
Sample Paper-03
SUMMATIVE ASSESSMENT-II
MATHEMATICS
Class – X

Time allowed: 3 hours

Maximum Marks: 90

General Instructions:

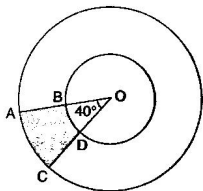
- a) All questions are compulsory.
- b) The question paper consists of 31 questions divided into four sections – A, B, C and D.
- c) Section A contains 4 questions of 1 mark each which are multiple choice questions, Section B contains 6 questions of 2 marks each, Section C contains 10 questions of 3 marks each and Section D contains 11 questions of 4 marks each.
- d) Use of calculator is not permitted.

Section A

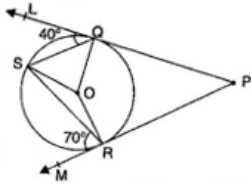
- 1. If $P(E) = 0.05$, what is the probability of 'not E'?
- 2. If the centroid of triangle formed by points $P(a, b)$, $Q(b, c)$ and $R(c, a)$ is at the origin, what is the value of $a + b + c$?
- 3. If n^{th} term of an AP is $\frac{3+n}{4}$, find its 8th term.
- 4. The length of the tangent to a circle from a point P, which is 25 cm away from the centre, is 24 cm. What is the radius of the circle?

Section B

- 5. Find the area of the shaded region in figure, if radii of the two concentric circles with centre O are 7 cm and 14 cm respectively and $\angle AOC = 40^\circ$.

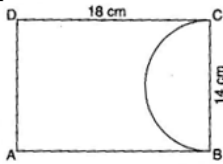


- 6. How many spherical lead shots of radius 2 cm can be made out of a solid cube of lead whose edge measures 44 cm?
 - 7. The rain water from a roof 22 m x 20 m drains into a cylindrical vessel having diameter of base 2 m and height 3.5 m of the vessel is just full, find the rainfall in cm.
 - 8. Find the discriminant of the quadratic equation: $3\sqrt{3}x^2 + 10x + \sqrt{3} = 0$
 - 9. Find the common difference and write the next two terms of the AP $1^2, 5^2, 7^2, 73, \dots$
 - 10. In figure, PQL and PRM are tangents to the circle with centre O at the points P and R respectively and S is a point on the circle such that $\angle SQL = 40^\circ$ and $\angle SRM = 70^\circ$. Then find $\angle QSR$.
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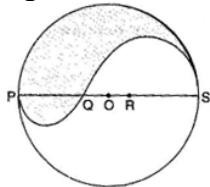


Section C

11. Find the area of the quadrilateral whose vertices taken in order are $(-4, -2)$, $(-3, -5)$, $(3, -2)$ and $(2, 3)$.
12. Find the centre of a circle passing through the points $(6, -6)$, $(3, -7)$ and $(3, 3)$.
13. A paper is in the form of a rectangle ABCD with $AB = 18$ cm and $BC = 14$ cm. A semi-circular portion with BC as diameter is cut off. Find the area of the remaining paper.



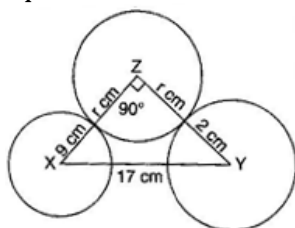
14. PQRS is a diameter of a circle of radius 6 cm. The lengths PQ, QR and RS are equal. Semi-circles are drawn on PQ and QS as diameters as shown in figure. Find the perimeter of the shaded region.



15. The radius of the base and the height of a solid right circular cylinder are in the ratio $2 : 3$ and its volume is 1617 cm^3 . Find the total surface area of the cylinder.
16. Solve the quadratic equation: $4x^2 + 4\sqrt{3}x + 3 = 0$
17. A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: Rs. 200 for the first day, Rs 250 for the second day, Rs 300 for the third day, etc., the penalty for each succeeding day being Rs 50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days?
18. A circle touching the side BC of a $\triangle ABC$ at P and touching AB and AC produced at Q and R respectively. Prove that: $AQ = \frac{1}{2} (\text{Perimeter of } \triangle ABC)$
19. An aeroplane flying horizontally at a height of 1.5 km above the ground is observed at a certain point on the earth to subtend an angle of 60° . After 15 seconds, its angle of elevation is observed to be 30° . Calculate the speed of aeroplane in km/hr.
20. A bag contains 5 red and some blue balls,
 - (i) If probability of drawing a blue ball from the bag is twice that of a red ball, find the number of blue balls in the bag.
 - (ii) If probability of drawing a blue ball from the bag is four times that of a red ball, find the number of blue balls in the bag.

Section D

21. Construct a triangle ABC in which $AB = 6.5$ cm, $\angle B = 60^\circ$ and $BC = 5.5$ cm. Also construct a triangle $AB'C'$ similar to $\triangle ABC$, whose each side is $\frac{3}{2}$ times the corresponding side of $\triangle ABC$.
22. The angle of elevation of the top Q of a vertical tower PQ from a point X on the ground is 60° . At a point Y, 40 m vertically above X, the angle of elevation is 45° , find the height of the tower PQ and the distance XQ.
23. A card is drawn at random from a well shuffled deck of playing cards. Find the probability that the card drawn is
- a card of spades or an ace
 - a red king
 - neither a king nor a queen
 - either a king or a queen
 - a face card
 - cards which is neither king nor a red card.
24. Prove that the point $(a, 0)$, $(0, b)$ and $(1, 1)$ are collinear, if $\frac{1}{a} + \frac{1}{b} = 1$
25. A right-angled triangle whose sides are 15 cm and 20 cm is made to revolve about its hypotenuse. Find the volume and surface area of the double cone so formed. (Use $\pi = 3.14$)
26. A bucket made up of a metal sheet in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm respectively. Find the cost of the bucket, if the cost of the metal sheet used is Rs.15 per 100 cm^2 . (Use $\pi = 3.14$)
27. A motorboat whose speed is 9 km/h in still water goes 12 km downstream and comes back in a total time of 3 h.
- Find the speed of the stream.
 - Explain the situation when speed of the stream is more than the speed of the boat in still water.
28. A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.
29. If a^2 , b^2 , c^2 are in AP, then prove that $\frac{1}{b+c}$, $\frac{1}{c+a}$, $\frac{1}{a+b}$ are in AP.
30. X and Y are centres of circles of radius 9 cm and 2 cm and $XY = 17$ cm. Z is the centre of a circle of radius r cm, which touches the above circles externally. Given that $\angle XZY = 90^\circ$, write an equation in r and solve it for r .



31. Prove that the lengths of tangents drawn from an external point to a circle are equal.
Using the above result, prove the following:
If a circle touches all the four sides of a parallelogram, show that the parallelogram is a rhombus.

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(Solutions)

SECTION-A

1. As we know that,

$$P(E) + P(\text{not } E) = 1$$

$$\therefore P(\text{not } E) = 1 - P(E) = 1 - 0.05 = 0.95$$

2. Centroid of $\Delta PQR = \left(\frac{a+b+c}{3}, \frac{b+c+a}{3} \right)$

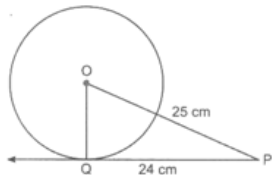
$$\text{Given } \left(\frac{a+b+c}{3}, \frac{b+c+a}{3} \right) = (0, 0)$$

$$\Rightarrow a + b + c = 0$$

3. $a_n = \frac{3+n}{4}$

$$a_8 = \frac{3+8}{4} = \frac{11}{4}$$

4. $OQ \perp PQ$



$$PQ^2 + OQ^2 = OP^2$$

$$25^2 = OQ^2 + 24^2$$

$$OQ = \sqrt{625 - 576}$$

$$= \sqrt{49} = 7$$

5. Area of shaded region = Area of sector OAC - Area of sector OBD

$$= \frac{40^\circ}{360^\circ} \times \frac{22}{7} \times (14)^2 - \frac{40^\circ}{360^\circ} \times \frac{22}{7} \times (7)^2$$

$$= \frac{40^\circ}{360^\circ} \times \frac{22}{7} \left[(14)^2 - (7)^2 \right]$$

$$= \frac{40^\circ}{360^\circ} \times \frac{22}{7} (14-7)(14+7)$$

$$= \frac{40^\circ}{360^\circ} \times \frac{22}{7} \times 7 \times 21$$

6. $r = \frac{14}{2} = 7 \text{ cm}$

Length of arc = $\frac{\theta}{360^\circ} 2\pi r$

$\Rightarrow \angle AED = \frac{180^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 7 = 22 \text{ cm}$

$\therefore$ Perimeter of given figure = $20 + 14 + 20 + 22 = 76 \text{ cm}$

7. Let n spherical lead shots be made, then

According to question,

$n \times \text{Volume of spherical lead shot} = \text{Volume of Solid Cube}$

$\Rightarrow n \cdot \frac{4}{3} \pi r^3 = (a)^3$

$\Rightarrow n \cdot \frac{4}{3} \times \frac{22}{7} \times (2)^3 = 44 \times 44 \times 44$

$\Rightarrow n = \frac{44 \times 44 \times 44 \times 7 \times 3}{4 \times 22 \times 8} = 2541$

8. Let the rainfall be $x \text{ cm}$.

Volume of rain water = Volume of cylindrical vessel

$\Rightarrow lbx = \pi r^2 h$

$\Rightarrow 22 \times 20 \times x = \frac{22}{7} \times 1 \times 1 \times 3.5$

$\Rightarrow x = \frac{22 \times 1 \times 3.5}{7 \times 22 \times 20} = 2.5 \text{ cm}$

9. $1^2, 5^2, 7^2, 73, \dots$

$\Rightarrow 1, 25, 49, 73, \dots$

$d = a_2 - a_1 = 25 - 1 = 24$

$d = 49 - 25 = 24$

$d = 73 - 49 = 24$

Hence, it is AP.

$a_5 = 73 + 24 = 97$

$a_6 = 97 + 24 = 121$

10. $\angle OQS = \angle OQL - \angle SQL = 90^\circ - 40^\circ = 50^\circ$

$\Rightarrow \angle OQS = 50^\circ$ (i)

$\angle ORS = \angle ORM - \angle SQM = 90^\circ - 70^\circ = 20^\circ$ (ii)

$\therefore \angle OSQ = \angle OQS = 50^\circ$ (iii) [From eq. (i)]

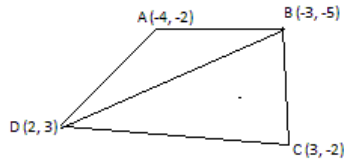
$\therefore OS = OR$

$\therefore \angle OSR = \angle ORS = 20^\circ$ (iv) [From eq. (ii)]

$\therefore \angle QSR = \angle ORS + \angle OSR$
 $= 50^\circ + 20^\circ = 70^\circ$

11. Area of Quadrilateral ABCD = Area of Triangle ABD + Area of Triangle BCD ... (1)

Using formula to find area of triangle:



$$\begin{aligned}
 \text{Area of } \triangle ABD &= \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] \\
 &= \frac{1}{2} [-4(-5 - 3) - 3\{3 - (-2)\} + 2\{-2 - (-5)\}] \\
 &= \frac{1}{2} (32 - 15 + 6) = \frac{1}{2} (23) = 11.5 \text{ sq units} \dots (2)
 \end{aligned}$$

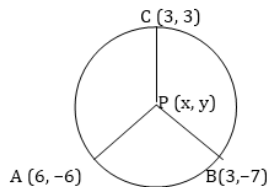
Again using formula to find area of triangle:

$$\begin{aligned}
 \text{Area of } \triangle BCD &= \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] \\
 &= \frac{1}{2} [-3(-2 - 3) + 3\{3 - (-5)\} + 2\{-5 - (-2)\}] \\
 &= \frac{1}{2} (15 + 24 - 6) = \frac{1}{2} (33) = 16.5 \text{ sq units} \dots (3)
 \end{aligned}$$

Putting (2) and (3) in (1), we get

$$\text{Area of Quadrilateral ABCD} = 11.5 + 16.5 = 28 \text{ sq units}$$

12. Let $P(x, y)$, be the centre of the circle passing through the points $A(6, -6)$, $B(3, -7)$ and $C(3, 3)$. Then $AP = BP = CP$.



Taking $AP = BP$

$$\begin{aligned}
 \Rightarrow AP^2 &= BP^2 \\
 \Rightarrow (x - 6)^2 + (y + 6)^2 &= (x - 3)^2 + (y + 7)^2 \\
 \Rightarrow x^2 - 12x + 36 + y^2 + 12y + 36 &= x^2 - 6x + 9 + y^2 + 14y + 49 \\
 \Rightarrow -12x + 6x + 12y - 14y + 72 - 58 &= 0 \\
 \Rightarrow -6x - 2y + 14 &= 0 \\
 \Rightarrow 3x + y - 7 &= 0 \dots\dots\dots (i)
 \end{aligned}$$

Again, taking $BP = CP$

$$\begin{aligned}
 \Rightarrow BP^2 &= CP^2 \\
 \Rightarrow (x - 3)^2 + (y + 7)^2 &= (x - 3)^2 + (y - 3)^2
 \end{aligned}$$

$$\Rightarrow x^2 - 6x + 9 + y^2 + 14y + 49 = x^2 - 6x + 9 + y^2 - 6y + 9$$

$$\Rightarrow -6x + 6x + 14y + 6y + 58 - 18 = 0$$

$$\Rightarrow 20y + 40 = 0$$

$$\Rightarrow y = -2$$

Putting the value of y in eq. (i),

$$3x + y - 7 = 0$$

$$\Rightarrow 3x = 9$$

$$\Rightarrow x = 3$$

Hence, the centre of the circle is $(3, -2)$.

13. Area of remaining paper =

Area of rectangle - Area of semi-circle

$$= l \times b - \frac{1}{2} \pi r^2$$

$$= 18 \times 14 - \frac{1}{2} \times \frac{22}{7} \times \frac{14}{2} \times \frac{14}{2}$$

$$= 252 - 77$$

$$= 175 \text{ cm}^2$$

14. $PQ = QR = RS = \frac{2 \times 6}{3} = 4 \text{ cm}$

$$\therefore \text{Perimeter of the shaded region} = \overline{PQ} + \overline{PS} + \overline{QS}$$

$$= \pi \times 2 + \pi \times 6 + \pi \times 4$$

$$= 12\pi \text{ cm}$$

15. Let the radius and height be $2k$ cm and $3k$ cm respectively. Then,

$$\text{Volume of cylinder} = \pi r^2 h$$

$$\Rightarrow 1617 = \pi (2k)^2 (3k)$$

$$\Rightarrow k = \frac{7}{2} \text{ cm}$$

$$\therefore r = 7 \text{ cm and } h = \frac{21}{2} \text{ cm}$$

$$\therefore \text{Total surface area of cylinder} = 2\pi r(r + h)$$

$$= 2 \times \frac{22}{7} \times 7 \left(7 + \frac{21}{2} \right)$$

$$= 770 \text{ cm}^2$$

16. $4x^2 + 4\sqrt{3}x + 3 = 0$

$$\Rightarrow (2x)^2 + 2(2x)(\sqrt{3}) + (\sqrt{3})^2 = 0$$

$$\Rightarrow (2x + \sqrt{3})^2 = 0$$

$$\Rightarrow (2x + \sqrt{3})(2x + \sqrt{3}) = 0$$

$$\Rightarrow 2x = -\sqrt{3}, 2x = -\sqrt{3}$$

$$\Rightarrow x = \frac{-\sqrt{3}}{2}, \frac{-\sqrt{3}}{2}$$

17. Penalty for first day = Rs 200, Penalty for second day = Rs 250

Penalty for third day = Rs 300

It is given that penalty for each succeeding day is Rs 50 more than the preceding day.

It makes it an arithmetic progression because the difference between consecutive terms is constant.

We want to know how much money the contractor has to pay as penalty, if he has delayed the work by 30 days.

So, we have an AP of the form 200, 250, 300, 350...30 terms

First term = $a = 200$, Common difference = $d = 50$, $n = 30$

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP, we get

$$S_n = \frac{30}{2}[400 + (30-1)50] \quad \Rightarrow \quad S_n = 15(400 + 29 \times 50)$$

$$\Rightarrow S_n = 15(400 + 1450) = 27750$$

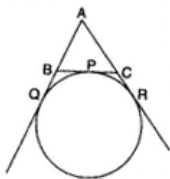
Therefore, penalty for 30 days is Rs. 27750.

18. $\therefore$ Tangent segments from an external point to a circle are equal in length.

$$\therefore AQ = AR \quad BP = BQ \quad CP = CR$$

$$\begin{aligned} \therefore \text{Perimeter of } \triangle ABC &= AB + BC + AC \\ &= AB + BP + CP + AC \\ &= AB + BQ + CR + AC \\ &= AQ + AR = AQ + AQ = 2AQ \end{aligned}$$

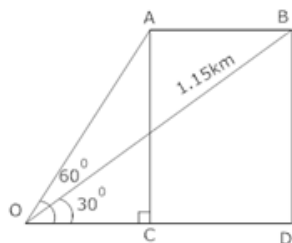
$$\Rightarrow AQ = \frac{1}{2} (\text{Perimeter of } \triangle ABC)$$



19. Let O be the observation point.

Let A be the position of aeroplane such that $\angle AOC = 60^\circ$ and $AC = 1.5 \text{ km} = BD$

Let B be the position of aeroplane after 15 seconds. $\angle BOD = 30^\circ$, $OC = x \text{ km}$, $CD = y \text{ km}$



$$\text{In right } \triangle OCA, \frac{x}{1.5} = \cot 60^\circ = \frac{1}{\sqrt{3}}$$

$$\Rightarrow x = \frac{1.5}{\sqrt{3}}$$

$$\Rightarrow x = \frac{15}{10\sqrt{3}} = \frac{\sqrt{3}}{2} \dots\dots(i)$$

In right $\triangle ADB$,

$$\frac{x+y}{1.5} = \cot 30^\circ = \sqrt{3}$$

$$\Rightarrow x+y = \sqrt{3}(1.5)$$

$$\Rightarrow x+y = \sqrt{3} \times \frac{3}{2} \dots\dots(ii)$$

eq.(ii) – eq.(i)

$$y = \frac{3\sqrt{3}}{2} - \frac{\sqrt{3}}{2} = (3-1)\frac{\sqrt{3}}{2} = \sqrt{3}$$

$\therefore$ Distance covered in 15 seconds $y = \sqrt{3} \text{ km}$

$$\text{Distance covered in 1 second} = \frac{\sqrt{3}}{15} \text{ km}$$

$$\text{Distance covered in 3600 seconds} = \frac{\sqrt{3}}{15} \times 3600 = 240\sqrt{3} \text{ km}$$

20. Let number of blue balls = x

Total number of balls = $5+x$

$$\text{Probability of red ball} = \frac{5}{5+x}$$

$$\text{Probability of blue ball} = \frac{x}{5+x}$$

By given condition,

$$(i) \frac{x}{5+x} = 2 \cdot \frac{5}{5+x}$$

$$\Rightarrow x = 10$$

No. of blue balls = 10

$$(ii) \text{ Here, } \frac{5}{5+x} = 4 \times \frac{x}{5+x}$$

$$\Rightarrow x = 20$$

Hence, number of blue balls = 20

21. **Steps of construction:**

(a) Draw a right angled triangle ABC with given measurements.

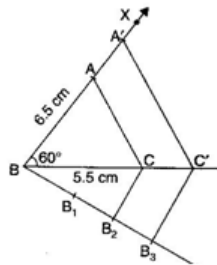
(b) Draw any ray BY making an acute angle with BC on the side opposite to the vertex A.

(c) Locate 3 points B_1 , B_2 and B_3 on BY so that $BB_1 = B_1B_2 = B_2B_3$.

(d) Join B_2 to C and draw a line through B_3 parallel to B_2C , intersecting the extended line segment BC at C' .

(e) Draw a line through C' parallel to CA intersecting the extended line segment BA at A' .

The $A'BC'$ is the required triangle.

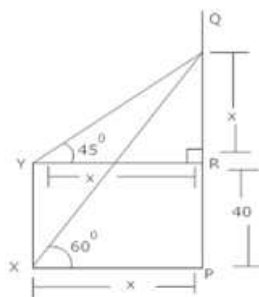


22. In right $\triangle QRY$,

$$\frac{QR}{x} = \tan 45^\circ$$

$$\Rightarrow \frac{QR}{x} = \frac{1}{1}$$

$$\Rightarrow QR = x$$



In right $\triangle QPX$,

$$\frac{x + 40}{x} = \tan 60^\circ$$

$$\Rightarrow \frac{x + 40}{x} = \frac{\sqrt{3}}{1}$$

$$\Rightarrow \sqrt{3}x = x + 40$$

$$\Rightarrow x = \frac{40}{\sqrt{3} - 1}$$

$$\Rightarrow x = \frac{40}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1}$$

$$\Rightarrow x = 20(\sqrt{3} + 1) = 54.64$$

$$PQ = x + 40$$

$$= 54.64 + 40 = 94.64 \text{ m}$$

In right $\triangle QPX$,

$$\frac{x+40}{XQ} = \sin 60^\circ$$

$$\Rightarrow \frac{54.64+40}{XQ} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow XQ = 109.3 \text{ m}$$

23. Total possible outcomes = 52

(i) No. of spades = 13

No. of ace = 4

1 card is common [ace of spade]

Favourable outcomes = $13+4 - 1=16$

$$\therefore \text{Required probability} = \frac{16}{52} = \frac{4}{13}$$

(ii) No. of red kings = 2

Favourable outcomes = 2

$$\therefore \text{Required probability} = \frac{2}{52} = \frac{1}{26}$$

(iii) No. of king and queen = $4+4=8$

Favourable outcomes = $52 - 8=44$

$$\text{Required probability} = \frac{44}{52} = \frac{11}{13}$$

(iv) No. of king and queen = $4+4=8$

$$\text{Required probability} = \frac{8}{52} = \frac{2}{13}$$

(v) No. of face cards = $4+4+4=12$ [Jack, queen and king are face card]

$$\text{Required probability} = \frac{12}{52} = \frac{3}{13}$$

(vi) No. of cards which are neither red card nor king = $52 - (26+4 - 2)$
 $= 52 - 28=24$

$$\therefore \text{Required probability} = \frac{24}{52} = \frac{6}{13}$$

24. Since $(a,0), (0,b)$ and $(1,1)$ are collinear

Area = 0

$$\frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$$

$$\Rightarrow \frac{1}{2} [a(b-1) + 0(1-0) + 1(0-b)] = 0$$

$$\Rightarrow ab - a - b = 0$$

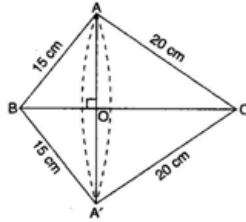
$$\Rightarrow ab = a + b$$

Dividing by ab ,

$$\frac{ab}{ab} = \frac{a}{ab} + \frac{b}{ab}$$

$$\frac{1}{a} + \frac{1}{b} = 1$$

25. $BC = \sqrt{15^2 + 20^2} = \sqrt{225 + 400} = \sqrt{625} = 25 \text{ cm}$



$$\triangle AOB \sim \triangle CAB \quad [\text{AA similarity criterion}]$$

$$\therefore \frac{AO}{20} = \frac{15}{25}$$

$$\Rightarrow AO = 12 \text{ cm}$$

And $\frac{BO}{15} = \frac{15}{25}$

$$\Rightarrow BO = 9 \text{ cm}$$

$$\Rightarrow CO = 25 - 9 = 16 \text{ cm}$$

Now, Volume of the double cone = $\frac{1}{3}\pi r^2 h + \frac{1}{3}\pi r_1^2 h_1$

$$= \frac{1}{3} \times 3.14 \times (12)^2 \times 9 + \frac{1}{3} \times 3.14 \times (12)^2 \times 16$$

$$= \frac{1}{3} \times 3.14 \times 144 (9 + 16)$$

$$= 3.14 \times 48 \times 25 = 3768 \text{ cm}^3$$

Surface area of the double cone = $\pi r l + \pi r_1 l_1$

$$= 3.14 \times 12 \times 15 + 3.14 \times 12 \times 20$$

$$= 3.14 \times 12 (15 + 20)$$

$$= 3.14 \times 12 \times 35 = 1318.8 \text{ cm}^2$$

26. $h = 16 \text{ cm}$, $r_1 = 20 \text{ cm}$, $r_2 = 8 \text{ cm}$

$$l = \sqrt{h^2 + (r_1 - r_2)^2} = \sqrt{(16)^2 + (20 - 8)^2} = \sqrt{256 + 144} = 20 \text{ cm}$$

Total surface area of bucket = $\pi(r_1 + r_2)l + \pi r_2^2$

$$= 3.14(20 + 8)20 + 3.14(8)^2$$

$$= 3.14(28 \times 20 + 64)$$

$$= 3.14(560 + 64)$$

$$= 3.14 \times 624$$

$$= 1959.36 \text{ cm}^2$$

$$\begin{aligned}\therefore \text{Cost of bucket} &= \frac{15}{100} \times 1959.36 \\ &= \text{Rs. } 293.90 \text{ (approx.)}\end{aligned}$$

27. (i) Let speed of the stream be x km/h

Speed of motorboat = 9 km/h

Speed of the motorboat to cover 12 km in downstream = $(9 + x)$ km/h

Speed of the motorboat to cover 12 km in upstream = $(9 - x)$ km/h

$$\text{Time taken to cover 12 km in downstream} = \frac{12}{(9 + x)} \text{ h}$$

$$\text{Time taken to cover 12 km in upstream} = \frac{12}{(9 - x)} \text{ h}$$

According to the question,

$$\begin{aligned}\frac{12}{9 + x} + \frac{12}{9 - x} &= 3 \\ \Rightarrow 12 \left[\frac{9 - x + 9 + x}{81 - x^2} \right] &= 3 & \Rightarrow 12 \left[\frac{18}{81 - x^2} \right] &= 3 \\ \Rightarrow 81 - x^2 &= \frac{12 \times 18}{3} & \Rightarrow -x^2 &= 72 - 81 \\ \Rightarrow x^2 &= 9 & \Rightarrow x &= \pm 3\end{aligned}$$

Since the speed of stream cannot be negative.

$\therefore$ the speed of stream is 3 km/h.

(ii) Logically if speed of stream is more than the speed of boat in still water, then the boat will not sail.

28. Let the speed of the train be x km/h.

$$\text{Time taken by the train with this speed for a journey of 360 km} = \frac{360}{x} \text{ h}$$

Increased speed of the train = $(x + 5)$ km/h

$$\text{Time taken by train with increased speed for a journey of 360 km} = \frac{360}{x + 5} \text{ h}$$

According to the question,

$$\begin{aligned}\frac{360}{x} - \frac{360}{x + 5} &= 1 \\ \Rightarrow 360 \left(\frac{x + 5 - x}{x^2 + 5x} \right) &= 1 & \Rightarrow x^2 + 5x &= 1800 \\ \Rightarrow x^2 + 5x - 1800 &= 0 & \Rightarrow x^2 + 45x - 40x - 1800 &= 0 \\ \Rightarrow x(x + 45) - 40(x + 45) &= 0 & \Rightarrow (x + 45)(x - 40) &= 0 & \Rightarrow x = -45, 40\end{aligned}$$

$x = -45$ is inadmissible as x is the speed which cannot be negative.

Hence the speed of the train is 40 km/h.

29. Given a^2, b^2, c^2 are in AP

Then $2b^2 = a^2 + c^2$(i)

If $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in AP then

$$\frac{1}{c+a} - \frac{1}{b+c} = \frac{1}{a+b} - \frac{1}{c+a}$$

$$\frac{(b+c)-(c+a)}{(c+a)(b+c)} = \frac{(c+a)-(a+b)}{(a+b)(c+a)}$$

$$\frac{b-a}{b+c} = \frac{c-b}{a+b}$$

$$2b^2 = c^2 + a^2$$
.....(ii)

From (i) and (ii),

Hence proved.

30. $\therefore \angle XZY = 90^\circ$

$$\therefore (9+r)^2 + (2+r)^2 = (17)^2 \quad [\text{By Pythagoras theorem}]$$

$$\Rightarrow r^2 + 11r - 102 = 0$$

$$\Rightarrow r^2 + 17r - 6r - 102 = 0$$

$$\Rightarrow r(r+17) - 6(r+17) = 0$$

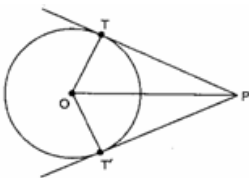
$$\Rightarrow (r+17)(r-6) = 0$$

$$\Rightarrow r = -17, 6$$

Radius cannot be negative.

Hence $r = 6$ cm

31. **First part:** Given : A circle with centre O and a point P outside the circle. PT and PT' are tangents from P to the circle.



To Prove: We need to prove that $PT = PT'$

Construction: Joined OP, OT and OT'

Proof $\because$ OT is a radius and PT is a tangent.

$$\therefore \angle OTP = 90^\circ$$

Similarly, $\angle OT'P = 90^\circ$

Now in right triangles OTP and OT'P,

$$OT = OT' \quad \text{..... (Radii of the same circle)}$$

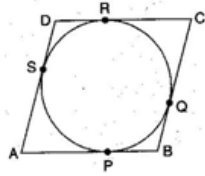
$$\text{and } OP = OP \quad \text{.....(Common)}$$

$$\therefore \triangle OTP \cong \triangle OT'P \quad \text{.....(RHS congruency)}$$

Hence, $PT = PT'$

Second part: Using the above, we get,

$$AP = AS, BP = BQ, CR = CQ, DR = DS$$



On adding, we get,

$$(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)$$

$$\Rightarrow AB + CD + AD + BC$$

$$\Rightarrow 2AB = 2BC \quad [\because ABCD \text{ is a parallelogram}]$$

$$\Rightarrow AB = BC$$

$$\Rightarrow AB = BC = CD = DA$$

$$[\because ABCD \text{ is a parallelogram}]$$

$$\Rightarrow ABCD \text{ is a rhombus.}$$
