CHAPTER 12

Boolean Logic

LEARNING OBJECTIVES

- What is Boolean Logic?
- Solving questions through decision tree analysis
- Approach to solve the questions

Boolean logic is a form of algebra in which all values are reduced to either true or false. It was developed by English mathematician George Boole in the mid-19th century. Its rules govern logical functions (true/false) and are the foundation of all electronic circuits in the computer. As add, subtract, multiply and divide are the primary operations of arithmetic, AND, OR and NOT are the primary operations of Boolean logic. Boolean logic is turned into logic gates on the chip, and the logic gates make up logic circuits that perform functions such as how to add two numbers together.

However, we would be concerned here with the logical application of Boolean logic in True/False situations only. Boolean logic is, sometimes, also referred as Binary Logic.

Let us see a Boolean Logic situation:

On a fictional island, all inhabitants are either knights, who always tell the truth, or knaves, who always lie. The question set involves a stranger to the island who meets small groups of inhabitants there. Usually, the aim is for the visitor to conclude the inhabitants' type from their statements, but some questions of this type ask for other facts to be deduced. The question may also be to determine a Yes/No question which the visitor can ask in order to discover what he needs to know.

Basic Question Type

There are three inhabitants referred as A, B and C. The visitor asks A what type he is, but does not hear A's answer. B then says "A said that he is a knave" and C says "Don't believe B: he is lying!"

To solve the puzzle, understand that no inhabitant can say that he is a knave. Reason for his can be given as—If somebody is a Knight, then he speaks truth always and of course he cannot say that 'I am knave." Similarly, if somebody is a knave, then he speaks false always. Now, if somebody speaks false only, he cannot say that I always speak false as this amounts to contradiction.

Coming back to the original question, hence B's statement must be untrue, so he is a knave, and C's statement must be true, so he is a knight. Since B is a knave, he will always lie. Therefore B was lying when he said that A said he was a knave. Therefore A must have said he was a Knight.

More Examples

A large number of elementary Boolean Logic questions can be solved by using the simple logic or elementary Boolean Algebra (or, logic truth tables). To increase the familiarity with Boolean logic and its simplification process, let us see some more basic questions.

Jain and Baid are residents of the island of knights and knaves.

Question 1

Jain says: We are both knaves.

Who is who?

Solution This is what Jain is saying in a more extended form:

"Jain is a knave and Baid is a knave."

If Jain was a knight, he would not be able to say that he was a knave since he would be lying. Therefore the statement "Jain is a knave" must be true.

Since knaves lie, and one statement is true, hence to make the statement given by Jain false, the other statement must be false. Therefore the statement "Baid is a knave" must be false which leads to the conclusion that Baid is a knight.

The solution is that Jain is a knave and Baid is a knight.

Alternatively, we can use Boolean algebra to find out who's who as follows:

Let J be true if Jain is a knight and let B be true if Baid is a knight. Now, either Jain is a knight and what he said was true, or Jain is not a knight and what he said was false. Translating that into Boolean algebra, we get:

Therefore Jain is a knave and Baid is a knight. Although we can do this question without using Boolean algebra very easily.

Here is one of the most famous Boolean Logic questions:

Question 2

A prince visits an island inhabited by Knights and Knaves. Knights always tell the truth, and knaves always lie.

The prince comes to a fork in the road. He needs to know which road leads to the jungle so as to rescue the princess. (Although the prince doesn't know it, the south road leads to the jungle and the north road leads to the monster.)

Standing at this fork in the road is a knight and a knave, but the prince can't tell who is who. What question should he ask to find the road to the jungle so that he can save the princess?

Solution Simply asking which road leads to the jungle won't help. The answer won't tell us who is lying and who is telling the truth. However, we really only need to talk to *one* of them. The trick is to ask a question where the response will be the same from both of them: a question that incorporates how a knight or a knave *not* answering would respond to the same question.

For example, what if we say to one of them, "If I asked a member of the type you *don't* belong to which road I should take to get to the jungle, what would he say?"

1. If we ask a truth-teller, the response will be: "He would say to take the north road." The road to the castle is the south road so the liar will tell us to take the north road, and the truth teller will faithfully report this to us.
2. If we ask a liar, the response will be: "He would say to take the north road." The road to the jungle is the south road and the truth teller will tell us to take the south road, but the liar will *not* report this faithfully to us - he will say the opposite.

In both cases we'll get the same response. We should do the opposite of what we have been told because, regardless of whether we are speaking to a liar or a truth teller, our question will always produce the wrong answer to which road we should take.

Variations in Problems

In some variants, inhabitants may also be alternators, who alternate between lying and telling the truth, or normal person, who can say whatever they want (as we can see in the case of Knight/Knave/Spy puzzles). A further complication can be brought by bringing the situation where the inhabitants may answer yes/no questions in their own language, and the visitor knows that "bal" and "da" mean "yes" and "no" but does not know which is which. So, this question will now lead to two Y-junctions.

Knights always tell the truth.

Knaves always lie.

Spies can either lie or tell the truth.

Normally, we encounter a group of three people, A, B and C and one of them is a Knight, one of them is a Knave and the left one is a spy, but we don't know who is which. However, they all know the identity of each other.

Example

A says: I am a knight.

B says: That is true.

C says: I am the spy.

Solution We can understand that neither B nor C can be the knight. (B is saying that somebody else is knight and C says that he is a spy). Hence A is a knight. What B is saying is true so he cannot be the knave. Hence he is a spy. So, C is knave.

A says: B is the spy.

B says: No, C is the spy.

C says: No, B is definitely the spy.

Solution: B cannot be the spy, as in that case both a knave and a knight would be accusing him of being the spy. And if B is not the spy, then in that case neither A nor C can be the knights since they would not be telling the truth. Hence B is a knight, as a result A and C are knave and spy respectively.

There can be only six possibilities of Knights, knaves and spy that we can have for any particular set:

Knight Knave Spy, or, Knight Spy Knave, Knave Spy Knight, Spy Knight Knave, Knave Knight Spy, Spy Knave Knight.

Any statement made by any person in a question can be classified into various possibilities. For example – the statement that 'I am a knave' cannot be said made by i. Knight, ii. Knave. Hence anybody making this statement has to be a spy. Similarly, the statement 'I am a spy' cannot be made by a knight. Hence anybody making this statement has to be either a knave or a spy.

However, there can be some useless statements too like 'I am a knight'. It can be seen that this statement can be made by anybody—a knight or a knave or a spy.

PRACTICE EXERCISES

Directions for questions 1 to 5: ***Read the following paragraph and answer the questions that follow:***

There are two types of people living in what is land—X type and Y type. The X type of inhabitants always speak the truth and the Y type of inhabitants always lie.

Q.1 Munu says, "I always lie". Which type of an inhabitant is she?
(a) X
(b) Y
(c) Either X or Y
(d) given statement is infeasible

Q.2 Chunmun says, "According to Munu, I always speak the truth." Which of the following is a correct conclusion?
(a) Chunmun has to be of type X
(b) Chunmun has to be of type Y
(c) Munu has to be of type X
(d) Munu has to be of type Y

Q.3 Bhasker says, "Sharma and I are of the same type." Which of the following is a correct conclusion?
(a) Bhasker and Sharma are necessarily of the same type.
(b) Sharma has to be of type X.
(c) Bhasker and Sharma cannot be of the same type.
(d) The given statement is infeasible

Q.4 Rahul says "Pallavi and I are of different types." Which of the following is a correct conclusion?
(a) Rahul and Pallavi are of type Y and type X respectively.
(b) Rahul and Pallavi cannot be of the same type.
(c) Pallavi has to be of Type Y.
(d) None of these

Q.5 Booker says, "At least one person among Shane and I always lie." What types are Booker and Shane respectively?
(a) Y, X
(b) Y, Y
(c) X, Y
(d) It is not possible to deduce

Directions for questions 6 to 8: ***Read the passage below and solve the questions based on it.***

On the Island of who went where, there are only two kinds of people. Type NO are those who, when they ask a question, must always get a 'No' for an answer and type Yes are those who must always get a Yes for an answer to every question they ask.

Q.6 Victor and Trish are married. Victor asks you: "Are both of us of the type No?" You can conclude that
(a) It is impossible for him to have asked such a question
(b) Victor is a No
(c) Trish is a No
(d) His type cannot be identified

Q.7 Jay, Ajay and Vijay all approach you. Jay asks, "Are at least two of us of the type No?" You can infer that
(a) Jay is a No
(b) Jay is a Yes
(c) None of them is No
(d) Cannot be determined.

Q.8 Abhay, Lokesh and Rituraj approach you. Abhay asks, "Is it true that neither Lokesh nor Rituraj can be yes?" you can infer that
(a) Both Lokesh nd Rituraj are Nos.
(b) Abhay is a No
(c) Abhay is a yes
(d) None of the above

Directions for questions 9 to 11: ***Read the information given below and solve the questions based on it.***

There are four members in a family—Kitto, Litto, Mitto and Nitto. Among these four persons, there is one couple, their son and their daughter. When asked about their relationships, the following were their replies:

Kitto – Nitto is my husband. Mitto is my daughter.

Litto – Kitto is my mother. Mitto is my son.

Mitto – Kitto and Litto are of the same gender. Litto is my sister.

Nitto – Litto is of the same gender as I. Mitto is my son.

If was known that only one of them always speaks the truth.

Q.9 Among them, if there are two persons who always lie then who always speaks truth?
(a) Kitto
(b) Mitto
(c) Nitto
(d) cannot be determined

Q.10 Among the four, who cannot be the truth teller?
(a) Litto
(b) Mitto
(c) Nitto
(d) cannot be determined

Q.11 Among them if there are two persons who always alternate between truth and lie, then who speaks the truth always?
(a) Kitto
(b) Mitto
(c) Nitto
(d) cannot be determined

Directions for questions 12 to 16: ***Read the information given below and solve the questions based on it.***

There are three friends—A, R and U—in a group. Out of these three friends, one always speaks truth, one always lies and another one alternates between truth and lies. Exactly one of them is the owner of facebook.com, the other one is the owner of orkut.com and the third one is the owner of terrificmail.com, in no particular order.

In a recent interview, each of them was asked—Which website do you own? Following is their reply pertaining to the above given question:

A – I own facebook.com. U owns orkut.com.
U – I own facebook.com. R owns terrificmail.com.
R – A owns orkut.com. U owns facebook.com.

Q.12 Who among the three is a liar?
(a) A
(b) R
(c) U
(d) cannot be determined

Q.13 Who among the following owns terrificmail.com?
(a) A
(b) R
(c) U
(d) cannot be determined

Q.14 Which website is owned by U?
(a) facebook.com
(b) orkut.com
(c) terrificmail.com
(d) cannot be determined

Q.15 Who among the three is a truth teller?
(a) A
(b) R
(c) U
(d) cannot be determined

Q.16 Who among the three is an alternator?
(a) A
(b) R
(c) U
(d) cannot be determined

Q.17 A puzzle is defined by the statements of three people. Out of those three people, one is a knight who always tells the truth, one is a knave who always lies, and one is a spy who can either lie or tell the truth. If there is no assignment of 'knight knave spy' which is valid, the puzzle is a paradox. How many of the given sets will end up in paradox?

I. A: I am a Knight
B: I am a Knave
C: I am a Knave
II. A: I am a Knight
B: I am a Knave
C: A is a Knight
III. A: I am a Knight
B: A is not a Knave
C: A is not a Knave

(a) 0 (b) 1
(c) 2 (d) 3

Directions for questions 18 to 20: ***Read the passage below and solve the questions based on it.***

Nobody has ever imagined that the level of politics in India will stoop so low that people will become indifferent and totally uninterested in Indian Politics. And as a result, they will forget even the name of their Prime Minister. And even the situation of the parliamentarians was no better. To hide the fact that they are unaware of the name of their Prime Minister, whenever they are asked the name of Prime Minister, they make two statements - one of which is true and the other one is false(If you cannot convince, then confuse them). Following is the recorded transcript of the interview of three parliamentarians:

Rakhi – The Prime Minister(PM) claims that he is the PM. I am the PM.
Sameera – I am the PM. Rakhi is the PM.
Mallika – I am the PM. Sameera knows who is the PM.

Q.18 Using the statements of whom of the three, can we determine the PM?
(a) Sameera's + Rakhi's
(b) Sameera's + Mallika's
(c) Mallika's + Rakhi's
(d) Cannot be determined

Q.19 Who is the PM?
(a) Sameera (b) Mallika
(c) Rakhi (d) None of these three

Q.20 Whose first statement is false?
(a) Sameera (b) Mallika
(c) Rakhi (d) None of these three

ANSWER KEYS

1. (d)	**2.** (c)	**3.** (b)	**4.** (c)	**5.** (c)	**6.** (b)	**7.** (d)	**8.** (d)	**9.** (c)	**10.** (a)
11. (a)	**12.** (b)	**13.** (b)	**14.** (b)	**15.** (a)	**16.** (c)	**17.** (d)	**18.** (a)	**19.** (a)	**20.** (b)

Answers to Q.1 to 5:

1. Munu cannot be X type person, as then her given statement would contradict it. If she is type Y person, then her statement is true which will prove that she is not type Y person. So, the given statement is infeasible.

Hence, option (d) is the correct answer.

2. We are given what Munu said about Chunmun. So, we should analyse it according to Chunmum being type X or type Y person. If Chunmun is type X person, then the statement given by Chunmun must be correct & so Munu is also type X person. If Chunmun is type Y person then the statement given by Chunmun must be wrong, so it means then Munu must have said that Chunmun lies which again shows that Munu is type X person. So, in either case Munu must be type X person. So, we can conclude that Munu has to be of type X.

Hence, option (c) is the correct answer.

3. We do not know whether Bhaskar is saying truth or he is lying. If we assume that Bhaskar is type X person, then he must be true & it proves that Sharma is also type X person. If Bhaskar is type Y person, then the statement must be false & so Bhaskar & Sharma must be of different type. So, in that case, Sharma must be type X person. So, in either case, Sharma is type X person. So, we can conclude that Sharma has to be of type X.

Hence, option (b) is the correct answer.

4. We do not know whether Rahul is saying truth or he is lying. If we assume that Rahul is type X person, then he must be true & it proves that Pallavi is type Y person. If Rahul is type Y person, then the statement must be false & so Rahul & Pallavi must be of same type. So, in that case, Pallavi must be type Y person. So, in either case, Pallavi is type Y person. So, we can conclude that Pallavi has to be of type Y.

Hence, option (c) is the correct answer.

5. We do not know whether Booker is saying truth or he is lying. If we assume that Booker is type X person, then he must be true & it proves that Shane is type Y person, as only Shane could lie among Booker & Shane. If Booker is type Y person, then the statement must be infeasible as the statement must be true but it cannot go along the fact that Booker is type Y person. So, only possibility is that Booker is type X & Shane is type Y person.

Hence, option (c) is the correct answer.

Answers to Q.6 to 8:

6. We do not know that whether Victor is type Yes or type No person. If we assume him to be type Yes person, then the answer must be no (as at least he won't be type No person then) but if answer No is given, he will become type No person. So, he must not be type Yes person. If he is type No person then the answer he receive must be no & it is possible in Trish is type Yes person (Victor still being type No person). So, we can conclude that Victor must be type No & Trish must be type Yes person.

Hence, option (b) is the correct answer.

7. We do not know that whether Jay is type Yes or type No person. If we assume Jay to be type Yes person, then the answer may be yes (if he is type Yes person, then Ajay & Vijay may be type No person) or no (if either Ajay or Vijay or both are type Yes person). So, it is feasible as it can give Yes as answer if Jay is also yes person.

If we assume Jay to be type No person, then the answer may be yes (if Ajay or Vijay or both are type No person) or no (if both Ajay & Vijay are type Yes person). So, it is feasible as it can give No as answer if Jay is also No person. So, we cannot conclude whether Jay is Yes type person or No type person. So, it cannot be determined.

Hence, option (d) is the correct answer.

8. We do not know that whether Abhay is type Yes or type No person. If we assume Abhay to be type Yes person, then the answer will be yes ,then Lokesh & Rituraj are type No person. If we assume Abhay to be type No person, then the answer will be no, then at least one among Lokesh & Rituraj is type Yes person (there will be 3 possibilities- Lokesh is type Yes & Ritiraj is type No, Lokesh is type No & Rituraj is type Yes or both are type Yes). So, we cannot conclude anything about Abhay, Lokesh & Rituraj. So, it cannot be determined.

Hence, option (d) is the correct answer.

Answers to Q.9 to 11:

We know there are 4 people- Kitto, Litto, Mitto & Nitto. Among these people there is a couple, their son & their daughter. We also know only 1 of them always speak truth (i.e. both the statements must be true only for 1 person)

If we check the statements by Litto, it is clear that at least 1 of his/her statement must be false as there are 2 generation & not 3 generations of this family.

If we assume both the statement of Kitto is correct. Then, Nitto & Kitto are husband-wife. Mitto & Litto are their daughter & son respectively.

If we assume both the statement of Mitto is correct. Then, Nitto & Kitto are husband-wife. Litto & Mitto are their daughter & son respectively.

If we assume both the statement of Nitto is correct. Then, Kitto & Nitto are husband-wife. Litto & Mitto are their daughter & son respectively.

9. We are given that 2 person always lie. We cannot choose any 2 from Kitto, Mitto & Nitto to be those 2 who always lie (say both Kitto & Mitto always lie or both Kitto & Nitto always lie or both Nitto & Mitto always lie). As then the 3rd person cannot be the truth teller (whose both statements will be correct) & we also know that 1 from these 3 must be the truth teller as Litto cannot be the truth teller.

So, 1 person who always speak lie is Litto. Other person is 1 from Kitto, Mitto & Nitto. The truth teller is 1 from Kitto, Mitto & Nitto. One of the people who always lie is Litto, so both statements by him/her must be false. So, neither Mitto is his/her son nor his/her mother is Kitto. We can conclude that Litto & Mitto are siblings & their parents are Nitto & Kitto. So, if Kitto is not mother he must be the father. So, Kitto is the husband & his wife is Nitto

Among Kitto, Mitto & Nitto only Nitto said that Kitto & Nitto are husband wife. So, only Nitto can be the truth teller.

Hence, option (c) is the correct answer.

10. It is observed in general solution of 9-11, that at least 1 statement of Litto must be false. So, Litto cannot be the truth teller.

Hence, option (a) is the correct answer.

11. We are given that 2 persons always alternate between truth & lie, so 1 of the statement must be true & other must be false. Litto cannot be the truth teller, but he/she may be the person who alternate between truth & lie. The truth teller is still 1 among the Kitto, Mitto & Nitto. So, we can assume any one from these 3 to be the truth teller & check if we can find 2 people such that both are having 1 statement correct & one statement false. If we assume both the statement of Kitto is correct. Then, Nitto & Kitto are husband-wife. Mitto & Litto are their daughter & son respectively. Litto's 1st statement is correct but 2nd is false. Mitto's both statements are false. Nitto's 1st statement is correct & 2nd is false. So, we can conclude that Kitto must be the truth teller.

Hence, option (a) is the correct answer.

Note: students must check it by assuming Mitto & Nitto as truth teller too. It must not give exactly 2 persons who always alternate between truth & lie.

Answers to Q.12 to 16:

We know there are 3 friends- A, R & U. Among these friends, one is truth teller (always speak truth, so both statements by that person must be correct), one always lie (so both statements by that person must be false) & one always alternate between truth & lie (so this person's one statement must be correct & one statement must be false). Exactly one of them is owner of facebook.com, other one is owner of orkut.com & 3rd one is owner of terrificmail.com.

Case 1: If we assume that A is truth teller, then A is owner of facebook.com, U is owner of orkut.com & R is owner of terrificmail.com.

Further, 1st statement by U is false & the 2nd statement is correct. Similarly, both statements of R are false.

So, A is truth teller, R always lie & U alternate between truth & lie.

Case 2: If we assume that U is truth teller, then U is owner of facebook.com, A is owner of orkut.com & R is owner of terrificmail.com.

Further, both statements of R are correct. Similarly, both statements of A are false.

So, both U & R are truth tellers & A always lie.

Case 3: If we assume that R is truth teller, then U is owner of facebook.com, A is owner of orkut.com & R is owner of terrificmail.com.

Further, both statements of U are correct. Similarly, both statements of A are false.

So, both U & R are truth tellers & A always lie.

We are given exactly 1 is truth teller (always speak truth), one always lie & 3rd one alternate between truth & lie.

So, only case 1 is feasible. Therefore A is truth teller, R always lie & U alternate between truth & lie.

12. R is a liar

Hence, option (b) is the correct answer.

13. R is the owner of terrificmail.com

Hence, option (b) is the correct answer.

14. The website orkut.com is owned by U

Hence, option (b) is the correct answer.

15. A is a truth teller

Hence, option (a) is the correct answer.

16. U is an alternator

Hence, option (c) is the correct answer.

17. If we check I, Knave will not say he is Knave & so B or C must not be Knave. Further any 1 of them is spy. If A is Knave then B or C cannot be Knight too as Knight must say he is Knight. So, this does not give valid assignment of Knight, Knave & Spy.

If we check II, Knave will not say he is Knave & so B must not be Knave & he cannot be Knight too, so he is spy. Knight must say he is Knight. So, A is spy. Now, C cannot be Knave. So, this does not give valid assignment of Knight, Knave & Spy.

If we check III, if A is Knight then there cannot be any Knave (as both B & C are also correct). If A is Knave, then there cannot be any Knight (as both B & C will be wrong). If A is spy then there is no Knave (as both B & C are correct). So, this does not give valid assignment of Knight, Knave & Spy.

So, all 3 sets end up in a paradox. Correct option is (d)

Answers to Q.18 to 20:

We are given 3 parliamentarians Rakhi, Sameera & Mallika. They all are alternators (each of them gave 2 statements, out of which 1 is correct & other is false).

If we check all 3 of them claimed to be PM. So, it means PM must have claimed that he/she is PM. So, Rakhi's 1st statement is correct. So, 2nd statement must be false & so Rakhi is not PM. Now, Sameera's 2nd statement is false (as Rakhi is not PM). So, Sameera's 1st statement is correct. So, Sameera is PM. Mallika's 1st statement is false & 2nd statement is correct

18. Sameera's + Rakhi's statements are sufficient to decide that Sameera is PM. (we have to start with 1st statement of Rakhi as shown above)

Hence, option (a) is the correct answer.

19. Sameera is the PM

Hence, option (a) is the correct answer.

20. Mallika's 1st statement is false

Hence, option (b) is the correct answer.