Parallelograms Exercise 16A

What is a Parallelogram?

- Opposite sides are parallel
- Opposite sides are congruent
- Opposite angles are equal
- Consecutive angles supplementary
- Diagonals bisect eachother

Туре	Properties
Parallelogram	 Opposite sides are equal and parallel Opposite angles are equal
Rectangle	Opposite sides are equal and parallel All angles are right angles (90°)
Square	Opposite sides are parallel All sides are equal All angles are right angles (90°)
Rhombus	Opposite sides are parallel All sides are equal Opposite angles are equal Diagonals bisect each other at right angles (90°)
Trapezoid	One pair of opposite sides is parallel
Kite	 Two pairs of adjacent sides are equal One pair of opposite sides are equal One diagonal bisects the other Diagonals intersect at right angle (90°)

Q1

Answer:

It is given that ABCD is a parallelogram in which $\angle A$ is equal to 110° . Sum of the adjacent angles of a parallelogram is 180° .

$$\therefore \angle A + \angle B = 180^{\circ}$$

$$\Rightarrow 110^{\circ} + \angle B = 180^{\circ}$$

$$\Rightarrow \angle B = (180^{\circ} - 110^{\circ})$$

$$\Rightarrow \angle B = 70^{\circ}$$

$$\therefore \angle B = 70^{\circ}$$

Also, $\angle B + \angle C = 180^{\circ}$ ⇒ $70^{\circ} + \angle C = 180^{\circ}$ ⇒ $\angle C = (180^{\circ} - 70^{\circ})$ ⇒ $\angle C = 110^{\circ}$ ∴ $\angle C = 110^{\circ}$ Further, $\angle C + \angle D = 180^{\circ}$ ⇒ $\angle D = (180^{\circ} - 110^{\circ})$ ⇒ $\angle D = 70^{\circ}$ ∴ $\angle D = 70^{\circ}$

Q2

Answer:

Let the required angle be x° .

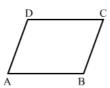
As the adjacent angles are equal, we have:

x+x=180 (s ince the sum of adjacent angles of a parallelogram is 180°) $\Rightarrow 2x=180$ $\Rightarrow x=\frac{180}{2}$ $\Rightarrow x=90°$

Hence, the measure of each of the angles is 90°.

Q3

Answer:



Let ABCD be the parallelogram.

Then, $\angle A$ and $\angle B$ are its adjacent angles.

Let
$$\angle A = (4x)^{\circ}$$

 $\angle B = (5x)^{\circ}$

 $\therefore \ \angle A + \angle B = 180^{\circ} \quad \ [s \, \text{ince sum of} \ the \ \text{adjacent angles of a parallelogram is} \ 180^{\circ}]$

$$\Rightarrow 4x + 5x = 180$$

$$\Rightarrow 9x = 180$$

$$\Rightarrow x = \frac{180}{9}$$

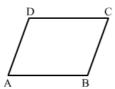
$$\Rightarrow x = 20$$

$$\therefore \angle A = (4 \times 20)^{\circ} = 80^{\circ}$$

 $\angle B = (5 \times 20)^{\circ} = 100^{\circ}$

 ${\cal O}\,{\rm pposite}$ angles of parallelogram are equal.

$$\therefore \angle C = \angle A = 80^{\circ}$$
$$\angle D = \angle B = 100^{\circ}$$



Let ABCD be a parallelogram.

$$L \text{ et } \angle A = (3x-4)^{\circ}$$

 $\angle B = (3x + 16)^{\circ}$

 $\therefore \angle A + \angle B = 180^{\circ}$ $[s\, {\rm ince} \, \ {\rm the} \, \, {\rm sum} \, \, \, {\rm of} \, \, {\rm adjacent} \, \, {\rm angles} \, \, {\rm of} \, \, {\rm a} \, \, {\rm parallelogram} \, \, {\rm is} \, \, 180^\circ]$

$$\Rightarrow 3\pmb{x}-4+3\pmb{x}+16=180$$

$$\Rightarrow 3x - 4 + 3x + 16 = 180$$

$$\Rightarrow 6x + 12 = 180$$

$$\rightarrow 0x + 12 = 10$$

$$\Rightarrow 6x = 168$$

$$\Rightarrow x = \frac{168}{6}$$

$$\Rightarrow x = 28$$

$$\therefore \angle A = (3 \times 28 - 4)^{\circ}$$

$$=(84-4)^{\circ}$$

$$= 80^{\circ}$$

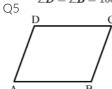
$$\angle B = ((3 \times 28) + 16)^{\circ}$$

= $(84 + 16)^{\circ}$

The opposite angles of a paralleleogram are equal.

$$\therefore \angle C = \angle A = 80^{\circ}$$

$$\angle D = \angle B = 100^{\circ}$$



Let ABCD be a parallelogram and let the sum of its opposite angles be 130°.

$$\angle A + \angle C = 130^{\circ}$$

T he opposite angles are equal in a parallelogram.

$$\therefore \angle A = \angle C = x^{\circ}$$

$$\Rightarrow x + x = 130$$

$$\Rightarrow 2x = 130$$

$$\Rightarrow x = \frac{130}{2}$$

$$\Rightarrow x = 65$$

 $\therefore \angle A = 65^{\circ} \text{ and } \angle C = 65^{\circ}$

 $\angle A + \angle B = 180^{\circ}$ [s ince the sum of adjacent angles of a parallelogram is 180°]

$$\Rightarrow$$
 65° + $\angle B$ = 180°

$$\Rightarrow \angle B = (180 - 65)^{\circ}$$

$$\Rightarrow \angle B = 115^{\circ}$$

$$\angle D = \angle B = 115^{\circ}$$
 opposite angles of parallelogram are equal

06

Answer:

Let the lengths of two sides of the parallelogram be 5x cm and 3x cm, respectively.

Then, its perimeter = 2(5x + 3x) cm

$$=16x$$
 cm

$$16x = 64$$

$$\Rightarrow x = \frac{64}{16}$$

$$\Rightarrow x = 4$$

 \therefore One side \Rightarrow (5 × 4) cm = 20 cm

Other side \Rightarrow (3 × 4) cm = 12 cm

07

Answer:

Let the lengths of two sides of the parallelogram be x cm and (x+10) cm, respectively. Then, its perimeter = 2[x + (x + 10)] cm

$$=2[\pmb{x}+\pmb{x}+10]~\mathbf{cm}$$

$$=2[2\pmb{x}+10]~\mathbf{cm}$$

$$=4x + 20$$
 cm

$$4x + 20 = 140$$

$$\Rightarrow 4x = 140 - 20$$

$$\Rightarrow 4x = 120$$

$$\Rightarrow x = \frac{120}{4}$$

$$\Rightarrow x = \frac{1}{4}$$

Length of one side =30 cm

Length of the other side \Rightarrow (30 + 10) cm = 40 cm

Answer:

Refer to the figure given in the book.

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In \triangle BMC and \triangle DNA:

\angle DNA = \angle BMC = 90^{\circ}

\angle BCM = \angle DAN (alternate angles)

BC = DA (opposite sides)

By AAS congruency criteria:

\triangle BMC \cong \triangle DNA

Yes, it is true that BM is equal to DN.

(by corresponding parts of congruent triangles BMC and DNA)
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09

Refer to the figure of the book.

$$\angle A = \angle C \qquad \text{(opposite angles of a parallelogram are equal)}$$

$$\Rightarrow \frac{1}{2} \angle A = \frac{1}{2} \angle C$$

$$=> \angle EAD = \angle FCB \qquad \left(AE \text{ and } CF \text{ bisect the angles A and C, respectively}\right)$$

$$In \ \triangle ADE \text{ and } \triangle CBF :$$

$$\angle B = \angle D \qquad \text{(opposite angles of a parallelogram are equal)}$$

$$\angle EAD = \angle FCB \qquad \left(p \text{ proved above}\right)$$

$$AD = BC \qquad \text{(opposite sides of a parallelogram are equal)}$$

$$By \text{ AAS concruency criteria:}$$

$$\triangle ADE \cong \triangle BCF \qquad \text{(opposite sides of a parallelogram are equal)}$$

$$By \text{ CD} = BF \qquad \text{(opposite sides of a parallelogram are equal)}$$

$$CD = AB \qquad \text{(opposite sides of a parallelogram are equal)}$$

$$Also, \ CD - DE = AB - BF \qquad \Rightarrow CE = AF \qquad BCD \text{ is a parallelogram.}$$

$$\therefore CD \parallel AB \qquad \text{(opposite sides of a parallelogram are parallel)}$$

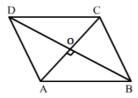
$$=> CE \parallel AF \qquad \text{(opposite sides of a parallelogram are parallel)}$$

If one pair of sides of a quadrilateral is parallel and equal, then it is a parallelogram. Therefore, AECF is a parallelogram.

∴ AE || CF

Q10

Answer:



Let ABCD be a rhombus.

 $L\,\mathrm{et}\,\,AC$ and BD be the diagonals of the rhombus intersecting at a point O.

Let AC = 16 cm

BD = 12 cm

We know that the diagonals of a rhombus bisect each other at right angles.

$$\therefore AO = \frac{1}{2} AC$$

$$= \left(\frac{1}{2} \times 16\right) \text{ cm}$$

$$= 8 \text{ cm}$$

$$BO = \frac{1}{2} BD$$

$$= \left(\frac{1}{2} \times 12\right) \text{ cm}$$

$$= 6 \text{ cm}$$

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From the right \triangle AOB: AB^2 = AO^2 + BO^2
= \left\{ (8)^2 + (6)^2 \right\} \text{ cm}^2
= (64 + 36) \text{ cm}^2
= 100 \text{ cm}^2
\Rightarrow AB = \sqrt{100} \text{ cm}
= 10 \text{ cm}
Hence, the length of the side AB \text{ is } 10 \text{ cm}.
AB = BC = CD = DA = 10 \text{ cm}
(all sides of a rhombus are equal)
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Q11

Answer:

Refer to the figure given in the book.

In $\triangle ADC$:

$$DA = DC$$
 (all sides of a square are equal)
 $\Rightarrow \angle ACD = \angle CAD$

Let $\angle ACD = \angle CAD = x^{\circ}$ [Angle opposite to the equal sides are equal]
 $x + x + 90 = 180$ [since the sum of the angles of a triangle is 180°]
 $\Rightarrow 2x + 90 = 180$ $\Rightarrow 2x = 90$ $\Rightarrow x = \frac{90}{2}$ $\Rightarrow x = 45$

Q12

Answer:

∴ ∠*CAD* = 45°

Let the length of two sides of the rectangle be 5x cm and 4x cm, respectively. Then, its perimeter = 2(5x+4x) cm

$$=18x$$
 cm

$$\therefore 18x = 90$$

$$\Rightarrow x = \frac{90}{18}$$

$$\Rightarrow x = 5$$

Length of one side \Rightarrow (5 \times 5) cm = 25 cm

Length of the other side \Rightarrow (4 × 5) cm = 20 cm

... Length of the rectangle = 25 cm

Breadth = 20 cm

Q13

Answer:

- (i) The diagonals are equal and the adjacent sides are unequal. Hence, the given parallelogram is a rectangle.
- (ii) The diagonals are equal and the adjacent sides are equal. Hence, the given parallelogram is a square.
- (iii) The diagonals are unequal and the adjacent sides are equal. Hence, the given parallelogram is a rhombus.
- (iv) All the sides are equal and one angle is 60°. Hence, the given parallelogram is a rhombus.
- (v) All the sides are equal and one angle is 90°. Hence, the given parallelogram is a square.
- (vi) All the angles are equal and the adjacent sides are unequal. Hence, the given parallelogram is a rectangle.

Answer:

(i) The given statement is false.

The diagonals of a parallelogram bisect each other, but they are not equal in length.

(ii) The given statement is false.

The diagonals of a rectangle are equal and bisect each other, but they are not perpendicular.

(iii) The given statement is false.

All the sides of a rhombus are equal, but the diagonals are not equal.

- (iv) The given statement is true.
- (v) The given statement is false.

Every square is a rectangle, but every rectangle is not a square.

- (vi) The given statement is true.
- (vii) The given statement is true.
- (viii) The given statement is true.
- (ix) The given statement is false.

A rectangle is a special type of parallelogram, but every parallelogram is not a rectangle.

(x) The given statement is true.

Parallelograms Exercise 16B

Q1

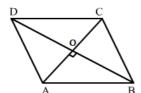
Answer:

(c) rhombus

In a rhombus, the two diagonals are not necessarily equal.

Q2

(c) 10 cm



Let ABCD be a rhombus.

Let AC and BD be the diagonals of the rhombus intersecting at a point O.

$$AC = 16$$
 cm

$$BD = 12$$
 cm

We know that the diagonals of a rhombus bisect each other at right angles.

$$\begin{split} \therefore \ AO &= \tfrac{1}{2} \, AC \\ &= \left(\tfrac{1}{2} \times 16 \right) \, \mathrm{cm} \\ &= 8 \, \, \mathrm{cm} \\ BO &= \tfrac{1}{2} \, BD \\ &= \left(\tfrac{1}{2} \times 12 \right) \, \mathrm{cm} \\ &= 6 \, \, \mathrm{cm} \end{split}$$

From the right $\triangle AOB$:

$$AB^{2} = AO^{2} + BO^{2}$$

$$= \{(8)^{2} + (6)^{2}\} \text{ cm}^{2}$$

$$= (64 + 36) \text{ cm}^{2}$$

$$= 100 \text{ cm}^{2}$$

$$\Rightarrow AB = \sqrt{100} \text{ cm}$$

$$= 10 \text{ cm}$$

Hence, the length of the side ABis10 cm.

Therefore, the length of each side of the rhombus is 10 cm because all the sides of a rhombus are equal.

Q3

Answer:

We know that the sum of adjacent angles of a parallelogram is 180°.

$$\Rightarrow 2x + 25 + 3x - 5 = 180$$

$$\Rightarrow 5x + 20 = 180$$

$$\Rightarrow 5x = 180 - 20$$

$$\Rightarrow 5x = 160$$

$$\Rightarrow x = \frac{160}{5}$$

$$\Rightarrow x = 32$$

Therefore, the value of x is 32.

Q4

Answer:

(a) parallelogram

In a parallelogram, the diagonals do not necessarily intersect at right angles.

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Answer:
                 (c) 70 cm
                 Let ABCD be a rectangle and let the diagonal AC be 25 cm, length AB be 4x cm and
                  breadth BC be 3x cm.
                 Each angle of a rectangle is a right angle.
                 ∴ ∠ABC = 90°
                 From the right \triangle ABC:
                 AC^2 = AB^2 + BC^2
                 \Rightarrow (25)^2 = (4x)^2 + (3x)^2
                 \Rightarrow 625 = 16\boldsymbol{x}^2 + 9\boldsymbol{x}^2
                 \Rightarrow 625 = 25x^2
                 x^2 = \frac{625}{25} = 25
                 \Rightarrow \boldsymbol{x} = 5
                  \therefore Length = 4 × 5 = 20 cm
                 \textbf{Breadth} = 3 \times 5 = 15 \ \textbf{cm}
                 \therefore Perimeter of the rectangle = 2(20+15) cm
   Q6
                                               = 70 cm
    Answer:
    The bisectors of any two adjacent angles of a parallelogram intersect at 90^{\circ}.
   Q7
    Answer:
    (b) 72°
    Let x^{\circ} be the angle of the parallelogram.
    Sum of the adjacent angles of a parallelogram is 180\,^{\circ}.
    \therefore x + \left(\frac{2}{3} \times x\right) = 180
     \Rightarrow x + \frac{2x}{3} = 180
     \Rightarrow \left(x + \frac{2x}{3}\right) = 180
     \Rightarrow \frac{5x}{3} = 180
     \Rightarrow x = \left(180 \times \frac{3}{5}\right)
    Hence, one angle of the parallelogram is 108^{\circ}.
    Its adjacent angle = (180 - 108)^{\circ} = 72^{\circ}
    Therefore, the smallest angle of the parallelogram is 72°.
   Q8
   Answer:
   (a) r ectangle
   In a rectangle, the diagonals do not necessarily bisect the interior angles at the vertices.
   Q9
   Q10
   Answer:
   (d) 8
   All the sides of a square are equal.
   AB = BC
    \Rightarrow 2x + 3 = 3x - 5
    \Rightarrow 3+5=3x-2x
    \Rightarrow 8 = x
    Therefore, the value of x is 8.
Answer:
(c) 112°
Let x^{\circ} be the smallest angle of the parallelogram.
The sum of adjacent angles of a parallelogram is 180^{\circ}.
\therefore x + 2x - 24 = 180
 \Rightarrow 3x - 24 = 180
\Rightarrow 3x = 180 + 24
 \Rightarrow 3x = 204
 \Rightarrow x = \frac{204}{3}
 \Rightarrow x = 68
 .. Smallest angle = 68°
L \, \text{argest angle} = (180 - 68)^{\circ} = 112^{\circ}
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