1

Vedic Mathematics

1.01 Introduction :

In the previous class we have studied that Swami Bharati Krishna ji Tirth meditated hard for eight years while residing at Sringeri.During his meditation in a highly enlightened state, he realized through introspection the Sutras or Aphorisms of Vedic Mathematics. According toVedic scholars Vedic classics is neither man made nor the knowledge of Vedas it is the result of man's mental thinking or mental labour. It is the knowledge in the form of mantras received after his realization of the Divine power. In this context the Sutras reconstructed by Swami ji are Vedic Mathematical Sutras.

1.02 Importance of Vedic Mathematics :

Regular mental practice of Vedic Mathematcal sutras imporves both the concentration and memory of the human being and enhances analytical and synthetical acumen of thought. The easy, interesting and attractive approach of Vedic Mathematics arouses an attitude of curiosity in the individual. This curiosity makes the man sentient.

The consciousness of the man is awakened which develops the neocortex and personality of a man.

1.03 Revision and Extension of Basic Operations :

(i) Addition :

In the previous class we studied the addition operation of whole numbers and measure unit distance (Km.-m.) by the Sutra Ekadhikena Purvena. In fact every question of addition of measurments like – money, weight, capacity, distance, time and whole numbers, decimal numberts etc. can be performed by this Sutra.

Precautions before Addition :

1. Keep the number of coloumns of sub unit according to the rule i.e. to write 05 paisa for 5 paisa in the sub unit of money, or 084 metres for 84 metres in the sub unit of distance.
2. After writing the figures columnwise add accordingly by the Sutra.
3. In the time unit, while adding keep the base (sum) = 10 in first column of second and minute. Keep base (sum) = 6 in the second column of minute and second. In hour's column base (sum) = 10 is always chosen.

   Let us see the following example.

Example 1 : Adding the following :

kg	gm
112	065 ↓
360	$\dot{0}\dot{8}5$
$\dot{2}\dot{8}\dot{9}$	$\dot{8}72$
156	345
918	367

Steps :

(i) Write 065 gms. for 65 gms and 085 gms for 85 gms.

(ii) Start adding from unit column.

(iii) $5+5=10$,
hence Ekadhika dot on digit 8 prior to 5,
Remainder $=10-10=0$

(iv) Further $0+2+5=7$, write at answer's place.

(v) Proceed accordingly.

(ii) Addition by Mental Process :

(Sutra Ekadhikena Purvena + Shunyant Number Method)

By a little pratice of above process addition of big numbers can be preformed very fast. Concept of a Shunyat number process is a speciality of Ancient Indian Mathematics which is very easy and effective in addition operation. In this method two digits at unit and ten's place of the numbers can also be added, if needed.

Method :- Out of two numbers make one a shunyat number. Complement its deficiency by the second number. Now add both the new numbers. The sum if so obtained is greater than 100, mark an Ekadhika dot on the previous number. Add the remainder to the next number. In the end write the last remainder on the ansewer's place. For the remaining columns repeat the process as carried before. The method will be clear from the follwing examples.

Example 2. Add 35 and 58.

Solution : To make 58 a shuyant number 60, 2 is needed. This deficiency of two is covered form the number 35. Hence

$$35+58=33+2+58=33+60=93$$

Example 3. Add 19 and 65.

Solution : $19+65=19+1+64=20+64=84$

Note: Any numbers can be added by this method.

Example 4. Add the following :

$$\begin{array}{r} 4\,9\,9\,8 \\ \dot{0}\,6\,\dot{7}\,8\,9 \\ 5\,\dot{7}\,1\,5 \\ \dot{0}\,4\,8\,3\,7 \\ \dot{0}\,8\,\dot{9}\,7\,6 \\ \hline 3\,1\,3\,1\,5 \\ \hline \end{array}$$

Steps :

(i) $98+89=98+2+87=100+87=187$
Hence mark an Ekadhika dot on 7 before 89.

(ii) Rest $87+15=87+3+12=90+12=102$
Hence mark an Ekadhika dot on 7 before 15.

(iii) Rest $02+37=39$. Now

$$39+76=35+4+76=35+80$$
$$=15+20+80=115$$

(iv) Hence mark an Ekadhika dot on 9 before 76 write 15 at answer's place.

(v) The remaining addition process will be completed accordingly.

Example 5. Add the following :

$$\begin{array}{r} 7\,5\,3\,4 \\ 2\,4\,5\,9 \\ \dot{0}\,1\,\dot{9}\,3\,2 \\ 6\,5\,4\,7 \\ \hline 1\,8\,4\,7\,2 \\ \hline \end{array}$$

Steps :

(i) $34+59=33+1+59=33+60=93$

(ii) $93+32=93+7+25=100+25=125$
hence Ekadhika dot on 9.

(iii) Remaining $25+47=22+3+47=22+50=72$ write at answer's place.

(iv) The remaining addition process can be completed as above.

(iii) Subtraction

In the previous class we studied two Vedic methods of substraction as given below :

1. Sutra Ekadhikena Purvena + Param mitra Aunka method
2. Sutra Ekanyunea Purvena + Param mitra Aunka method

By the first method every question of substraction of an measure unit or whole numbers can be performed. Let us revised this same method again. We know that the two numbers are called best friends fo each other if their sum is equal to ten. Vyojya and Viyojak are also known to us. This method will be clear by the following illustrations.

Examples 6. By Vedic method subtract the follwing :

$$\begin{array}{r} 8\,0\,0 \\ -\,\dot{2}\,\dot{6}\,3 \\ \hline 5\,3\,7 \\ \hline \end{array}$$

Steps :

(i) 3 can not be subtracted form 0, hence add param mitra aunka i.e. best friend of 3 to 0 $\therefore$ 0 + 7 = 7. Write it at answer's place. Also mark an Ekadhika dot on 6 prior to 3.

(ii) $\dot{6} = 7$ can not be subtracted form 0 hence add param mitra aunka of 7 to 0 i.e. 0 + 3 = 3. Write it at answer's place. Also mark an Ekadhika dot on 2 prior to 6.

(iii) $8 - \dot{2} = 5$, write it at answer's place. Hence Remainder = 537

Example 7. Subtract the following by Vedic method.

km.	m.	cm.
3 7	4 6 7	3 5
$\dot{2}$ 8	$\dot{3}$ 7 $\dot{5}$	$\dot{4}$ 6
0 9	0 9 1	8 9

Steps :

(i) Columns of cm and metre are arranged accordingly.

(ii) In cm column 6 is subtracted form 5. Hence add param mitra aunka of 6 to 5.

(iii) Sum = 4 + 5 = 9, write it at anwer's place and make an Ekadhika dot on 4 prior 6.

(iv) $\dot{4}$ = 5 can not be subtracted from 3 hence add best friend of 5 to 3i.e. 5 + 3 = 8.

(v) Write 8 at answer's place. Also mark an Ekadhika dot on 5of metre's unit place.

(vi) 7 - $\dot{5}$ = 1, write it at answer's place.

(vii) 7 can not be subtractedfor 6, hence add best friend of 7 to 6, i.e, 3 + 6 = 9 write 9 at the answer's place and mark an Ekahdika dot on 3.

(viii) $4 - \dot{3} = 0$, write it at answer's place.

(ix) Proceed further in the same way.

Finally Remainder = 9 km. 91 m. 89 cm.

(iv) Multiplication :

In the previous class we studied in details the multiplication methods based on three important sutras. We should practice these methods extensively so that just by throwing a glance on any question, we may able to choose the best method for a quick solution.

Sutra Urdhva Triyagbhyam

Any question of multiplication can be done orally by Sutra Urdha Triyagbhyam. Only a line space is required for writing the answer. Sutra can be applied from both sides i.e. left or right.

(a) Meaning :

Sutra is composed of two words 'Urdhva' and 'Triyank' Its meaning is 'Vertically and crosswise'. Urdha means vertically or straight. Its symbol is ↑ or ↓ or ↕ and its function is to multiply the digits written vertically. The word Triyak means crosswise. Its symbol is ↗ or ↖ or ⤧ and its function is to multiply the digits written at cross.

(b) Applications:

(i) Multiplication :

Method : Firstly the digits of numbers in question are arranged in columns. The deficiency of the digits is fulfilled by introducing zeroes at those places. Groups are formed by these columns.

No. of Groups = No. of columns × 2 – 1 = an odd number always

Now in groups symbols are marked. According to these symbols multiplication is carried out. Lastly the products are written in a special arrangement. Finally their sum gives the product of two numbers.

Note: (1) In any groups the total number of symbols is always equal to the number of columns of that group.

(2) All these symbols of a group pass through a common point.

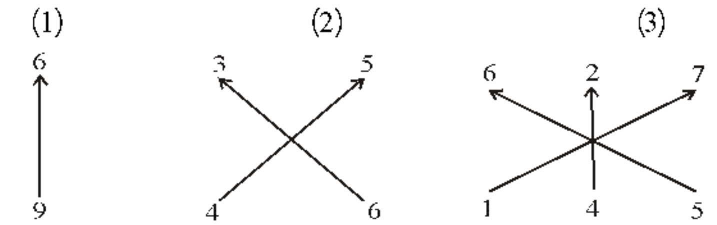
(3) A group formed on even place consists of cross symbols in pairs.

(4) A group formed at odd place consists of one vertical symbol only which is marked in the first or last group. The remaining symbols of this group are cross symbols in pairs.

(5) The middle group is always the largest one and is equal to the question itself. The method is explained by the following examples.

(6) Multiply by Sutra vertically and crosswise:

Example 8 : Calculate the products by marking symbols of the following groups.

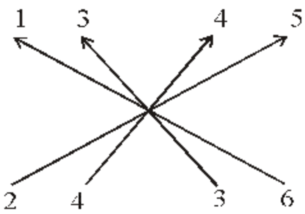


(1) $= 6 \times 9$ $= 54$ Vetically

(2) $= 6 \times 3 + 4 \times 5$ $= 38$ Cross in pair

(3) $= 5 \times 6 + 1 \times 7 + 4 \times 2$ $= 45$ Two cross pairs + Vertically

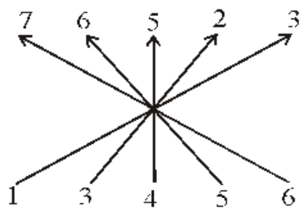
(4)



$= (6\times1+2\times5)$
$+ (4\times4+3\times3)$
$= 41$

Two cross pairs

(5)



$= (6\times7+1\times3)$
$+ (5\times6+3\times2)$
$+ 4\times5$
$= 101$

Two cross pairs + vertically

Find : 147 x 28

		1	4	7
	×	0	2	8
0	2	6	6	6
	1	4	5	
=	4	1	1	6

Five groups formed

V	IV	III	II	I
1	14	147	47	7
↑	✕	✕↑	✕	↑
0	02	028	28	8
= 0	= 2	= 16	= 46	= 56

Addition of all above five products are as follows :

(1) In 56 write 6 as unit place and 5 in second line of ten's place.

(2) From 46, write 6 in first line of ten's place and 4 write in second line of hundred's place.

(3) Similary 16, 2, 0 and 1 adjusted in I and II line as shown above. Sum all terms.

Example 9. Choose the best sutra based method in finding the product of 588×512 .

I Solution : In this multiplication let us examine first method based on the Sutra Ekadhikena Purvena.

, Sum of the digits of unit and ten's place $= 88+12=100$

Rest Nikhilam digit is equal to each other $= 5$, hence Sutra Ekadhikena Purvena is effective.

According to Sutra

$588\times512 = 5\times6/88\times12$ (Four digits in R.H.S.)

$= 301056$

II Solution : Sutra Nikhilam (Sub-base) method is applicable in this question

588×512

$= \begin{matrix} 588 & +88 \\ \times512 & +12 \end{matrix}$

$= 5(588+12)/88\times12$

$= 5\times600/_{10}56$

$= 3000/_{10}56 = 301056$

Steps :

(i) Base $= 100$

(ii) Sub Base $= 100\times5$

(iii) Upadhar aunka $= 5$

(iv) Deviations are +12 and +88.

(v) Two digits in R.H.S.

III Solution : Sutra Eka nyunena Purvena is not applicable in this question as out if these two numbers one must be composed of digit 9 only.

IV Solution : Product of 512 and 588 can be found out by Sutra Urdhva Tiriyak method. There are three columns in the question hence five groups are to be evaluated. Find groupwise products and write them in a special way. In the end add them to get the final product.

$$\begin{array}{r} 5\ 8\ 8 \\ \times 5\ 1\ 2 \\ \hline 2\ 5\ 5\ 8\ 4\ 6 \\ 4\ 5\ 2\ 1 \\ \hline = 3\ 0\ 1\ 0\ 5\ 6 \end{array}$$

Results

1. Final product = 301056.
2. In the firtst solution, the product is quickly recieved by Sutra Ekadhikena Purvena and this is the best sutra out of all.

Example 10. Choose the best sutra based method in finding the product of 842×858 .

I Solution : Sutra Ekadhikena Purvena is not effective in this case as in the R.H.S. the product 42×58 cannot be obtained easily.

II Solution : Sutra Nikhilam (Base) method is also not effective as deviations will be - 158 and - 142 w.r.t. base 1000. Nikhilam Sub base method is also not effective as the deviations w.r.t. sub base = 800 will be 42 and 58.

III Solution : Sutra Ekanyunena Purvena is not applicable in this case.

IV Solution : Sutra Urdhva Tiriyak is very effective in all types of multiplication. The calculations can be hard as the numbers are composed of big digits. Hence new alternative is to be searched out.

V Solution : In finding out the product of 842×858 start with the Sutra Ekadhikena Purvena and finish it up by Sutra Urdhva Triyak.

842×858

$= 8 \times 9 / 42 \times 58$

$= 72 / 2436$

$= 722436$

Steps :

(i) By sutra Urdhva Triyak get the product mentally

$$\begin{array}{r} 42 \\ \times 58 \\ \hline 2436 \\ \hline \end{array}$$

1.04 Squaring (Sutra Nikhilam – Base, Sub-Base)

We have studied the multiplication of two numbers by Sutra Nikhilam - Base, Sub-Base methods. When both the numbers are equal to each other, it becomes a square operation.The squaring of the numbers is explained by the following examples.

Base method : Formula : $(\text{Number})^2$ = Number + deviation / $(\text{deviation})^2$

1. $17^2 = 17 \times 17,$ Base $= 10$, deviation $= +7$

$=17+7$
$17+7$

$17+7/49$

$=24/_{4}9=289$

or

$17^2=17+7/7^2$
$=24/_{4}9$

$=289$

2. $98^2=98\times98,$ Base $=100,$ deviation $=-02$

$=98-02/(-02)^2$
$=9604$

3. $104^2 \;=104+04/(04)^2$

$=10816$

4. $115^2=115+15/15^2$
$=130/_{2}25$
$=13225$

Sub-Base Method :

Formula :– $(\text{Number})^2=\text{Sub base digit }(\text{number}+\text{deviation})/(\text{deviation})^2$

5. $23^2=23\times23,$ Base $=10,$ Sub-Base $=10\times2,$ Deviation $=+3$

Nikhilam sub base method or Squaring in one line-

number deviation $\qquad 23^2=2(23+3)/3^2=529$

$=23 \quad +3$

$23 \quad +3$

$=2(23+3)/3^2$

$=529$

6. $64^2,$ Base $=10$, Sub-base $=10\times6,$ Deviation $=+04$

$=6(64+4)/4^2$
$=408/_{1}6$
$=4096$

7. $308^2,$ Base $=100$, Sub-base $=100\times3,$ Deviation $=+08$

$=3(308+08)/(08)^2$
$=94864$

1.05 Cube (Sutra Nikhilam - Base, Sub-Base)

We have studied the methods of multiplication of three numbers by Sutra Nikhilam Base, Sub-Base When all the three numbers are equal then cube can be obtained by the also method.

Base method :

Formula : $(\text{Number})^3$ = Number + 2 x Deviation / 3 x $(\text{Deviation})^2$ / $(\text{Deviation})^3$

1. 12^3, Base $=10$, Deviation $=+2$

$$=12+2\times(2)\,/\,3\times(2)^2\,/\,(2)^3$$

$$=16\,/\,12\,/\,8=1728$$

2. 105^3, Base $=100$, Deviation $=+05$

$$=105+2\times(05)\,/\,3\times(05)^2\,/\,(05)^3$$

$$=115\,/\,75\,/_{1}\,25=1157625$$

3. 98^3, Base $=100$, Deviation $=-02$ (Here deviation is negative)

$$=98+2\times(-02)\,/\,3\times(-02)^2\,/\,(-02)^3$$

$$=94\,/\,12\,/\,-08$$ (taking 1 from 12 in third section)

$$=94\,11\,92$$ (in third section value of 1 is 100, ∵ base = 100)

Sub-Base Method Formula of cubing a number

Formula : $(\text{Number})^3$ = $(\text{Sub base digit})^2$ (number + 2 x deviation)
sub-base x 3 x $(\text{deviation})^2$ / $(\text{deviaton})^3$

4. 35^3 , Base $=10$, Upadhar Unka $=3$, deviaton $=5$

$$=3^2(35+2\times5)\,/\,3\times3\times(5)^2\,/\,5^3$$

$$=9\times45\,/\,9\times25\,/\,125$$

$$=405\,/_{22}\,5\,/_{12}\,5=42875$$

5. 497^3

$$=5^2\{497+2\times(-03)\}\,/\,5\times3\times(-03)^2\,/\,(-03)^3$$

$$=25\times491\,/\,5\times27\,/\,-27$$

$$=12275 \Big/_{1} 35 \Big/ (-27)$$

$$=12276 \Big/ 34 \Big/ 100-27$$

$$=122763473$$

1.06 Division Operation :

In this class we will studied the division operation based on the following three sutras :

1. Sutra Nikhilam
2. Sutra Paravartya Yojeyeta
3. Sutra Urdhva Tiriyak

Sutra Nikhilam based division method is effective only when the digits of the divisor are greater than 5 and with respect to base 10 or any power of 10 the complement of the divisor can be found out. In this method the main operation is done by the complement of the divisor and digit 1 on its extreme left.

If the divisor contains digits lower than five or it can be reduced into a number of digits lower than five with digit 1 on its left side and deviations can be calculated w.r.t. base 10 or power of 10, the division based on Sutra paravartya Yojeyeta can be performed. This is the only method which can be applied in Algebra.

Dhwajanka Method of Division : (Sutra Urdhva Tiriyak) Every question of division can be performed by this method. Selection of the Mukhyanka and Dhwajanka of the divisor is important. There can be any number of digits in Dhwajanka as well as in Mukhyanka but division by Mukhyanka is a necessary condition. So many digits form unity are written in third part as there are digits in the Dhwajanka. The method is explained here with by the following examples.

Division (Sutra Nikhilam) :

Sutra Nikhilam based method is always suitable in case the each digit of divisor is greater than 5.

Method to write the question

Divide the required place in three parts by drawing two vertical lines. From the left in the first place write the divisor and its complementary number beneath it. Write as many digits of dividend from unit place in the third part as there are zero's in the base. Write the remainder digits of dividend in the middle part.

Nikhilam method:

From left hand side write the first digit of dividend down at the sum's place below the horizontal line. Multiply this number by the complementary number and write the product in the middle part below the second digit of the dividend. If there are two digits in the complementary number, write the product below the third digit of the dividend also. Add only second place digits upper and lower and write it down at the sum's place don't add the digits of third place. Multiply the second digit of sum's by the complementary number and write the product in the middle part below the third digit of the divident and add. Repeat this process until the product may cover the last digit i.e. unit digit of the dividend in the third part. Add, again lastly the sum so achieved in the middle part is equal to the quotient and sum achieved in the third part is equal to the remainder.

If the remaidner is greater than the divisor than subtract the divisior from it and get the final quoteint and final remainder.

The method is explained by the following examples.

Example 11 : (i) $311 \div 8$, Base $= 10$

First	Second		Third
8	3	1	1
2		6	–
			14
	3	7	15
		+1	–8
	3	8	7

Steps

(i) Quotient = 37, Remainder = 15

(ii) Remainder 15 > divisor 8

(iii) Adjustment is necessary

(iv) New Quotient = 38
Remainder = 7

(ii) $10025 \div 88$, Base $= 100$

8	8	1	0	0	2	5
1	2		1	2	–	–
				1	2	–
					3	6
		1	1	3	8	1

Steps

(i) Complementary number = 1000 – 88 = 12

(ii) In the middle part write 1 beneath 1 as shown. Now 1 x 12 = 12 is written in middle part beneath the next digits as shown.

(iii) 0 + 1 = 1 is written in middle part as shown.

(iv) 1 x 12 = 12 is written in middle and third part beneath the digits as shown.

(v) 0 + 2 + 1 = 3 is written in middle part.

(vi) 3 x 12 = 36 is written in the third part digits as shown on adding.

(vii) Quotient = 113

(vii) Remainder = 81

Sutra Paravartya Yojayet

Sutra is widely employed in several fields like solution of equations, formation of magic squares and also in division specially in Algebra.

(a) Meaning:

Meaning of Sutra Paravartya yojayet is "Transpose and adjust", e.g. in case of transposition the sign are changed into opposite one i.e. $(+)$ into $(-)$, $(-)$ into $(+)$, $(\times)$ into $(\div)$, and $(\div)$ into $(\times)$. Similalry in case of magic squares after every last line or last column, there starts the formation of a new number in a coming line or column.

(b) Application :

Division : Divisor based on Sutra Paravartya Yojayet is convenient only when the divisor is very close to the base = 10 or power of 10 and its first digit from left is one. When the first digit is not one but it can be adjusted into one even then this method is applicable.

Method:

(1) Subtract the nearest base from divisor and get the deviation. If the digit of deviation is greater than 5, convert it into small digit by Vinculum operation. Now transpose the sign of every digit of deviation.

(2) (i) Divide the place of division in three parts.

(ii) In the first part from the L.H.S. write the divisor, below it the deviation, and finally the transposed digits below the deviation. With a slight practice the transposed digits can be directly written below the divisor.

(iii) According to the number of deviation digits or the number of zeroes in the base the third part is fulfilled by the dividend digits. Lastly the second part is fulfilled by the remaining dividend digits.

(iv) The further steps are like that of Sutra Nikhilam.

The method is explained by the following examples:

Example 12 : $1358 \div 113$, Base $= 100$

1 1 3	1 3	5 8
1 3	–1	–3 –
–1 –3		–2 –6
	1 2	0 2

Steps

(i) Write dividend digit 1 of the second part at the answer's place.

(ii) Multiply transposed digits –1–3 by this dividend digit 1.
Write the product –1–3 below 3 and 5.

(iii) 3 – 1 = 2, Again write the product –2 – 6 below 5 and 8

∵ $2[-1-3] = -2-6$

By adding Quotient = 12, Remainder = 02

(2) $395166 \div 1321$, Base $= 1000$

1 3 2 1	3 9 5	1 6 6
$\overline{3}$ $\overline{2}$ $\overline{1}$	$\overline{9}$ $\overline{6}$	$\overline{3}$ – –
	0	0 0 –
		3 2 1
	3 0 $\overline{1}$	1 8 7

Quotient $= 30\overline{1} = 299$

Remainder $= 187$

(Dhvajanka Sutra)

Any big question of division can be solved with minimum calculations by this universal method based on Dhvajanka Sutra. In this method divisor is divided into two parts (i) main divisor i.e. Mukhyanka (ii) Top flag digit i.e. Dhvajanka. The unit digit of divisor or unit digit with more digits with more digits of divisor are written on the top of the flag and known as Dhvajanka. The remainder part of the divisor is known as main divisor (Mukhyanka) and is written on the base place. It conducts the whole divisior process.

Method :

(1) Divide the place of division in three parts.

(2) In the first part from L.H.S., write Mukhyanka at base and Dhvajanka at the power place.

(3) Write so many digits of dividend as that of Dhvajanka in the third part starting from the unit place.

(4) Write now the remaining digits of dividend in the middle or second part.

More details of the method are given in the following illustrations.

Example 13 :

(1) $23754 \div 74$ (Dhvajanka method)

4	2	3	7	5	4
7		2	1		0
		3	2	1	$4-1\times4=0$

Steps

(i) $23 \div 7$, Quotient digit$_1 = 3$, write it below the horizontal line.
Write Remainder $= 2$ below and before dividend digit 7.

(ii) New Remainder $= 27$, Adjust it by the formula,
Dividend adjusted = New Dividend − Quotient digit × Dhvajanka
$= 27 - 3 \times 4 = 15$

(iii) $15 \div 7$, Quotient digit $_2 = 2$, Remainder $= 1$
Write it according to step no. (i)

(iv) New Dividend $= 15$, Dividend adjusted $= 15 - 2 \times 4 = 7$

(v) $7 \div 7$, Quotient digit $_3 = 1$, Remainder $= 0$

(vi) Last Remainder $= 04 - 1 \times 4 = 0$

$\therefore$ Quotient $= 321$, Remainder $= 0$

(2) $21112 \div 812$ (Dhvajanka method)

1 2	2	1	1	1 2
8		5		1
		2	6	112-100-12=0

Steps

(i) $21 \div 8$, Quotient digit $_1 = 2$, Remainder $= 5$

(ii) New Dividend $= 51$, Dividend adjusted $= 51 - 2 \times 1 = 49$

(iii) $49 \div 8$, Quotient digit $_2 = 6$, Remainder $= 1$

(iv) Dividend adjusted or Last Remainder

$$= 112 - (6 \times 1 + 2 \times 2)10 - 6 \times 2$$
$$= 112 - 100 - 12 = 0$$

Final Quotient $= 26$, Remainder $= 0$

Note: (1) Dhvajanka 1 2

Quotient 2 6

Three groups are formed by Dhvajanka and Quotient.

Groups

1	1 2	2
↑	⤫	↑
2	2 6	6
1×2	$(6 \times 1 + 2 \times 2)$	6×2
$= 2$	$= 10$	$= 12$

(2) (i) Product of I group = 2 subtracted from 51

(ii) Product of II group $= 10 \times 10 = 100$ subtracted from 112

(iii) Product of III group = 12 also subtracted from 112

(3) In the end as soon as the remainder enter into third part, division process is over.

Example 14. **$98765 \div 87$ (Dhwajanka method)**

7	9	8	7	6	5
8		1	3	6	5
	1	1	3	5	$55 - 5 \times 7 = 20$

Steps :

(i) Divisor $= 87$, Mukhyanka $= 8$, Dhwajanka $= 7$

(ii) In III part only one digit of dividend is kept i.e. $= 5$

(iii) $9 \div 8$, Quotient first digit $= 1$, Remainder $= 1$

(iv) New Dividend $= 18$, Modified dividend $= 18 - 1 \times 7 = 11$

(v) $11 \div 8$, Quotient II digit $= 1$, Remainder $= 3$

(vi) New Dividend $= 37$, Modified dividend $= 37 - 1 \times 7 = 30$

(vii) $30 \div 8$, Quotient III digit $= 3$, Remainder $= 6$

(viii) New Dividend 66, Modified dividend $= 66 - 3 \times 7 = 45$

(ix) $45 \div 8$, Quotient IV digit $= 5$, Remainder $= 5$

(x) New Dividend 55,

Modified dividend or last Remainder $= 55 - 5 \times 7 = 20$

$\therefore$ Quotient $= 1135$, Remainder $= 20$

Example 15. $13579 \div 975$ (Dhwajanka method)

75	13 5	79
9	4	11
	1 3	$1179-260-15=904$

Steps:

(i) Divisor = 975, Mukhyanka = 9 and Dhwajanka = 75, hence two digits 79 are written in III part.

(ii) $13 \div 9$, Quotient I digit = 1, Remainder = 4

(iii) New dividend = 45, Modified dividend $= 45 - 1 \times 7 = 38$

(iv) $38 \div 9$, Quotient II digit $= 4$, Remainder $= 2$

(v) New dividend = 27,

Modified dividend $= 27 - (4 \times 7 + 1 \times 5) = 27 - 33 = -6$

As the modified dividend is negative, hence quotient II digit 4 is rejected and 3 will be more suitable ($\neq 4$).

* Therefore steps nos. (iv) and (v) are liable to be rejected.

(vi) Again $38 \div 9$, Quotient II digit = 3, Remainder = 11

(vii) New dividend $= 1179$, Modified dividend or last Remainder

$$= 1179 - (3 \times 7 + 1 \times 5) \times 10 - 3 \times 5 = 1179 - 260 - 15 = 904$$

Quotient = 13, Remainder = 904

Exmple 16. $21015 \div 879$ (Dhwajanka method)

Solution : The big digits of divisor 879 are changed into smaller digit by Vincular method. $879 = \dot{8}\bar{2}\bar{1} = 9\bar{2}\bar{1}$

Hence Mukhyanka = 9 and Dhwajanka = $\bar{2}\bar{1}$

$\bar{2}\bar{1}$	21 0	15
9	3	7
	2 3	$715+80+3=798$

Steps :

(i) $21 \div 9$, Quotient first digit = 2, Remainder = 3

(ii) New dividend = 30, Modified dividend $= 30 - 2 \times \bar{2} = 34$

(iii) $34 \div 9$, Quotient II digit = 3, Remainder = 7

(iv) New divided = 715, Modified dividend or

last Remainder $= 715 - (3 \times \bar{2} + 2 \times \bar{1})10 - 3 \times \bar{1}$

$= 715 + 80 + 3 = 798$

$\therefore$ Quotient = 23, Remainder = 798

Example 17. $7453 \div 79$

$\bar{1}$	74 5	3
8	2	2
	9 4	$23+4=27$

Steps :

(i) Divisor $79 = 8\bar{1}$, Mukhyanka = 8, Dhwajanka = $\bar{1}$

(ii) $74 \div 8$, Quotient first digit = 9, Remainder = 2

(iii) New dividend = 25, Modified dividend $= 25 + 9 = 34$

(iv) $34 \div 8$, Quotient II digit = 4, Remainder = 2

(v) New dividend or last Remainder $= 23 + 4 = 27$

$\therefore$ Quotient = 94, Remainder = 27

Attention :

1. When the unit digit of the divisor = 9 is converted into Dhwajanka $\bar{1}$, see that
   Modified dividend = New dividend + Previous quotient digit.
2. When unit digit of the divisor is = 1 i.e. Dhwajanka is also equal = 1, then
   Modified diviednd = New dividend - Previoud Quotient digit.
3. In both these cases writing of steps can be avoided.

Example 18. $43758972 \div 81$

Mukhyanka = 8, Dhwajanka = 1

1	4	3	7	5	8	9	7	2
8			3	0	1	2	3	2
		5	4	0	2	3	4	$22 - 4 = 18$

Quotient = 540234 Remainder = 18

1.07 Algebra

Solution of Simple Equations (Vedic Method)

Simple equation can be solved quickly by the sutras Paravartya Yojayet and Sunyam Samya samuccaye. The application of these sutras are very simple and based on mental calculation.

Sutra Paravartya Yojayet

The sutra means "transformation and adjustment".Swami Bhartikrishna Ji Teerth has discussed appliation of the sutra. All these sutras give answers by mere metnal arithmetic calculation in one line.

First Application

If $ax + b = cx + d$ then $x = \dfrac{d-b}{a-c}$ (algebraic formula)

Second Application

If $\dfrac{ax+b}{p} = \dfrac{cx+d}{q}$ then $x = \dfrac{dp-bq}{aq-cp}$ (algebraic formula)

Third Application

If $(x+a)(x+b) = (x+c)(x+d)$ then

$$x = \frac{cd-ab}{a+b-c-d} \text{ (algebraic formula)}$$

Example 19 : Solve the simple equation :

$$(x+1)(x+2)=(x-3)(x-4)$$

Solution : By algebraic formula $x=\frac{12-2}{1+2+3+4}=\frac{10}{10}=1$

Fourth Application

If $\frac{m}{x+a}+\frac{n}{x+b}=0$ then $x=-\frac{mb+na}{m+n}$ (algebraic formula)

Example 20 : Solve the eqution $\frac{4}{x+2}+\frac{3}{x+5}=0$

Solution : By algebric formula $x=-\frac{(20+6)}{4+3}=-\frac{26}{7}$

Sutra Sunyam Samyasamuccaye

The meaning of sutra is "when the samuccaya is the same, that samuccaya is zero", i.e., it should be equated to 'zero'. Samuccaya is a technical term which has several meaning under different context and we shall explain them one by one.

First meaing and Application

In x is a common factor in all the terms concerned, then $x=0$ (algebraic formula).

Example 21. Solve the equation $12x+3x=4x+5x$.

Solution : By algebric formula $x=0$.

Example 22. Solve : $2(x+1)=7(x+1)$

Solution : $x+1$ is a common factor, hence $x+1=0$

$\therefore \quad x=-1$

Second meaning and Application

If in a linear equation, the independent terms in both the sides have the same value then value of the variable is 'zero'.

Example 23. Solve : $(x+3)+(2x+5)+4=2(x+6)$.

Solution : Independent terms in both the sides of the eqaution are same = 12, hence x = 0.

Example 24. Solve $(x+1)(x+9)=(x+3)(x+3)$.

Solution : Independent terms in both the sides of the equation are same = 9, hence x = 0.

Third meaning and Application

If in the equation, both the fractions have the same numerical numerator, then the solution of the equation is obtained by putting the sum of denominator equal to 'zero'.

Example 25. Solve $\frac{1}{x+a}+\frac{1}{x+b}=0$.

Solution : Here, the numerator in both the fractions are same = 1

Hence $x+a+x+b=0 \quad \therefore \ x=-\frac{a+b}{2}$.

Example 26. Solve $\frac{m}{2x+1}+\frac{m}{3x+4}=0$

Solution : Here, the numerator in both the fractions are same $=m$

Hence $2x+1+3x+4=0$

or $5x+5=0 \quad \therefore \quad x=-1$

Fourth meaning and Application

If the sum of the numerators and sum of the denominators of the fractions are same or in a definate ratio, then that sum equal to 'zero' will give the solution of the equation.

Example 27. Solve the equation $\frac{2x+3}{2x+5}=\frac{2x+5}{2x+3}$

Solution : Sum of the numerators $=2x+3+2x+5=4x+8$

Sum of the denominators $=4x+8$

Sum of the numenators and sum of the denominators same, therefore, by the formula $4x+8=0, \ \therefore \ x=-2$

Example 28. Solve the equation $\frac{3x+4}{6x+7}=\frac{x+1}{2x+3}$

Solution : Sum of the numerators $=3x+4+x+1=4x+5 \quad \cdots$ (i)

Sum of the denominators $=6x+7+2x+3=8x+10 \quad \cdots$ (ii)

Sum of the numerators and sum of the denominators is in the ratio = 1 : 2

Hence, by the formula either $4x+5=0$ or $8x+10=0$

$\therefore \quad x=-\frac{5}{4}$

Fifth meaning and Application

If the difference of numerator and denominator of one fraction be the same as that of second fraction or their differences are in a definate ratio then that difference is equal to 'zero' give the solution of the equation.

Example 29. Solve the equation $\frac{3x+4}{2x+1}=\frac{x-8}{2x-5}$

Solution : Difference of the numerator and denominator of the L.H.S. fraction

$=3x+4-2x-1=x+3 \ \cdots$ (i)

Difference of the numerator and denominator of the R.H.S. fraction

$=2x-5-x+8=x+3 \ \cdots$ (ii)

Differences of numerator and denominator in (i) and (ii) are in the ratio = 2 : 1

Hence either $x+3=0$ and $2x+6=0 \quad \therefore \ x=-3$

Example 30. Solve the equation $\frac{x-8}{3x-2}=\frac{3x+4}{2x+1}$

Solution : Difference of the numerator and denominator of the L.H.S. fraction

$$=3x-2-x+8=2x+6 \cdots \text{(i)}$$

Difference of the numerator and denominator of the R.H.S. fraction

$$=3x+4-2x-1=x+3 \cdots \text{(ii)}$$

Difference of numerator and denominator in (i) and in (ii) are in the ratio = 2 : 1

Hence either $x+3=0$ or $2x+6=0$ $\therefore x=-3$

Note : By the fourth and fifth application of the sutra "Sunyam Samyasamuccaye"' we can obtain both the root of a quadratic equation. For example, the equation $\frac{3x+4}{6x+7}=\frac{5x+6}{2x+3}$ has two roots $x=-\frac{5}{4}$ and $x=-1$ obtained by the fourth and fifth application respectively.

Sixth meaning and Application

If in a equation each side contains two fraction and numerator in each fraction be the same, the sum of the denominators in the L.H.S. fraction and sum of the denominator in the R.H.S fraction are the same then that equal to 'zero' will give the solution of the equation.

Example 31. Solve the equation $\frac{1}{x+7}+\frac{1}{x+9}=\frac{1}{x+6}+\frac{1}{x+10}$

Solution : Sum of the denominators in L.H.S.

$$=x+7+x+9=2x+16$$

Sum of the denominator in R.H.S

$$=x+6+x+10=2x+16$$

Hence by the formula $2x+16=0$ $\therefore$ $x=-8$

Example 32. Solve the equation $\frac{1}{x-8}+\frac{1}{x-9}=\frac{1}{x-5}+\frac{1}{x-12}$

Solution : Sum of the denominator in L.H.S. $=2x-17$

Hence by the formula $2x-17=0$ $\therefore$ $x=\frac{17}{2}=8\frac{1}{2}$

Example 33. Solve the equation $\frac{1}{x+1}-\frac{1}{x+3}=\frac{1}{x+2}-\frac{1}{x+4}$

Solution : Transferring the negative fraction of both the sides, we have

$$\frac{1}{x+1}+\frac{1}{x+4}=\frac{1}{x+2}+\frac{1}{x+3}$$

Hence by the formula $2x+5=0$ $\therefore$ $x=-\frac{5}{2}=-2\frac{1}{2}$

Methods of checking the results

There are two methods of checking the results received from any operation.

(a) Navanka method

(b) Ekadashanka Method

(a) Navanka method :

In the navanka method we find the beejanka of any number by taking base as digit 9. After subtracting 9 from the digits of a number or sum of the digits of a number remaining single digit is know as Beejanka of that number e.g. Beejanka of 947 = 2.

Checking of the Division results:

$4283 \div 7$, Quotient $= 611$, Remainder $= 6$

We prove that

= Beejanka of Quotient × Beejanka of divisor + Beejanka of Remainder

i.e. $8 = 7 \times 8 + 6$

or $= 62$

or $= 8$

Hence answer is correct

Note: 1. If two digits of any row or two digits of any column interchange their places the error cannot be spotted.

2. In Vedic Mathematics there are several methods to solve a question. The result can be verified by Ekadashanka Method.

(b) Ekadashanka Method or Difference Method:

In this method the difference of the sumof digits at odd places and the sum of digits at even places is called the Beejanka of the number e.g. Beejanka of 63254

$= 4 - 5 + 2 - 3 + 6 = 4$

Checking of the Divisions results:

$6789 \div 12$, Quotient = 565, Remainder = 9

Beejanka of Dividend $= 9 - 8 + 7 - 6 = 2$

Beejanka of Quotient $= 5 - 6 + 5 \quad = 4$

Beejanka of Divisor $= 2 - 1 \quad = 1$

Beejanka of Remainder $= 9 \quad = 9$

L.H.S. = Divisor × Quotient + Remainder

$= 4 \times 1 + 9 = 13$

$\therefore$ Beejanka of L.H.S. $= 3 - 1 = 2$

Beejanka of dividend = Beejanka of R.H.S. = 2.

Both are equal, hence answer is correct.

Exercise 1

Divide by the Nikhilam method:

1. $1245 \div 97$ 2. $311 \div 8$ 3. $1013 \div 88$

Square the following numbers:

4. 103 5. 95 6. 204 7. 225

Find the cube of the following numbers:

8. 15 9. 91 10. 32 11. 208

Divide by Dhvajanka method:

12. $4532 \div 112$ 13. $1234 \div 42$ 14. $98765 \div 87$ 15. $2101532 \div 879$

Divide by Paravartya Yojayet method:

16. $1154 \div 103$ 17. $1358 \div 113$ 18. $1432 \div 88$ 19. $14885 \div 123$

Answers

	1.	2.	3.
Quotient →	12	38	11
Remainder →	8	7	45

4. 10609 5. 9025 6. 41616 7. 50625

8. 3375 9. 753571 10. 32768 11. 8998912

	12.	13.	14.	15.
Quotient →	40	29	1135	2390
Remainder →	52	16	20	722

	16.	17.	18.	19.
Quotient →	11	12	16	121
Remainder →	21	2	24	2

❑