CBSE Board

Class X Mathematics (Standard)

Sample Paper - 4

Term 2 - 2022

Time: 2 hours **Total Marks: 40**

General Instructions:

1. The question paper consists of 14 questions divided into 3 sections A, B, C.
2. All questions are compulsory.
3. Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.
4. Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.
5. Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.

Section A

Q1 – Q6 are of 2 mark each.

1. Find the 20^{th} term from the last term of the AP 3, 8, 13, ……., 253

OR

Check whether – 150 is a term of the AP 11, 8, 5, 2,…

2. The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.

3. Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

4. A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of Rs 500 per m^2.

5. 100 surnames were randomly picked up from a local telephone directory and the frequency distribution of the number of letters in the English alphabets in the surnames was obtained as follows:

Number of letters	1 – 4	4 – 7	7 – 10	10 – 13	13 – 16	16 – 19
Number of surnames	6	30	40	6	4	4

Find the mean number of letters in the surnames.

6. Sum of the areas of two squares is 468 m^2. If the difference of their perimeters is 24 m, find the sides of the two squares.

OR

The sum of the reciprocals of Rehman's ages, (in years) 3 years ago and 5 years from now is $\frac{1}{3}$. Find his present age.

Section B

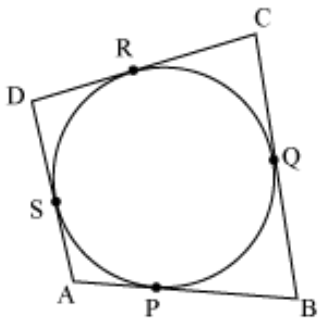
Q7 – Q10 are of 3 mark each.

7. Find the following tables gives the distribution of the life time of 400 neon lamps:

Life time (in hours)	**Number of lamps**
1500 – 2000	14
2000 – 2500	56
2500 – 3000	60
3000 – 3500	86
3500 – 4000	74
4000 – 4500	62
4500 – 5000	48

Find the median life time of a lamp.

8. A quadrilateral ABCD is drawn to circumscribe a circle. Prove that AB + CD = AD + BC.



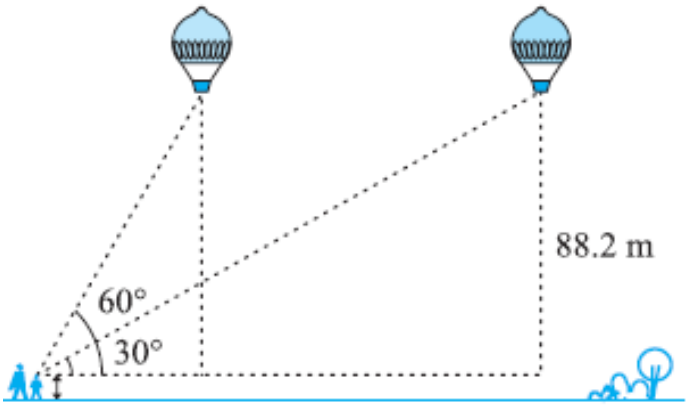
9. The following frequency distribution gives the monthly consumption of electricity of 68 consumers of a locality. Find the mode of the data.

Monthly consumption (in units)	**Number of consumers**
65 – 85	4
85 – 105	5
105 – 125	13
125 – 145	20
145 – 165	14
165 – 185	8
185 – 205	4

10. As observed from the top of a 75 m high lighthouse from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships.

OR

A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60°. After some time, the angle of elevation reduces to 30°. Find the distance travelled by the balloon during the interval.

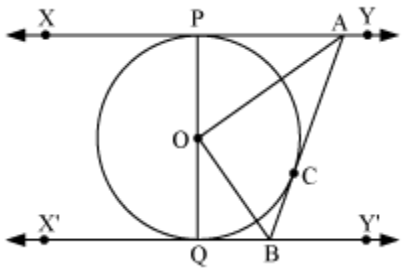


Section C

Q11 – Q14 are of 4 mark each.

11. A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm.

12. In the given figure XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY and A and X'Y' at B. Prove that $\angle AOB=90^\circ$.



OR

Prove that the parallelogram circumscribing a circle is a rhombus.

13. Harsh is standing between two buildings having height 10m and 15m. Now the angle of elevation from the point where harsh is standing, to the top of 10m building is 45^o. Whereas the angle of elevation from the same point to the top of 15m building is 60^o. Using the given data, answer the following questions.

i) Draw a labelled figure on the basis of the given information and find the distance between Harsh and the 10m building.
ii) Find the distance between Harsh and the 15m building.

14. Shaurya wants to participate in a 200m race. He can currently run that distance in 51 seconds and with each day of practice, it takes him 2 seconds less. But he want to complete that distance in 31 seconds. Based on the above information, answer the following questions.

i) Form an AP using the given data and find how many minimum number of days he needs to practice till his goal is achieved.
ii) Find the total number of seconds he ran till his goal was achieved.

Solution

Section A

1. Given AP is

3, 8, 13,….. …, 253

Common difference for this AP is 5.

Therefore, this AP can be written in reverse order as

253, 248, 243, …, 13, 8, 5

For this AP,

$a = 253$

$d = 248 - 253 = -5$

$n = 20$

$a_{20} = a + (20 - 1)\, d$

$a_{20} = 253 + (19)\,(-5)$

$a_{20} = 253 - 95$

$a = 158$

Therefore, 20th term from the last term is 158.

OR

For this AP,

$a = 11$

$d = a_2 - a_1 = 8 - 11 = -3$

Let –150 be the n^{th} term of this AP

We know that,

$$a_n = a + (n - 1)d$$

$$-150 = 11 + (n - 1)(-3)$$

$$-150 = 11 - 3n + 3$$

$$-164 = -3n$$

$$n = \frac{164}{3}$$

Clearly, n is not an integer.

Therefore, –150 is not a term of this AP

2. Let the larger and smaller number be x and y respectively.
According to the given question,

$x^2 - y^2 = 180$ and $y^2 = 8x$

$\Rightarrow x^2 - 8x = 180$

$\Rightarrow x^2 - 8x - 180 = 0$

$\Rightarrow x^2 - 18x + 10x - 180 = 0$

$\Rightarrow x(x - 18) + 10(x - 18) = 0$

$\Rightarrow (x - 18)(x + 10) = 0$

$\Rightarrow x = 18, -10$

However, the larger number cannot be negative as 8x will be negative and the square of the smaller number will be negative which is not possible.
Therefore, the larger number will be 18 only.

$x = 18$

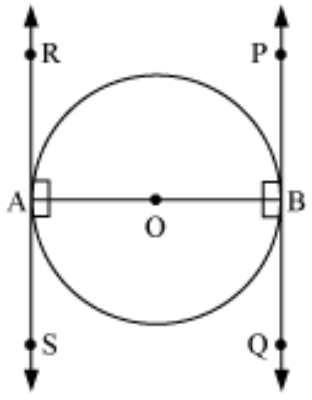
$\therefore y^2 = 8x = 8 \times 18 = 144$

$\Rightarrow y = \pm\sqrt{144} = \pm 12$

$\therefore$ Smaller number $= \pm 12$

Therefore, the numbers are 18 and 12 or 18 and –12.

3.



Let AB be a diameter of circle. Two tangents PQ and RS are drawn at the end points of the diameter AB.

It is known that the radius is perpendicular to tangent at the point of contact.

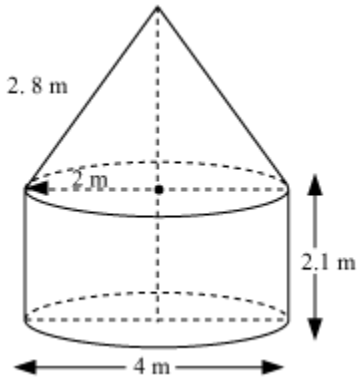
Therefore, $\angle$ OAR = 90°, $\angle$ OAS = 90°, $\angle$ OBP = 90° and $\angle$ OBQ = 90°

$\angle$ OAR = $\angle$ OBQ (alternate interior angles)

$\angle$ OAS = $\angle$ OBP (alternate interior angles)

Since, alternate interior angles are equal, lines PQ and RS will be parallel.

4.



Given that

Height (h) of the cylindrical part = 2.1m

Diameter of the cylindrical part = 4m

So, radius of the cylindrical part = 2m

Slant height (l) of conical part = 2.8m

Area of canvas used = CSA of conical part + CSA of cylindrical part

$$= \pi rl + 2\pi rh$$

$$= \pi \times 2 \times 2.8 + 2\pi \times 2 \times 2.1$$

$$= 2\pi\left[2.8 + 2 \times 2.1\right] = 2\pi\left[2.8 + 4.2\right] = 2 \times \frac{22}{7} \times 7$$

$$= 44m^2$$

Cost of $1m^2$ canvas = Rs.500

Cost of $44m^2$ canvas = 44 × 500 = 22000

So, it will cost Rs.22000 for making such a tent.

5. Now we can find class marks of given class intervals by using relation

$$\text{Class mark} = \frac{\text{Upper class limit} + \text{Lower class limit}}{2}$$

Taking 11.5 as assumed mean (a) we can find d_i, u_i and f_iu_i according to step deviation method as below.

Number of letters	Number of surnames	x_i	$x_i - a$	$u_i = \frac{x_i - a}{3}$	f_iu_i
1 – 4	6	2.5	–9	–3	–18
4 – 7	30	5.5	–6	–2	–60
7 – 10	40	8.5	–3	–1	–40
10 – 13	16	11.5	0	0	0
13 – 16	4	14.5	3	1	4
16 – 19	4	17.5	6	2	8
Total	100				–106

$\sum f_iu_i = -106$

$\sum f_i = 100$

Mean $\bar{x} = a + \left(\frac{\sum f_iu_i}{\sum f_i} \right) h$

$= 11.5 + \left(\frac{-106}{100} \right) \times 3$

= 11.5 – 3.18 = 8.32

6. Let the sides of the two squares be x m and y m. Therefore, their perimeter will be 4x and 4*y* respectively and their areas will be x^2 and y^2 respectively.

It is given that, $4x - 4y = 24$

$x - y = 6$

$x = y + 6$

Also, $x^2 + y^2 = 468$

$\Rightarrow (6 + y)^2 + y^2 = 468$

$\Rightarrow 36 + y^2 + 12y + y^2 = 468$

$\Rightarrow 2y^2 + 12y - 432 = 0$

$\Rightarrow y^2 + 6y - 216 = 0$

$\Rightarrow y^2 + 18y - 12y - 216 = 0$

$\Rightarrow y(y + 18) - 12(y + 18) = 0$

$\Rightarrow (y + 18)(y - 12) = 0$

$\Rightarrow y = -18 \text{ or } 12.$

However, side of a square cannot be negative.

Hence, the sides of the squares are 12 m and (12 + 6) m = 18 m

OR

Let the present age of Rehman be x years.

Three years ago, his age was $(x - 3)$ years.

Five years hence, his age will be $(x + 5)$ years.

It is given that the sum of the reciprocals of Rehman's ages 3 years ago and 5 years from now is $\frac{1}{3}$.

$$\therefore \frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3}$$

$$\frac{x+5+x-3}{(x-3)(x+5)} = \frac{1}{3}$$

$$\frac{2x+2}{(x-3)(x+5)} = \frac{1}{3}$$

$$\Rightarrow 3(2x+2) = (x-3)(x+5)$$

$$\Rightarrow 6x + 6 = x^2 + 2x - 15$$

$$\Rightarrow x^2 - 4x - 21 = 0$$

$$\Rightarrow x^2 - 7x + 3x - 21 = 0$$

$$\Rightarrow x(x-7) + 3(x-7) = 0$$

$$\Rightarrow (x-7)(x+3) = 0$$

$$\Rightarrow x = 7, -3$$

However, age cannot be negative.

Therefore, Rehman's present age is 7 years.

Section B

7. We can find cumulative frequencies with their respective class intervals as below -

Life time	**Number of lamps (f_i)**	**Cumulative frequency**
1500 – 2000	14	14
2000 – 2500	56	14 + 56 = 70
2500 – 3000	60	70 + 60 = 130
3000 – 3500	86	130 + 86 = 216

3500 – 4000	74	216 + 74 = 290
4000 – 4500	62	290 + 62 = 352
4500 – 5000	48	352 + 48 = 400
Total (*n*)	400	

Now we may observe that cumulative frequency just greater th

$\frac{n}{2}\left(\text{i.e. } \frac{400}{2} = 200\right)$ is 216 belonging to class interval 3000 – 3500.

Median class = 3000 – 3500

Lower limit (*l*) of median class = 3000

Frequency (*f*) of median class = 86

Cumulative frequency (*cf*) of class preceding median class = 130

Class size (*h*) = 500

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$= 3000 + \left(\frac{200 - 130}{86}\right) \times 500$$

$$= 3000 + \frac{70 \times 500}{86}$$

= 3406.976

So, median life time of lamps is 3406.98 hours.

8. It can be observed that:

DR = DS (tangents from point D)

CR = CQ (tangents from point C)

BP = BQ (tangents from point B)

AP = AS (tangents from point A)

Adding the above four equations,

DR + CR + BP + AP = DS + CQ + BQ + AS

(DR + CR) + (BP + AP) = (DS + AS) + (CQ + BQ)

CD + AB = AD + BC

9.

Monthly consumption (in units)	**Number of consumers (f_i)**
65 – 85	4
85 – 105	5
105 – 125	13
125 – 145	20
145 – 165	14
165 – 185	8
185 – 205	4

Now from table it is clear that maximum class frequency is 20 belonging to class interval 125 – 145.

Modal class = 125 – 145

Lower limit (l) of modal class = 125

Class size (h) = 20

Frequency (f_1) of modal class = 20

Frequency (f_0) of class preceding modal class = 13

Frequency (f_2) of class succeeding the modal class = 14

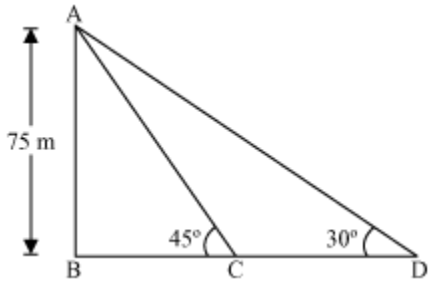
$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$$

$$= 125 + \left[\frac{20 - 13}{2(20) - 13 - 14} \right] \times 20$$

$$= 125 + \frac{7}{13} \times 20$$

$$= 125 + \frac{140}{13} = 135.76$$

10.



Let AB be the lighthouse and the two ships be at point C and D respectively.

In ΔABC,

$$\frac{AB}{BC} = \tan 45°$$

$$\frac{75\ m}{BC} = 1$$

$$BC = 75\ m$$

In ΔABD,

$$\frac{AB}{BD} = \tan 30°$$

$$\frac{75\ m}{BC + CD} = \frac{1}{\sqrt{3}}$$

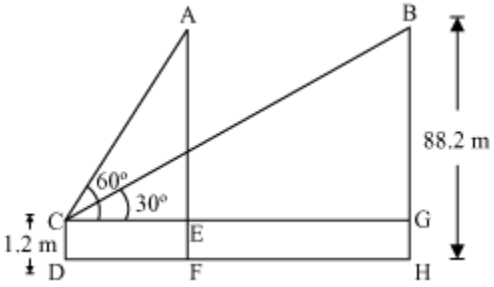
$$\frac{75\ m}{75\ m + CD} = \frac{1}{\sqrt{3}}$$

$$75\sqrt{3}\ m = 75\ m + CD$$

$$CD = 75\left(\sqrt{3} - 1\right)\ m$$

Thus, the distance between the two ships is $75\left(\sqrt{3} - 1\right)$ m.

OR



Let A be the initial position of the balloon and the position changes to B after some time and CD is the girl.

In ΔACE,

$\frac{AE}{CE} = \tan 60°$

$\frac{AF - EF}{CE} = \tan 60°$

$\frac{88.2 - 1.2}{CE} = \sqrt{3}$

$\frac{87}{CE} = \sqrt{3}$

$CE = \frac{87}{\sqrt{3}} = 29\sqrt{3}$

In ΔBCG,

$\frac{BG}{CG} = \tan 30°$

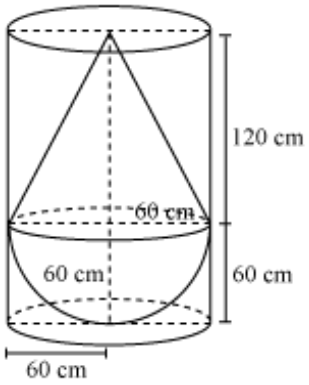
$\frac{88.2 - 1.2}{CG} = \frac{1}{\sqrt{3}}$

$87\sqrt{3} = CG$

Distance travelled by balloon = EG = CG – CE $= 87\sqrt{3} - 29\sqrt{3} = 58\sqrt{3}$ m

Section C

11.



Radius (r) of hemispherical part = radius (r) of conical part = 60cm

Height (h_2) of conical part of solid = 120cm

Height (h_1) of cylinder = 180cm

Radius (r) of cylinder = 60cm

Volume of water left = volume of cylinder – volume of solid

$= \text{volume of cylinder} - (\text{volume of cone} + \text{volume of hemisphere})$

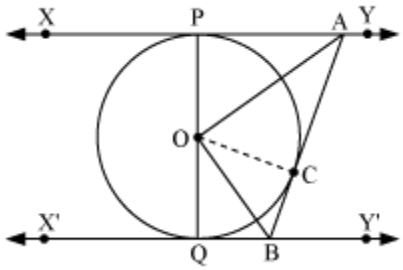
$= \pi r^2 h_1 - \left(\frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3 \right)$

$= \pi (60)^2 (180) - \left(\frac{1}{3}\pi (60)^2 \times 120 + \frac{2}{3}\pi (60)^3 \right)$

$= \pi (60)^2 \left[(180) - (40 + 40) \right]$

$= \pi (3,600)(100) = 3,60,000\pi \text{ cm}^3 = 1131428.57 \text{cm}^3 = 1.131 \text{m}^3$

12.



Join OC.

In $\triangle$ OPA and $\triangle$ OCA,

OP = OC	(Radius of the same circle)
AP = AC	(tangents from point A)
AO = AO	(common)
$\triangle$ OPA $\cong$ $\triangle$ OCA	(SSS congruence rule)
$\therefore \angle$ POA = $\angle$ COA	… (1)

Similarly $\triangle$ OQB $\cong$ $\triangle$ OCB

$\therefore \angle$ QOB = $\angle$ COB … (2)

Since POQ is a diameter of circle we can say it is a straight line.

So, $\angle$ POA + $\angle$ COA + $\angle$ COB + $\angle$ QOB = 180º

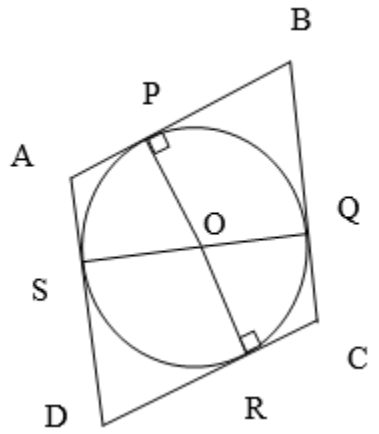
Now from equations (1) and (2),

2$\angle$ COA + 2 $\angle$ COB = 180º

($\angle$ COA + $\angle$ COB) = 90º

$\angle$ AOB = 90°

OR



Since, ABCD is a parallelogram,

AB = CD (i)

BC = AD (ii)

Now, it can be observed that:

DR = DS (tangents on circle from point D)

CR = CQ (tangents on circle from point C)

BP = BQ (tangents on circle from point B)

AP = AS (tangents on circle from point A)

Adding all the above four equations,

DR + CR + BP + AP = DS + CQ + BQ + AS

(DR + CR) + (BP + AP) = (DS + AS) + (CQ + BQ)

CD + AB = AD + BC (iii)

From equation (i) (ii) and (ii):

2AB = 2BC

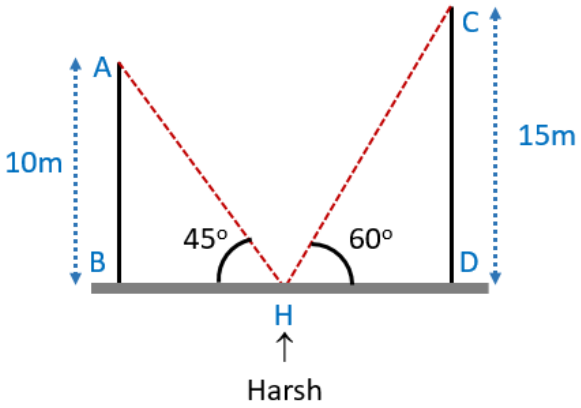
AB = BC

$\therefore$AB = BC = CD = DA

Hence, ABCD is a rhombus.

13.

i) Let AB and CD denote the 10m and 15m buildings respectively and H denote the position of Harsh



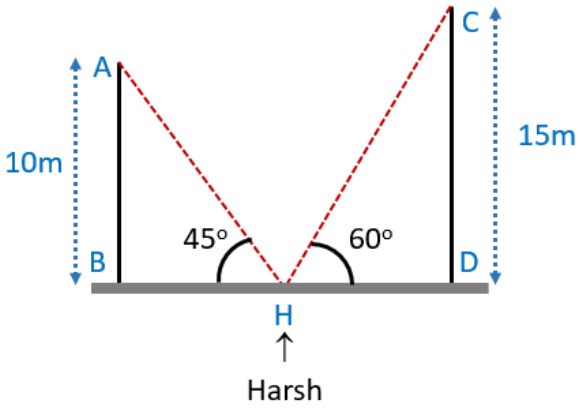
In $\triangle ABH$

$$\tan 45 = \frac{AB}{BH}$$

$$\therefore 1 = \frac{AB}{BH}$$

$$\therefore AB = BH = 10m$$

ii)



In $\triangle CDH$

$$\tan 60 = \frac{15}{DH}$$

$$\therefore \sqrt{3} = \frac{15}{DH}$$

$$\therefore DH = 5\sqrt{3}m$$

14.

i) The required AP is 51, 49, 47, ……..

Here, d = −2, a = 51 and n^{th} term = 31

n^{th} term of an AP = a + (n – 1) d

$\Rightarrow$ 31 = 51 + (n – 1)(–2)

$\Rightarrow$ –2n + 2 + 51 = 31

$\Rightarrow$ –2n + 53 = 31

$\Rightarrow$ –2n = –22

$\Rightarrow$ n = 11

ii)

$$S_n = \frac{n}{2}(a + l)$$

$$= \frac{11}{2}(51 + 31)$$

$$= 451 \text{ seconds.}$$