PROBABILITY
18

CONCEPT AND IMPORTANCE OF PROBABILITY

Probability is one of the most important mathematical concepts that we use/ come across in our day-to-day life. Particularly important in business and economic situations, probability is also used by us in our personal lives. For a lot of students who are not in touch with Mathematics after their Xth/XIIth classes, this chapter, along with permutations and combinations, is seen as an indication that XIIth standard Mathematics appear in the MBA entrance exams. This leads to students taking negative approach while tacking/preparing for the Mathematics section. Students are advised to remember that the Math asked in MBA entrance is mainly logical while studying the chapter.

As I set out to explain the basics of this chapter, I intend to improve your concepts of probability to such an extent that you feel in total control of this topic.

For those who are reasonably strong, my advice would be to use this chapter both for revisiting the basic concepts as well as for extensive practice.

Probability means the chance of the occurrence of an event. In layman terms, we can say that it is the likelihood that something—that is defined as the event—will or will not occur. Thus, probabilities can be estimated for each of the following events in our personal lives:

(a) The probability that an individual student of B.Com will clear the CAT

(b) The chance that a candidate chosen at random will clear an interview

(c) The chance that you will win a game of flush in cards if you have a trio of twos in a game where four people are playing

(d) The likelihood of India's winning the football World Cup in 2014

(e) The probability that a bulb will fuse in its first day of operation, and so on

The knowledge of these estimations helps individuals decide on the course of action they will take in their day-to-day life. For instance, your estimation/judgement of the probability of your chances of winning the card game in event (c) above will influence your decision about the amount of money you will be ready to invest in the stakes for the particular game. The application of probability to personal life helps in improving our decision making.

However, the use of probability is much more varied and has far reaching influence on the world of economics and business. Some instances of these are:

(a) The estimation of the probability of the success of a business project

(b) The estimation of the probability of the success of an advertising campaign in boosting the profits of a company

(c) The estimation of the probability of the death of a 25-year old man in the next ten years and that of the death of a 55-year old man in the next ten years leading to the calculation of the premiums for life insurance

(d) The estimation of the probability of the increase in the market price of the share of a company, and so on

UNDERLYING FACTORS FOR REAL-LIFE ESTIMATION OF PROBABILITY

The factors underlying an event often affect the probability of that event's occurrence. For instance, if we estimate the probability of India winning the 2027 cricket world cup as 0.14 based on certain expectations of outcomes, then this probability will definitely improve if we know that Virat Kohli will score 800 runs in that particular World Cup.

As we now move towards the mathematical aspects of the chapter, one underlying factor that recurs in every question of probability is that whenever one is asked the question, what is the probability? The immediate question that arises/ should arise in one's mind is the probability of what?

The answer to this question is the probability of the EVENT.

The EVENT is the cornerstone or the bottom-line of probability. Hence, the first objective while trying to solve any question in probability is to define the event.

The event whose probability is to be found out is described in the question and the task of the student in trying to solve the problem is to define it.

In general, the student can either define the event narrowly or broadly. Narrow definitions of events are the building blocks of any probability problem and whenever there is a doubt about a problem, the student is advised to get into the narrowest form of the event definition.

The *difference* between the narrow and broad definition of event can be explained through an example:

Example: What is the probability of getting a number greater than 2, in a throw of a normal unbiased dice having six faces?

The broad definition of the event here is getting a number greater than 2 and this probability is given by 4/6. However, this event can also be broken down into its more basic definitions as:

The event is defined as getting 3 or 4 or 5 or 6. The individual probabilities of each of these are 1/6,1/6,1/6 and 1/6 respectively.

Hence, the required probability is 1/6 + 1/6 + 1/6 + 1/6 = 4/6 = 2/3.

Although in this example it seems highly trivial, the narrow event-definition approach is very effective in solving difficult problems on probability.

In general, event definition means breaking up the event to the most basic building blocks, which have to be connected together through the two English conjunctions—AND and OR.

Use of the Conjunction AND (Tool No. 9)

Refer *Back to School* section. Whenever we use AND as the natural conjunction joining two separate parts of the event definition, we replace the AND by the multiplication sign.

Thus, if A AND B have to occur, and if the probability of their occurrence are $P(A)$ and $P(B)$ respectively, then the probability that A AND B occur is obtained by connecting $P(A)$ AND $P(B)$. Replacing the AND by multiplication sign we get the required probability as:

$$P(A) \times P(B)$$

Example: If we have the probability of A hitting a target as 1/3 and that of B hitting the target as 1/2, then the probability that both hit the target if one shot is taken by both of them is obtained by

Event Definition: A hits the target AND B hits the target.

$$\rightarrow P(A) \times P(B) = 1/3 \times 1/2 = 1/6$$

Note: That since we use the conjunction AND in the definition of the event here, we multiply the individual probabilities that are connected through the conjunction AND.

Use of the Conjunction OR (Tool No. 10)

Refer *Back to School* section. Whenever we use OR as the natural conjunction joining two separate parts of the event definition, we replace the OR by the addition sign.

Thus, if A OR B have to occur, and if the probability of their occurrence are $P(A)$ and $P(B)$ respectively, then the probability that A OR B occur is obtained by connecting

$P(A)$ OR $P(B)$. Replacing the OR by addition sign, we get the required probability as

$$P(A) + P(B)$$

Example: If we have the probability of A winning a race as 1/3 and that of B winning the race as 1/2, then the probability that either A or B win a race is obtained by

Event Definition: A wins OR B wins.

$$\rightarrow P(A) + P(B) = 1/3 + 1/2 = 5/6$$

Note: that since we use the conjunction OR in the definition of the event here, we add the individual probabilities that are connected through the conjunction OR.

Combination of AND and OR

If two dice are thrown, what is the chance that the sum of the numbers is not less than 10?

Event Definition: The sum of the numbers is not less than 10 if it is either 10 OR 11 OR 12.

This can be done by

(6 AND 4) OR (4 AND 6) OR (5 AND 5) OR (6 AND 5) OR (5 AND 6) OR (6 AND 6)

that is, $1/6 \times 1/6 + 1/6 \times 1/6 + 1/6 \times 1/6 + 1/6 \times 1/6 + 1/6 \times 1/6 + 1/6 \times 1/6 = 6/36 = 1/6$

The bottom-line is that no matter how complicated the problem on probability is, it can be broken up into its narrower parts, which can be connected by ANDs and ORs to get the event definition.

Once the event is defined, the probability of each narrow event within the broad event is calculated and all the narrow events are connected by multiplication (for AND) or by addition (for OR) to get the final solution.

Example: In a four-game match between Kasporov and Anand, the probability that Anand wins a particular game is 2/5 and that of Kasporov winning a game is 3/5. Assuming that there is no probability of a draw in an individual game, what is the chance that the match is drawn (Score is 2–2)?

For the match to be drawn, two games have to be won by each of the players. If A represents the event that Anand won a game and K represents the event that Kasporov won a game; the event definition for the match to end in a draw can be described as: [The student is advised to look at the use of narrow event definition.]

(A&A&K&K) OR (A&K&A&K) OR (A&K&K&A)

OR (K&K&A&A) OR (K&A&K&A) OR (K&A&A&K)

This further translates into

$$(2/5)_2(3/5)_2 + (2/5)_2(3/5)_2 + (2/5)_2(3/5)_2 + (2/5)_2(3/5)_2$$
$$+ (2/5)_2(3/5)_2 + (2/5)_2(3/5)_2$$

$$= (36/625) \times 6 = 216/625$$

After a little bit of practice, you can also think about this directly as:

$${}_4C_2 \times (2/5)_2 \times {}_2C_2 \times (3/5)_2 = 6 \times 1 \times 36/625 = 216/625$$

After a little bit of practice, you can also think about this directly as:

$$_4C_2 \times (2/5)^2 \times {_2C_2} \times (3/5)^2 = 6 \times 1 \times 36/625 = 216/625$$

Where, $4C_2$ gives us the number of ways in which Anand can win two games and $2C_2$ gives us the number of ways in which Kasporov can win the remaining 2 games (obviously, only one).

BASIC FACTS ABOUT PROBABILITY

For every event that can be defined, there is a corresponding non-event, which is the opposite of the event. The relationship between the event and the non-event is that they are mutually exclusive, that is, if the event occurs then the non-event does not occur and vice versa.

The event is denoted by E; the number of ways in which the event can occur is defined as $n(E)$ and the probability of the occurrence of the event is $P(E)$.

The non-event is denoted by E'; the number of ways in which the non-event can occur is defined as $n(E')$ while the probability of the occurrence of the event is $P(E')$.

The following relationships hold true with respect to the event and the non-event.

$n(E) + n(E')$ = sample space representing all the possible events that can occur related to the activity.

$$P(E) + P(E') = 1$$

This means that if the event does not occur, then the non-event occurs.

$$\rightarrow P(E) = 1 - P(E')$$

This is often very useful for the calculation of probabilities of events where it is easier to describe and count the non-event rather than the event.

Illustration

The probability that you get a total more than three in a throw of two dice

Here, the event definition will be a long and tedious task, which will involve long counting. Hence, it would be more convenient to define the non-event and count the same.

Therefore, here the non-event will be defined as

A total not more than 3 → 2 or 3 → (1&1) OR (1&2) OR (2&1) = 1/36 + 1/36 + 1/36 = 3/36 = 1/12.

However, a word of caution especially for students not comfortable at Mathematics: Take care while defining the non-event. Beware of a trap like → event definition: Total > 10 in two throws of a dice does not translate into a non-event of < 10 but instead into the non-event of ≤ 10.

SOME IMPORTANT CONSIDERATIONS WHILE DEFINING EVENT

Random Experiment An experiment whose outcome has to be among a set of events that are completely known but whose exact outcome is unknown is a random experiment (e.g. throwing of a dice, tossing of a coin). Most questions on probability are based on random experiments.

Sample Space This is defined in the context of a random experiment and denotes the set representing all the possible equally likely outcomes of the random experiment. [e.g. sample space when a coin is tossed is (head, tail). Sample space when a dice is thrown is (1, 2, 3, 4, 5, 6).]

Event The set representing the desired outcome of a random experiment is called the event. Note that the event is a subset of the sample space.

Non-event The outcome that is opposite to the desired outcome is the non-event. Note that if the event occurs, the non-event does not occur and vice versa.

Impossible Event An event that can never occur is an impossible event. The probability of an impossible event is 0. (e.g. probability of the occurrence of 7 when a dice with six faces numbered 1–6 is thrown).

Mutually Exclusive Events A set of events is mutually exclusive when the occurrence of any one of them means that the other events cannot occur. (If head appears on a coin, tail will not appear and vice versa.)

Equally Likely Events If two events have the same probability or chance of occurrence, they are called equally likely events. (e.g. in a throw of a dice, the chance of 1 showing on the dice is equal to 2 is equal to 3 is equal to 4 is equal to 5 is equal to 6 appearing on the dice.)

Exhaustive Set of Events A set of events that includes all the possibilities of the sample space is said to be an exhaustive set of events. (e.g. in a throw of a dice, the number is less than three or more than or equal to three.)

Independent Events An event is described as such if the occurrence of an event has no effect on the probability of the occurrence of another event. (e.g. if the first child of a couple is a boy, there is no effect on the chances of the second child being a boy.)

Conditional Probability It is the probability of the occurrence of an event A given that the event B has already occurred. This is denoted by $P(A|B)$. (e.g. the probability that in two throws of a dice, we get a total of 7 or more, given that in the first throw of the dice, the number 5 had occurred.)

Concept of Odds For and Odds Against

Sometimes, probability is also viewed in terms of *odds* for and *odds against* an event.

Odds in favour of an event E are defined as: $\frac{P(E)}{P(E)'}$

Odds against an event are defined as: $\frac{P(E)'}{P(E)}$

Expectation: The expectation of an individual is defined as

Probability of winning × Reward of winning

Illustration

A man holds 20 out of the 500 tickets to a lottery. If the reward for the winning ticket is ₹1000, find the expectation of the man.

Solution: Expectation = Probability of winning × Reward of winning = $\frac{20}{500} \times 1000$ = ₹40.

ANOTHER APPROACH TO LOOK AT THE PROBABILITY PROBLEMS

The probability of an event is defined as

$$\frac{\text{Number of ways in which the event occurs}}{\text{Total number of outcomes possible}}$$

This means that the probability of any event can be obtained by counting the numerator and the denominator independently.

Hence, from this approach, the concentration shifts to counting the numerator and the denominator.

Thus, for the example used above, the probability of a number > 2 appearing on a dice is:

$$\frac{\text{Number of ways in which the event occurs}}{\text{Total number of outcomes possible}} = \frac{4}{6}$$

The counting is done through any of the following:

(a) The physical counting as illustrated above

(b) The use of the concept of permutations

(c) The use of the concept of combinations

(d) The use of the MNP rule

[*Refer to the chapter on Permutations and Combinations to understand (b), (c) and (d)s above.*]

WORKED-OUT PROBLEMS

Problem 18.1 In a throw of two dice, find the probability of getting one prime and one composite number.

Solution: The probability of getting a prime number when a dice is thrown is 3/6 = 1/2. (This occurs when we get 2, 3 or 5 out of a possibility of getting 1, 2, 3, 4, 5 or 6.)

Similarly, in a throw of a dice, there are only two possibilities of getting composite numbers, viz: 4 or 6 and this gives a probability of 1/3 for getting a composite number.

Now, let us look at defining the event. The event is—getting one prime and one composite number.

This can be obtained as:

The first number is prime, and the second is composite OR the first number is composite and the second is prime.

$$= (1/2) \times (1/3) + (1/3) \times (1/2) = 1/3$$

Problem 18.2 Find the probability that a leap year chosen at random will have 53 Sundays.

Solution: A leap year has 366 days. Fifty-two complete weeks will have 364 days. The 365th day can be a Sunday (probability = 1/7) OR the 366th day can be a Sunday (probability = 1/7). Answer = 1/7 + 1/7 = 2/7.

Alternatively, you can think of this as: The favourable events will occur when we have Saturday and Sunday or Sunday and Monday as the 365th and 366th, days respectively. (i.e. two possibilities of the event occurring). Besides, the total number of ways that can happen are Sunday and Monday OR Monday and Tuesday … OR Friday and Saturday OR Saturday and Sunday.

Problem 18.3 There are two bags containing white and black balls. In the first bag, there are eight white and six black balls and in the second bag, there are four white and seven black balls. One ball is drawn at random from any of these two bags. Find the probability of this ball being black.

Solution: The event definition here is: first bag and black ball OR second Bag and Black Ball. The chances of picking up either the first OR the second bag are 1/2 each.

Besides, the chance of picking up a black ball from the first bag is 6/14 and the chance of picking up a black ball from the second bag is 7/11.

Thus, using these values and the ANDs and Ors, we get:

$$(1/2) \times (6/14) + (1/2) \times (7/11) = (3/14) + (7/22) =$$
$$(66 + 98)/308) = 164/308 = 41/77$$

Problem 18.4 The letters of the word LUCKNOW are arranged among themselves. Find the probability of always having NOW in the word.

Solution: The required probability will be given by the equation

= Number of words having NOW/Total number of words

= 5!/7! = 1/42 [See the chapter of Permutations and Combinations to understand the logic behind these values.]

Problem 18.5 A person has three children with at least one boy. Find the probability of having at least two boys among the children.

Solution: The event is occurring under the following situations:

(a) Second is a boy and third is a girl OR

(b) Second is a girl and third is a boy OR

(c) Second is a boy and third is a boy

This will be represented by: $(1/2) \times (1/2) + (1/2) \times (1/2) + (1/2) \times (1/2) = 3/4$

Problem 18.6 Out of thirteen applicants for a job, there are five women and eight men. Two persons are to be selected for the job. The probability that at least one of the selected persons will be a woman is:

Solution: The required probability will be given by

First is a woman and second is a man OR

First is a man and second is a woman OR

First is a woman and second is a woman

i.e. $(5/13) \times (8/12) + (8/13) \times (5/12) + (5/13) \times (4/12) = 100/156 = 25/39$

Alternatively, we can define the non-event as: There are two men and no women. Then, probability of the non-event is

$(8/13) \times (7/12) = 56/156$

Hence, $P(E) = (1 - 56/156) = 100/156 = 25/39$

> ***Note:*** This is a case of probability calculation where repetition is not allowed.

Problem 18.7 The probability that A can solve the problem is 2/3 and B can solve it is 3/4. If both of them attempt the problem, then what is the probability that the problem gets solved?

Solution: The event is defined as:

A solves the problem AND B does not solve the problem

OR

A does not solve the problem AND B solves the problem

OR

A solves the problem AND B solves the problem

Numerically, this is equivalent to:

$$(2/3) \times (1/4) + (1/3) \times (3/4) + (2/3) \times (3/4)$$

$$= (2/12) + (3/12) + (6/12) = 11/12$$

Problem 18.8 Six positive numbers are taken at random and are multiplied together. Then what is the probability that the product ends in an odd digit other than 5.

Solution: The event will occur when all the numbers selected are ending in 1, 3, 7 or 9.

If we take numbers between 1 to 10 (both inclusive), we will have a positive occurrence if each of the six numbers selected are 1, 3, 7 or 9.

The probability of any number selected being either of these 4 is 4/10 (four positive events out of ten possibilities).

> ***Note:*** If we try to take numbers between 1 to 20, we will have a probability of 8/20 = 4/10. Hence, we can extrapolate up to infinity and say that the probability of any number selected ending in 1, 3, 7 or 9 so as to fulfil the requirement is 4/10.

Hence, answer = $(0.4)^6$

Problem 18.9 The probability that Arjit will solve a problem is 1/5. What is the probability that he solves at least one problem out of ten problems?

Solution: The non-event is defined as:

He solves no problems, i.e. he does not solve the first problem and he does not solve the second problem … and he does not solve the tenth problem.

Probability of non-event = $(4/5)^{10}$

Hence, probability of the event is $1-(4/5)^{10}$.

Problem 18.10 A carton contains twenty-five bulbs, eight of which are defective. What is the probability that if a sample of four bulbs is chosen, exactly two of them will be defective?

Solution: The probability that exactly two balls are defective and exactly two are not defective will be given by $({}^4C_2) \times (8/25) \times (7/24) \times (17/23) \times (16/22)$.

Problem 18.11 Out of forty consecutive integers, two are chosen at random. Find the probability that their sum is odd.

Solution: Forty consecutive integers will have twenty odd and twenty even integers. The sum of two chosen integers will be odd, only if

(a) First is even and second is odd OR

(b) First is odd and second is even

Mathematically, the probability will be given by:

P(first is even) × P(second is odd) + P(first is odd) × P(second is even)

$$= (20/40) \times (20/39) + (20/40) \times (20/39)$$

$$= (2 \times 20_2/40 \times 39) = 20/39$$

Problem 18.12 An integer is chosen at random from the first 100 integers. What is the probability that this number will not be divisible by 5 or 8?

Solution: For a number from 1 to 100 to not be divisible by 5 or 8, we need to remove all the numbers that are divisible by 5 or 8.

Thus, we remove 5, 8, 10, 15, 16, 20, 24, 25, 30, 32, 35, 40, 45, 48, 50, 55, 56, 60, 64, 65, 70, 72, 75, 80, 85, 88, 90, 95, 96, and 100.

i.e. 30 numbers from the 100 are removed.

Hence, answer is 70/100 = 7/10 (required probability).

Alternatively, we could have counted the numbers as number of numbers divisible by 5 + number of numbers divisible by 8 – number of numbers divisible by both 5 or 8.

$$= 20 + 12 - 2 = 30$$

Problem 18.13 From a bag containing eight green and five red balls, three are drawn one after the other. Find the probability of all three balls being green if

(a) The balls drawn are replaced before the next ball is picked

(b) The balls drawn are not replaced

Solution:

(a) When the balls drawn are replaced, we can see that the number of balls available for drawing out will be the same for every draw. This means that the probability of a green ball appearing in the first draw and a green ball appearing in the second draw as well as one appearing in the third draw are equal to each other.

Hence, answer to the question above will be:

Required probability = $\frac{8}{13} \times \frac{8}{13} \times \frac{8}{13}$ = $(8^3/13^3)$

(b) When the balls are not replaced, the probability of drawing any colour of ball for every fresh draw changes. Hence, the answer here will be:

Required probability = $\frac{8}{13} \times \frac{7}{12} \times \frac{6}{11}$

LEVEL OF DIFFICULTY (I)

1. In throwing a fair dice, what is the probability of getting the number '3'?

   (a) $\frac{1}{3}$

   (b) $\frac{1}{6}$

2. What is the chance of throwing a number greater than four with an ordinary dice whose faces are numbered from 1 to 6?

   (a) $\frac{1}{3}$

   (b) $\frac{1}{6}$

   (c) $\frac{1}{9}$

(d) $\frac{1}{8}$

3. Find the chance of throwing at least one ace in a simple throw with two dice.

(a) $\frac{1}{12}$

(b) $\frac{1}{3}$

(c) $\frac{1}{4}$

(d) $\frac{11}{36}$

4. Find the chance of drawing two blue balls in succession from a bag containing five red and seven blue balls, if the balls are not being replaced.

(a) $\frac{3}{13}$

(b) $\frac{21}{64}$

(c) $\frac{7}{22}$

(d) $\frac{21}{61}$

5. From a pack of 52 cards, two are drawn at random. Find the chance that one is a knave and the other a queen.

(a) $\frac{8}{663}$

(b) $\frac{1}{6}$

(c) $\frac{1}{9}$

(d) $\frac{1}{12}$

6. If a card is picked up at random from a pack of 52 cards. Find the probability that it is

(i) A spade

(a) $\frac{1}{9}$

(b) $\frac{1}{6}$

(c) $\frac{1}{4}$

(d) $\frac{2}{9}$

(ii) A king or queen

(a) $\frac{3}{13}$

(b) $\frac{2}{13}$

(c) $\frac{7}{52}$

(d) $\frac{1}{169}$

(iii) 'A spade' or 'a king' or 'a queen'

(a) $\frac{21}{52}$

(b) $\frac{5}{13}$

(c) $\frac{19}{52}$

(d) $\frac{15}{52}$

7. Three coins are tossed. What is the probability of getting

(i) Two tails and one head?

(a) $\frac{1}{4}$

(b) $\frac{3}{8}$

(c) $\frac{2}{3}$

(d) $\frac{1}{8}$

(ii) One tail and two heads?

(a) $\frac{3}{8}$

(b) 1

(c) $\frac{2}{3}$

(d) $\frac{3}{4}$

8. Three coins are tossed. What is the probability of getting?

(i) Neither three heads nor three tails?

(a) $\frac{1}{2}$

(b) $\frac{1}{3}$

(c) $\frac{2}{3}$

(d) $\frac{3}{4}$

(ii) Three heads?

(a) $\frac{1}{8}$

(b) $\frac{1}{4}$

(c) $\frac{1}{2}$

(d) $\frac{2}{3}$

9. For the above question, the probability that there is at least one tail is

(a) $\frac{2}{3}$

(b) $\frac{7}{8}$

(c) $\frac{3}{8}$

(d) $\frac{1}{2}$

10. Two fair dice are thrown. Find the probability of getting the sum as:

(i) A number divisible by 2 or 4

(a) $\frac{1}{2}$

(b) $\frac{3}{4}$

(c) $\frac{1}{3}$

(d) $\frac{2}{3}$

(ii) A number divisible by 2 and 4

(a) $\frac{1}{3}$

(b) $\frac{1}{4}$

(c) $\frac{3}{4}$

(d) $\frac{5}{7}$

(iii) A prime number less than 8

(a) $\frac{11}{13}$

(b) $\frac{1}{13}$

(c) $\frac{1}{4}$

(d) $\frac{13}{36}$

11. A bag contains three green and seven white balls. Two balls are drawn from the bag in succession without replacement. What is the probability that

(i) Both are white?

(a) $\frac{1}{7}$

(b) $\frac{5}{11}$

(c) $\frac{7}{11}$

(d) $\frac{7}{15}$

(ii) They are of different colour?

(a) $\frac{7}{15}$

(b) $\frac{7}{9}$

(c) $\frac{5}{11}$

(d) $\frac{7}{11}$

12. One-hundred students appeared for two examinations. Sixty passed the first, fifty passed the second and thirty passed both. Find the probability that a student selected at random has failed in both the examinations.

(a) $\frac{1}{5}$

(b) $\frac{1}{7}$

(c) $\frac{5}{7}$

(d) $\frac{5}{6}$

13. What is the probability of throwing a number greater than two with a fair dice?

(a) $\frac{2}{3}$

(b) $\frac{2}{5}$

(c) 1

(d) $\frac{3}{5}$

14. Three cards numbered 2, 4 and 8 are put into a box. If a card is drawn at random, what is the probability that the card drawn is

(i) A prime number?

(a) 1

(b) $\frac{1}{3}$

(c) $\frac{4}{5}$

(d) $\frac{5}{7}$

(ii) An even number?

(a) 1

(b) $\frac{2}{3}$

(c) $\frac{1}{2}$

(d) $\frac{3}{5}$

(iii) An odd number?

(a) 1

(b) 0

(c) $\frac{1}{3}$

(d) $\frac{2}{3}$

15. Two fair coins are tossed. Find the probability of obtaining

(i) Two heads

(a) 1

(b) $\frac{2}{3}$

(c) $\frac{1}{2}$

(d) $\frac{1}{4}$

(ii) One head and one tail

(a) $\frac{1}{2}$

(b) 1

(c) $\frac{1}{3}$

(d) $\frac{2}{3}$

(iii) Two tails

(a) 1

(b) $\frac{1}{4}$

(c) $\frac{2}{3}$

(d) $\frac{1}{2}$

16. In rolling two dice, find the probability that

(i) There is at least one '6'

(a) $\frac{11}{36}$

(b) $\frac{22}{36}$

(c) $\frac{15}{36}$

(d) $\frac{29}{36}$

(ii) The sum is 5

(a) $\frac{1}{4}$

(b) $\frac{1}{9}$

(c) $\frac{1}{2}$

(d) $\frac{1}{6}$

17. From a bag containing four white and five black balls a man draws three at random. What are the odds against these being all black?

(a) $\frac{5}{37}$

(b) $\frac{37}{5}$

(c) $\frac{11}{13}$

(d) $\frac{13}{37}$

18. Amit throws three dice in a special game of Ludo. If it is known that he needs 15 or higher in this throw to win then find the chance of his winning the game.

(a) $\frac{5}{54}$

(b) $\frac{17}{216}$

(c) $\frac{13}{216}$

(d) $\frac{15}{216}$

19. Find out the probability of forming 187 or 215 with the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 when only numbers of three digits are formed and when

(i) Repetitions are not allowed

(a) $\frac{12}{504}$

(b) $\frac{18}{504}$

(c) $\frac{2}{504}$

(d) $\frac{24}{504}$

(ii) Repetitions are allowed

(a) $\frac{2}{729}$

(b) $\frac{6}{729}$

(c) $\frac{11}{729}$

(d) $\frac{4}{729}$

20. In a horse race there were eighteen horses numbered 1–18. The probability that horse 1 would win is 1/6, that 2 would win is 1/10 and that 3 would win is 1/8. Assuming that a tie is impossible, find the chance that one of the three will win.

(a) $\frac{47}{120}$

(b) $\frac{119}{120}$

(c) $\frac{11}{129}$

(d) $\frac{1}{5}$

21. Two balls are to be drawn from a bag containing eight grey and three blue balls. Find the chance that they will both be blue.

(a) $\frac{1}{5}$

(b) $\frac{3}{55}$

(c) $\frac{11}{15}$

(d) $\frac{14}{45}$

22. Two fair dice are thrown. What is the probability of

(i) Throwing a double?

(a) $\frac{1}{6}$

(b) 1

(c) $\frac{2}{3}$

(d) $\frac{1}{2}$

(ii) The sum is greater than 10?

(a) $\frac{2}{3}$

(b) $\frac{2}{5}$

(c) $\frac{1}{6}$

(d) $\frac{1}{12}$

(iii) The sum is less than 10?

(a) $\frac{5}{6}$

(b) $\frac{2}{5}$

(c) $\frac{3}{5}$

(d) $\frac{2}{3}$

23. In a certain lottery the prize is one crore and 5000 tickets have been sold. What is the expectation of a man who holds ten tickets?

(a) ₹20,000

(b) ₹25,000

(c) ₹30,000

(d) ₹15,000

24. Two letters are randomly chosen from the word LIME. Find the probability that the letters are *L* and *M*.

(a) $\frac{1}{2}$

(b) $\frac{1}{4}$

(c) $\frac{1}{3}$

(d) $\frac{1}{6}$

Directions for Questions 25 to 27: Read the following passage and answer the questions based on it.

The Bangalore office of Infosys has 1200 executives. Of these, 880 subscribe to the *Time* magazine and 650 subscribe to the *Economist*. Each executive may subscribe to either the *Time* or the *Economist* or both. If an executive is picked at random, answer questions 25–27.

25. What is the probability that

(i) He has subscribed to the *Time* magazine?

(a) $\frac{11}{15}$

(b) $\frac{11}{12}$

(c) $\frac{7}{15}$

(d) $\frac{7}{11}$

(ii) He has subscribed to the *Economist*?

(a) $\frac{13}{21}$

(b) $\frac{13}{20}$

(c) $\frac{13}{24}$

(d) $\frac{12}{30}$

26. He has subscribed to both magazines?

(a) $\frac{22}{40}$

(b) $\frac{11}{40}$

(c) $\frac{12}{20}$

(d) $\frac{4}{20}$

27. If among the executives who have subscribed to the *Time* magazine, an executive is picked at random. What is the probability that he has also subscribed to the *Economist*?

(a) $\frac{3}{8}$

(b) $\frac{5}{8}$

(c) $\frac{2}{3}$

(d) $\frac{1}{8}$

28. A bag contains four black and five red balls. If three balls from the bag are chosen at random, what is the chance that they are all black?

(a) $\frac{1}{21}$

(b) $\frac{1}{20}$

(c) $\frac{2}{23}$

(d) $\frac{1}{9}$

29. If a number of two digits is formed with the digits 2, 3, 5, 7, 9 without repetition of digits, what is the probability that the number formed is 35?

(a) $\frac{1}{10}$

(b) $\frac{1}{20}$

(c) $\frac{2}{11}$

(d) $\frac{1}{11}$

30. From a pack of 52 playing cards, three cards are drawn at random. Find the probability of drawing a king, a queen and jack.

(a) $\frac{16}{5525}$

(b) $\frac{1}{13^3}$

(c) $\frac{1}{14^3}$

(d) $\frac{1}{15^3}$

31. A bag contains twenty balls marked 1 to 20. One ball is drawn at random. Find the probability that it is marked with a number multiple of 5 or 7.

(a) $\frac{3}{10}$

(b) $\frac{7}{10}$

(c) $\frac{1}{11}$

(d) $\frac{2}{3}$

32. A group of investigators took a fair sample of 1972 children from the general population and found that there are 1000 boys and 972 girls. If the investigators claim that their research is so accurate that the sex of a new born child can be predicted based on the ratio of the sample of the population, then what is the expectation in terms of the probability that a new child born will be a girl?

(a) $\frac{243}{250}$

(b) $\frac{250}{257}$

(c) $\frac{9}{10}$

(d) $\frac{243}{493}$

33. A bag contains three red, six white and seven black balls. Two balls are drawn at random. What is the probability that both are black?

(a) $\frac{1}{8}$

(b) $\frac{7}{40}$

(c) $\frac{12}{40}$

(d) $\frac{13}{40}$

34. A bag contains six red, four white and eight blue balls. If three balls are drawn at random, find the probability that

(i) All the three balls are of the same colour

(a) $\frac{17}{240}$

(b) $\frac{5}{51}$

(c) $\frac{31}{204}$

(d) None of these

(ii) All the three balls are blue

(a) $\frac{8}{51}$

(b) $\frac{50}{51}$

(c) $\frac{7}{102}$

(d) $\frac{13}{51}$

35. If $P(A) = 1/3, P(B) = 1/2, P(A \cap B) = 1/4$ then find $P(A' \cup B')$.

(a) $\frac{1}{3}$

(b) $\frac{2}{5}$

(c) $\frac{2}{3}$

(d) $\frac{3}{4}$

36. A and B are two candidates seeking admission to the IIMs. The probability that A is selected is 0.5 and the probability that both A and B are selected is at most 0.3. Is it possible that the probability of B getting selected is 0.9?

(a) No

(b) Yes

(c) Either (a) or (b)

(d) Cannot say

37. The probability that a student will pass in Mathematics is 3/5 and the probability that he will pass in English is 1/3. If the probability that he will pass in both Mathematics and English is 1/8, what is the probability that he will pass in at least one subject?

(a) $\frac{97}{120}$

(b) $\frac{87}{120}$

(c) $\frac{53}{120}$

(d) $\frac{120}{297}$

38. The odds in favour of standing first of three students, Amit, Vikas and Vivek appearing at an examination are 1:2. 2:5 and 1:7 respectively. What is the probability that either of them will stand first (assume that a tie for the first place is not possible)?

(a) $\frac{168}{178}$

(b) $\frac{122}{168}$

(c) $\frac{5}{168}$

(d) $\frac{125}{168}$

(d) $\frac{1}{13}$

40. A and B are two mutually exclusive events of an experiment. If $P(A') = 0.65$, $P(A \cup B) = 0.65$ and $P(B) = p$, find the value of p.

(a) 0.25

(b) 0.3

(c) 0.1

(d) 0.2

41. A bag contains four white and two black balls. Another contains three white and five black balls. If one ball is drawn from each bag, find the probability that

(i) Both are white

(a) $\frac{1}{3}$

(b) $\frac{2}{3}$

(c) $\frac{1}{4}$

(d) $\frac{3}{4}$

(ii) Both are black

(a) $\frac{3}{24}$

(b) $\frac{1}{24}$

(c) $\frac{3}{12}$

(d) $\frac{5}{24}$

(iii) One is white and one is black

(a) $\frac{13}{24}$

(b) $\frac{15}{24}$

(c) $\frac{11}{21}$

(d) $\frac{1}{2}$

42. The odds against an event are 5:3 and the odds in favour of another independent event are 7:5. Find the probability that at least one of the two events will occur.

(a) $\frac{52}{96}$

(b) $\frac{69}{96}$

(c) $\frac{71}{96}$

(d) $\frac{13}{96}$

43. Kamal and Monica appeared for an interview for two vacancies. The probability of Kamal's selection is 1/3 and that of Monica's selection is 1/5. Find the probability that only one of them will be selected.

(a) $\frac{2}{5}$

(b) $\frac{1}{5}$

(c) $\frac{5}{9}$

(d) $\frac{2}{3}$

44. A husband and a wife appear in an interview for two vacancies for the same post. The probability of husband's selection is (1/7) and that of the wife's selection is 1/5. What is the probability that

(i) Both of them will be selected?

(a) $\frac{1}{35}$

(b) $\frac{2}{35}$

(c) $\frac{3}{35}$

(d) $\frac{1}{7}$

(ii) One of them will be selected?

(a) $\frac{1}{7}$

(b) $\frac{3}{7}$

(c) $\frac{2}{7}$

(d) $\frac{5}{7}$

(iii) None of them will be selected?

(a) $\frac{24}{35}$

(b) $\frac{20}{35}$

(c) $\frac{21}{35}$

(d) $\frac{2}{7}$

(iv) At least one of them will be selected?

(a) $\frac{12}{35}$

(b) $\frac{11}{35}$

(c) $\frac{16}{35}$

(d) $\frac{1}{5}$

LEVEL OF DIFFICULTY (II)

1. Two fair dice are thrown. Given that the sum of the dice is less than or equal to four, find the probability that only one dice shows two.

(a) $\frac{1}{4}$

(b) $\frac{1}{2}$

(c) $\frac{2}{3}$

(d) $\frac{1}{3}$

2. *A* can hit the target three times in six shots, *B* two times in six shots and *C* four times in six shots. They fire a volley. What is the probability that at least two shots hit?

(a) $\frac{1}{2}$

(b) $\frac{1}{3}$

(c) $\frac{2}{3}$

(d) $\frac{3}{4}$

3. There are two bags, one of them contains five red and seven white balls and the other three red and twelve white balls, and a ball is to be drawn from one or the other of the two bags. Find the chance of drawing a red ball.

(a) $\frac{37}{120}$

(b) $\frac{30}{120}$

(c) $\frac{11}{120}$

(d) None of these

4. In two bags there are to be put altogether five red and twelve white balls, neither bag being empty. Apart from this, there is no constraint of placing the balls. Once the balls are placed, a person will draw one ball from either of the two bags. Find the probability for the following conditions to be met:

(i) The least chance of drawing a red ball?

(a) $\frac{3}{35}$

(b) $\frac{5}{32}$

(c) $\frac{7}{32}$

(d) $\frac{1}{16}$

(ii) The greatest chance of drawing a red ball?

(a) $\frac{3}{4}$

(b) $\frac{2}{3}$

(c) $\frac{5}{8}$

(d) $\frac{5}{7}$

5. If eight coins are tossed, what is the chance that one and only one will turn up head?

(a) $\frac{1}{16}$

(b) $\frac{3}{35}$

(c) $\frac{3}{32}$

(d) $\frac{1}{32}$

6. What is the chance that a leap year, selected at random, will contain 53 Sundays?

(a) $\frac{2}{7}$

(b) $\frac{3}{7}$

(c) $\frac{1}{7}$

(d) $\frac{5}{7}$

7. Out of all the two-digit integers between 1 to 200, a two-digit number has to be selected at random. What is the probability that the selected number is not divisible by 7?

(a) $\frac{11}{90}$

(b) $\frac{33}{90}$

(c) $\frac{55}{90}$

(d) $\frac{77}{90}$

8. A child is asked to pick up two balloons from a box containing ten blue and fifteen red balloons. What is the probability of the child picking, at random, two balloons of different colours?

(a) $\frac{1}{2}$

(b) $\frac{2}{3}$

(c) $\frac{1}{3}$

(d) $\frac{3}{5}$

9. Tom and Dick are running in the same race; the probability of their winning is 1/5 and ½ respectively. Find the probability that

(i) Either of them will win the race

(a) $\frac{7}{10}$

(b) $\frac{3}{10}$

(c) $\frac{1}{5}$

(d) $\frac{7}{9}$

(ii) Neither of them will win the race

(a) $\frac{7}{10}$

(b) $\frac{3}{10}$

10. Two dice are thrown. If the total on the faces of the two dices are six, find the probability that there are two odd numbers on the faces.

(a) $\frac{2}{5}$

(b) $\frac{1}{5}$

(c) $\frac{5}{9}$

(d) $\frac{3}{5}$

11. Amarnath appears in an exam that has four subjects. The chance he passes an individual subject's test is 0.8. What is the probability that he will

(i) Pass in all the subjects?

(a) 0.8^4

(b) 0.3^4

(c) 0.7^3

(d) None of these

(ii) Fail in all the subjects?

(a) 0.4^2

(b) 0.2^4

(c) 0.3^4

(d) None of these

(iii) Pass in at least one of the subjects?

(a) 0.99984

(b) 0.9984

(c) 0.0016

(d) None of these

12. A box contains two tennis, three cricket and four squash balls. Three balls are drawn in succession with replacement. Find the probability that

(i) All are cricket balls

(a) $\frac{1}{27}$

(b) $\frac{2}{27}$

(c) $\frac{25}{27}$

(d) $\frac{1}{8}$

(ii) The first is a tennis ball, the second is a cricket ball, and the third is a squash ball

(a) $\frac{8}{243}$

(b) $\frac{5}{243}$

(c) $\frac{4}{243}$

(d) $\frac{11}{243}$

(iii) All three are of the same type

(a) $\frac{11}{81}$

(b) $\frac{1}{9}$

(c) $\frac{13}{81}$

(d) $\frac{17}{81}$

13. With the data in the above question, answer the questions when the balls are drawn in succession without replacement.

(i)

(a) $\frac{3}{84}$

(b) $\frac{1}{84}$

(c) $\frac{5}{84}$

(d) None of these

(ii)

(a) $\frac{2}{21}$

(b) $\frac{4}{21}$

(c) $\frac{1}{21}$

(d) $\frac{1}{9}$

(iii)

(a) $\frac{3}{84}$

(b) $\frac{1}{84}$

(c) $\frac{5}{84}$

(d) $\frac{11}{84}$

14. In the Mindworkzz library, there are eight books by Stephen Covey and one book by Vinay Singh in shelf A. At the same time, there are five books by Stephen Covey in shelf B. One book is moved from shelf A to shelf B. A student picks up a book from shelf B. Find the probability that the book by Vinay Singh

(i) Is still in shelf A

(a) $\frac{1}{3}$

(b) $\frac{8}{9}$

(c) $\frac{3}{4}$

(d) None of these

(ii) Is in shelf B

(a) $\frac{3}{54}$

(b) $\frac{4}{54}$

(c) $\frac{5}{54}$

(d) None of these

(iii) Is taken by the student

(a) $\frac{3}{54}$

(b) $\frac{1}{54}$

(c) $\frac{2}{27}$

(d) None of these

15. The ratio of number of men and ladies in the Scorpion Squadron and in the Gunners Squadron are 3:1 and 2:5, respectively. An individual is selected to be the chairperson of their association. The chance that this individual is selected from the Scorpions is 2/3. Find the probability that the chairperson will be a men.

(a) $\frac{25}{42}$

(b) $\frac{13}{43}$

(c) $\frac{11}{43}$

(d) $\frac{7}{42}$

16. A batch of fifty transistors contains three defective ones. Two transistors are selected at random from the batch and put into a radio set. What is the probability that

(i) Both the transistors selected are defective?

(a) $\frac{4}{1225}$

(b) $\frac{3}{1225}$

(c) $\frac{124}{1224}$

(d) None of these

(ii) Only one is defective?

(a) $\frac{141}{1225}$

(b) $\frac{121}{1225}$

(c) $\frac{123}{1224}$

(d) None of these

(iii) Neither is defective?

(a) $\frac{1082}{1224}$

(b) $\frac{1081}{1225}$

(c) $\frac{1081}{1224}$

(d) None of these

17. The probability that a man will be alive in 35 years is $\frac{3}{5}$ and the probability that his wife will be alive is $\frac{3}{7}$. Find the probability that after 35 years

(i) Both will be alive

(a) $\frac{2}{35}$

(b) $\frac{9}{35}$

(c) $\frac{6}{35}$

(d) $\frac{3}{35}$

(ii) Only the man will be alive

(a) $\frac{12}{35}$

(b) $\frac{11}{35}$

(c) $\frac{13}{35}$

(d) $\frac{8}{35}$

(iii) Only the wife will be alive

(a) $\frac{2}{35}$

(b) $\frac{3}{35}$

(c) $\frac{6}{35}$

(d) $\frac{11}{35}$

(iv) At least one will be alive

(a) $\frac{27}{35}$

18. *A* speaks the truth three out of four times, and *B* five out of six times. What is the probability that they will contradict each other in stating the same fact?

(a) $\frac{2}{3}$

(b) $\frac{1}{3}$

(c) $\frac{5}{6}$

(d) None of these

19. A party of *n* persons sit at a round table. Find the odds against two specified persons sitting next to each other.

(a) $\frac{n+1}{2}$

(b) $\frac{n-3}{2}$

(c) $\frac{n+3}{2}$

(d) None of these

20. If four whole numbers are taken at random and multiplied together, what is the chance that the last digit in the product is 1, 3, 7 or 9?

(a) $\frac{15}{653}$

(b) $\frac{12}{542}$

(c) $\frac{16}{625}$

(d) $\frac{17}{625}$

21. In four throws with a pair of dice, what is the chance of throwing a double twice?

(a) $\frac{11}{216}$

(b) $\frac{25}{216}$

(c) $\frac{35}{126}$

(d) $\frac{41}{216}$

22. A life insurance company insured 25,000 young boys, 14,000 young girls and 16,000 young adults. The probability of death within ten years of a young boy, young girl and a young adult are 0.02, 0.03 and 0.15 respectively. One of the insured persons dies. What is the probability that the dead person is a young boy?

(a) $\frac{36}{165}$

(b) $\frac{25}{166}$

(c) $\frac{26}{165}$

(d) $\frac{32}{165}$

23. Three groups of children consist of three girls and one boy, two girls and two boys, and one girl and two boys, respectively. One child is selected at random from each group. The probability that the three selected consist of one girl and two boys is

(a) $\frac{3}{8}$

(b) $\frac{1}{5}$

(c) $\frac{5}{8}$

(d) $\frac{3}{5}$

24. A locker at the world famous WTC building can be opened by dialling a fixed three-digit code (between 000 and 999). Don, a policeman, only knows that the number is a three-digit number and has only one six. Using this information, he tries to open the locker by dialling three digits at random. The probability that he succeeds in his endeavour is

(a) $\frac{1}{243}$

(b) $\frac{1}{900}$

(c) $\frac{1}{1000}$

(d) $\frac{3}{216}$

25. In a bag, there are twelve black and six white balls. Two balls are chosen at random and the first one is found to be black. The probability that the second one is also black is

(a) $\frac{11}{17}$

(b) $\frac{12}{17}$

(c) $\frac{13}{18}$

(d) None of these

26. In the above question, what is the probability that the second one is white?

(a) $\frac{3}{17}$

(b) $\frac{6}{17}$

(c) $\frac{5}{17}$

(d) $\frac{1}{17}$

27. A fair dice is tossed six times. Find the probability of getting a third six on the sixth throw.

(a) $\frac{{}^5C_2\, 5^2}{6^2}$

(b) $\frac{{}^5C_2\, 5^3}{6^6}$

(c) $\frac{{}^5C_3\, 5^2}{6^3}$

(d) $\frac{{}^5C_3\, 5^2}{6^6}$

28. In shuffling a pack of cards, four are accidentally dropped. Find the chance that the dropped cards should be one from each suit.

(a) $\frac{13^4}{^{52}C_4}$

(b) $\frac{12^4}{^{52}C_2}$

(c) $\frac{13^2}{^{34}C_2}$

(d) $\frac{12^2}{^{22}C_3}$

29. Three of the six vertices of a regular hexagon are chosen at random. The probability that the triangle with these vertices is equilateral is

(a) $\frac{1}{10}$

(b) $\frac{3}{10}$

(c) $\frac{1}{5}$

(d) $\frac{4}{10}$

30. There are five red shoes and four black shoes in a sale. They have got all mixed up with each other. What is the probability of getting a matched shoe if two shoes are drawn at random?

(a) $\frac{6}{9}$

(b) $\frac{4}{9}$

(c) $\frac{2}{9}$

(d) $\frac{5}{9}$

31. A person draws a card from a pack of 52, replaces it and shuffles it. He continues doing it until he draws a heart. What is the probability that he has to make three trials?

(a) $\frac{9}{64}$

(b) $\frac{3}{64}$

(c) $\frac{5}{64}$

(d) $\frac{1}{64}$

32. For the above problem, what is the probability if he does not replace the cards?

(a) $\frac{274}{1700}$

(b) $\frac{123}{1720}$

(c) $\frac{247}{1700}$

(d) $\frac{234}{1500}$

33. An event X can happen with probability P, and event Y can happen with probability P'. What is the probability that exactly one of them happens?

(a) $P + P' - 2PP'$

(b) $2PP' - P' + P$

(c) $P - P' + 2PP'$

(d) $2P'P - P' + P$

34. In the above question, what is the probability that at least one of them happens?

(a) $P + P' + PP'$

(b) $P + P' - PP'$

(c) $2PP' - P' - P$

(d) $P + P' - 2PP'$

35. Find the probability that a year chosen at random has 53 Mondays.

(a) $\frac{2}{7}$

(b) $\frac{5}{28}$

(c) $\frac{1}{28}$

(d) $\frac{3}{28}$

36. There are four machines and it is known that exactly two of them are faulty. They are tested one by one in a random order till both the faulty machines are identified. Then the probability that only two tests are needed is

(a) $\frac{2}{3}$

(b) $\frac{1}{6}$

(c) $\frac{1}{3}$

(d) $\frac{5}{6}$

37. For the above question, the probability that exactly three tests will be required to identify the two faulty machines is

(a) $\frac{1}{2}$

(b) 1

(c) $\frac{1}{3}$

(d) $\frac{2}{3}$

38. Seven white balls and three black balls are randomly placed in a row. Find the probability that no two black balls are placed adjacent to each other.

(a) $\frac{7}{15}$

(b) $\frac{2}{15}$

(c) $\frac{3}{7}$

(d) $\frac{2}{7}$

39. A fair coin is tossed repeatedly. If head appears on the first four tosses then the probability of appearance of tail on the fifth toss is

(a) $\frac{1}{7}$

(b) $\frac{1}{2}$

(c) $\frac{3}{7}$

(d) $\frac{2}{3}$

40. The letters of the word ARTICLE are arranged at random. Find the probability that the vowels may occupy the even places.

(a) $\frac{2}{35}$

(b) $\frac{1}{35}$

(c) $\frac{3}{36}$

(d) $\frac{2}{34}$

41. What is the probability that four *S* come consecutively in the word MISSISSIPPI?

(a) $\frac{4}{165}$

(b) $\frac{2}{165}$

(c) $\frac{3}{165}$

(d) $\frac{1}{165}$

42. Eleven books, consisting of five Engineering books, four Mathematics books and two Physics books, are arranged in a shelf at random. What is the probability that the books of each kind are all together?

(a) $\frac{5}{1155}$

(b) $\frac{2}{1155}$

(c) $\frac{3}{1155}$

(d) $\frac{1}{1155}$

43. Three students appear in an examination of Mathematics. The probability of their success is $1/3$, $1/4$, $1/5$, respectively. Find the probability of success of at least two.

(a) $\frac{1}{6}$

(b) $\frac{2}{5}$

(c) $\frac{3}{4}$

(d) $\frac{3}{5}$

44. A bag contains eight white and four red balls. Five balls are drawn at random. What is the probability that two of them are red and three are white?

(a) $\frac{12}{44}$

(b) $\frac{14}{33}$

(c) $\frac{14}{34}$

(d) $\frac{15}{34}$

45. A team of four is to be constituted out of five girls and six boys. Find the probability that the team may have three girls.

(a) $\frac{4}{11}$

(b) $\frac{3}{11}$

(c) $\frac{5}{11}$

(d) $\frac{2}{11}$

46. Twelve persons are seated around a round table. What is the probability that two particular persons sit together?

(a) $\frac{2}{11}$

(b) $\frac{1}{6}$

(c) $\frac{3}{11}$

(d) $\frac{3}{15}$

47. Six boys and six girls sit in a row randomly. Find the probability that all the six girls sit together.

(a) $\frac{3}{22}$

(b) $\frac{1}{132}$

(c) $\frac{1}{1584}$

(d) $\frac{1}{66}$

48. From a group of seven men and four women, a committee of six persons is formed. What is the probability that the committee will consist of exactly two women?

(a) $\frac{5}{11}$

(b) $\frac{3}{11}$

(c) $\frac{4}{11}$

(d) $\frac{2}{11}$

49. A bag contains five red, four green and three black balls. If three balls are drawn out of it at random, find the probability of drawing exactly two red balls.

(a) $\frac{7}{22}$

(b) $\frac{10}{33}$

(c) $\frac{7}{12}$

(d) $\frac{7}{11}$

50. A bag contains 100 tickets numbered 1, 2, 3, ..., 100. If a ticket is drawn out of it at random, what is the probability that the ticket drawn has the digit 2 appearing on it?

(a) $\frac{19}{100}$

(b) $\frac{21}{100}$

(c) $\frac{32}{100}$

(d) $\frac{23}{100}$

LEVEL OF DIFFICULTY (III)

1. Out of a pack of 52 cards, one is lost from the remainder of the pack; two cards are drawn and are found to be spades. Find the chance that the missing card is a spade.

(a) $\frac{11}{50}$

(b) $\frac{11}{49}$

(c) $\frac{10}{49}$

(d) $\frac{10}{50}$

2. A and B throw one dice for a stake of ₹11, which is to be won by the player who first throws a six. The game ends when the stake is won by A or B. If A has the first throw, what are their respective expectations?

(a) 5 and 6

(b) 6 and 5

(c) 11 and 0

(d) 9 and 2

3. Counters marked 1, 2, 3 are placed in a bag and one of them is withdrawn and replaced. The operation being repeated three times, what is the chance of obtaining a total of six in these three operations?

(a) $\frac{11}{27}$

(b) $\frac{7}{27}$

(c) $\frac{1}{27}$

(d) $\frac{5}{14}$

4. A speaks the truth three times out of four, while B speaks seven times out of ten. They both assert that a white ball is drawn from a bag containing six balls, all of different colours. Find the probability of the truth of the assertion.

(a) $\frac{12}{49}$

(b) $\frac{3}{10}$

(c) $\frac{21}{40}$

(d) None of these

5. In a shirt factory, processes A, B and C respectively manufacture 25%, 35% and 40% of the total shirts. Of their respective productions, 5%, 4% and 2% of the shirts are defective. A shirt is selected at random from the production of a particular day. If it is found to be defective, what is the probability that it is manufactured by the process C?

(a) $\frac{16}{69}$

(b) $\frac{25}{69}$

(c) $\frac{28}{69}$

(d) $\frac{27}{44}$

6. A pair of fair dice is rolled together till a sum of either 5 or 7 is obtained. The probability that 5 comes before 7 is

(a) 0.45

(b) 0.4

(c) 0.5

(d) 0.7

7. For the above problem, the probability of 7 coming before 5 is:

(a) $\frac{3}{5}$

(b) 0.55

(c) 0.4

(d) 0.7

8. For the above problem, the probability of 4 coming before either 5 or 7 is:

(a) $\frac{3}{13}$

(b) $\frac{7}{13}$

(c) $\frac{11}{13}$

(d) $\frac{10}{13}$

9. The probability of a bomb hitting a bridge is half and two direct hits are needed to destroy it. The least number of bombs required so that the probability of the bridge being destroyed is greater than 0.9 is

(a) 7 bombs

(b) 3 bombs

(c) 8 bombs

(d) 9 bombs

10. What is the probability of the destruction of the bridge if only five bombs are dropped?

(a) 62.32%

(b) 81.25%

(c) 45.23%

(d) 31.32%

11. Sanjay writes a letter to his friend Kesari from IIT, Kanpur. It is known that one out of '*n*' letters that are posted does not reach its destination. If Sanjay does not receive the reply to his letter, then what is the probability that Kesari did not receive Sanjay's letter? It is certain that Kesari will

definitely reply to Sanjay's letter if he receives it.

(a) $\frac{n}{(2n-1)}$

(b) $\frac{n-1}{n}$

(c) $\frac{1}{n}$

(d) None of these

12. A word of six letters is formed from a set of sixteen different letters of the English alphabet (with replacement). Find out the probability that exactly two letters are repeated.

(a) $\frac{225 \times 224 \times 156}{16^6}$

(b) $\frac{18080}{16^6}$

(c) $\frac{15 \times 224 \times 156}{16^6}$

(d) None of these

13. A number is chosen at random from the numbers 10 to 99. By seeing the number, a man will sing if the product of the digits is 12. If he chooses three numbers with replacement, then the probability that he will sing at least once is:

(a) $1-\left(\frac{43}{45}\right)^3$

(b) $\left(\frac{43}{45}\right)^3$

(c) $1-\frac{48 \times 86}{90^3}$

(d) None of these

14. In a bag, there are ten black, eight white and five red balls. Three balls are chosen at random and one is found to be black. Find the probability that the remaining two balls are white.

(a) $\frac{8}{23}$

(b) $\frac{4}{33}$

(c) $\frac{10 \times 8 \times 7}{23 \times 22 \times 21}$

(d) $\frac{5}{23}$

15. In the above question, find the probability that the remaining two balls are red.

(a) $\frac{10}{231}$

(b) $\frac{12}{231}$

(c) $\frac{12}{363}$

(d) None of these

16. Ten tickets are numbered 1, 2, 3..., 10. Six tickets are selected at random, one at a time with replacement. The probability that the largest number appearing on the selected ticket is 7 is

(a) $\frac{(7^6 - 1)}{10^6}$

(b) $\frac{7^6 - 6^6}{10^6}$

(c) $\frac{6^6}{10^6}$

(d) None of these

17. A bag contains 15 tickets numbered 1 to 15. A ticket is drawn and replaced. Then one more ticket is drawn and replaced. The probability that first number drawn is even and second is odd is

(a) $\frac{56}{225}$

(b) $\frac{26}{578}$

(c) $\frac{57}{289}$

(d) None of these

18. Six blue balls are put in three boxes. The probability of putting balls in the boxes in equal numbers is

(a) $\frac{1}{21}$

(b) $\frac{1}{8}$

(c) $\frac{1}{28}$

(d) $\frac{1}{7}$

19. AMS employs eight professors on their staff. Their respective probability of remaining in employment for ten years is 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, and 0.9. The probability that after ten years at least six of them still work in AMS is

(a) 0.19

(b) 1.22

(c) 0.1

(d) None of these

20. A person draws a card from a pack of 52, replaces it and shuffles it. He continues doing so until he draws a heart. What is the probability that he has to make at least three trials?

(a) $\frac{3}{17}$

(b) $\frac{8}{19}$

(c) $\frac{2}{17}$

(d) $\frac{11}{16}$

21. Hilips, the largest white goods producer in India, uses a quality-check scheme on produced items before they are sent to the market. The plan is as follows: A set of twenty articles is readied and four of them are chosen at random. If any one of them is found to be defective then the whole set is put under 100% screening again. If no defectives are found, the whole set is sent to the market. Find the probability that a box containing four defective articles will be sent to the market.

(a) $\frac{364}{969}$

(b) $\frac{364}{963}$

(c) $\frac{96}{969}$

(d) $\frac{343}{969}$

22. In the above question, what is the probability that a box containing only one defective will be sent back for screening?

(a) $\frac{2}{3}$

(b) $\frac{1}{5}$

(c) $\frac{2}{5}$

(d) $\frac{4}{5}$

23. If the integers m and n are chosen at random from 1 to 100, then the probability that a number of the form $7m + 7n$ is divisible by 5 is

(a) $\frac{1}{4}$

(b) $\frac{1}{2}$

(c) $\frac{1}{16}$

(d) $\frac{1}{6}$

24. Three numbers are chosen at random without replacement from (1, 2, 3 ..., 10). The probability that the minimum number is 3 or the maximum number is 7 is

(a) $\frac{12}{37}$

(b) $\frac{11}{40}$

(c) $\frac{13}{35}$

(d) $\frac{14}{35}$

25. An unbiased dice with face values 1, 2, 3, 4, 5 and 6 is rolled four times. Out of the four face values obtained, find the probability that the minimum face value is not less than two and the maximum face value is not greater than five.

(a) $\frac{16}{81}$

(b) $\frac{14}{6^4}$

(c) $\frac{16}{80}$

(d) None of these

26. Three faces of a dice are yellow, two faces are red and one face is blue. The dice is tossed three times. Find the probability that the colours yellow, red and blue appear in the first, second and the third toss respectively.

(a) $\frac{1}{18}$

(b) $\frac{1}{12}$

(c) $\frac{1}{9}$

(d) $\frac{1}{36}$

27. If from each of three boxes containing three white and one black, two white and two black, one white and three black balls, one ball is drawn at random, then the probability that two white and one black ball will be drawn is

(a) $\frac{13}{32}$

(b) $\frac{12}{14}$

(c) $\frac{12}{25}$

(d) $\frac{3}{13}$

28. Probabilities that Rajesh passes in Maths, Physics and Chemistry are m, p and c, respectively. Of these subjects, Rajesh has a 75% chance of passing in at least one, 50% chance of passing in at least two and 40% chance of

passing in exactly two. Find which of the following is true.

(a) $p + m + c = \frac{19}{20}$

(b) $p + m + c = \frac{27}{20}$

(c) $pmc = \frac{1}{20}$

(d) $pmc = \frac{1}{8}$

29. There are five envelopes corresponding to five letters. If the letters are placed in the envelopes at random, what is the probability that all the letters are not placed in the right envelopes?

(a) $\frac{119}{120}$

(b) $\frac{59}{60}$

(c) $\frac{23}{24}$

(d) $\frac{4^5}{5^5}$

30. For the above question, what is the probability that no single letter is placed in the right envelope?

(a) $\frac{12}{35}$

(b) $\frac{11}{30}$

(c) $\frac{12}{25}$

(d) $\frac{3}{12}$

31. An urn contains four tickets having numbers 112, 121, 211, and 222 written on them. If one ticket is drawn at random and Ai (i = 1, 2, 3) be

the event that the i_{th} digit from left of the number on ticket drawn is 1, which of these can be said about the events A_1, A_2 and A_3?

(a) They are mutually exclusive

(b) A_1 and A_3 are not mutually exclusive to A_2

(c) A_1 and A_3 are mutually exclusive

(d) Both (b) and (c)

32. The probability that a contractor will get a plumbing contract is 2/3 and the probability that he will get an electric contract is 5/9. If the probability of getting at least one contract is 4/5, what is the probability that he will get both the contracts?

(a) $\frac{19}{45}$

(b) $\frac{13}{45}$

(c) $\frac{12}{35}$

(d) $\frac{11}{23}$

33. If $P(A) = 3/7$, $P(B) = 1/2$ and $P(A' \cap B') = 1/14$, then are A and B are mutually exclusive events?

(a) No

(b) Yes

(c) Either yes or no

(d) Cannot be determined

34. Six boys and six girls sit in a row at random. Find the probability that the boys and girls sit alternately.

(a) $\frac{1}{132}$

(b) $\frac{1}{462}$

(c) $\frac{1}{623}$

(d) $\frac{1}{231}$

35. A problem on Mathematics is given to three students whose chances of solving it are 1/2, 1/3 and 1/4 respectively. What is the chance that the problem will be solved?

(a) $\frac{2}{3}$

(b) $\frac{3}{4}$

(c) $\frac{1}{3}$

(d) $\frac{1}{2}$

36. *A* and *B* throw a pair of dice alternately. *A* wins if he throws 6 before *B* throws 5, and *B* wins if he throws 5 before *A* throws 6. Find *B*'s chance of winning if *A* makes the first throw.

(a) $\frac{1}{2}$

(b) $\frac{5}{12}$

(c) $\frac{1}{3}$

(d) $\frac{5}{11}$

37. Two persons A and B toss a coin alternately till one of them gets head and wins the game. Find B's chance of winning if A tosses the coin first.

(a) $\frac{1}{3}$

(b) $\frac{2}{3}$

(c) $\frac{1}{2}$

(d) None of these

38. A bag contains three red and four white balls and another bag contains four red and three white balls. A dice is cast and if the face 1 or 3 turns up, a ball is taken from the first bag and if any other face turns up, a ball is taken from the second bag. Find the probability of drawing a red ball.

(a) $\frac{11}{20}$

(b) $\frac{12}{21}$

(c) $\frac{2}{11}$

(d) $\frac{11}{21}$

39. Three groups of children contain respectively three girls and one boy, two girls and two boys, and one girl and three boys. One child is selected at random from each group. The probability that the three selected consists of one girl and two boys is

(a) $\frac{13}{32}$

(b) $\frac{12}{32}$

(c) $\frac{15}{32}$

(d) $\frac{11}{32}$

40. The probabilities of A, B and C solving a problem are, $\frac{1}{3}, \frac{2}{7}$ and $\frac{3}{8}$ respectively. If all the three try to solve the problem simultaneously, find the probability that exactly one of them will solve it.

(a) $\frac{26}{65}$

(b) $\frac{25}{56}$

(c) $\frac{52}{65}$

(d) $\frac{25}{52}$

41. A bag contains five black and three red balls. A ball is taken out of the bag and is not returned to it. If this process is repeated three times, then what is the probability of drawing a black ball in the next draw of a ball?

(a) 0.7

(b) 0.625

(c) 0.1

(d) None of these

42. For question 41, what is the probability of drawing a red ball?

(a) 0.375

(b) 0.9

(c) 0.3

(d) 0.79

43. One bag contains five white and four black balls. Another bag contains seven white and nine black balls. A ball is transferred from the first bag to the second and then a ball is drawn from the second bag. Find the probability that the ball drawn is white.

(a) $\frac{7}{18}$

(b) $\frac{5}{9}$

(c) $\frac{4}{9}$

(d) $\frac{11}{18}$

44. V Anand and Gary Kasparov play a series of five chess games. The probability that V Anand wins a game is 2/5 and the probability of Kasparov winning a game is 3/5. There is no probability of a draw. The series will be won by the person who wins three matches. Find the probability that Anand wins the series. (The series ends the moment when any of the two wins threematches.)

(a) $\frac{992}{3125}$

(b) $\frac{273}{625}$

(c) $\frac{1021}{3125}$

(d) $\frac{1081}{3125}$

45. There are ten pairs of socks in a cupboard from which four individual socks are picked at random. The probability that there is at least one pair is.

(a) $\frac{195}{323}$

(b) $\frac{99}{323}$

(c) $\frac{198}{323}$

(d) $\frac{185}{323}$

46. A fair coin is tossed ten times. Find the probability that two heads do not occur consecutively.

(a) $\frac{1}{2^4}$

(b) $\frac{1}{2^3}$

(c) $\frac{1}{2^5}$

(d) None of these

47. In a room, there are seven persons. The chance that two of them were born on the same day of the week is

(a) $\frac{1080}{7^5}$

(b) $\frac{2160}{7^5}$

(c) $\frac{540}{7^4}$

(d) None of these

48. In a hand at a game of bridge, what is the chance that the four kings are held by a specified player?

(a) $\frac{10}{4165}$

(b) $\frac{11}{4165}$

(c) $\frac{110}{4165}$

(d) None of these

49. One-hundred identical coins, each with probability P of showing up heads are tossed once. If $0 < P < 1$ and the probability of heads showing on 50 coins is equal to that of heads showing on 51 coins, then value of P is

(a) $\frac{1}{21}$

(b) $\frac{49}{101}$

(c) $\frac{50}{101}$

(d) $\frac{51}{101}$

50. Two small squares on a chess board are chosen at random. Find the probability that they have a common side.

(a) $\frac{1}{12}$

(b) $\frac{1}{18}$

(c) $\frac{2}{15}$

(d) $\frac{3}{14}$

ANSWER KEY

Level of Difficulty (I)

1. (b)

2. (a)

3. (d)

4. (c)

5. (a)

6(i). (c)

6(ii). (b)

6(iii). (c)

7(i). (b)

7(ii). (a)

8(i). (d)

8(ii). (a)

9. (b)

10(i). (a)

10(ii). (b)

10(iii). (d)

11(i). (d)

11(ii). (a)

12. (a)

13. (a)

14(i). (b)

14(ii). (a)

14(iii). (b)

15(i). (d)

15(ii). (a)

15(iii). (b)

16(i). (a)

16(ii). (b)

17. (b)

18. (a)

19(i). (c)

19(ii). (a)

20. (a)

21. (b)

22(i). (a)

22(ii). (d)

22(iii). (a)

23. (a)

24. (d)

25(i). (a)

25(ii). (c)

26. (b)

27. (a)

28. (a)

29. (b)

30. (a)

31. (a)

32. (d)

33. (b)

34(i). (b)

34(ii). (c)

35. (d)

36. (a)

37. (a)

38. (d)

39. (a)

40. (b)

41(i). (c)

41(ii). (d)

41(iii). (a)

42. (c)

43. (a)

44(i). (a)

44(ii). (c)

44(iii). (a)

44(iv). (b)

Level of Difficulty (II)

1. (d)

2. (a)

3. (a)

4(i) (b)

4(ii) (c)

5. (d)

6. (a)

7. (d)

8. (a)

9(i) (a)

9(ii) (b)

10. (d)

11(i). (a)

11(ii). (b)

12(i). (a)

12(ii). (a)

12(iii). (a)

13(i). (b)

13(ii). (c)

13(iii). (c)

14(i). (b)

14(ii). (c)

14(iii). (b)

15. (a)

16(i). (b)

16(ii). (a)

16(iii). (b)

17(i). (b)

17(ii). (a)

17(iii). (c)

17(iv). (a)

18. (b)

19. (b)

20. (c)

21. (b)

22. (b)

23. (a)

24. (a)

25. (a)

26. (b)

27. (b)

28. (a)
29. (a)
30. (b)
31. (a)
32. (c)
33. (a)
34. (b)
35. (a)
36. (b)
37. (c)
38. (a)
39. (b)
40. (b)
41. (a)
42. (d)
43. (a)
44. (b)
45. (d)
46. (a)
47. (b)
48. (a)
49. (a)
50. (a)

Level of Difficulty (III)

1. (a)
2. (b)
3. (b)
4. (c)

5. (a)
6. (b)
7. (a)
8. (a)
9. (a)
10. (b)
11. (a)
12. (a)
13. (a)
14. (b)
15. (a)
16. (b)
17. (a)
18. (c)
19. (a)
20. (d)
21. (a)
22. (b)
23. (a)
24. (b)
25. (a)
26. (d)
27. (a)
28. (b)
29. (a)
30. (b)

31. (b)
32. (a)
33. (b)
34. (b)
35. (b)
36. (d)
37. (a)
38. (d)
39. (b)
40. (b)
41. (a)
42. (a)
43. (c)
44. (a)
45. (b)
46. (b)
47. (b)
48. (b)
49. (d)
50. (b)

Solutions and Shortcuts

Level of Difficulty (I)

1. Out of a total of six occurrences, three is one possibility = 1/6.

2. 5 or 6 out of a sample space of

   1, 2, 3, 4, 5 or 6 = 2/6 = 1/3

3. Event definition is:

(1 and 1) or (1 and 2) or (1 and 3) or (1 and 4) or (1 and 5) or (1 and 6) or (2 and 1) or (3 and 1) or (4 and 1) or (5 and 1) or (6 and 1)

Total 11 out of 36 possibilities = 11/36

4. Event definition: First is blue and second is blue = 7/12 × 6/11 = 7/22.

5. Knave and Queen or Queen and Knave →

$$4/52 \times 4/51 + 4/52 \times 4/51 = 8/663$$

6. (i) 13/52 = 1/4

   (ii) Four kings and Four queens out of 52 cards.

   Thus, 8/52 = 2/13.

   (iii) 13 spades + 3 kings + 3 queens → 19/52

7. (i) Event definition is: *T* and *T* and *H* or *T* and *H* and *T* or *H* and *T* and *T* = 3 × 1/8 = 3/8.

   (ii) Same as above = 3/8

8. (i) Probability of three heads = 1/8

   Also, probability of three tails = 1/8

   Required probability = 1– (1/8 + 1/8) = 6/8 = 3/4

   (ii) *H* and *H* and *H* = 1/8

9. At least one tail is the non-event for all heads.

   Thus, *P* (at least 1 tail) = 1 – *P* (all heads)

$$= 1 - 1/8 = 7/8.$$

10. (i) Positive outcomes are 2(1 way), 4(3 ways), 6(5 ways) 8(5 ways), 10 (3 ways), 12 (1way).

Thus, 18/36 = 1/2

(ii) Positive outcomes are: 4, 8 and 12

4 (3 ways), 8(5 ways) and 12(1 way)

Gives us 9/36 = 1/4

(iii) Positive outcomes are 2(1 way), 3(2 ways), 5(4 ways), 7(6 ways). Total of 13 positive outcomes out of 36

Thus, 13/36

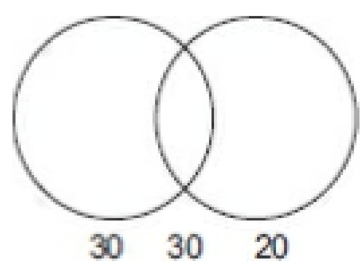
11. (i) First is white and second is white 7/10 × 6/9 = 7/15

(ii) White and green or green and white

7/10 × 3/9 + 3/10 × 7/9

42/90 = 7/15

12.



From the figure, it is evident that 80 students passed at least one exam. Thus, twenty failed both and the required probability is 20/100 = 1/5.

13. 3 or 4 or 5 or 6 = 4/6 = 2/3

14. (i) Since 2 is the only prime number out of the three numbers, the answer would be 1/3.

(ii) Since all the numbers are even, it is sure that the number drawn out is an even number. Hence, the required probability is 1.

(iii) Since there are no odd numbers amongst 2, 4 and 8, the required probability is 0.

15. (i) The event would be head and head → $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

(ii) The event would be head and tail OR tail and head → $= \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} = 1/2$

(iii) The event would be tail and tail → $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

16. (i) With a six on the first dice, there are six possibilities of outcomes that can appear on the other dice (viz. 6 & 1, 6 & 2, 6 & 3, 6 & 4, 6 & 5 and 6 & 6). At the same time with 6 on the second dice, there are five more possibilities for outcomes on the first dice: (1 & 6, 2 & 6, 3 & 6, 4 & 6, 5 & 6).

Also, the total outcomes are 36. Hence, the required probability is 11/36.

(ii) Out of 36 outcomes, five can come in the following ways – 1 + 4; 2 + 3; 3 + 2 or 4 + 1 → 4/36 = 1/9.

17. Odds against an event = $\frac{p(E')}{p(E)}$

In this case, the event is: all black, i.e. first is black and second is black and third is black.

$P(E) = 5/9 \times 4/8 \times 3/7 = 60/504 = 5/42$

Odds against the event = 37/5

18. Event definition is: 15 or 16 or 17 or 18

15 can be obtained as: 5 and 5 and 5 (one way)

Or

6 and 5 and 4 (Six ways)

Or

6 and 6 and 3 (3 ways)

Total ten ways

16 can be obtained as: 6 and 6 and 4 (3 ways)

Or

6 and 5 and 5 (3 ways)

Total six ways

17 has three ways and 18 has one way of appearing.

Thus, the required probability is: (10 + 6 + 3 + 1)/216 = 20/216 = 5/54.

19. (i) Positive outcomes = 2 (187 or 215)

Total outcomes = 9 × 8 × 7

Required probability = 2/504 = 1/252

(ii) = 2/729

20. 1/6 + 1/10 + 1/8 = 47/120

21. The event definition would be given by:

First is blue and second is blue = 3/11 × 2/10 = 3/55

22. (i) There are six doubles (1, 1; 2 & 2; 3 & 3; 4 & 4; 5 & 5; 6 & 6) out of a total of 36 outcomes → 6/36 = 1/6

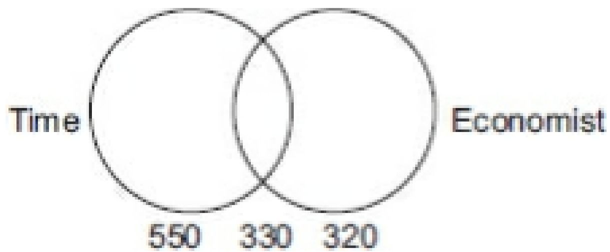
(ii) Sum greater than 10 means 11 or 12 → 3/36 = 1/12

(iii) Sum less than 10 is the non-event for the case sum is 10 or 11 or 12. There are three ways of getting 10, two ways of getting 11 and one way of getting a sum of 12 in the throw of two dice. Thus, the required probability would be 1 – 6/36 = 5/6.

23. Expectation = Probability of winning × Reward of winning = (10/5000) × 1 crore = (1 crore/500) = 20000.

24. $1/{}_4C_2 = 1/6$

25.



(i) 880/1200 = 11/15

(ii) 650/1200 = 13/24

26. 330/1200 = 11/40

27. 330/880 = 3/8

28. Black and black and black = 4/9 × 3/8 × 2/7

= 24/504 = 1/21

29. $1/{}_5P_2 = 1/20$

30. 6 × (4/52) × (4/51) × (4/50) = 16/5525

31. Positive outcomes are: 5, 7, 10, 14, 15 or 20

Thus, 6/20 = 3/10

32. 972/1972 = 243/493

33. Black and black = (7/16) × 6/15 = 7/40

34. (i) The required probability would be given by:

All are red OR all are white OR all are blue

= (6/18) × (5/17) × (4/16) + (4/18) × (3/17) × (2/16) + (8/18) × (7/17) × (6/16)

= 480/(18 × 17 × 16)

(ii) All blue = (8 × 7 × 6)/(18 × 17 × 16) = 7/102

35. The required value of the union of the two non-events (of *A* and *B*) would be 1 – 1/4 = ¾

36. *P* (both are selected) = *P*(*A*) × *P*(*B*)

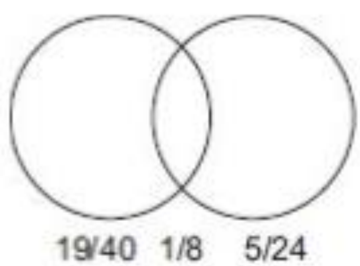
Since *P*(*A*) = 0.5, we get

0.3 = 0.5 × 0.6

The maximum value of *P*(*B*) = 0.6

Thus, *P*(*B*) = 0.9 is not possible.

37.



We have: 19/40 + 1/8 + 5/24 = 97/120

38. *P* (Amit) = 1/3

P (vikas) = 2/7

P (vivek) = 1/8

Required probability = 1/3 + 2/7 + 1/8 = 125/168

39. $P(A) + P(B) + P(C) = 1 \rightarrow 2P(B)/3 + P(B) + P(B)/2 = 1 \rightarrow 13P(B)/6 = 1 \rightarrow P(B) =$ 6/13. Hence, $P(A) = 4/13$

40. $P(A) = 1 - 0.65 = 0.35$

Hence, $P(B) = 0.65 - 0.35 = 0.3$

41. (i) The required probability would be given by the event:

White from first bag and white from second bag = (4/6) × (3/8) = ¼

(ii) The required probability would be given by the event:

Black from first bag and black from second bag = (2/6) × (5/8) = 10/48 = 5/24

(iii) This would be the non-event for the event [both are white OR both are black).

Thus, the required probability would be:

1 – 1/4 – 5/24 = 13/24

42. $P(E_1)$ 3/8

$P(E_2) = 7/12$

Event definition is: E_1 occurs and E_2 does not occur or E_1 occurs and E_2 occurs or E_2 occurs and $E1$ does not occur

(3/8) × (5/12) + (3/8) × (7/12) + (5/8) × (7/12) = 71/96

43. Kamal is selected and Monica is not selected or Kamal is not selected and Monica is selected → (1/3) × (4/5) + (2/3) × (1/5) = (6/15) = (2/5).

44. (i) 1/5 × 1/7 = 1/35

(ii) (1/5) × (6/7) + (4/5) × (1/7) = 10/35 = 2/7

(iii) (4/5) × (6/7) = 24/35

(iv) Both selected or one selected = 1/35 + 2/7 = 11/35

Level of Difficulty (II)

1. The possible outcomes are:

$$(1, 1); (1, 2); (2, 1), (2, 2); (3, 1); (1, 3)$$

Out of six cases, in two cases there is exactly one '2'.

Thus, the correct answer is 2/6 = 1/3.

2. Event definition is *A* hits, *B* hits and *C* hits OR any two of the three hits.

$$\frac{3}{6}\times\frac{2}{6}\times\frac{4}{6}+\frac{3}{6}\times\frac{2}{6}\times\frac{4}{6}+\frac{3}{6}\times\frac{4}{6}\times\frac{4}{6}+\frac{3}{6}\times\frac{2}{6}\times\frac{2}{6}=\frac{1}{2}$$

3. The event can be defined as:

First bag is selected and red ball is drawn.
Or second bag is selected and red ball is drawn.

1/2 × 5/12 + ½ × 3/15 = (5/24) + (3/30) = 37/120

4. (a) For the least chance of drawing a red ball, the distribution has to be five red + eleven white in one bag and one white in the second bag. This gives us

$$\frac{1}{2}\times\frac{5}{16}+\frac{1}{2}\times 0=\frac{5}{32}$$

(b) For the greatest chance of drawing a red ball the distribution has to be one red in the first bag and four red + twelve white balls in the second bag. This gives us

$$\frac{1}{2}\times 1+\frac{1}{2}\times\frac{4}{16}=\frac{5}{8}$$

5. One head and seven tails would have eight positions where the head can come.

   Thus, $8 \times (1/2)^8 = (1/32)$

6. A leap year has 366 days – which means 52 completed weeks and two more days. The last two days can be (Sunday, Monday) or (Monday, Tuesday) or ……. (Saturday, Sunday).

   Two scenarios out of seven have a 53rd Sunday.

   Thus, 2/7 is the required answer.

7. The count of the event will be given by:

   The number of all two digit integers – the number of all two digit integers divisible by 7 = 90 – 13 = 77. Hence, Required probability = 77/90.

8. Blue and red or red and blue

   = (10/25) × (15/24) + (15/25) × (10/24) = (1/2)

9. (i) 1/5 + ½ = 7/10

   (ii) 1 – (7/10) = (3/10)

10. A total of six can be made in any of the following ways (1 + 5, 2 + 4, 3 + 3, 4 + 2, 5 + 1)

    Required probability = 3/5

11. The event definitions are:

    (a) Passes the first AND passes the second AND passes the third AND passes the fourth

(b) Fails the first AND fails the second AND fails the third AND fails the fourth

(c) Fails all is the non-event

12. (i) First is cricket and second is cricket and third is cricket → (3/9) × (3/9) × (3/9) = (1/27)

(ii) (2/9) × (3/9) × (4/9) = (8/243)

(iii) All are cricket or all are tennis or all are squash

$$(3/9)^3 + (2/9)^3 + (4/9)^3 = (99/729) = (11/81)$$

13. (i) (3/9) × (2/8) × (1/7) = (1/84)

(ii) (2/9) × (3/8) × (4/7) = (1/21)

(iii) (3/9) × (2/8) × (1/7) + 0 + (4/9) × (3/8) × (2/7)

30/504 = 5/84

14. The event definitions are:

(i) The book transferred is by Stephen Covey

(ii) The book transferred is by Vinay Singh AND the book picked up is by Stephen Covey

(iii) The book transferred is by Stephen Covey AND the book picked up is by Vinay Singh

15. (2/3) × (3/4) + (1/3) × (2/7) = (1/2) + (2/21) = (25/42)

16. (i) (3/50) × (2/49) = (3/1225)

(ii) (3/50) × (47/49) + 47/50 × (3/49) = (141/1225)

(iii) (47/50) × (46/49) = (1081/1225)

17. (i) $(3/5) \times (3/7) = (9/35)$

(ii) $(3/5) \times 4/7) = (12/35)$

(iii) $(2/5) \times (3/7) = (6/35)$

(iv) $(3/5) \times (4/7) + (2/5) \times (3/7) + (3/5) \times (3/7) = 27/35$

18. They will contradict each other if: A is true and B is false or A is false and B is true.

$$(3/4) \times (1/6) + (1/4) \times (5/6) = 1/3$$

19. For the counting of the number of events, think of it as a circular arrangement with $n - 1$ people (by considering the two specified persons as one). This will give you $n(E) = (n - 2)! \times (2)!$

The total number of ways of them sitting without constraint = $(n - 1)!$. Hence, Probability of the event i.e. Probability of both sitting next to each other $P(E) = (n - 2)! \times 2/(n - 1)!$.

Probability of the non-event $P(E') = 1 - P(E)$.
Required odds against = $P(E')/P(E) = (n - 3)/2$

20. The whole numbers selected can only be 1, 3, 7 or 9 and cannot contain 2, 4, 6, 8, 0 or 5.

Required probability = $(2/5)^4$

21. ${}^4C_2 \times (6/36)^2 \times (30/36)^2 = 6 \times (1/36) \times (25/36) = 25/216$.

22. The required probability will be given by the expression:

$$\frac{\text{The number of young boys who will die}}{\text{The total number of people who will die}}$$

$$\frac{500}{(500 + 420 + 2400)} = \frac{25}{166}$$

23. Girl and boy and boy or boy and girl and boy

Or

Boy and boy and girl

= (3/4) × (2/4) × (2/3) + (1/4) × (2/4) × (2/3) + (1/4) × (2/4) × (1/3) = 18/48 = 3/8

24. $n(E)$ = 1

$n(S) = {}_3C_1 \times 9 \times 9 = 243$

25. 11/17 (if the first one is black, there will be 11 black balls left out of 17)

26. 6/17

27. There must have been two sixes in the first five throws. Thus, the answer is given by:

$$^5C_2 \times \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6}$$

28. $\frac{^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1}{^{52}C_4}$

29. There will be ${}_6C_3$ triangles formed overall. Out of these, there will be 2 equilateral triangles. Hence, the required probability is 2/20 = 1/10.

30. (5/9) × (4/8) + (4/9) × (3/8) = 32/72 = 4/9

As matched shoes means both red or both black

31. The event definition here is that first is not a heart and second is not a heart and third is a heart = (3/4) × (3/4) × (1/4) = 9/64.

32. (39/52) × (38/51) × (13/50) = 247/1700

33. The event definition will be: Event X happens and Y does not happen Or Y happens and X does not happen.

34. X happens and Y does not happen or X does not happen and Y happens or X happens and Y happens

$P \times (1 - P') + (1 - P) \times P' + P \times P' = P + P' - PP'$

35. Same logic as question 6.

36. (2/4) × (1/3) = 1/6 (Faulty and faulty)

37. Faulty and not faulty and faulty or not faulty and faulty and faulty = (2/4) × (2/3) × (1/2) + (2/4) × (2/3) × (1/2) = 1/3

38. When you put the seven balls with a gap between them in a row, you will have eight spaces. The required probability will be: $_8C_3/(10!/7! \times 3!)$ = 7/15

39. The appearance of head or tail on a toss is independent of previous occurrences.

Hence, 1/2.

40. $\frac{3! \times 4!}{7!}$ = 1/35 = 1/35

41. $P = \frac{\text{No. of arrangements with four S together}}{\text{Total No. of arrangements}}$

$= \frac{[8!(4! \times 2!)]}{[11!(4! \times 4! \times 2!)]}$

= 8! × 4!/11! = 24/990 = 4/165

42. $\frac{(5! \times 4! \times 2! \times 3!)}{11!} = \frac{24 \times 2 \times 6}{11 \times 10 \times 9 \times 8 \times 7 \times 6}$ = 1/1155

43. (1/3) × (1/4) × (4/5) + (1/3) × (3/4) × (1/5) + (2/3) × (1/4) × (1/5) + (1/3) × (1/4) × (1/5)

$= 10/60 = 1/6$

44. ${}_5C_3 \times [(8/12) \times (7/11) \times (6/10) \times (4/9) \times (3/8)] = 14/33$

45. ${}_5C_3 \times {}_6C_1/{}_{11}C_4 = 2/11$

46. $P = \dfrac{\text{Total no of ways in which two people sit together}}{\text{Total No. of ways}}$

$= (10! \times 2!)/11!$

47. Consider the six girls to be one person. Then the number of arrangements satisfying the condition is given by $n(E) = 7! \times 6!$.

48. ${}_6C_2 \times [(7/11) \times (6/10) \times (5/9) \times (4/8) \times (4/7) \times (3/6)] = 5/11$.

49. The event definition is red AND red AND not red OR red AND not red AND red OR not red AND red AND red.

50. The numbers having 2 in them are: 2, 12, 22, 32....92 and 21, 23, 24, 25....29. Hence, $n(E) = 19$.

Level of Difficulty (III)

1. This problem has to be treated as if we are selecting the third card out of the 50 remaining cards.

   Eleven of these are spades.

   Hence, 11/50.

2.

	Chance that A will win	*Chance that B will win*
First throw	$\frac{1}{6}$	$\frac{5}{6} \times \frac{1}{6}$
Second throw	$\frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}$	$\frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}$
Third throw	$\frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}$	$\frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}$

The chance that A will win is an infinite GP with a = $\frac{1}{6}$ and $r = \frac{25}{36}$

Similarly, the chance that B will win is an infinite GP with $a = \frac{5}{36}$ and $r = \frac{25}{36}$.

3. A total of six can be obtained by either (1 + 2 + 3) or by (2 + 2 + 2).

4. A is true and B is true = (3/4) × (7/10) = (21/40).

5. The required probability will be given by:

$$\frac{\text{Percentage of defective from } C}{\text{Percentage of defectives from } A, B \text{ and } C}$$

6. and 7.

We do not have to consider any sum other than 5 or 7 occurring.

A sum of 5 can be obtained by any of [4 + 1, 3 + 2, 2 + 3, 1 + 4]

Similarly a sum of 7 can be obtained by any of [6 + 1, 5 + 2, 4 + 3, 3 + 4, 2 + 5, 1 + 6]

For 6: $n(E) = 4, n(S) = 6 + 4$

$P = 0.4$

For 7: $n(E) = 6$

$n(S) = 6 + 4$ $P = 0.6$

8. The number of ways for a sum of 4 = 3 i.e. [3 + 1, 2 + 2, 1 + 3]

9. Try to find the number of ways in which 0 or 1 bomb hits the bridge if n bombs are thrown.

The required value of the number of bombs will be such that the probability of 0 or 1 bomb hitting the bridge should be less than 0.1.

10. The required probability will be given by: $1 - [{}_nC_0 + {}_nC_1]\left(\frac{1}{2}\right)^n$ for $n = 5$.

11. The required answer will be given by:

$$\frac{P(\text{Kesari does not receive the letter})}{P(\text{Kesari does not receive the letter}) + P(\text{Kesari replied and Sanjay did not receive the reply})}$$

12. $n(E) = {}_6C_2 \times 16 \times 15 \times 14 \times 13 \times 12$

$n(S) = 16_6$

13. The number of events for the condition that he will sing = 4 [34, 43, 26, 62].

The number of events in the sample = 90.

Probability that he will sing at least once

= 1 – probability that he will not sing

14. The required probability would be given by the event definition:

First is white and second is white = 8/22 × 7/21 = 4/33

15. The required probability would be given by the event definition:

First is red and second is red = 5/22 × 4/21 = 10/231

16. The number of ways in which six tickets can be selected from ten tickets is 10_6.

The number of ways in which the selection can be done so that the condition is satisfied = $7_6 - 6_6$.

17. In the first draw, we have seven even tickets out of fifteen and in the second we have eight odd tickets out of fifteen.

Thus, $(7/15) \times (8/15) = 56/225$.

18. The total number of ways in which the six identical balls can be distributed amongst the three boxes such that each box can get 0, 1, 2, 3, 4, 5 or 6 balls is given by the formula: ${}^{n+r-1}C_r$ where n is the number of identical balls and r is the number of boxes. This will give the sample space.

19. Event definition:

Any six of them work AND four leave OR any seven work AND three leave OR any eight work AND two leave OR any nine work AND one leaves OR all ten work.

20. "To make at least 3 trials to draw a heart" implies that he did not get a heart in the first two trials.

21. A box containing four defectives would get sent to the market if all the four articles selected are not defective.

Thus, ${}^{16}C_4/{}^{20}C_4 = 364/969$.

22. If the box contains only one defective, it would be sent back if the defective is one amongst the four selected for testing.

To ensure one of the four is selected, the number of ways is ${}^{19}C_3$, while the total number of selections of four out of 20 is ${}^{20}C_4$.

Thus, ${}^{19}C_3/{}^{20}C_4 = 1/5$.

23. For divisibility by 5, we need the units' digit to be either 0 or 5.

The units digit in the powers of 7 follow the pattern – 7, 9, 3, 1, 7, 9, 3, 1, 7, 9…….

Hence, divide 1 to 100 into four groups of 25 elements each as follows.

A = 1, 5, 9, ------- → 25 elements

B = 2, 6, 10, ------ → 25 elements

C = 3, 7, 11, ------ → 25 elements

D = 4, 8, 12, ------ → 25 elements

Check the combination values of m and n so that $7_m + 7_n$ is divisible by 5.

24. P(minimum 3) or P (maximum 7)
    P(minimum 3) = ${}_7C_2/{}_{10}C_3$ = 21/120

    P(max 7) = ${}_6C_2/{}_{10}C_2$ = 15/120

    (**Note:** The logic for this can be explained for minimum three conditions as: Since the minimum value has to be 3, the remaining two numbers have to be selected from 4 to 10. This can be done in ${}_7C_2$ ways.)

25. For the event to occur, the dice should show values from 2, 3, 4 or 5. This is similar to selection with repetition.

26. Yellow and red and blue = (3/6) × (2/6) × (1/6) = (6/216) = 1/36.

27. Two white and one black can be obtained only through the following three sequences:

    Balls drawn from A and B are white and the ball drawn from C is black.
    or

    Balls drawn from A and C are white and the ball drawn from B is black.
    or

    Balls drawn from B and C are white and the ball drawn from A is black.

Balls drawn from A and C are white and the ball drawn from B is black.

or

Balls drawn from B and C are white and the ball drawn from A is black.

28. At least one means (exactly one + exactly two + exactly three)

At least two means (exactly two + exactly three)
The problem gives the probabilities for passing in at least one, at least two and exactly two.

29. All four are not in the correct envelopes means that at least one of them is in a wrong envelope. A little consideration will show that one letter being placed in a wrong envelope is not possible, since it will have to be interchanged with some other letter.

Since, there is only one way to put all the letters in the correct envelopes, we can say that the event of not all four letters going into the correct envelopes will be given by

$5! - 1 = 119$

30. $n(E) = 44$

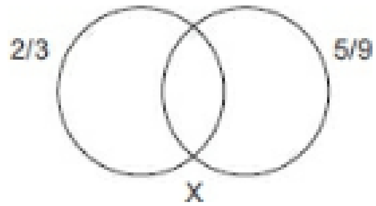
$n(S) = 120$

31. $A_1 = (112 \text{ or } 121)$

$A_2 = (112 \text{ or } 211)$

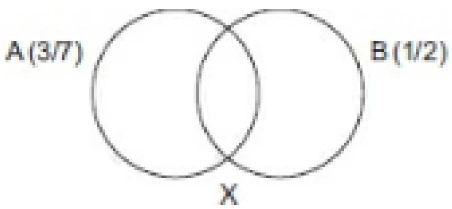
$A_3 = (121 \text{ or } 211)$

Option (b) is correct.

32. 

From the Venn diagram, we get:

$(2/3) + (5/9) - x = 4/5 \rightarrow x = (2/3) + (5/9) - (4/5) = 19/45$

33. 

Also, $P(A \cup B) = 1 - P(A' \cap B')$

$(3/7) + (1/2) - x = 13/14 \rightarrow x = 0$

Thus, there is no interference between A and B as $P(A \cap B) = x = 0$. Hence, A and B are mutually exclusive.

34. The required probability will be given by $\frac{2 \times 6! \times 6!}{12!}$.

35. The non-event in this case is that the problem is not solved.

Solutions for Questions 36 and 37:

Are similar to Question No. 2 of LOD III.

38. The event definition will be:

The first bag is selected AND a red ball is selected. or

The second bag is selected AND a red ball is selected.

39. The event definition is:

A girl is selected from the first group and one boy each is selected from the second and third groups. OR A girl is selected from the second group and one boy each is selected from the first and third groups. OR A girl is selected from the third group and one boy each is selected from the first and second groups.

40. The event definition will be:

A solves the problem and B and C do not solve the problem OR B solves the problem and A and C do not solve the problem OR C solves the problem and A and B do not solve the problem

41. The three balls that are taken out can be either three black balls or two black and one red ball or one black and two red balls or three red balls.

Each of these will give their own probabilities of drawing a black ball.

42. Three blacks and fourth is red or two blacks and one red and fourth is red or one black and two reds and fourth is red

$= (5/8) \times (4/7) \times (3/6) \times (3/5) + {}^3C_1 \times (5/8) \times (4/7)$

$\times (3/6) \times (2/5) + {}^3C_1 \times (5/8) \times (3/7) \times (2/6) \times (1/5)$

$= \frac{180 + 360 + 90}{1680} = 630/1680 = 3/8 = 0.375$

43. The event definition would be: Ball transferred is white and white ball drawn

Or

Ball transferred is black and white ball is drawn.

The answer will be given by:

$(5/9) \times (8/17) + (4/9) \times (7/17) = 68/153 = 4/9$

44. Solve this on a similar pattern to the example given in the theory of this chapter.

45. Required probability = 1 – probability that no pair is selected

46. We can have a maximum of five heads.

For 0 heads → $P(E) = (1/2^{10}) \times 1$

For 1 heads → $P(E) = (1/2^{10}) \times 1$

For 2 heads and for them not to occur consecutively, we will need to see the possible distribution of eight tails and two heads.

Since the two heads do not need to occur consecutively, this would be given by (all – two heads together)

→ $({}^{10}C_8 - 9)$

$P(E) = \frac{({}^{10}C_8 - 9)}{2^{10}}$

Solving in this fashion, we would get $1/2^3$.

47. This can be obtained by taking the number of ways in which exactly two people are born on the same day divided by the total number of ways in which seven people can be born in seven days of a week. For the first part, select two people from 7 in ${}^7C_2$ ways and select a day from the week on which they have to be born in ${}^7C_1$ ways and for the remaining five people select five days out of the remaining six days of the week and then the number of arrangements of these five people in five days-thus a total of ${}^7C_2 \times {}^7C_1 \times {}^6C_5 \times 5!$ ways. Also, the number of ways in which seven people can be born on seven days would be given by 7^7. Hence, the answer is given by: $({}^7C_2 \times {}^7C_1 \times {}^6C_5\ 5!/7^7) = 21 \times 7 \times 6 \times 120/7^7 = 3 \times 6 \times 120/7^5 = 2160/7^5$.

48. This can be obtained by defining the number of ways in which the player can get a deal of 13 cards if he gets all four kings divided by the number

of ways in which the player can get a deal of 13 cards without any constraints from 52 cards:

The requisite value would be given by: $^{48}C_9/^{52}C_{13}$ = [48!/39! × 9!] × [13! × 39!/52!] = (48! × 13! × 39!)/(39! × 9! × 52!) = 13 × 12 × 11 × 10/52 × 51 × 50 × 49 = 1 × 11/17 × 17 × 5 × 49 = 11/4165

49. P of heads showing on 50 coins = $^{100}C_{50} \times P^{50}(1-p)^{50}$

P of heads showing on 51 coins = $^{100}C_{51} \times P^{51}(1-p)^{49}$

Both are equal

$^{100}C_{50} \times P^{50}(1-P)^{50} = {}^{100}C_{51} \times P^{51}(1-P)^{49}$

or $\frac{100 \times 49 \times \ldots \times 52 \times 51}{1 \times 2 \times \ldots \times 49 \times 50} \times (1-P) \times (1-P)$

$= \frac{100 \times 99 \times \ldots \times 53 \times 52}{1 \times 2 \times \ldots \times 48 \times 49} \times P \times P$

or $\frac{51}{50} \times (1-P) = P$

or $51 \times (1-P) = 50P$

or $51 - 51P = 50P$

or $51 = 101P$

$\therefore P = \frac{51}{101}$

50. The common side could be horizontal or vertical. Accordingly, the number of ways the event can occur is.

$n(E) = 8 \times 7 + 8 \times 7 = 112$

$n(S) = {}^{64}C_2$

$= \frac{2 \times 8 \times 7 \times 2}{64 \times 63} = \frac{1}{18}$