.30. A cutoff voltage is applied between the cathode and the anode of a thermionic valve ("minus" at the anode and "plus" at the cathode). The cathode temperature, however, is sufficient for thermionic emission to manifest itself. If the direction of the electric field is reversed by applying between the cathode and the anode a voltage at which saturation current will flow through the valve, will the temperature of the cathode maintained in the cutoff direction of the field remain the same?

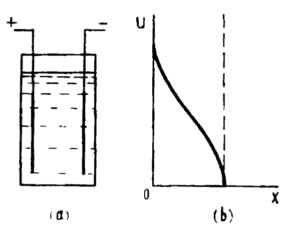


Fig. 4.31

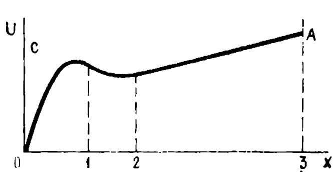


Fig. 4.32

4.31. For a current passing through an electrolyte (Figure (a)), the distribution of potential between the electrodes is shown in Figure (b). Why, notwithstanding the fact that the electrodes are flat and the distance between them is much smaller than their linear dimensions, is the field between the electrodes nonuniform?

4.32. The distribution of potential between the cathode and anode in a glow discharge is shown in the figure accompanying the problem (the distance from the cathode is laid off on the horizontal axis). Within which regions of space (see the numbers on the horizontal axis) is there a positive volume charge, a negative volume charge, and a volume charge that is practically zero?

4.33. In the plasma of a gas discharge, the concentration of electrons and that of positive ions are practically the same. Does this mean that the current densities created by the motion of electrons and ions are also the same? Will an ammeter connected in series with the gas discharge gap show the sum of the electron and ion currents or their difference?

4.34. A negatively charged particle is accelerated in its motion from a cathode C to an anode A, passes through an aperture in the latter, and moves toward a Faraday cylinder F that is at the same potential as the anode (Figure (a)). For the sake of simplicity it will be assumed that the particle moves from A to F with a constant velocity. Determine the moment of time when a measuring device G in the circuit will register a current (the time is reckoned from the moment when the particle leaves the anode) and the form of the current, that is, whether the current is in the form of a pulse when the particle leaves the anode (Figure (b)) or whether it is a pulse when the particle enters the Faraday cylinder (Figure (c)) or whether there are two pulses (one when the particle leaves the anode and the other when the particle enters the Faraday cylinder; see Figure (d)) or whether the current is steady over the entire motion of the particle from the anode to the Faraday cylinder (Figure (e)).

4.35. The behavior of the potential energy of an electron inside and outside a metal is shown for two metals in Figures (a) and (b). The same figures indicate the limiting kinetic energies W_F of electrons in the metals (the Fermi levels) at $T = 0$ K. If the metals are brought into contact, what will be the values of the internal and external contact potential differences? In which metal will the electron concentration be higher?

4.36. The energy distribution function for electrons in a metal at absolute zero can be written as follows:

$$f(W) = CW^{1/2}, \qquad (4.36.1)$$

where C is a constant coefficient that is a combination of universal constants. This function terminates at W_F, which is the limiting energy, or the Fermi level. Using

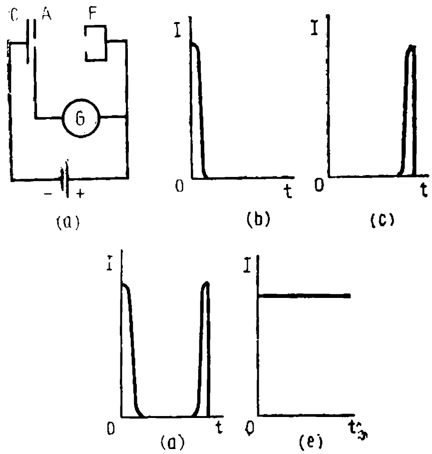


Fig. 4.34

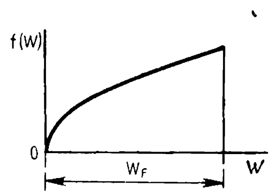


Fig. 4.36

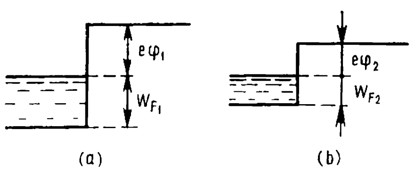


Fig. 4.35

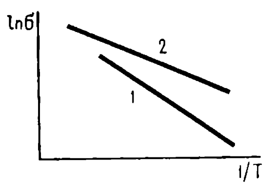


Fig. 4.37

(4.36.1), establish how the limiting energy depends on electron concentration.

4.37. The dependence of the logarithm of conductivity, $\ln\sigma$, on T^{-1}, where T is the temperature, for two semiconductors is shown in the figure. In which of the two semiconductors is the gap (the forbidden band) between the valence band and the conduction band wider?

4.38. The dependence of the logarithm of conductivity, $\ln\sigma$, on $1/T$ for two semiconductors is shown schematically in the figure. In which respect do these semiconductors differ?

4.39. The distribution of potential near the boundary between two semiconductors with different types of conduction depends on the direction of the applied external

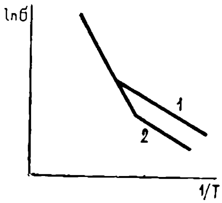


Fig. 4.38

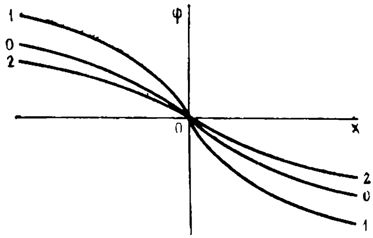


Fig. 4.39

voltage. Which distribution corresponds to the blocking direction and which, to conduction? To what semiconductors do the left and right branches of the curves in the figure belong?

4.40. The current-voltage characteristic of a semiconductor diode based on the properties of the *p-n* junction

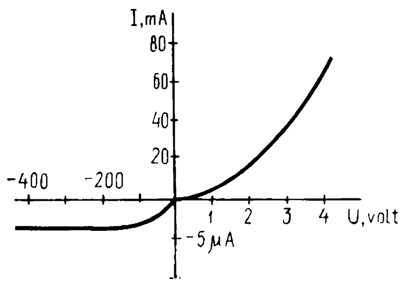


Fig. 4.40

has two branches: the upper right branch and the lower left branch. Since the right branch corresponds to small voltages and the left branch to considerably higher voltages (with the currents in the conductive direction being much higher than the currents in the blocking direction), the two branches are constructed using different scales. What is the explanation for the existence of the left branch and in what manner does the current in the blocking direction depend on the temperature of the diode?

4.41. The phenomenon of secondary electron emission consists in the following. When electrons bombard a solid surface, the surface emits secondary electrons (and partially reflects the primary electrons, which impinge on the surface). Secondary electron emission is characterized by the secondary emission coefficient σ, which is the

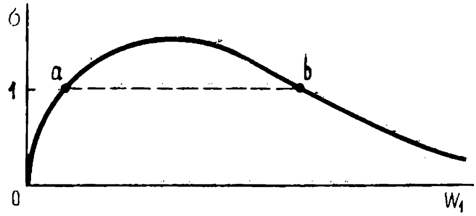


Fig. 4.41

ratio of the secondary electron current to the primary current. The dependence of the secondary emission coefficient on the primary electron energy W_1 for a certain dielectric is depicted in the figure. At $\sigma = 1$ the surface of the dielectric does not change its potential under electron bombardment, since the number of electrons leaving the surface every second is equal in this case to the number of electrons bombarding the surface every second. The two points a and b on the σ vs. W_1 curve correspond to $\sigma = 1$. At which point is the process stable and at which is it unstable?

4.42. Under secondary electron emission (see Problem 4.41), the energy distribution function $F(W_2)$ for

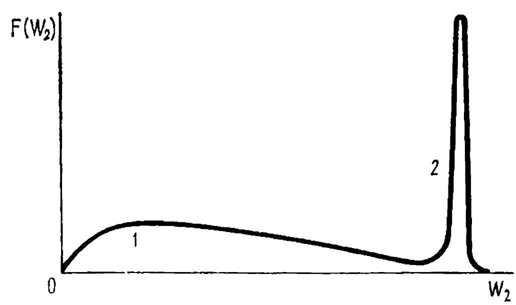


Fig. 4.42

secondary electrons is represented sufficiently well by two curves (*1* and *2*) shown in the figure accompanying the problem. Which of the two curves represents the primary electrons and which, the "true" secondary electrons?

4. Direct Current

4.1. The two conductors, *1-3-5* and *2-4-6*, have different potentials, with the result that when key K is closed, a current will flow from *3* to *4*, while the currents passing through the resistors will flow from *1* to *3*, from *5* to *3*, from *4* to *6*, and from *4* to *2*. The closing of the key leads to an increase in the current flowing through the ammeter. If the resistances of the conductors *1-3-5*, *2-4-6*, and *3-4* are extremely low, then the sections *1-2* and *5-6* of the resistors will be shorted for all practical purposes.

4.2. Prior to closing the key, the circuit consists of two resistors connected in parallel (the resistance of each resistor being $3R$). This means that the total resistance of the circuit is $1.5R$. After the key has been closed, the circuit consists of two sections connected in series, each of which has two resistors connected in parallel. The resistance R' of each section is given by the formula

$$\frac{1}{R'} = \frac{1}{R} + \frac{1}{2R}$$

and is equal to $2R/3$. The resistance of the entire circuit is $4R/3$. The current measured by the ammeter is higher than that measured prior to closing the key.

4.3. If R is the resistance of the whole potentiometer and R_V is the resistance of the voltmeter, the total resistance of section ab of the potentiometer is

$$R_{ab} = \frac{R_V (R/2)}{R_V + R/2} = \frac{R}{2(1+R/2R_V)} < \frac{R}{2}.$$

The resistance of section bc is equal to $R/2$. The voltage applied to the potentiometer will not be distributed evenly. Since the resistance of ab is less than that of bc, the voltage applied to the first section is lower than that applied to the second. The higher the resistance of the voltmeter, the closer the readings of the voltmeter are to one-half of the applied voltage.

4.4. Since the voltage applied to the "black box" is supplied by a DC source, it is natural to assume that there are only resistances inside the "box". The simplest way to lower voltage is to use a potentiometer (see the figure accompanying the answer). However, there is not a single circuit employing only resistances that can raise voltage. As the figure accompanying the answer demonstrates, from the voltage applied to terminals *1* and *2* one can always "take" a certain fraction, e.g. 127 V, while the 127 V applied to terminals *3* and *4* will yield the same 127 V on terminals *1* and *2*. The remark (made in the problem) that concerns the role of the measuring device is important since a voltmeter, which always has a finite resistance, redistributes the resistances in the circuit and, hence, changes the voltages (see Problem 4.3).

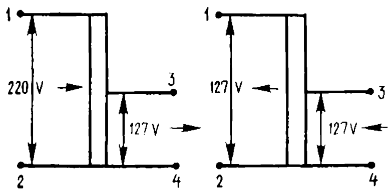


Fig. 4.4

4.5. Let us assume, for the sake of simplicity, that the resistances of the two potentiometers are the same. When the sliding contact of each potentiometer is in the middle, the total resistance of the circuit is $R_0/2$, where R_0 is the resistance of each potentiometer. If the sliding contact of the second potentiometer is in the extreme (left or right) position, we have two resistances, R_0 and $R_0/2$, connected in parallel (assuming that the wires have no resistance), so that the total resistance is $R = R_0/3$. The reading of the ammeter proves to be greater than when the sliding contact of the second potentiometer was in the middle position by a factor of 1.5. Thus, when the sliding contact of the second potentiometer is moved from one extreme position to the other, the readings of the ammeter pass through a minimum.

4.6. If x is the resistance of the potentiometer between point a and the sliding contact, the total resistance between a and the sliding contact is $rx/(r+x)$, while the resistance of the entire circuit is $R - x + rx/(r+x)$. The current supplied by the DC source is

$$I = \frac{U_0}{R - x + rx/(r+x)} .$$

The potential difference between the sliding contact and point a is

$$U=\frac{U_0rx}{(R-x)(r+x)+rx}=\frac{U_0rx}{Rx-x^2+Rr}.$$

The current passing through the ammeter is

$$I_a=\frac{U_0r}{Rx-x^2+Rr}. \tag{4.6.1}$$

To find the extremum, we take the derivative

$$\frac{dI_a}{dx}=U_0r\left[\frac{-R+2x}{(Rx-x^2+rx)^2}\right]. \tag{4.6.2}$$

Nullifying (4.6.2) yields

$$x = R/2. \tag{4.6.3}$$

If we substitute (4.6.3) into (4.6.1) we find the minimal current:

$$I_{min}=\frac{U_0r}{R(r+R/4)}.$$

Thus, as the sliding contact is moved, the current through the ammeter passes through a minimum, and the

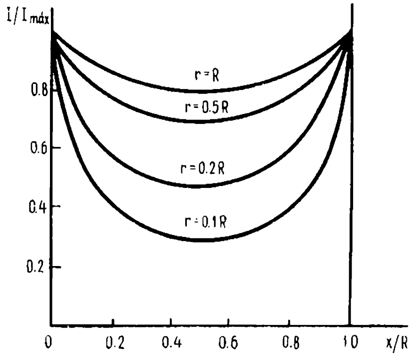


Fig. 4.6

smaller the r the deeper the minimum. At $x = 0$ and $x = R$, a current of $I_{max} = U_0/R$ passes through the ammeter. The ratio of I_{min} to I_{max} is equal to $(r/R)(1 + r/R)^{-1}$. The I_a/I_{max} vs. x/R curves for several values of r/R are shown in the figure accompanying the answer.

4.7. The exact value of $\mathscr{E}_x$ can be determined if one measures exactly the potential difference between points a and b provided that the current passing through the source in question is zero. This can be achieved by selecting a proper ratio of resistances between points a and b and points b and c using the resistance box. Knowing the resistance R between points a and b and the current I, measured by the ammeter, we find the sought emf:

$$\mathscr{E}_x = IR.$$

4.8. Since the currents in the resistors R_1 and R_2 are the same, we can write

$$U_{Aa}/R_1 = U_{aB}/R_2.$$

The charges on capacitors connected in series are the same, which means that

$$U_{Ab}C_1 = U_{bB}C_2.$$

Since

$$U_{Aa}/U_{aB} = R_1/R_2,\ U_{Ab}/U_{bB} = C_2/C_1,\ U_{Aa} = U_{Ab},$$

$$U_{aB} = U_{bB},$$

we have

$$R_1/R_2 = C_2/C_1.$$

The resistances and capacitances are in inverse ratio.

Just as in Problem 3.27, where a DC source generates a potential difference between points A and B, the solution holds true only if the (active) resistances of the capacitors are infinitely large.

4.9. The current remains unchanged on the entire section from one junction to another. A junction is a point in a circuit where more than three conductors meet. There are seven such sections in the figure accompanying the problem. If there are n junctions in a circuit, then Kirchhoff's first law yields $n - 1$ independent equations. There are four junctions in the circuit in question (*1*, *4*, *5* and *7*). Thus, to determine seven currents we are lacking four equations, which Kirchhoff's second law will yield. The simplest way to employ Kirchhoff's second law is to use loops that do not overlap, namely, *1-2-3-4-1*, *1-4-5-1*, *1-5-7-1*, and *5-6-7-5*.

4.10. There are eight junctions in the circuit. Since Kirchhoff's first law yields only $n - 1$ independent equations for n junctions, we have seven such equations. If the circuit is transformed onto a plane (see the figure accompanying the answer), there are five nonoverlapping loops, while the loop *1-4-8-5-1* overlaps all other loops and therefore can be obtained from these.

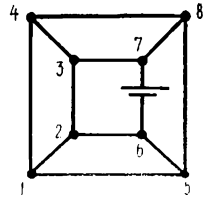


Fig. 4.10

4.11. The current flowing in the circuit is $I = \mathscr{E}/(R + r)$. The power output in the external circuit is

$$P = I^2R = \mathscr{E}^2 \frac{R}{(R+r)^2} .$$

The maximal power output can be found from the condition $\mathrm{d}P/\mathrm{d}R = 0$, or

$$\frac{\mathrm{d}P}{\mathrm{d}R} = \mathscr{E}^2 \frac{(R+r)^2 - 2(R+r)R}{(R+r)^4} = 0,$$

whence $R = r$. The fact that the resistances are equal means that the power outputs must be equal, too:

$$I^2r = I^2R.$$

Hence, the efficiency is equal to 0.5.

4.12. The current is maximal when the circuit is shorted, or when the external resistance is zero:

$$I_{\mathrm{m}} = \mathscr{E}/r.$$

Thus, in both cases the ratio of the emf to the internal resistance is the same.

Maximal useful power output (the power output of the external resistance) is achieved when the exernal resistance is made equal to the internal resistance (see the answer to Problem 4.11), that is, when the current is one-half the maximal current. This power output is

$$P = \frac{\mathscr{E}^2}{4R} = \frac{\mathscr{E}^2}{4r} .$$

Since the ratio $\mathscr{E}/r$ is the same in both cases, a double useful power output is achieved at a double electromotive force (for equal currents). Note that the internal resistance of the DC source must also be doubled if we want the ratio to remain unchanged.

4.13. If in one position of the sliding contact the rheostat has a resistance R_1 and in the other, a resistance R_2, the current is $\mathscr{E}/(R_1 + r)$ in the first case and $\mathscr{E}/(R_2 + r)$ in the other. Correspondingly, the power output in the external circuit (the same in both cases) is

$$P = \frac{\mathscr{E}^2 R_1}{(R_1 + r)^2} = \frac{\mathscr{E}^2 R_2}{(R_2 + r)^2}.$$

Dividing this expression by $\mathscr{E}^2$ and solving for r, we find that

$$r = \sqrt{R_1 R_2}.$$

By hypothesis, in one case $R_1 = xR$ and in the other, $R_2 = (1 - x) R$. Whence

$$r = R\sqrt{x(1 - x)}.$$

4.14. The likely circuit, apparently, consists of a combination of cells connected in parallel and in series. There are two possibilities here: several parallel groups of cells connected in series or several in-series groups of cells connected in parallel. First, it can be shown that the two variants are equivalent. Indeed, in the first variant, the potentials at the points a_1, a_2, a_3, etc. coincide, i.e.

$$U_{a_1} = U_{a_2} = U_{a_3} = \ldots;$$

the same is true of the potentials at the points b_1, b_2, b_3, etc., i.e.

$$U_{b_1} = U_{b_2} = U_{b_3} = \ldots .$$

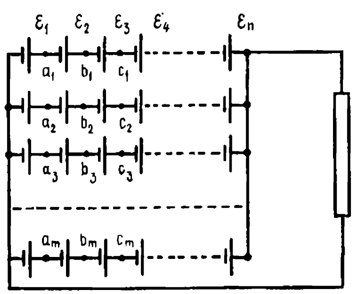


Fig. 4.14

This line of reasoning can be continued. The respective points can be interconnected, and the entire circuit will be transformed into the second variant. Suppose the overall number of cells is N. We connect these cells in such a manner that groups of n cells that form $m = N/n$ parallel groups are connected in series. In this case the current in the external circuit is

$$I = \frac{\mathcal{E} n}{R+rn/m} = \frac{\mathcal{E} Nn}{RN+rn^2}.$$

The power output in the external circuit is

$$P = I^2R = (\mathcal{E} N)^2 R \frac{n^2}{(RN+rn^2)^2}.$$

To find the maximal value, we nullify the derivative of P with respect to n:

$$\frac{\mathrm{d}P}{\mathrm{d}n} = \frac{2\mathcal{E}^2N^2n}{(RN+rn^2)^2}(RN-rn^2) = 0.$$

Whence

$$n = \sqrt{RN/r}. \qquad (4.14.1)$$

But this does not solve the problem completely. The number n should be one of the cofactors of N. To find a practical value of n, we must compare the power outputs for two values of n that are closest to the one given by (4.14.1), that is, one must be smaller than the calculated value and the other must be greater, and yet the two must be cofactors of N. Here is an example. Suppose $N = 400$, $R = 16\ \Omega$ and $r = 9\ \Omega$. The calculated value is

$$n = \sqrt{\frac{400\times 16}{9}} = 26.7.$$

The closest cofactors of N are 25 and 40. In this example the greater power output is at $n = 25$. Thus, the circuit consists of 16 parallel groups of 25 cells connected in series in each group.

4.15. Since the displacement current is defined as

$$I_{\text{dis}} = S\frac{\mathrm{d}D}{\mathrm{d}t},$$

after performing certain manipulations we can write

$$I_{\text{dis}} = \frac{\varepsilon_0\varepsilon}{l}\frac{\mathrm{d}U}{\mathrm{d}t} = \frac{\mathrm{d}Q}{\mathrm{d}t},$$

where Q is the charge on the capacitor. Thus, the displacement current may be made constant over a definite time interval if the capacitor is charged (or discharged) by a direct current. For this in the circuit of the capacitor being charged we must have a device that restricts the current flowing through it within broad voltage limits (Figure (a)). A diode operating in the saturation mode may serve as such a device. For the case of a thermionic valve (or diode) the appropriate circuit is shown in Figure (b), while for the case of a semiconductor diode the cir-

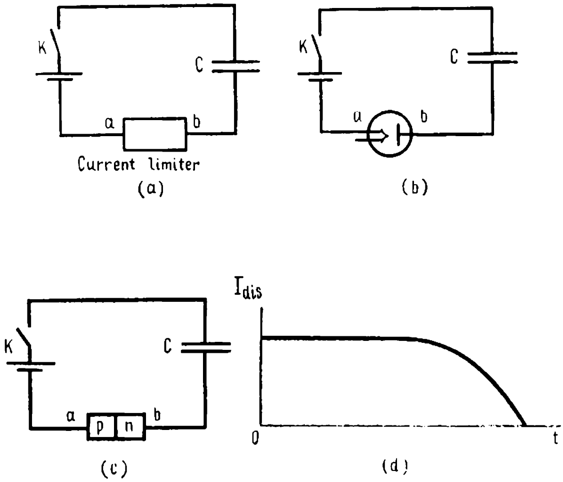


Fig. 4.15

cuit is shown in Figure (c). The diode is introduced into the circuit in the cut-off direction, and the voltage across the diode is

$$U_d = \mathscr{E}_0 - U_C.$$

As long as U_d remains within the saturation region, the current through the diode (and, hence, the charging current) remains constant. The displacement current remains constant in the process. After a certain time interval has elapsed (the lower the charging current the longer the interval), the charging current rapidly falls off to zero. The time dependence of the displacement current is illustrated schematically in Figure (d).

4.16. At each moment of time the capacitor voltage is equal to the potential drop across the resistor:

$$U = IR.$$

Bearing in mind that $U = Q/C$ and $I = -\mathrm{d}Q/\mathrm{d}t$ (the minus sign shows that the capacitor's charge decreases), we get

$$\frac{Q}{C} = -R\frac{\mathrm{d}Q}{\mathrm{d}t},$$

or

$$\frac{\mathrm{d}Q}{Q} = -\frac{1}{RC}\,\mathrm{d}t. \qquad (4.16.1)$$

Integrating (4.16.1) from the initial charge Q_0 to Q and from the initial moment $t = 0$ to time t, we get

$$Q = Q_0 \exp\left(-\frac{t}{RC}\right) = U_0 C \exp\left(-\frac{t}{RC}\right).$$

Accordingly, the current varies with time as follows:

$$I = I_0 \exp\left(-\frac{t}{RC}\right), \qquad (4.16.2)$$

with $I_0 = U_0/R$. Taking logs, we can write (4.16.2) as follows

$$\ln I = \ln I_0 - \left(\frac{1}{RC}\right) t.$$

Thus, the time dependence of $\ln I$ is represented by a straight line with a negative slope, whose absolute value is $1/RC$. The resistance R determines the current at the first moment of discharge and the initial capacitor voltage, which is equal to the emf of the source. The value of R determined in this manner and the slope of the straight line fix the value of C.

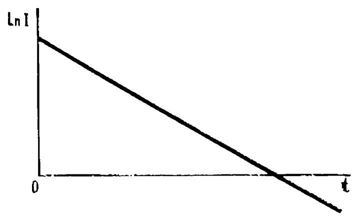


Fig. 4.16

4.17. As shown in the answer to Problem 4.16, the discharge current varies with time as

$$I = I_0 \exp\left(-\frac{t}{RC}\right), \quad \text{or} \quad \ln I = \ln I_0 - \left(\frac{1}{RC}\right) t.$$

Initially, i.e. at $t = 0$, both currents are the same (see the figure accompanying the problem). For a fixed capacitance C this is possible if the other two parameters, U

and R, change simultaneously. Since by hypothesis only one parameter changes, we conclude that the capacitance C varies. The fact that the slope of the straight line representing the $\ln I$ vs. t dependence decreases means that the capacitance C increases.

4.18. The time variation of the current proceeds as follows:

$$\ln I = \ln I_0 - \left(\frac{1}{RC}\right) t.$$

The fact that the two straight lines, *1* and *2*, are parallel indicates that the product RC must be constant. Since by hypothesis the discharge processes differ only in the value of one parameter, both R and C remain constant. What is different is the initial capacitor voltage, and since for straight line *1* the initial current is higher than for straight line *2*, so is the initial capacitor voltage.

4.19. The current flowing through the resistor with resistance R will generate during a time interval dt the following amount of heat:

$$dq = I^2 R dt.$$

The time variation of the discharge current of the capacitor is

$$I = I_0 \exp\left(-\frac{t}{RC}\right).$$

Thus,

$$dq = I_0^2 R \exp\left(-\frac{2t}{RC}\right) dt.$$

Integrating this expression with respect to t from $t = 0$ to $t = \infty$, we get

$$q = I_0^2 RC/2. \qquad (4.19.1)$$

At the first moment the discharge current is

$$I_0 = U_0/R. \qquad (4.19.2)$$

Substituting (4.19.2) into (4.19.1), we obtain the initial energy stored by the charged capacitor:

$$q = U_0^2 C/2.$$

4.20. According to Kirchhoff's second law, at each moment of time the emf of the DC source is equal to the sum of the potential drop across the resistor and the capacitor voltage:

$$\mathscr{E} = IR + U.$$

Bearing in mind that $I = \mathrm{d}Q/\mathrm{d}t$ and $U = Q/C$, we get

$$\mathscr{E} = R\frac{\mathrm{d}Q}{\mathrm{d}t} + \frac{Q}{C}, \quad \text{or} \quad \frac{\mathrm{d}Q}{Q-\mathscr{E}C} = -\frac{1}{RC}\,\mathrm{d}t.$$

Integration from $Q = 0$ to Q and from $t = 0$ to t and appropriate transformations yield

$$Q = \mathscr{E}C\left[1 - \exp\left(-\frac{t}{RC}\right)\right],$$

whence

$$I = \frac{\mathscr{E}}{R}\exp\left(-\frac{t}{RC}\right) = I_0\exp\left(-\frac{T}{RC}\right).$$

The amount of heat generated by the current in the resistor R in the course of $\mathrm{d}t$ is

$$\mathrm{d}q = I^2R\,\mathrm{d}t = I_0^2R\exp\left(-\frac{2t}{RC}\right).$$

Integration from $t = 0$ to $t = \infty$ yields

$$q = I_0^2R^2C/2 = \mathscr{E}^2C/2.$$

The same amount of energy is stored by the capacitor when the latter is charged to a voltage equal to the source's emf. The total energy used up by the source,

$$\int_0^\infty \mathscr{E}I\,\mathrm{d}t = \mathscr{E}I_0\int_0^\infty e^{-t/(RC)}\,\mathrm{d}t,$$

is equal to $\mathscr{E}^2C$, which is the sum of two equal quantities $\mathscr{E}^2C/2$.

4.21. The energy stored by a charged capacitor can be written in the form

$$W_0 = Q^2/2C.$$

After the second (uncharged) capacitor is connected to the first, the total charge does not change while the capacitance doubles. Thus, the total energy stored by this system becomes

$$W = Q^2/4C,$$

which is one-half of the initial energy. So where did the other half go to? As the charge is redistributed between the two capacitors, a current flows through the conductors connecting them and generates heat. In addition, there is always a magnetic field around a conductor with current, and this magnetic field carries energy, just as an electric field does. If the resistance of the conductors is low (zero in the case of superconductors), the difference between the initial and the final energy will go to the magnetic field. Eventually the second capacitor will become fully charged while the first capacitor will become completely discharged and the current will cease. Then the second capacitor will begin to discharge, and charge will flow to the first capacitor. This process will continue, that is, there will appear electromagnetic oscillations in which the energy will alternate between that of the electric field and that of the magnetic.

4.22. For an electron that is inside the disk at a distance r from the axis to move along a circle, there should be a force pulling it to the axis. According to Newton's second law,

$$F = m\omega^2 r.$$

This force is generated by a radial electric field caused by the redistribution of the electrons in the disk and is such that the force acting on the electron is

$$F = eE = m\omega^2 r.$$

If we substitute $-d\varphi/dt$ for E and integrate from φ_1 to φ_2 and from 0 to R, where R is the radius of the disk, we get

$$\int_{\varphi_1}^{\varphi_2} \frac{d\varphi}{dt} = -\frac{m\omega^2}{e} \int_0^R r\, dr.$$

As a result, we get the potential difference between the center of the disk and the edge:

$$U = \varphi_1 - \varphi_2 = \frac{m\omega^2 R^2}{2e} = \frac{mv^2}{2e}, \qquad (4.22.1)$$

where v is the linear velocity of points at the edge of the disk. Theoretically formula (4.22.1) can be used to determine the electron's charge-to-mass ratio. But actually this constitutes a problem, as shown by an estimate of the potential difference between the axis and the edge. The electron charge is 1.6×10^{-19} C and the electron mass is 9.1×10^{-31} kg. We set the electron linear velocity on the edge at 300 m/s. The potential difference then proves to be less than 10^{-9} V. It is extremely difficult to measure such a quantity in such a rotating system.

4.23. Moving together with the cylinder, the electrons in the wire have a momentum mv each. When the cylinder is braked, the electrons continue to move, but the generated potential difference creates a braking electric field of strength E. The force acting on every electron in the wire is

$$F = eU/l,$$

with U the instantaneous potential difference. According to Newton's second law,

$$\Delta mv = \frac{e}{l} \int_0^t U\,dt,$$

where Δmv is the momentum lost by an electron during the entire braking time, which quantity is equal to the initial momentum mv. The charge-to-mass ratio for the electron is then

$$\frac{e}{m} = \frac{vl}{\int_0^\infty U\,dt}.$$

The integral in the denominator can be evaluated by calculating the area under the voltage oscillogram.

4.24. The heating of the conductor will result in the electron diffusing into the neighborhood of section ab, with the potential of the conductor somewhat increasing. The current flowing in the conductor will have to overcome a potential barrier at point a. This requires additional energy, which will be taken from the metal. On the other hand, when passing through the conductor at point b, the current goes to a region with a lower potential, and in this place energy will be released to the metal. As a result, the point where the temperature is at a maximum will shift in the direction of current flow.

4.25. Prior to cooling, the resistance of the wire was the same over the entire length of the wire (precisely, the resistivity was the same at all points of the wire). When the fan is switched on, the resistance of the section that is being cooled will lower. This leads to a redistribution of the potential between the cooled and uncooled sections, with the greater voltage applied to the latter section, as a result of which its temperature increases. This phenomenon is enhanced by the fact that the resistance of the uncooled section somewhat grows with temperature, which leads to a still greater inhomogeneity in the distribution of the potential in both sections.

4.26. The resistances of bulbs with the same rated voltage are in inverse proportion to the rated wattages. Hence, the resistance of the bulb with the lower wattage is six times the resistance of the bulb with the higher wattage. When the bulbs are connected in series with the DC source, the current is the same and six-sevenths of the total voltage of 220 V, or 189 V, is applied to the first (25 W) bulb and one-seventh, to the second (150 W) bulb. Actually the difference is still greater because the resistance of the first bulb will increase due to overheating, while that of the second will decrease. Hence, the 25-W bulb must burn out.

4.27. An increase in voltage will lead to an increase in the currents passing through the conductor and semiconductor, and this will lead to an increase in temperature of both. As a result the resistance of the conductor will increase and that of the semiconductor will decrease. Hence, the current through the semiconductor will increase greater than in proportion to the voltage, while the current through the conductor will increase lesser than in proportion to the applied voltage, with the result that the ammeter in the semiconductor circuit will register a higher current than the ammeter in the conductor circuit.

4.28. Prior to an increase in voltage, the resistances of the semiconductor and the conductor were equal. When the voltage is increased, the current in the circuit increases, too, and so does the temperatures of the semiconductor and conductor. This leads to a drop in the resistance of the semiconductor and an increase in the resistance of the conductor. The voltage between the semiconductor and conductor will redistribute in such a manner that the voltmeter connected to the conductor will register a higher voltage than the voltmeter connected to the semiconductor.

4.29. The electrons leaving the filament, or cathode, create a negative space charge whose field does not let all the emitted electrons into the region. According to the Child-Langmuir theory developed for parallel plane electrodes on the assumption that the initial velocity of the electrons is zero, the current density between the electrodes is

$$j = \frac{4\sqrt{2}\,\varepsilon_0}{9}\sqrt{\frac{e}{m}}\,\frac{U^{3/2}}{d^2}$$

(the three-halves power law). Here e and m are the electron charge and electron mass, U is the voltage drop across the electrodes, and d is the distance between the electrodes. On the current-voltage characteristic, the initial segment of the curve agrees with the three-halves power law. Then, as the electron cloud is dissipated, the current gradually reaches a plateau and saturation sets in, with the saturation current being the total flux of electrons that the cathode can deliver at a given temperature. The temperature dependence of the current density is given by the Richardson-Dushman equation

$$j_{\text{sat}} = A'T^2 \exp\left(-\frac{P}{kT}\right).$$

The quantity P in the numerator of the exponent is the so-called work function, or the work that an electron must do to leave the metal. The other quantities in the equation are as follows: T the thermodynamic temperature, k the Boltzmann constant, $A' = 6.02 \times 10^5$ A/m$^2\cdot$K^2 is a constant that is a combination of universal constants. The difference in the curves in the figure accompanying the problem lies in the temperature of the cathode, which is higher for curve *2*.

4.30. When thermoelectric current flows from the cathode to the anode, the electrons leaving the cathode carry away an energy required for overcoming the potential barrier that exists at the metal-vacuum interface (the work function of the electrons), with the result that the cathode cools off. To maintain a constant cathode temperature, the filament current must be increased.

4.31. When the potential difference between the electrodes is nil, the concentration of positive and negative ions (cations and anions) is the same in practically the entire volume. When an external voltage is applied, a current generated by the motions of cations to the cathode and anions to the anode begins to flow in the electrolyte. As a result, the regions near the electrodes prove to be depleted of ions whose sign is that of the electrode. Cations leave the anode and anions leave the cathode. For this reason, near the anode an excess of negative charge is formed, while an excess of positive charge is formed in the region near the cathode. All this leads to a distortion in the electric field. The enhanced field near the electrodes imparts an enhanced velocity to the ions. This ensures the flow of current under lower charge carrier concentrations.

4.32. The sign of the volume charge is determined by the direction of convexity of the U vs. x curve.* The volume charge is positive where the curve is convex upward and negative where the curve is convex downward, while the volume charge is nil where the U vs. x dependence is represented by a straight line. Hence, the entire region between the cathode and the anode is divided, within the first approximation (i.e. ignoring certain details), into the cathode space (from point *0* to point *1* in the figure accompanying the problem) with a surplus positive charge, the Faraday dark space (from point *1* to point *2*) with a negative charge, and the region of the "positive column" (from point *2* to point *3*), which constitutes a plasma with practically equal concentrations of electrons and positive ions and, hence, with a net charge that is practically nil.

* See Problems 3.28 and 3.29.

4.33. The conduction-current density is given by the formula

$$j = e \sum_k n_k u_k Z_k, \qquad (4.33.1)$$

where e is the magnitude (without taking into account the sign) of the elementary charge (the electron charge), n_k the concentration of the given type of charge carriers, u_k the average directional velocity of the carriers, and Z_k the charge number, or valence, of the carriers. For an electron $Z = -1$, while for a positive doubly charged ion (say, He^{++}) $Z = +2$. Electron velocities exceed ion velocities by a factor of 10 or even 100, with the result that even at equal concentrations the electron current is much stronger than the ion current. Since in an electric field electrons and ions move in opposite directions, we can assume that the electron velocity is negative if the ion velocity is set positive. Since the number Z for electrons is negative, the signs of the products in (4.33.1) coincide, with the result that the ammeter in the gas discharge gap circuit will register the total current of electrons and ions.

4.34. As the particle moves from the anode to the Faraday cylinder, the field in the region between A and F constantly changes. When the particle leaves the anode (through the aperture) and is moving toward the Faraday

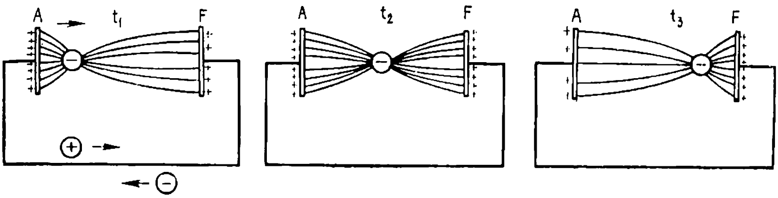


Fig. 4.34

cylinder, it induces positive charges on these electrodes, and the magnitude of these charges constantly changes. The density of these charges on the anode decreases while that on the Faraday cylinder increases (the variation in the distribution of electric charge for three moments in time is shown in the figure accompanying the answer). For this reason, in the region of space between A and F there appears a continuous displacement current, which means that an exact replica of this current appears in the circuit. The current in the circuit can be graphically represented as a consequence of the fact that in approaching the Faraday cylinder the particle repels, so to say, the electrons which, in effect, move toward the anode through the measuring device G. Thus, the current in the circuit exists during the entire time of motion of the particle between the anode and the Faraday cylinder, as shown in Figure (e) accompanying the problem.

4.35. If two metals are brought into contact, the limiting energies of the electrons will establish themselves at a common level (the common Fermi level; see the figure accompanying the answer). The difference between the height of a potential barrier and the Fermi level determines the external work function $e\varphi$. The difference between the two work functions (for the two barriers) is equal to the external contact potential difference. To transfer an electron from the surface of metal *2* to the surface of metal *1* requires performing an amount of work equal to $e\Delta\varphi$. The distance between the levels of minimal electron energy in the metals determines the internal contact potential difference ΔW_F. According to the quantum theory of metals, the Fermi level at 0 K is pinned at $(h^2/2m) \times (3n/\pi)^{2/3}$, where h is the Planck constant, m the electron mass, and n the electron concentration in a metal. Hence, the concentration of electrons in metal *1* is higher.

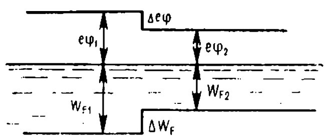


Fig. 4.35

4.36. The concentration of electrons whose energy ranges from W to $W + \Delta W$ is

$$dn = f(W)\,dW = CW^{1/2}dW,$$

in accordance with Eq. (4.36.1). Integrating this expression from zero to the limiting energy, we obtain the concentration of electrons in the entire energy range:

$$n = C\int_0^{W_F} W^{1/2}\,dW = \frac{2}{3}CW_F^{3/2}.$$

Hence, $W_F \propto n^{2/3}$. As is proved in the quantum theory of metals, W_F is given by the following formula (with due regard for universal constants):

$$W_F = \frac{h^2}{2m}\left(\frac{3n}{\pi}\right)^{2/3}.$$

4.37. The electrical conductivity (specific conductance) of a semiconductor depends on temperature according to the following law:

$$\sigma = \sigma_0 \exp\left(-\frac{W}{kT}\right), \quad \text{or} \quad \ln\sigma = \ln\sigma_0 - \frac{W}{kT},$$

where W is the width of the forbidden band. This law implies that the wider the forbidden band, the steeper the straight line representing the $\ln\sigma$ vs. T^{-1} dependence. Hence, semiconductor *1* has a wider forbidden band.

4.38. Since the upper sections of the curves for the two semiconductors coincide and the slopes of the lower sections are smaller than those of the upper, the intrinsic conductivities are the same. Also, since the lower sections of the curves slope in the same manner (i.e. the slopes are equal), the width of the forbidden band for the impurities is the same for both semiconductors. Thus, these diagrams can be interpreted as a characteristic of impurity semiconductors with different concentrations of the same impurities. For a fixed temperature, a higher conductivity corresponds to a higher located characteristic, *1*, and, hence, a higher concentration of impurities.

4.39. The diffusion of electrons from an n-type semiconductor into a p-type one and of holes from the p-type semiconductor to the n-type one leads to the appearance of a positive potential on the n-type semiconductor (the left branch of curve *0*) and a negative potential on the p-type semiconductor (the right branch of the same curve). If we now apply a positive potential to the n-type semiconductor and a negative potential to the p-type, the potential difference between the two semiconductors will increase (curve *1*), whence the boundary between the two semiconductors is depleted of charge carries as a result of electrons being drained to the n-type semiconductor and holes, to the p-type. Such a direction of the potential difference is the cut-off one. When the external voltage is applied in the reverse direction (curve *2*), the potential difference lowers, and it proves easier for the electrons to move to the p-type semiconductor and the holes, to the n-type. This direction is the conducting one.

4.40. Every semiconductor possesses intrinsic conduction in addition to extrinsic (or impurity) conduction. Intrinsic conduction is caused by the transfer of electrons from the valence band to the conduction band and by simultaneous formation of holes in the valence band. Intrinsic conduction is of a mixed nature for this reason, and because of this the n-type semiconductor carries a small number of holes while the p-type semiconductor carries a small number of electrons. When a voltage is applied in the cut-off direction, these charge carriers constitute the so-called reverse current. As the temperature of a semiconductor diode is increased, the electron and hole concentrations grow, as a result of which conductivity in the cut-off direction grows, too. The reverse current reaches a plateau when practically all the "alien" charge carriers (holes in the n-type semiconductor and electrons in the p-type) participate. Usually this current is several orders of magnitude less than the direct current, with the plateau reached at relatively high voltages. The direct current grows with voltage very rapidly, since as the voltage is increased, it becomes easier for the charge carriers to pass through the junction.

4.41. Let us suppose that there is a small deviation from the state with $\sigma = 1$. If σ drops at point a, the number of electrons impinging on the surface is smaller than the number of electrons leaving the surface, with the surface acquiring a negative potential, which brings down σ still further. And this leads to a further increase in the negative potential. The process continues until the current of primary electrons becomes totally cut off. If at the same point the value of σ increases somewhat, the surface acquires a positive potential, the velocity of the electrons increases, and the current continues to grow, which leads to an increase in σ, up to the maximum on the curve, and then to point b, where $\sigma = 1$, just as at point a. Similar reasoning leads us to the conclusion that small variations in σ at point b change the potential of the surface in such a way that σ returns to its initial value $\sigma = 1$. Thus, point a corresponds to an unstable state, while point b corresponds to a stable state. For a small deviation from the state of equilibrium corresponding to point a, the surface acquires a potential that either completely cuts off the current of the primary electrons or corresponds to that at point b.

4.42. The reflected electrons retain practically all their initial energy and, hence, correspond to curve *2* in the figure accompanying the problem. Secondary electrons, on the other hand, are freed from the solid bombarded with the primary electrons at the expense of the energy of the primary electrons, and this energy is distributed between the emitted electrons. The energy of the latter is, as a rule, considerably less than that of the primary electrons. Moreover, while all the reflected electrons have velocities that are concentrated within a narrow interval and have energies close to those of the primary electrons, the secondary electrons form a broad spectrum of velocities. The "true" secondary electrons are represented by curve *1* in the figure accompanying the problem.