Circles

1) If arcs AXB and CYD of a circle are congruent, find the ratio of AB and CD.
2) If the perpendicular bisector of a chord AB of a circle PXAQBY intersects the circle at P and Q, prove that arc PXA ≅ Arc PYB.
3) A, B and C are three points on a circle. Prove that the perpendicular bisectors of AB, BC and CA are concurrent.
4) AB and AC are two equal chords of a circle. Prove that the bisector of the angle BAC passes through the centre of the circle.
5) If a line segment joining mid-points of two chords of a circle passes through the centre of the circle, prove that the two chords are parallel.
6) ABCD is such a quadrilateral that A is the centre of the circle passing through B, C and D. Prove that

$$\angle CBD + \angle CDB = \frac{1}{2}\angle BAD$$

7) O is the circumcentre of the triangle ABC and D is the mid-point of the base BC. Prove that $\angle BOD = \angle A$.
8) On a common hypotenuse AB, two right triangles ACB and ADB are situated on opposite sides. Prove that $\angle BAC = \angle BDC$.
9) Two chords AB and AC of a circle subtends angles equal to 90° and 150°, respectively at the centre. Find $\angle BAC$, if AB and AC lie on the opposite sides of the centre.
10) If BM and CN are the perpendiculars drawn on the sides AC and AB of the triangle ABC, prove that the points B, C, M and N are concyclic.
11) If a line is drawn parallel to the base of an isosceles triangle to intersect its equal sides, prove that the quadrilateral so formed is cyclic.
12) If a pair of opposite sides of a cyclic quadrilateral are equal, prove that its diagonals are also equal.
13) The circumcentre of the triangle ABC is O. Prove that $\angle OBC + \angle BAC = 90°$.
14) A chord of a circle is equal to its radius. Find the angle subtended by this chord at a point in major segment.
15) In Fig.10.13, $\angle ADC = 130°$ and chord BC = chord BE. Find $\angle CBE$.

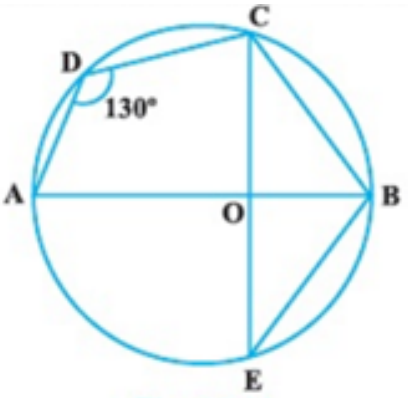


Fig. 10.13

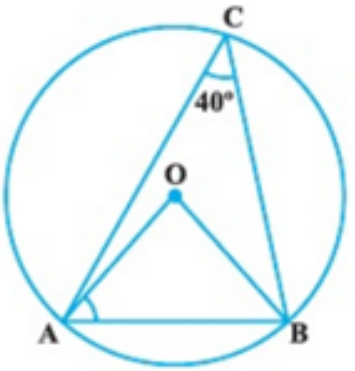


Fig. 10.14

16) In Fig.10.14, $\angle ACB = 40°$. Find $\angle OAB$.

17) A quadrilateral ABCD is inscribed in a circle such that AB is a diameter and $\angle ADC = 130°$. Find $\angle BAC$.

18) Two circles with centres O and O′ intersect at two points A and B. A line PQ is drawn parallel to OO′ through A(or B) intersecting the circles at P and Q. Prove that $PQ = 2\ OO'$.

19) In Fig.10.15, AOB is a diameter of the circle and C, D, E are any three points on the semi-circle. Find the value of $\angle ACD + \angle BED$.

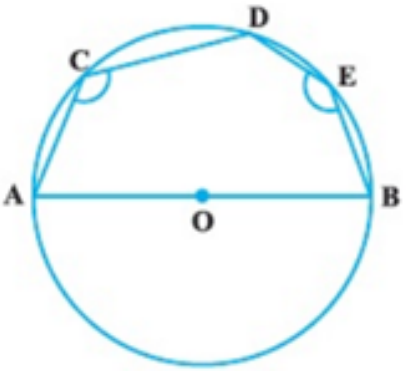
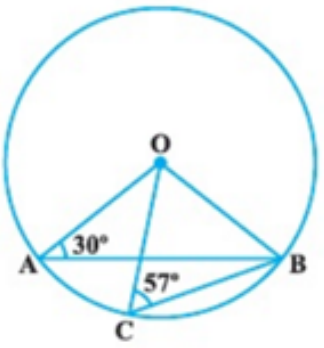


Fig. 10.15

20) In Fig. 10.16, $\angle OAB = 30°$ and $\angle OCB = 57°$. Find $\angle BOC$ and $\angle AOC$.



21) If two equal chords of a circle intersect, prove that the parts of one chord are separately equal to the parts of the other chord.

22) If non-parallel sides of a trapezium are equal, prove that it is cyclic.

23) If P, Q and R are the mid-points of the sides BC, CA and AB of a triangle and AD is the perpendicular from A on BC, prove that P, Q, R and D are concyclic.

24) ABCD is a parallelogram. A circle through A, B is so drawn that it intersects AD at P and BC at Q. Prove that P, Q, C and D are concyclic.

25) Prove that angle bisector of any angle of a triangle and perpendicular bisector of the opposite side if intersect, they will intersect on the circumcircle of the triangle.

26) If two chords AB and CD of a circle AYDZBWCX intersect at right angles (see Fig.10.18), prove that arc CXA + arc DZB = arc AYD + arc BWC = semi-circle.

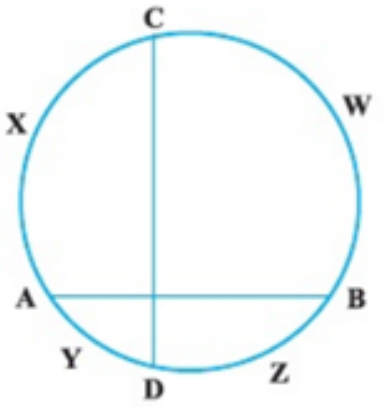


Fig. 10.18

27) If ABC is an equilateral triangle inscribed in a circle and P be any point on the minor arc BC which does not coincide with B or C, prove that PA is angle bisector of $\angle BPC$.

28) In Fig. 10.19, AB and CD are two chords of a circle intersecting each other at point E. Prove that $\angle AEC = \frac{1}{2}$ (Angle subtended by arc CXA at centre + angle subtended by arc DYB at the centre).

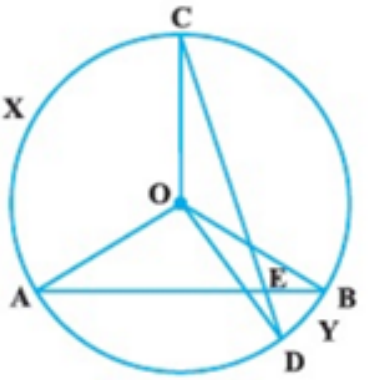


Fig. 10.19

29) If bisectors of opposite angles of a cyclic quadrilateral ABCD intersect the circle, circumscribing it at the points P and Q, prove that PQ is a diameter of the circle.

30) A circle has radius $\sqrt{2}$ cm. It is divided into two segments by a chord of length 2 cm. Prove that the angle subtended by the chord at a point in major segment is 45°.

31) Two equal chords AB and CD of a circle when produced intersect at a point P. Prove that PB = PD.

32) AB and AC are two chords of a circle of radius r such that AB = 2AC. If p and q are the distances of AB and AC from the centre, prove that $4q^2 = p^2 + 3r^2$.

33) In Fig. 10.20, O is the centre of the circle, $\angle BCO = 30°$. Find x and y.

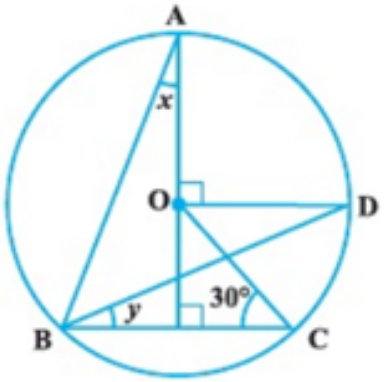


Fig. 10.20

34) In Fig. 10.21, O is the centre of the circle, BD = OD and CD ⊥ AB. Find ∠CAB.

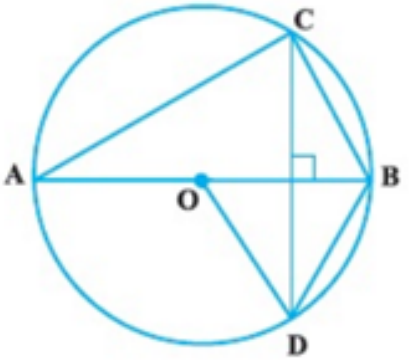


Fig. 10.21