THINK, DISCUSS AND WRITE

After showing $m\angle R = m\angle N = 70°$, can you find $m\angle I$ and $m\angle G$ by any other method?

3.4.6 Diagonals of a parallelogram

The diagonals of a parallelogram, in general, are not of equal length. (Did you check this in your earlier activity?) However, the diagonals of a parallelogram have an interesting property.

DO THIS

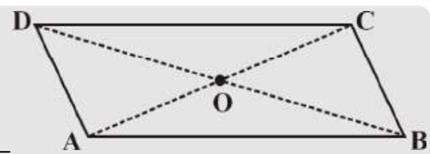


Fig 3.29

Take a cut-out of a parallelogram, say, ABCD (Fig 3.29). Let its diagonals $\overline{AC}$ and $\overline{DB}$ meet at O.

Find the mid point of $\overline{AC}$ by a fold, placing C on A. Is the mid-point same as O?

Does this show that diagonal $\overline{DB}$ bisects the diagonal $\overline{AC}$ at the point O? Discuss it with your friends. Repeat the activity to find where the mid point of $\overline{DB}$ could lie.

Property: *The diagonals of a parallelogram bisect each other (at the point of their intersection, of course!)*

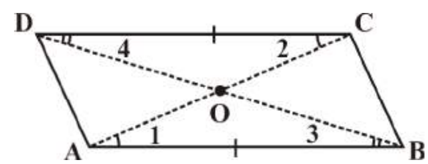


Fig 3.30

To argue and justify this property is not very difficult. From Fig 3.30, applying ASA criterion, it is easy to see that

Δ AOB $\cong$ Δ COD (How is ASA used here?)

This gives AO = CO and BO = DO

Example 6: In Fig 3.31 HELP is a parallelogram. (Lengths are in cms). Given that OE = 4 and HL is 5 more than PE? Find OH.

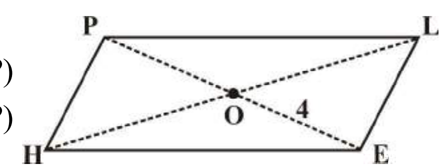


Fig 3.31

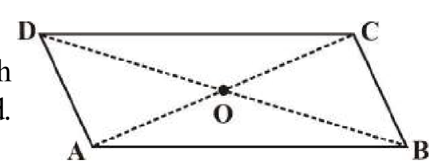
Solution : If OE = 4 then OP also is 4 (Why?)

So PE = 8, (Why?)

Therefore HL = 8 + 5 = 13

Hence $OH = \frac{1}{2} \times 13 = 6.5$ (cms)

EXERCISE 3.3

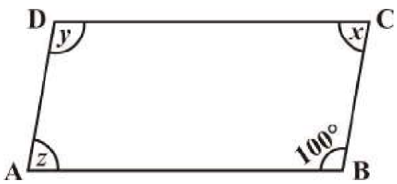


1. Given a parallelogram ABCD. Complete each statement along with the definition or property used.

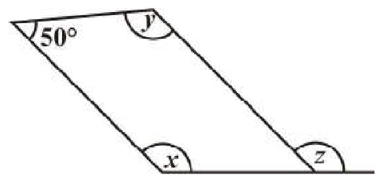
   (i) AD = ... (ii) $\angle$ DCB = ...

   (iii) OC = ... (iv) $m\angle DAB + m\angle CDA$ = ...

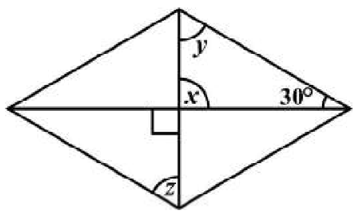
2. Consider the following parallelograms. Find the values of the unknowns x, y, z.



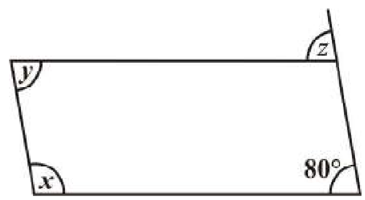
(i)



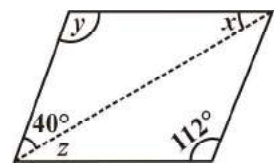
(ii)



(iii)



(iv)



(v)

3. Can a quadrilateral ABCD be a parallelogram if

(i) $\angle D + \angle B = 180°$? (ii) AB = DC = 8 cm, AD = 4 cm and BC = 4.4 cm?

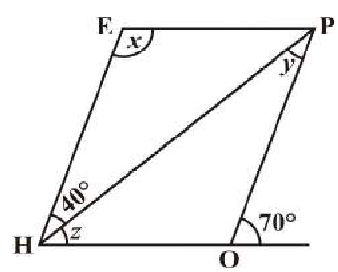
(iii) $\angle A = 70°$ and $\angle C = 65°$?

4. Draw a rough figure of a quadrilateral that is not a parallelogram but has exactly two opposite angles of equal measure.

5. The measures of two adjacent angles of a parallelogram are in the ratio 3 : 2. Find the measure of each of the angles of the parallelogram.

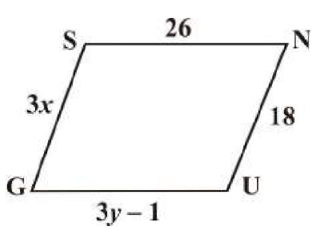
6. Two adjacent angles of a parallelogram have equal measure. Find the measure of each of the angles of the parallelogram.

7. The adjacent figure HOPE is a parallelogram. Find the angle measures x, y and z. State the properties you use to find them.

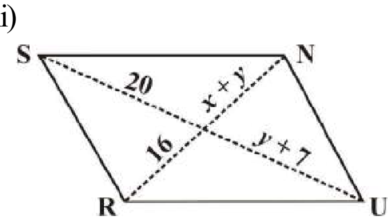


8. The following figures GUNS and RUNS are parallelograms. Find x and y. (Lengths are in cm)

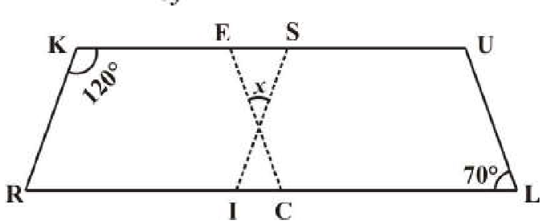
(i)



(ii)



9.



In the above figure both RISK and CLUE are parallelograms. Find the value of x.

10. Explain how this figure is a trapezium. Which of its two sides are parallel? (Fig 3.32)

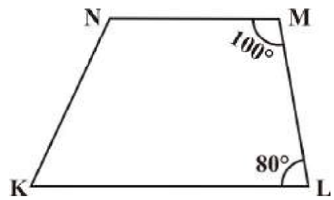


Fig 3.32

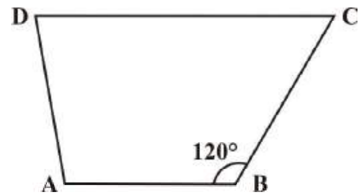


Fig 3.33

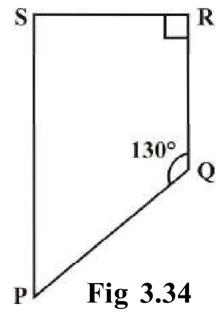


Fig 3.34

11. Find $m\angle C$ in Fig 3.33 if $\overline{AB} \parallel \overline{DC}$.

12. Find the measure of $\angle P$ and $\angle S$ if $\overline{SP} \parallel \overline{RQ}$ in Fig 3.34. (If you find $m\angle R$, is there more than one method to find $m\angle P$?)

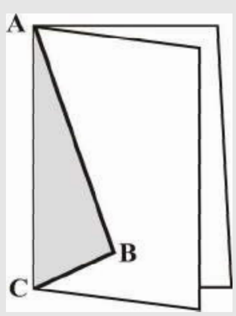
3.5 Some Special Parallelograms

3.5.1 Rhombus

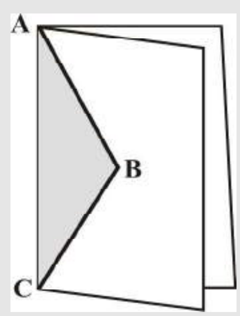
We obtain a Rhombus (which, you will see, is a parallelogram) as a special case of kite (which is not a a parallelogram).

DO THIS

Recall the paper-cut kite you made earlier.



Kite-cut



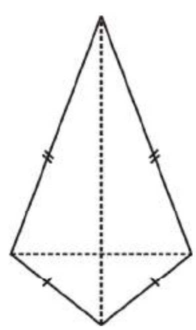
Rhombus-cut

When you cut along ABC and opened up, you got a kite. Here lengths AB and BC were different. If you draw AB = BC, then the kite you obtain is called a **rhombus**.

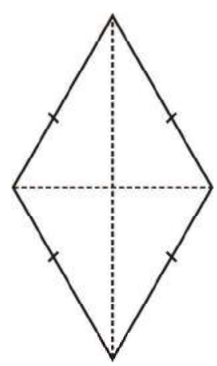
Note that the sides of rhombus are all of same length; this is not the case with the kite.

A rhombus is a quadrilateral with sides of equal length.

Since the opposite sides of a rhombus have the same length, it is also a parallelogram. So, *a rhombus has all the properties of a parallelogram and also that of a kite*. Try to list them out. You can then verify your list with the check list summarised in the book elsewhere.



Kite



Rhombus

The most useful property of a rhombus is that of its diagonals.

Property: *The diagonals of a rhombus are perpendicular bisectors of one another.*

DO THIS

Take a copy of rhombus. By paper-folding verify if the point of intersection is the mid-point of each diagonal. You may also check if they intersect at right angles, using the corner of a set-square.

Here is an outline justifying this property using logical steps.

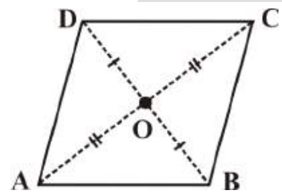


Fig 3.35

ABCD is a rhombus (Fig 3.35). Therefore it is a parallelogram too.

Since diagonals bisect each other, OA = OC and OB = OD.

We have to show that $m\angle AOD = m\angle COD = 90°$

It can be seen that by SAS congruency criterion

$$\Delta\,AOD \cong \Delta\,COD$$

Since AO = CO (Why?)
AD = CD (Why?)
OD = OD

Therefore, $m\,\angle AOD = m\,\angle COD$

Since $\angle AOD$ and $\angle COD$ are a linear pair,

$$m\,\angle AOD = m\,\angle COD = 90°$$

Example 7:

RICE is a rhombus (Fig 3.36). Find x, y, z. Justify your findings.

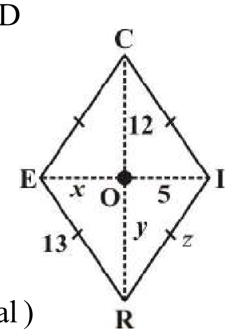


Fig 3.36

Solution:

x = OE
= OI (diagonals bisect)
= 5

y = OR
= OC (diagonals bisect)
= 12

z = side of the rhombus
= 13 (all sides are equal)

3.5.2 A rectangle

A rectangle is a parallelogram with equal angles (Fig 3.37).

Fig 3.37

What is the full meaning of this definition? Discuss with your friends.

If the rectangle is to be equiangular, what could be the measure of each angle?

Let the measure of each angle be $x°$.

Then $4x° = 360°$ (Why)?

Therefore, $x° = 90°$

Thus each angle of a rectangle is a right angle.

So, a rectangle is a parallelogram in which every angle is a right angle.

Being a parallelogram, the rectangle has opposite sides of equal length and its diagonals bisect each other.

In a parallelogram, the diagonals can be of different lengths. (Check this); but surprisingly the rectangle (being a special case) has diagonals of equal length.

Property: *The diagonals of a rectangle are of equal length.*

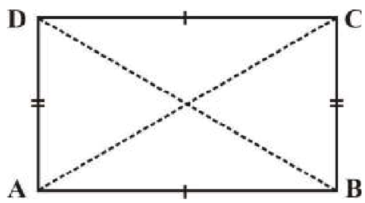


Fig 3.38

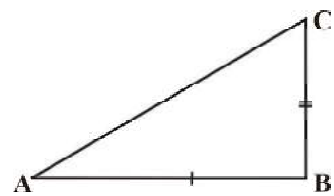


Fig 3.39

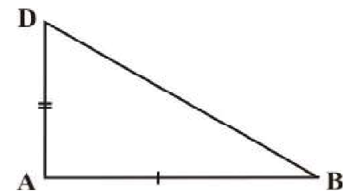


Fig 3.40

This is easy to justify. If ABCD is a rectangle (Fig 3.38), then looking at triangles ABC and ABD separately [(Fig 3.39) and (Fig 3.40) respectively], we have

$$\Delta\, ABC \cong \Delta\, ABD$$

This is because	$AB = AB$	(Common)
	$BC = AD$	(Why?)
	$m\,\angle A = m\,\angle B = 90°$	(Why?)

The congruency follows by SAS criterion.

Thus $\qquad AC = BD$

and *in a rectangle the diagonals, besides being equal in length bisect each other* (Why?)

Example 8: RENT is a rectangle (Fig 3.41). Its diagonals meet at O. Find x, if OR = $2x + 4$ and OT = $3x + 1$.

Solution: $\overline{OT}$ is half of the diagonal $\overline{TE}$,

$\overline{OR}$ is half of the diagonal $\overline{RN}$.

Diagonals are equal here. (Why?)

So, their halves are also equal.

Therefore $\qquad 3x + 1 = 2x + 4$

or $\qquad x = 3$

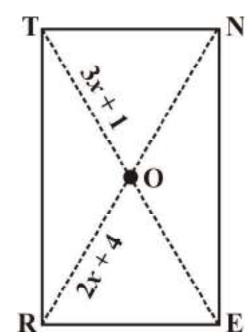


Fig 3.41

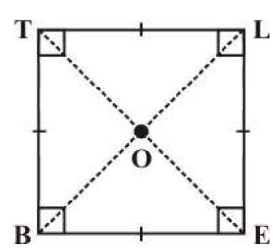
3.5.3 A square

A square is a rectangle with equal sides.

This means a square has all the properties of a rectangle with an additional requirement that all the sides have equal length.

The square, like the rectangle, has diagonals of equal length.

In a rectangle, there is no requirement for the diagonals to be perpendicular to one another, (Check this).



BELT is a square, BE = EL = LT = TB
$\angle B, \angle E, \angle L, \angle T$ are right angles.

BL = ET and $\overline{BL} \perp \overline{ET}$.

OB = OL and OE = OT.

In a square the diagonals.

(i) bisect one another (square being a parallelogram)

(ii) are of equal length (square being a rectangle) and

(iii) are perpendicular to one another.

Hence, we get the following property.

Property: *The diagonals of a square are perpendicular bisectors of each other.*

DO THIS

Take a square sheet, say PQRS (Fig 3.42).

Fold along both the diagonals. Are their mid-points the same?

Check if the angle at O is 90° by using a set-square.

This verifies the property stated above.

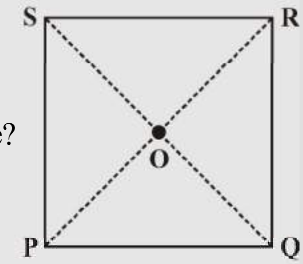


Fig 3.42

We can justify this also by arguing logically:

ABCD is a square whose diagonals meet at O (Fig 3.43).

OA = OC (Since the square is a parallelogram)

By SSS congruency condition, we now see that

Δ AOD $\cong$ Δ COD (How?)

Therefore, $m\angle AOD = m\angle COD$

These angles being a linear pair, each is right angle.

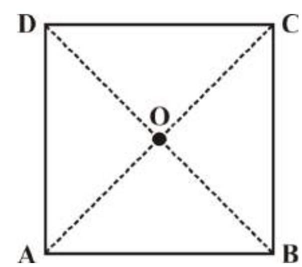


Fig 3.43

EXERCISE 3.4

1. State whether True or False.

(a) All rectangles are squares

(b) All rhombuses are parallelograms

(c) All squares are rhombuses and also rectangles

(d) All squares are not parallelograms.

(e) All kites are rhombuses.

(f) All rhombuses are kites.

(g) All parallelograms are trapeziums.

(h) All squares are trapeziums.

2. Identify all the quadrilaterals that have.

(a) four sides of equal length (b) four right angles

3. Explain how a square is.

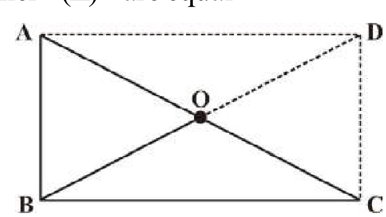
(i) a quadrilateral (ii) a parallelogram (iii) a rhombus (iv) a rectangle

4. Name the quadrilaterals whose diagonals.

(i) bisect each other (ii) are perpendicular bisectors of each other (iii) are equal

5. Explain why a rectangle is a convex quadrilateral.

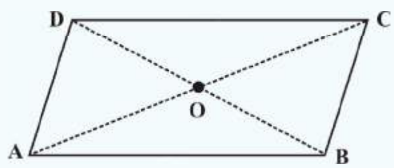
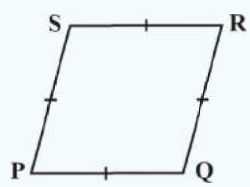
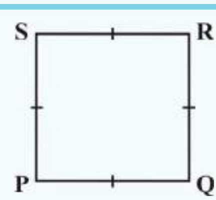
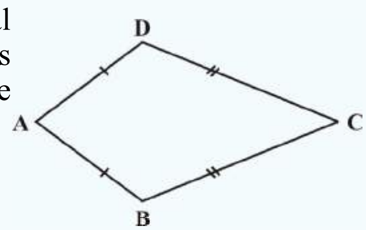
6. ABC is a right-angled triangle and O is the mid point of the side opposite to the right angle. Explain why O is equidistant from A, B and C. (The dotted lines are drawn additionally to help you).



THINK, DISCUSS AND WRITE

1. A mason has made a concrete slab. He needs it to be rectangular. In what different ways can he make sure that it is rectangular?
2. A square was defined as a rectangle with all sides equal. Can we define it as rhombus with equal angles? Explore this idea.
3. Can a trapezium have all angles equal? Can it have all sides equal? Explain.

WHAT HAVE WE DISCUSSED?

Quadrilateral		Properties
Parallelogram: A quadrilateral with each pair of opposite sides parallel.		(1) Opposite sides are equal. (2) Opposite angles are equal. (3) Diagonals bisect one another.
Rhombus: A parallelogram with sides of equal length.		(1) All the properties of a parallelogram. (2) Diagonals are perpendicular to each other.
Rectangle: A parallelogram with a right angle.		(1) All the properties of a parallelogram. (2) Each of the angles is a right angle. (3) Diagonals are equal.
Square: A rectangle with sides of equal length.		All the properties of a parallelogram, rhombus and a rectangle.
Kite: A quadrilateral with exactly two pairs of equal consecutive sides		(1) The diagonals are perpendicular to one another (2) One of the diagonals bisects the other. (3) In the figure $m\angle B = m\angle D$ but $m\angle A \neq m\angle C$.

CHAPTER

4

Practical Geometry

4.1 Introduction

You have learnt how to draw triangles in Class VII. We require three measurements (of sides and angles) to draw a unique triangle.

Since three measurements were enough to draw a triangle, a natural question arises whether four measurements would be sufficient to draw a unique four sided closed figure, namely, a quadrilateral.

DO THIS

Take a pair of sticks of equal lengths, say 10 cm. Take another pair of sticks of equal lengths, say, 8 cm. Hinge them up suitably to get a rectangle of length 10 cm and breadth 8 cm.

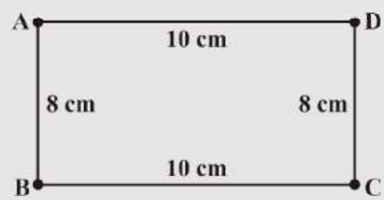


Fig 4.1

This rectangle has been created with the 4 available measurements.

Now just push along the breadth of the rectangle. Is the new shape obtained, still a rectangle (Fig 4.2)? Observe that the rectangle has now become a parallelogram. Have you altered the lengths of the sticks? No! The measurements of sides remain the same.

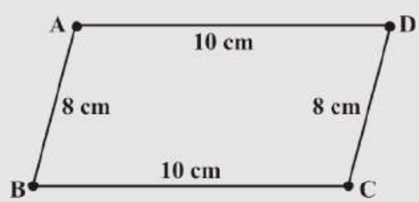


Fig 4.2

Give another push to the newly obtained shape in a different direction; what do you get? You again get a parallelogram, which is altogether different (Fig 4.3), yet the four measurements remain the same.

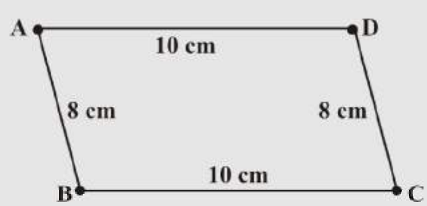


Fig 4.3

This shows that 4 measurements of a quadrilateral cannot determine it uniquely. Can 5 measurements determine a quadrilateral uniquely? Let us go back to the activity!

You have constructed a rectangle with two sticks each of length 10 cm and other two sticks each of length 8 cm. Now introduce another stick of length equal to BD and tie it along BD (Fig 4.4). If you push the breadth now, does the shape change? No! It cannot, without making the figure open. The introduction of the fifth stick has fixed the rectangle uniquely, i.e., there is no other quadrilateral (with the given lengths of sides) possible now.

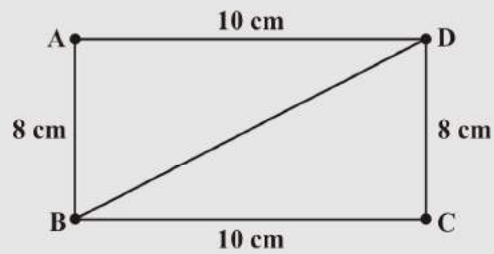


Fig 4.4

Thus, we observe that five measurements can determine a quadrilateral uniquely. But will any five measurements (of sides and angles) be sufficient to draw a unique quadrilateral?

THINK, DISCUSS AND WRITE

Arshad has five measurements of a quadrilateral ABCD. These are AB = 5 cm, ∠A = 50°, AC = 4 cm, BD = 5 cm and AD = 6 cm. Can he construct a unique quadrilateral? Give reasons for your answer.

4.2 Constructing a Quadrilateral

We shall learn how to construct a unique quadrilateral given the following measurements:

- When four sides and one diagonal are given.
- When two diagonals and three sides are given.
- When two adjacent sides and three angles are given.
- When three sides and two included angles are given.
- When other special properties are known.

Let us take up these constructions one-by-one.

4.2.1 When the lengths of four sides and a diagonal are given

We shall explain this construction through an example.

Example 1: Construct a quadrilateral PQRS where PQ = 4 cm,QR = 6 cm, RS = 5 cm, PS = 5.5 cm and PR = 7 cm.

Solution: [A rough sketch will help us in visualising the quadrilateral. We draw this first and mark the measurements.] (Fig 4.5)

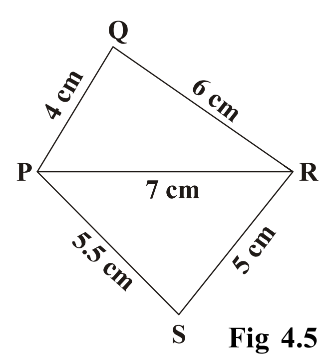


Fig 4.5

Step 1 From the rough sketch, it is easy to see that ΔPQR can be constructed using SSS construction condition. Draw ΔPQR (Fig 4.6).

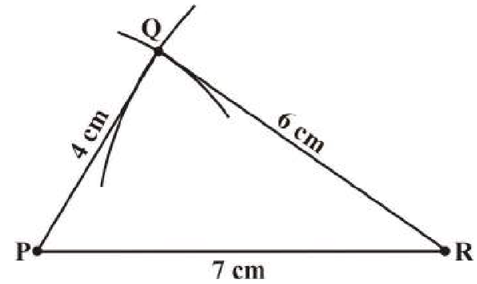


Fig 4.6

Step 2 Now, we have to locate the fourth point S. This 'S' would be on the side opposite to Q with reference to PR. For that, we have two measurements.

S is 5.5 cm away from P. So, with P as centre, draw an arc of radius 5.5 cm. (The point S is somewhere on this arc!) (Fig 4.7).

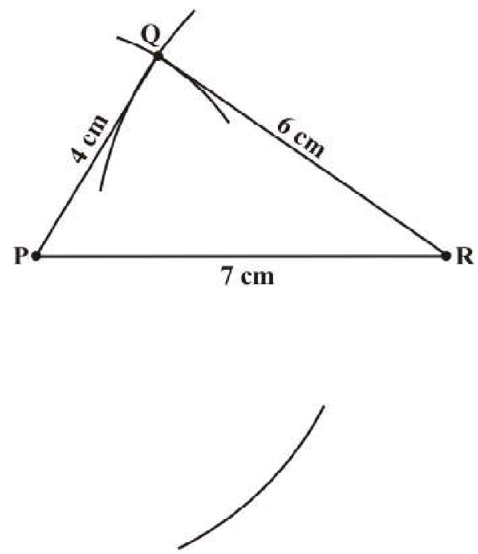


Fig 4.7

Step 3 S is 5 cm away from R. So with R as centre, draw an arc of radius 5 cm (The point S is somewhere on this arc also!) (Fig 4.8).

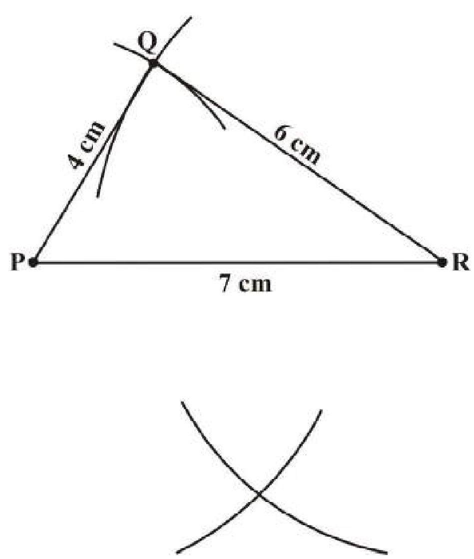


Fig 4.8

Step 4 S should lie on both the arcs drawn. So it is the point of intersection of the two arcs. Mark S and complete PQRS. PQRS is the required quadrilateral (Fig 4.9).

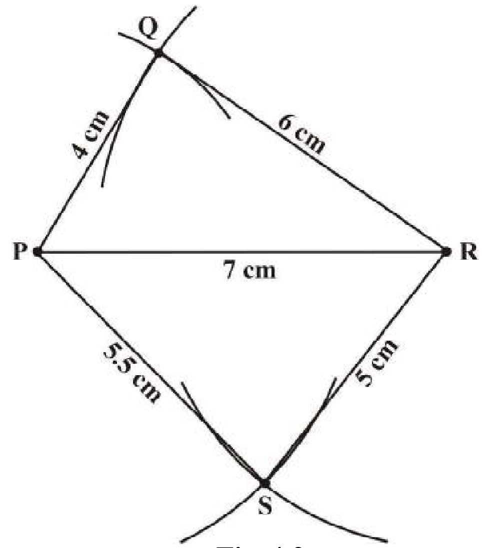


Fig 4.9

THINK, DISCUSS AND WRITE

(i) We saw that 5 measurements of a quadrilateral can determine a quadrilateral uniquely. Do you think any five measurements of the quadrilateral can do this?

(ii) Can you draw a parallelogram BATS where BA = 5 cm, AT = 6 cm and AS = 6.5 cm? Why?

(iii) Can you draw a rhombus ZEAL where ZE = 3.5 cm, diagonal EL = 5 cm? Why?

(iv) A student attempted to draw a quadrilateral PLAY where PL = 3 cm, LA = 4 cm, AY = 4.5 cm, PY = 2 cm and LY = 6 cm, but could not draw it. What is the reason?
[**Hint:** Discuss it using a rough sketch].

EXERCISE 4.1

1. Construct the following quadrilaterals.

(i) Quadrilateral ABCD.
AB = 4.5 cm
BC = 5.5 cm
CD = 4 cm
AD = 6 cm
AC = 7 cm

(ii) Quadrilateral JUMP
JU = 3.5 cm
UM = 4 cm
MP = 5 cm
PJ = 4.5 cm
PU = 6.5 cm

(iii) Parallelogram MORE
OR = 6 cm
RE = 4.5 cm
EO = 7.5 cm

(iv) Rhombus BEST
BE = 4.5 cm
ET = 6 cm

4.2.2 When two diagonals and three sides are given

When four sides and a diagonal were given, we first drew a triangle with the available data and then tried to locate the fourth point. The same technique is used here.

Example 2: Construct a quadrilateral ABCD, given that BC = 4.5 cm, AD = 5.5 cm, CD = 5 cm the diagonal AC = 5.5 cm and diagonal BD = 7 cm.

Solution:

Here is the rough sketch of the quadrilateral ABCD (Fig 4.10). Studying this sketch, we can easily see that it is possible to draw ΔACD first (How?).

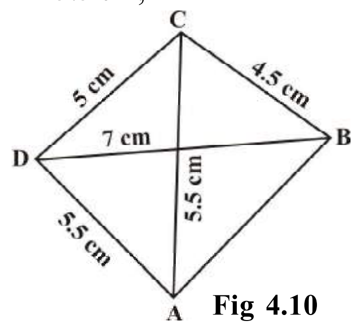


Fig 4.10

Step 1 Draw ΔACD using SSS construction (Fig 4.11).
(We now need to find B at a distance of 4.5 cm from C and 7 cm from D).

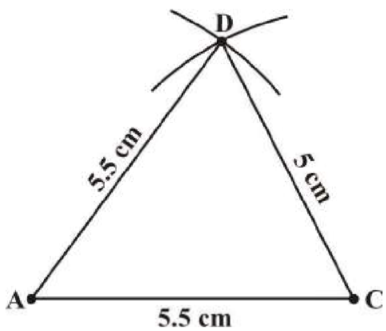


Fig 4.11

Step 2 With D as centre, draw an arc of radius 7 cm. (B is somewhere on this arc) (Fig 4.12).

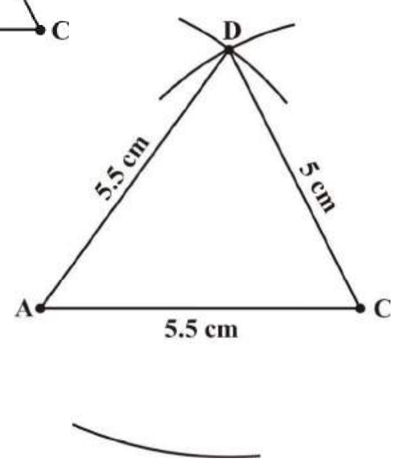


Fig 4.12

Step 3 With C as centre, draw an arc of radius 4.5 cm (B is somewhere on this arc also) (Fig 4.13).

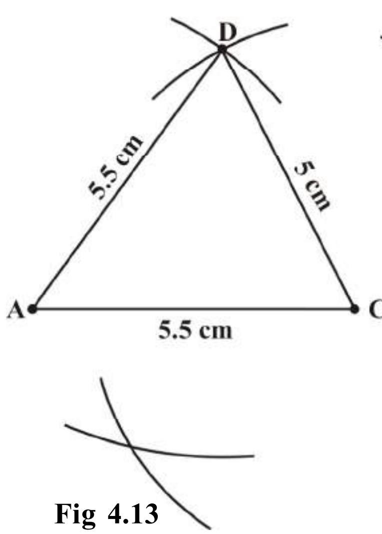


Fig 4.13

Step 4 Since B lies on both the arcs, B is the point intersection of the two arcs. Mark B and complete ABCD. ABCD is the required quadrilateral (Fig 4.14).

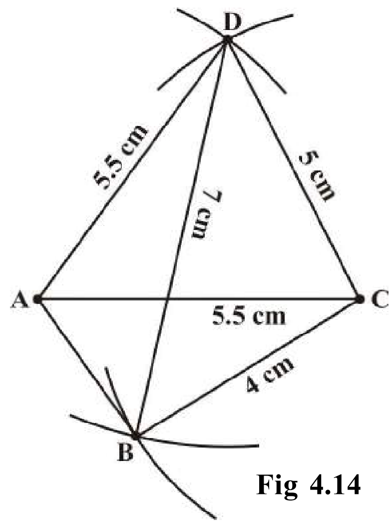


Fig 4.14

THINK, DISCUSS AND WRITE

1. In the above example, can we draw the quadrilateral by drawing ΔABD first and then find the fourth point C?
2. Can you construct a quadrilateral PQRS with PQ = 3 cm, RS = 3 cm, PS = 7.5 cm, PR = 8 cm and SQ = 4 cm? Justify your answer.

EXERCISE 4.2

1. Construct the following quadrilaterals.

(i) quadrilateral LIFT

LI = 4 cm

IF = 3 cm

TL = 2.5 cm

LF = 4.5 cm

IT = 4 cm

(ii) Quadrilateral GOLD

OL = 7.5 cm

GL = 6 cm

GD = 6 cm

LD = 5 cm

OD = 10 cm

(iii) Rhombus BEND

BN = 5.6 cm

DE = 6.5 cm

4.2.3 When two adjacent sides and three angles are known

As before, we start with constructing a triangle and then look for the fourth point to complete the quadrilateral.

Example 3: Construct a quadrilateral MIST where MI = 3.5 cm, IS = 6.5 cm, $\angle$M = 75°, $\angle$I = 105° and $\angle$S = 120°.

Solution:

Here is a rough sketch that would help us in deciding our steps of construction. We give only hints for various steps (Fig 4.15).

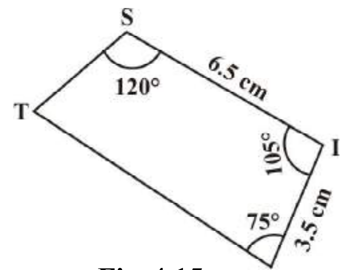


Fig 4.15

Step 1 How do you locate the points? What choice do you make for the base and what is the first step? (Fig 4.16)

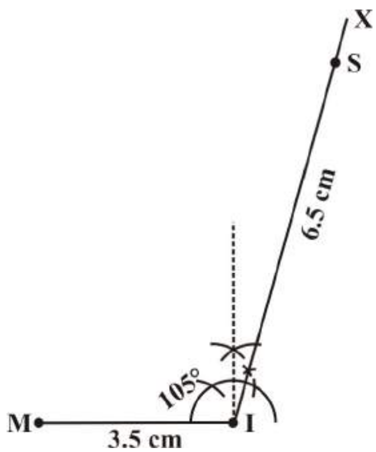


Fig 4.16

Step 2 Make ∠ISY = 120° at S (Fig 4.17).

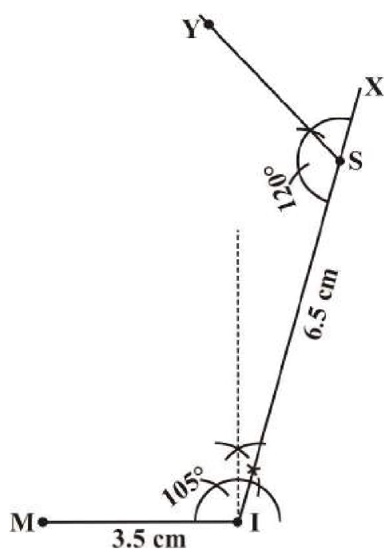


Fig 4.17

Step 3 Make $\angle IMZ = 75°$ at M. (where will SY and MZ meet?) Mark that point as T. We get the required quadrilateral MIST (Fig 4.18).

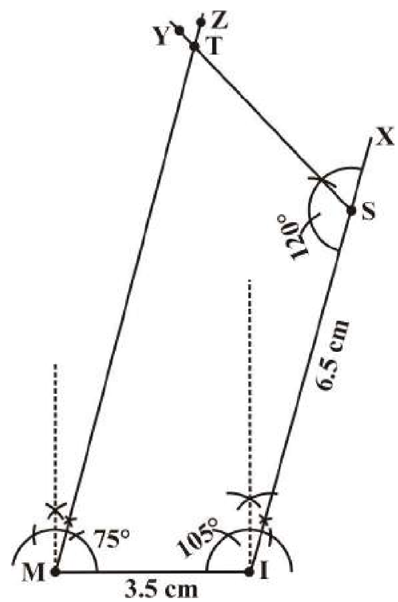


Fig 4.18

THINK, DISCUSS AND WRITE

1. Can you construct the above quadrilateral MIST if we have 100° at M instead of 75°?
2. Can you construct the quadrilateral PLAN if PL = 6 cm, LA = 9.5 cm, $\angle P = 75°$, $\angle L = 150°$ and $\angle A = 140°$? (**Hint:** Recall angle-sum property).
3. In a parallelogram, the lengths of adjacent sides are known. Do we still need measures of the angles to construct as in the example above?

EXERCISE 4.3

1. Construct the following quadrilaterals.

(i) Quadrilateral MORE
MO = 6 cm
OR = 4.5 cm
$\angle M = 60°$
$\angle O = 105°$
$\angle R = 105°$

(ii) Quadrilateral PLAN
PL = 4 cm
LA = 6.5 cm
$\angle P = 90°$
$\angle A = 110°$
$\angle N = 85°$

(iii) Parallelogram HEAR
HE = 5 cm
EA = 6 cm
$\angle R = 85°$

(iv) Rectangle OKAY
OK = 7 cm
KA = 5 cm

4.2.4 When three sides and two included angles are given

Under this type, when you draw a rough sketch, note carefully the "included" angles in particular.

Example 4: Construct a quadrilateral ABCD, where AB = 4 cm, BC = 5 cm, CD = 6.5 cm and ∠B = 105° and ∠C = 80°.

Solution:

We draw a rough sketch, as usual, to get an idea of how we can start off. Then we can devise a plan to locate the four points (Fig 4.19).

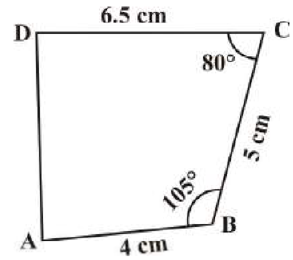


Fig 4.19

Step 1 Start with taking BC = 5 cm on B. Draw an angle of 105° along BX. Locate A 4 cm away on this. We now have B, C and A (Fig 4.20).

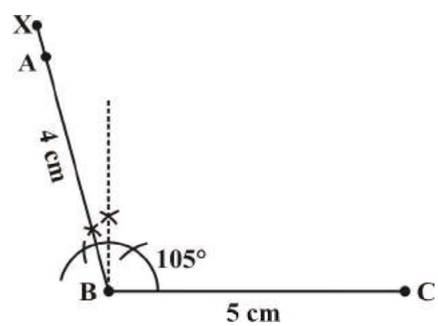


Fig 4.20

Step 2 The fourth point D is on CY which is inclined at 80° to BC. So make ∠BCY = 80° at C on BC (Fig 4.21).

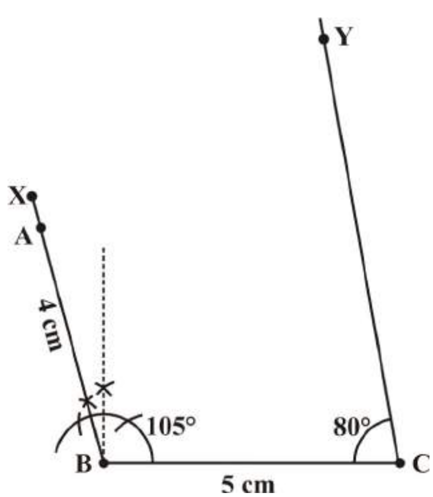


Fig 4.21

Step 3 D is at a distance of 6.5 cm on CY. With C as centre, draw an arc of length 6.5 cm. It cuts CY at D (Fig 4.22).

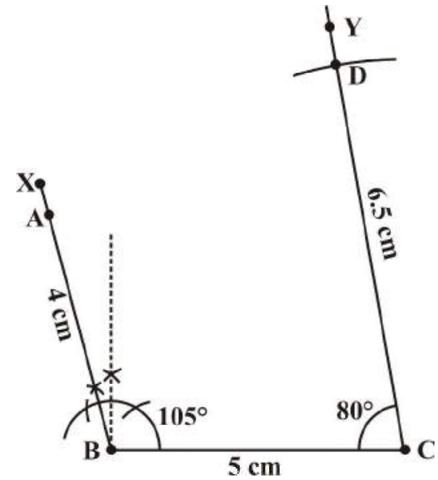


Fig 4.22

Step 4 Complete the quadrilateral ABCD. ABCD is the required quadrilateral (Fig 4.23).

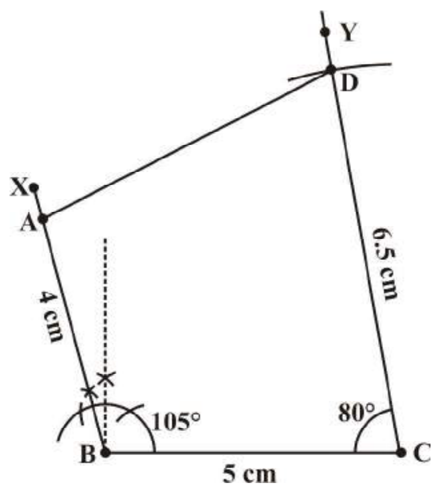


Fig 4.23

THINK, DISCUSS AND WRITE

1. In the above example, we first drew BC. Instead, what could have been be the other starting points?
2. We used some five measurements to draw quadrilaterals so far. Can there be different sets of five measurements (other than seen so far) to draw a quadrilateral? The following problems may help you in answering the question.
   (i) Quadrilateral ABCD with AB = 5 cm, BC = 5.5 cm, CD = 4 cm, AD = 6 cm and $\angle B = 80°$.
   (ii) Quadrilateral PQRS with PQ = 4.5 cm, $\angle P = 70°$, $\angle Q = 100°$, $\angle R = 80°$ and $\angle S = 110°$.

   Construct a few more examples of your own to find sufficiency/insufficiency of the data for construction of a quadrilateral.

EXERCISE 4.4

1. Construct the following quadrilaterals.

(i) Quadrilateral DEAR
DE = 4 cm
EA = 5 cm
AR = 4.5 cm
$\angle E = 60°$
$\angle A = 90°$

(ii) Quadrilateral TRUE
TR = 3.5 cm
RU = 3 cm
UE = 4 cm
$\angle R = 75°$
$\angle U = 120°$

4.3 Some Special Cases

To draw a quadrilateral, we used 5 measurements in our work. Is there any quadrilateral which can be drawn with less number of available measurements? The following examples examine such special cases.

Example 5: Draw a square of side 4.5 cm.

Solution: Initially it appears that only one measurement has been given. Actually we have many more details with us, because the figure is a special quadrilateral, namely a square. We now know that each of its angles is a right angle. (See the rough figure) (Fig 4.24)

This enables us to draw ΔABC using SAS condition. Then D can be easily located. Try yourself now to draw the square with the given measurements.

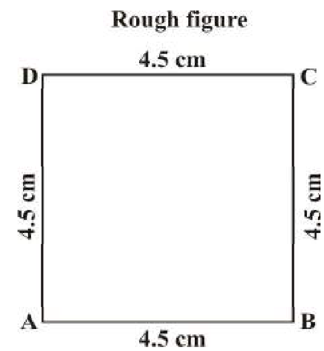


Fig 4.24

Example 6: Is it possible to construct a rhombus ABCD where AC = 6 cm and BD = 7 cm? Justify your answer.

Solution: Only two (diagonal) measurements of the rhombus are given. However, since it is a rhombus, we can find more help from its properties.

The diagonals of a rhombus are perpendicular bisectors of one another.

So, first draw AC = 7 cm and then construct its perpendicular bisector. Let them meet at 0. Cut off 3 cm lengths on either side of the drawn bisector. You now get B and D.

Draw the rhombus now, based on the method described above (Fig 4.25).

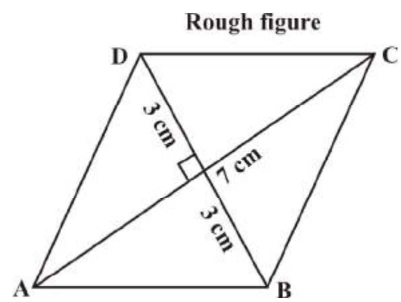


Fig 4.25

TRY THESE

1. How will you construct a rectangle PQRS if you know only the lengths PQ and QR?
2. Construct the kite EASY if AY = 8 cm, EY = 4 cm and SY = 6 cm (Fig 4.26). Which properties of the kite did you use in the process?

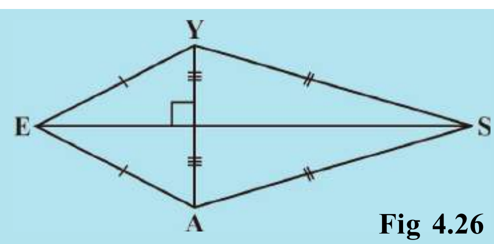


Fig 4.26

EXERCISE 4.5

Draw the following.

1. The square READ with RE = 5.1 cm.
2. A rhombus whose diagonals are 5.2 cm and 6.4 cm long.
3. A rectangle with adjacent sides of lengths 5 cm and 4 cm.
4. A parallelogram OKAY where OK = 5.5 cm and KA = 4.2 cm. Is it unique?

WHAT HAVE WE DISCUSSED?

1. Five measurements can determine a quadrilateral uniquely.
2. A quadrilateral can be constructed uniquely if the lengths of its four sides and a diagonal is given.
3. A quadrilateral can be constructed uniquely if its two diagonals and three sides are known.
4. A quadrilateral can be constructed uniquely if its two adjacent sides and three angles are known.
5. A quadrilateral can be constructed uniquely if its three sides and two included angles are given.

Data Handling

CHAPTER 5

5.1 Looking for Information

In your day-to-day life, you might have come across information, such as:

(a) Runs made by a batsman in the last 10 test matches.

(b) Number of wickets taken by a bowler in the last 10 ODIs.

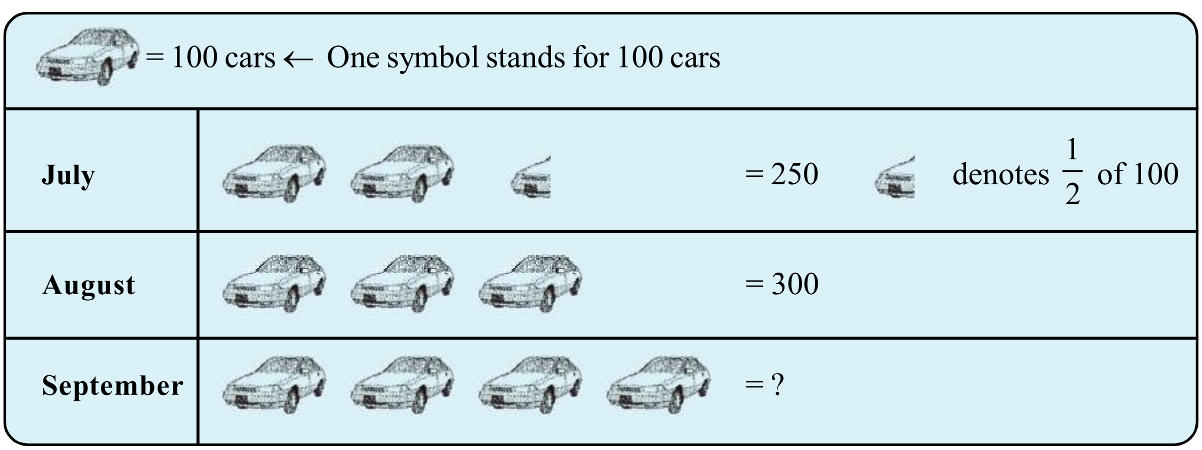
(c) Marks scored by the students of your class in the Mathematics unit test.

(d) Number of story books read by each of your friends etc.

The information collected in all such cases is called **data**. Data is usually collected in the context of a situation that we want to study. For example, a teacher may like to know the average height of students in her class. To find this, she will write the heights of all the students in her class, organise the data in a systematic manner and then interpret it accordingly.

Sometimes, data is represented **graphically** to give a clear idea of what it represents. Do you remember the different types of graphs which we have learnt in earlier classes?

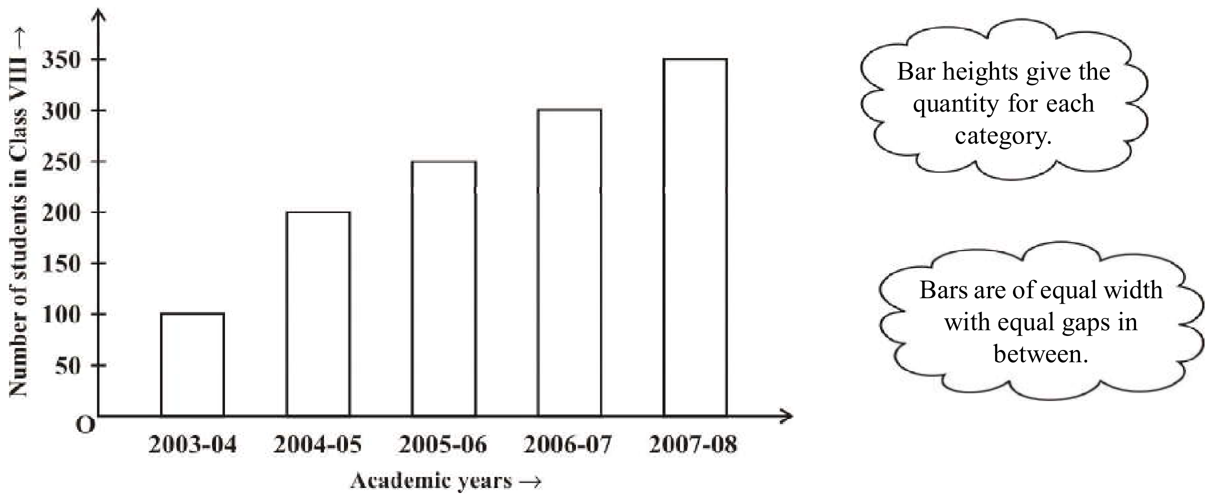
1. A Pictograph: Pictorial representation of data using symbols.



(i) How many cars were produced in the month of July?

(ii) In which month were maximum number of cars produced?

2. **A bar graph:** A display of information using bars of uniform width, their heights being proportional to the respective values.



(i) What is the information given by the bar graph?

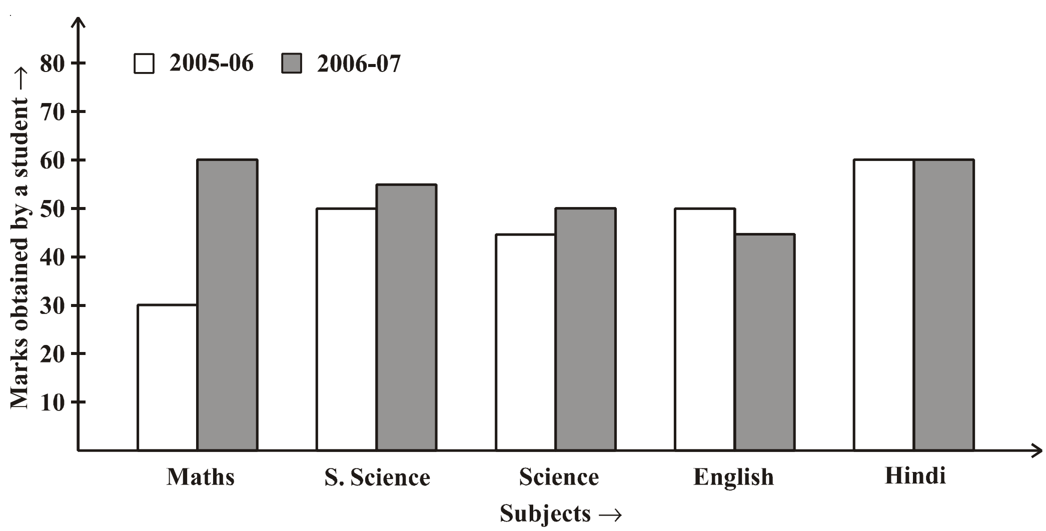
(ii) In which year is the increase in the number of students maximum?

(iii) In which year is the number of students maximum?

(iv) State whether true or false:

'The number of students during 2005-06 is twice that of 2003-04.'

3. **Double Bar Graph:** A bar graph showing two sets of data simultaneously. It is useful for the comparison of the data.



(i) What is the information given by the double bar graph?

(ii) In which subject has the performance improved the most?

(iii) In which subject has the performance deteriorated?

(iv) In which subject is the performance at par?

THINK, DISCUSS AND WRITE

If we change the position of any of the bars of a bar graph, would it change the information being conveyed? Why?

TRY THESE

Draw an appropriate graph to represent the given information.

1.

Month	July	August	September	October	November	December
Number of watches sold	1000	1500	1500	2000	2500	1500

2.

Children who prefer	School A	School B	School C
Walking	40	55	15
Cycling	45	25	35

3. Percentage wins in ODI by 8 top cricket teams.

Teams	From Champions Trophy to World Cup-06	Last 10 ODI in 07
South Africa	75%	78%
Australia	61%	40%
Sri Lanka	54%	38%
New Zealand	47%	50%
England	46%	50%
Pakistan	45%	44%
West Indies	44%	30%
India	43%	56%

5.2 Organising Data

Usually, data available to us is in an unorganised form called **raw data**. To draw meaningful inferences, we need to organise the data systematically. For example, a group of students was asked for their favourite subject. The results were as listed below:

Art, Mathematics, Science, English, Mathematics, Art, English, Mathematics, English, Art, Science, Art, Science, Science, Mathematics, Art, English, Art, Science, Mathematics, Science, Art.

Which is the most liked subject and the one least liked?

It is not easy to answer the question looking at the choices written haphazardly. We arrange the data in Table 5.1 using tally marks.

Table 5.1

Subject	Tally Marks	Number of Students
Art	𝍸 𝍷𝍷	7
Mathematics	𝍸	5
Science	𝍸 𝍷	6
English	𝍷𝍷𝍷𝍷	4

The number of tallies before each subject gives the number of students who like that particular subject.

This is known as the **frequency** of that subject.

Frequency gives the number of times that a particular entry occurs.

From Table 5.1, Frequency of students who like English is 4

Frequency of students who like Mathematics is 5

The table made is known as **frequency distribution table** as it gives the number of times an entry occurs.

TRY THESE

1. A group of students were asked to say which animal they would like most to have as a pet. The results are given below:
dog, cat, cat, fish, cat, rabbit, dog, cat, rabbit, dog, cat, dog, dog, dog, cat, cow, fish, rabbit, dog, cat, dog, cat, cat, dog, rabbit, cat, fish, dog.
Make a frequency distribution table for the same.

5.3 Grouping Data

The data regarding choice of subjects showed the occurrence of each of the entries several times. For example, Art is liked by 7 students, Mathematics is liked by 5 students and so on (Table 5.1). This information can be displayed graphically using a pictograph or a bargraph. Sometimes, however, we have to deal with a large data. For example, consider the following marks (out of 50) obtained in Mathematics by 60 students of Class VIII:

21, 10, 30, 22, 33, 5, 37, 12, 25, 42, 15, 39, 26, 32, 18, 27, 28, 19, 29, 35, 31, 24, 36, 18, 20, 38, 22, 44, 16, 24, 10, 27, 39, 28, 49, 29, 32, 23, 31, 21, 34, 22, 23, 36, 24, 36, 33, 47, 48, 50, 39, 20, 7, 16, 36, 45, 47, 30, 22, 17.

If we make a frequency distribution table for each observation, then the table would be too long, so, for convenience, we make groups of observations say, 0-10, 10-20 and so on, and obtain a frequency distribution of the number of observations falling in each group. Thus, the frequency distribution table for the above data can be.

Table 5.2

Groups	Tally Marks	Frequency
0-10	||	2
10-20	𝍸 𝍸	10
20-30	𝍸 𝍸 𝍸 𝍸 |	21
30-40	𝍸 𝍸 𝍸 ||||	19
40-50	𝍸 ||	7
50-60	|	1
	Total	**60**

Data presented in this manner is said to be **grouped** and the distribution obtained is called **grouped frequency distribution**. It helps us to draw meaningful inferences like –

(1) Most of the students have scored between 20 and 40.

(2) Eight students have scored more than 40 marks out of 50 and so on.

Each of the groups 0-10, 10-20, 20-30, etc., is called a **Class Interval** (or briefly a class).

Observe that 10 occurs in both the classes, i.e., 0-10 as well as 10-20. Similarly, 20 occurs in classes 10-20 and 20-30. But it is not possible that an observation (say 10 or 20) can belong simultaneously to two classes. To avoid this, we adopt the convention that the common observation will belong to the higher class, i.e., 10 belongs to the class interval 10-20 (and not to 0-10). Similarly, 20 belongs to 20-30 (and not to 10-20). In the class interval, 10-20, 10 is called the **lower class limit** and 20 is called the **upper class limit**. Similarly, in the class interval 20-30, 20 is the lower class limit and 30 is the upper class limit. Observe that the difference between the upper class limit and lower class limit for each of the class intervals 0-10, 10-20, 20-30 etc., is equal, (10 in this case). This difference between the upper class limit and lower class limit is called the **width** or **size** of the class interval.

TRY THESE

1. Study the following frequency distribution table and answer the questions given below.

Frequency Distribution of Daily Income of 550 workers of a factory

Table 5.3

Class Interval (Daily Income in ₹)	Frequency (Number of workers)
100-125	45
125-150	25

150-175	55
175-200	125
200-225	140
225-250	55
250-275	35
275-300	50
300-325	20
Total	**550**

(i) What is the size of the class intervals?
(ii) Which class has the highest frequency?
(iii) Which class has the lowest frequency?
(iv) What is the upper limit of the class interval 250-275?
(v) Which two classes have the same frequency?

2. Construct a frequency distribution table for the data on weights (in kg) of 20 students of a class using intervals 30-35, 35-40 and so on.

   40, 38, 33, 48, 60, 53, 31, 46, 34, 36, 49, 41, 55, 49, 65, 42, 44, 47, 38, 39.

5.3.1 Bars with a difference

Let us again consider the grouped frequency distribution of the marks obtained by 60 students in Mathematics test. (Table 5.4)

Table 5.4

Class Interval	Frequency
0-10	2
10-20	10
20-30	21
30-40	19
40-50	7
50-60	1
Total	**60**

This is displayed graphically as in the adjoining graph (Fig 5.1).

Is this graph in any way different from the bar graphs which you have drawn in Class VII? Observe that, here we have represented the groups of observations (i.e., class intervals)

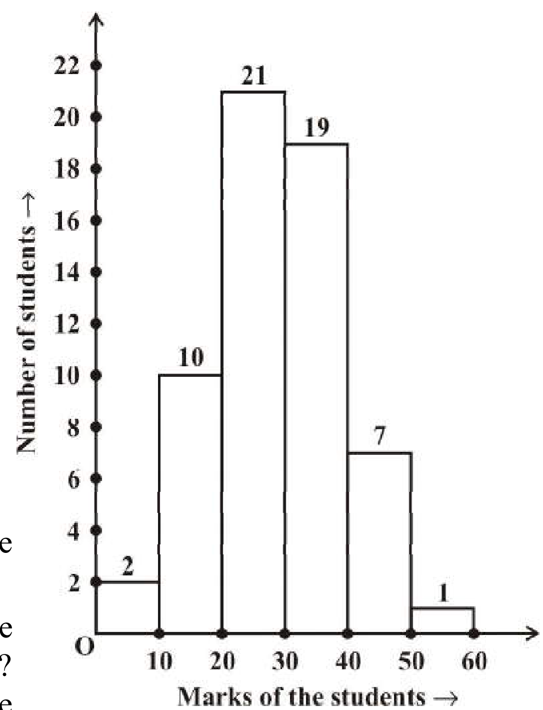


Fig 5.1

on the horizontal axis. The **height** of the bars show the **frequency** of the class-interval. Also, there is no gap between the bars as there is no gap between the class-intervals.

The graphical representation of data in this manner is called a **histogram**.

The following graph is another histogram (Fig 5.2).

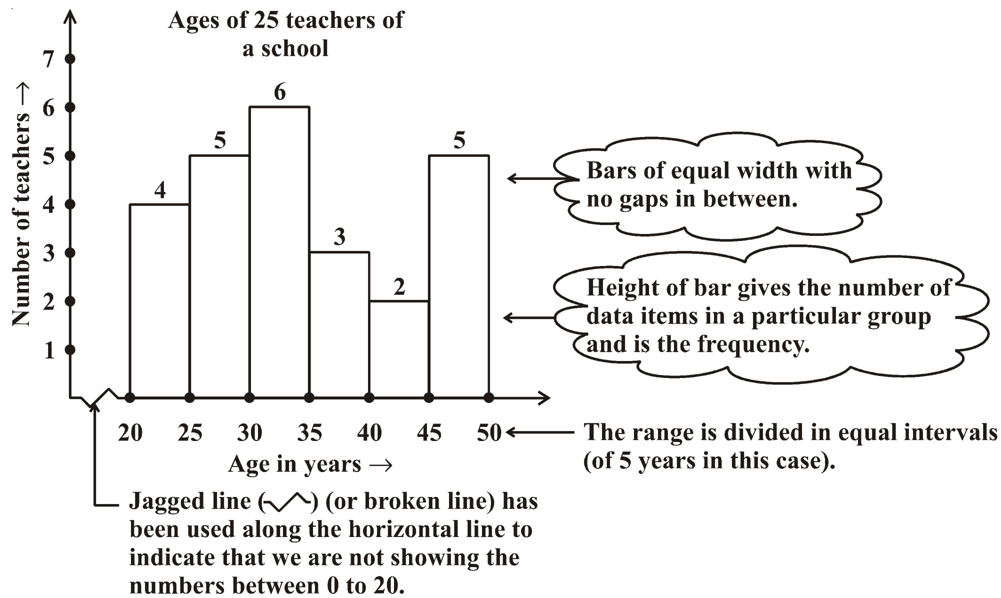


Fig 5.2

From the bars of this histogram, we can answer the following questions:

(i) How many teachers are of age 45 years or more but less than 50 years?

(ii) How many teachers are of age less than 35 years?

TRY THESE

1. Observe the histogram (Fig 5.3) and answer the questions given below.

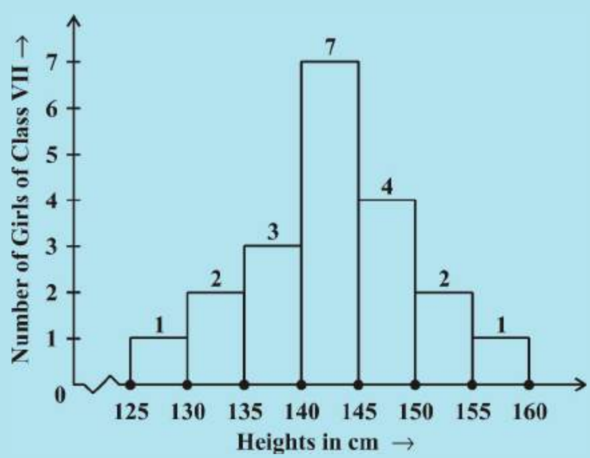


Fig 5.3

(i) What information is being given by the histogram?

(ii) Which group contains maximum girls?

(iii) How many girls have a height of 145 cms and more?

(iv) If we divide the girls into the following three categories, how many would there be in each?

150 cm and more — Group A
140 cm to less than 150 cm — Group B
Less than 140 cm — Group C

EXERCISE 5.1

1. For which of these would you use a histogram to show the data?

(a) The number of letters for different areas in a postman's bag.

(b) The height of competitors in an athletics meet.

(c) The number of cassettes produced by 5 companies.

(d) The number of passengers boarding trains from 7:00 a.m. to 7:00 p.m. at a station.

Give reasons for each.

2. The shoppers who come to a departmental store are marked as: man (M), woman (W), boy (B) or girl (G). The following list gives the shoppers who came during the first hour in the morning:

W W W G B W W M G G M M W W W W G B M W B G G M W W M M W W
W M W B W G M W W W W G W M M W W M W G W M G W M M B G G W

Make a frequency distribution table using tally marks. Draw a bar graph to illustrate it.

3. The weekly wages (in ₹) of 30 workers in a factory are.

830, 835, 890, 810, 835, 836, 869, 845, 898, 890, 820, 860, 832, 833, 855, 845, 804, 808, 812, 840, 885, 835, 835, 836, 878, 840, 868, 890, 806, 840

Using tally marks make a frequency table with intervals as 800–810, 810–820 and so on.

4. Draw a histogram for the frequency table made for the data in Question 3, and answer the following questions.

(i) Which group has the maximum number of workers?

(ii) How many workers earn ₹ 850 and more?

(iii) How many workers earn less than ₹ 850?

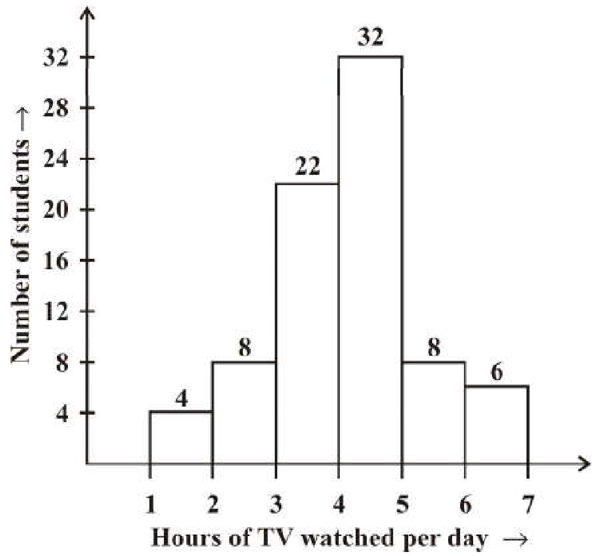
5. The number of hours for which students of a particular class watched television during holidays is shown through the given graph.

Answer the following.

(i) For how many hours did the maximum number of students watch TV?

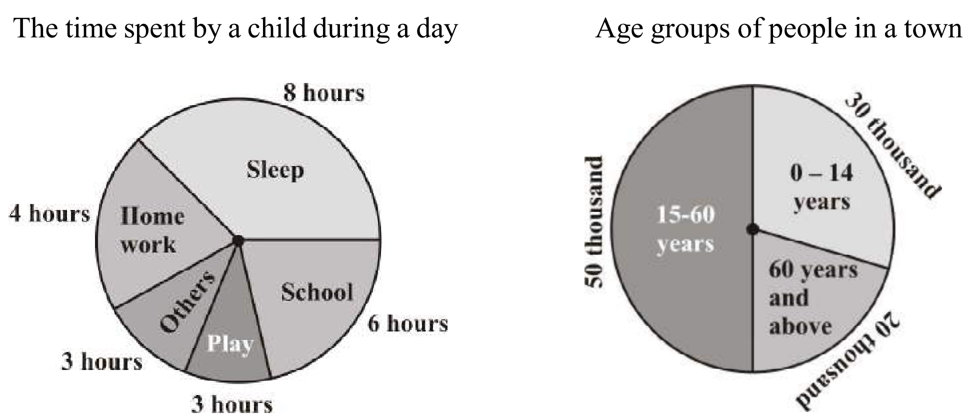
(ii) How many students watched TV for less than 4 hours?

(iii) How many students spent more than 5 hours in watching TV?



5.4 Circle Graph or Pie Chart

Have you ever come across data represented in circular form as shown (Fig 5.4)?



(i) **Fig 5.4** (ii)

These are called **circle graphs**. A circle graph shows the relationship between a whole and its parts. Here, the whole circle is divided into sectors. The size of each sector is proportional to the activity or information it represents.

For example, in the above graph, the proportion of the sector for hours spent in sleeping

$$= \frac{\text{number of sleeping hours}}{\text{whole day}} = \frac{8 \text{ hours}}{24 \text{ hours}} = \frac{1}{3}$$

So, this sector is drawn as $\frac{1}{3}$rd part of the circle. Similarly, the proportion of the sector

for hours spent in school $= \frac{\text{number of school hours}}{\text{whole day}} = \frac{6 \text{ hours}}{24 \text{ hours}} = \frac{1}{4}$

So this sector is drawn $\frac{1}{4}$th of the circle. Similarly, the size of other sectors can be found.

Add up the fractions for all the activities. Do you get the total as one?

A circle graph is also called a **pie chart**.

TRY THESE

1. Each of the following pie charts (Fig 5.5) gives you a different piece of information about your class. Find the fraction of the circle representing each of these information.

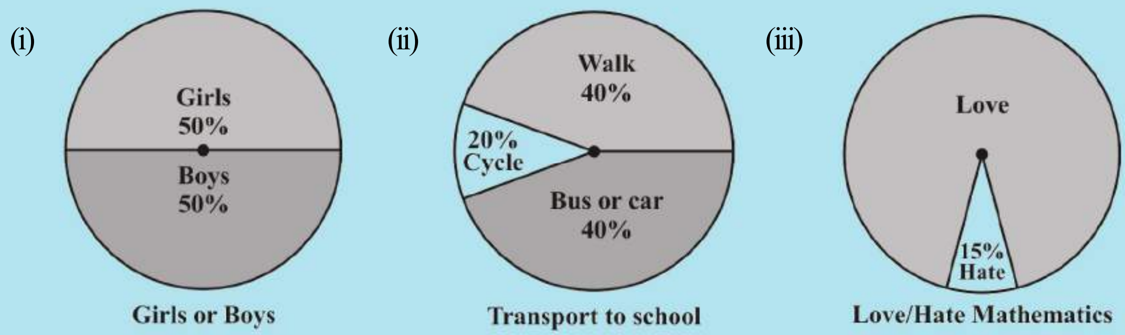
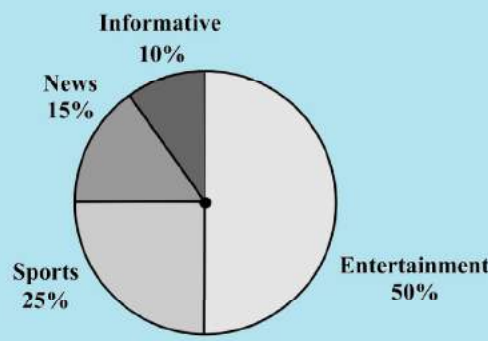


Fig 5.5

2. Answer the following questions based on the pie chart given (Fig 5.6).
   (i) Which type of programmes are viewed the most?
   (ii) Which two types of programmes have number of viewers equal to those watching sports channels?



Viewers watching different types of channels on T.V.

Fig 5.6

5.4.1 Drawing pie charts

The favourite flavours of ice-creams for students of a school is given in percentages as follows.

Flavours	Percentage of students Preferring the flavours
Chocolate	50%
Vanilla	25%
Other flavours	25%

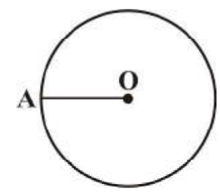
Let us represent this data in a pie chart.

The total angle at the centre of a circle is 360°. The central angle of the sectors will be a fraction of 360°. We make a table to find the central angle of the sectors (Table 5.5).

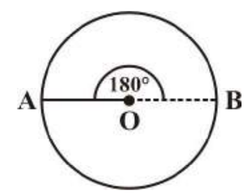
Table 5.5

Flavours	Students in per cent preferring the flavours	In fractions	Fraction of 360°
Chocolate	50%	$\frac{50}{100} = \frac{1}{2}$	$\frac{1}{2}$ of $360° = 180°$
Vanilla	25%	$\frac{25}{100} = \frac{1}{4}$	$\frac{1}{4}$ of $360° = 90°$
Other flavours	25%	$\frac{25}{100} = \frac{1}{4}$	$\frac{1}{4}$ of $360° = 90°$

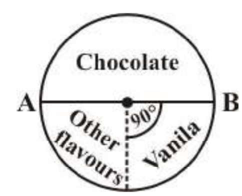
1. Draw a circle with any convenient radius. Mark its centre (O) and a radius (OA).



2. The angle of the sector for chocolate is 180°. Use the protractor to draw $\angle AOB = 180°$.



3. Continue marking the remaining sectors.



Example 1: Adjoining pie chart (Fig 5.7) gives the expenditure (in percentage) on various items and savings of a family during a month.

(i) On which item, the expenditure was maximum?

(ii) Expenditure on which item is equal to the total savings of the family?

(iii) If the monthly savings of the family is ₹ 3000, what is the monthly expenditure on clothes?

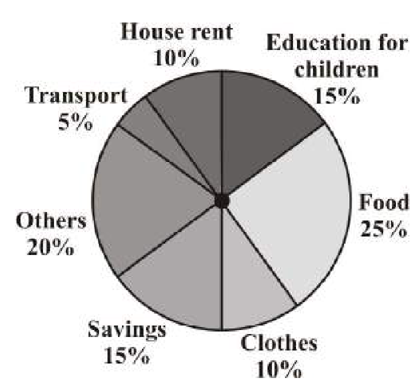


Fig 5.7

Solution:

(i) Expenditure is maximum on food.

(ii) Expenditure on Education of children is the same (i.e., 15%) as the savings of the family.

(iii) 15% represents ₹ 3000

Therefore, 10% represents ₹ $\frac{3000}{15} \times 10$ = ₹ 2000

Example 2: On a particular day, the sales (in rupees) of different items of a baker's shop are given below.

ordinary bread	:	320
fruit bread	:	80
cakes and pastries	:	160
biscuits	:	120
others	:	40
Total	**:**	**720**

Draw a pie chart for this data.

Solution: We find the central angle of each sector. Here the total sale = ₹ 720. We thus have this table.

Item	Sales (in ₹)	In Fraction	Central Angle
Ordinary Bread	320	$\frac{320}{720} = \frac{4}{9}$	$\frac{4}{9} \times 360° = 160°$
Biscuits	120	$\frac{120}{720} = \frac{1}{6}$	$\frac{1}{6} \times 360° = 60°$
Cakes and pastries	160	$\frac{160}{720} = \frac{2}{9}$	$\frac{2}{9} \times 360° = 80°$
Fruit Bread	80	$\frac{80}{720} = \frac{1}{9}$	$\frac{1}{9} \times 360° = 40°$
Others	40	$\frac{40}{720} = \frac{1}{18}$	$\frac{1}{18} \times 360° = 20°$

Now, we make the pie chart (Fig 5.8):

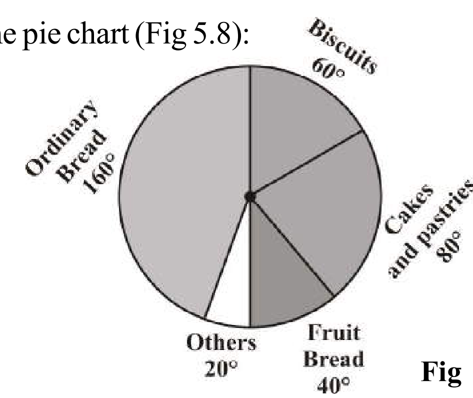


Fig 5.8

TRY THESE

Draw a pie chart of the data given below.

The time spent by a child during a day.

Sleep — 8 hours
School — 6 hours
Home work — 4 hours
Play — 4 hours
Others — 2 hours

THINK, DISCUSS AND WRITE

Which form of graph would be appropriate to display the following data.

1. Production of food grains of a state.

Year	2001	2002	2003	2004	2005	2006
Production (in lakh tons)	60	50	70	55	80	85

2. Choice of food for a group of people.

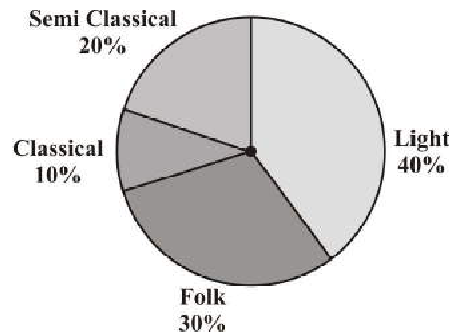
Favourite food	Number of people
North Indian	30
South Indian	40
Chinese	25
Others	25
Total	**120**

3. The daily income of a group of a factory workers.

Daily Income (in Rupees)	Number of workers (in a factory)
75-100	45
100-125	35
125-150	55
150-175	30
175-200	50
200-225	125
225-250	140
Total	**480**

EXERCISE 5.2

1. A survey was made to find the type of music that a certain group of young people liked in a city. Adjoining pie chart shows the findings of this survey.



From this pie chart answer the following:
   (i) If 20 people liked classical music, how many young people were surveyed?
   (ii) Which type of music is liked by the maximum number of people?
   (iii) If a cassette company were to make 1000 CD's, how many of each type would they make?

2. A group of 360 people were asked to vote for their favourite season from the three seasons rainy, winter and summer.
   (i) Which season got the most votes?
   (ii) Find the central angle of each sector.
   (iii) Draw a pie chart to show this information.

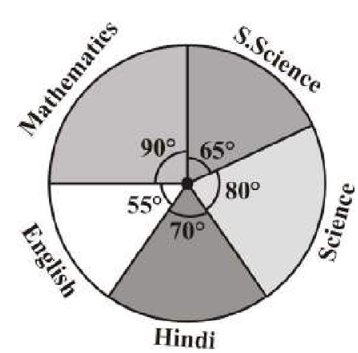
Season	No. of votes
Summer 	90
Rainy 	120
Winter 	150

3. Draw a pie chart showing the following information. The table shows the colours preferred by a group of people.

Colours	Number of people
Blue	18
Green	9
Red	6
Yellow	3
Total	**36**

Find the proportion of each sector. For example, Blue is $\frac{18}{36} = \frac{1}{2}$; Green is $\frac{9}{36} = \frac{1}{4}$ and so on. Use this to find the corresponding angles.

4. The adjoining pie chart gives the marks scored in an examination by a student in Hindi, English, Mathematics, Social Science and Science. If the total marks obtained by the students were 540, answer the following questions.
   (i) In which subject did the student score 105 marks?
   (**Hint:** for 540 marks, the central angle = 360°. So, for 105 marks, what is the central angle?)
   (ii) How many more marks were obtained by the student in Mathematics than in Hindi?
   (iii) Examine whether the sum of the marks obtained in Social Science and Mathematics is more than that in Science and Hindi.
   (**Hint:** Just study the central angles).



5. The number of students in a hostel, speaking different languages is given below. Display the data in a pie chart.

Language	Hindi	English	Marathi	Tamil	Bengali	Total
Number of students	40	12	9	7	4	72

5.5 Chance and Probability

Sometimes it happens that during rainy season, you carry a raincoat every day and it does not rain for many days. However, by chance, one day you forget to take the raincoat and it rains heavily on that day.

Sometimes it so happens that a student prepares 4 chapters out of 5, very well for a test. But a major question is asked from the chapter that she left unprepared.

Everyone knows that a particular train runs in time but the day you reach well in time it is late!

You face a lot of situations such as these where you take a chance and it does not go the way you want it to. Can you give some more examples? These are examples where the chances of a certain thing happening or not happening are not equal. The chances of the train being in time or being late are not the same. When you buy a ticket which is wait listed, you do take a chance. You hope that it might get confirmed by the time you travel.

We however, consider here certain experiments whose results have an equal chance of occurring.

5.5.1 Getting a result

You might have seen that before a cricket match starts, captains of the two teams go out to toss a coin to decide which team will bat first.

What are the possible results you get when a coin is tossed? Of course, Head or Tail.

Imagine that you are the captain of one team and your friend is the captain of the other team. You toss a coin and ask your friend to make the call. Can you control the result of the toss? Can you get a head if you want one? Or a tail if you want that? No, that is not possible. Such an experiment is called a **random experiment**. Head or Tail are the two **outcomes** of this experiment.

TRY THESE

1. If you try to start a scooter, what are the possible outcomes?
2. When a die is thrown, what are the six possible outcomes?

3. When you spin the wheel shown, what are the possible outcomes? (Fig 5.9) List them.

   (Outcome here means the sector at which the pointer stops).

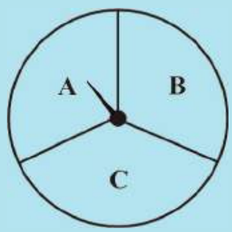


Fig 5.9

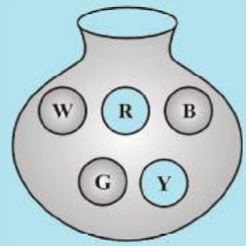


Fig 5.10

4. You have a bag with five identical balls of different colours and you are to pull out (draw) a ball without looking at it; list the outcomes you would get (Fig 5.10).

THINK, DISCUSS AND WRITE

In throwing a die:

- Does the first player have a greater chance of getting a six?
- Would the player who played after him have a lesser chance of getting a six?
- Suppose the second player got a six. Does it mean that the third player would not have a chance of getting a six?

5.5.2 Equally likely outcomes:

A coin is tossed several times and the number of times we get head or tail is noted. Let us look at the result sheet where we keep on increasing the tosses:

Number of tosses	Tally marks (H)	Number of heads	Tally mark (T)	Number of tails
50	卌 卌 卌 卌 卌 ||	27	卌 卌 卌 卌 |||	23
60	卌 卌 卌 卌 卌 |||	28	卌 卌 卌 卌 卌 卌 ||	32
70	...	33	...	37
80	...	38	...	42
90	...	44	...	46
100	...	48	...	52

Observe that as you increase the number of tosses more and more, the number of heads and the number of tails come closer and closer to each other.

This could also be done with a die, when tossed a large number of times. Number of each of the six outcomes become almost equal to each other.

In such cases, we may say that the different outcomes of the experiment are equally likely. This means that each of the outcomes has the same chance of occurring.

5.5.3 Linking chances to probability

Consider the experiment of tossing a coin once. What are the outcomes? There are only two outcomes – Head or Tail. Both the outcomes are equally likely. Likelihood of getting a head is one out of two outcomes, i.e., $\frac{1}{2}$. In other words, we say that the probability of getting a head $= \frac{1}{2}$. What is the probability of getting a tail?

Now take the example of throwing a die marked with 1, 2, 3, 4, 5, 6 on its faces (one number on one face). If you throw it once, what are the outcomes?

The outcomes are: 1, 2, 3, 4, 5, 6. Thus, there are six equally likely outcomes.

What is the probability of getting the outcome '2'?

It is $\dfrac{1 \leftarrow \text{Number of outcomes giving 2}}{6 \leftarrow \text{Number of equally likely outcomes.}}$

What is the probability of getting the number 5? What is the probability of getting the number 7? What is the probability of getting a number 1 through 6?

5.5.4 Outcomes as events

Each outcome of an experiment or a collection of outcomes make an **event**.

For example in the experiment of tossing a coin, getting a Head is an event and getting a Tail is also an event.

In case of throwing a die, getting each of the outcomes 1, 2, 3, 4, 5 or 6 is an event.

Is getting an even number an event? Since an even number could be 2, 4 or 6, getting an even number is also an event. What will be the probability of getting an even number?

It is $\frac{3}{6}$ ← Number of outcomes that make the event
← Total number of outcomes of the experiment.

Example 3: A bag has 4 red balls and 2 yellow balls. (The balls are identical in all respects other than colour). A ball is drawn from the bag without looking into the bag. What is probability of getting a red ball? Is it more or less than getting a yellow ball?

Solution: There are in all (4 + 2 =) 6 outcomes of the event. Getting a red ball consists of 4 outcomes. (Why?)

Therefore, the probability of getting a red ball is $\frac{4}{6} = \frac{2}{3}$. In the same way the probability of getting a yellow ball $= \frac{2}{6} = \frac{1}{3}$ (Why?). Therefore, the probability of getting a red ball is more than that of getting a yellow ball.

TRY THESE

Suppose you spin the wheel

1. (i) List the number of outcomes of getting a green sector and not getting a green sector on this wheel (Fig 5.11).
   (ii) Find the probability of getting a green sector.
   (iii) Find the probability of not getting a green sector.

Fig 5.11

5.5.5 Chance and probability related to real life

We talked about the chance that it rains just on the day when we do not carry a rain coat.

What could you say about the chance in terms of probability? Could it be one in 10 days during a rainy season? The probability that it rains is then $\frac{1}{10}$. The probability that it does not rain $= \frac{9}{10}$. (Assuming raining or not raining on a day are equally likely)

The use of probability is made in various cases in real life.

1. To find characteristics of a large group by using a small part of the group.

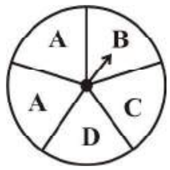
   For example, during elections 'an exit poll' is taken. This involves asking the people whom they have voted for, when they come out after voting at the centres which are chosen off hand and distributed over the whole area. This gives an idea of chance of winning of each candidate and predictions are made based on it accordingly.

2. Metrological Department predicts weather by observing trends from the data over many years in the past.

EXERCISE 5.3

1. List the outcomes you can see in these experiments.

(a) Spinning a wheel



(b) Tossing two coins together

2. When a die is thrown, list the outcomes of an event of getting

(i) (a) a prime number (b) not a prime number.

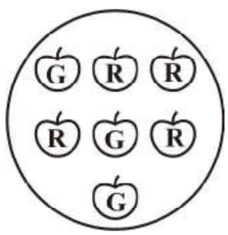
(ii) (a) a number greater than 5 (b) a number not greater than 5.

3. Find the.

(a) Probability of the pointer stopping on D in (Question 1-(a))?

(b) Probability of getting an ace from a well shuffled deck of 52 playing cards?

(c) Probability of getting a red apple. (See figure below)



4. Numbers 1 to 10 are written on ten separate slips (one number on one slip), kept in a box and mixed well. One slip is chosen from the box without looking into it. What is the probability of .

(i) getting a number 6?

(ii) getting a number less than 6?

(iii) getting a number greater than 6?

(iv) getting a 1-digit number?

5. If you have a spinning wheel with 3 green sectors, 1 blue sector and 1 red sector, what is the probability of getting a green sector? What is the probability of getting a non blue sector?

6. Find the probabilities of the events given in Question 2.

WHAT HAVE WE DISCUSSED?

1. Data mostly available to us in an unorganised form is called **raw data**.

2. In order to draw meaningful inferences from any data, we need to organise the data systematically.

3. **Frequency** gives the number of times that a particular entry occurs.
4. Raw data can be 'grouped' and presented systematically through 'grouped frequency distribution'.
5. Grouped data can be presented using **histogram**. Histogram is a type of bar diagram, where the class intervals are shown on the horizontal axis and the heights of the bars show the frequency of the class interval. Also, there is no gap between the bars as there is no gap between the class intervals.
6. Data can also presented using **circle graph** or **pie chart**. A circle graph shows the relationship between a whole and its part.
7. There are certain experiments whose outcomes have an equal chance of occurring.
8. A **random experiment** is one whose outcome cannot be predicted exactly in advance.
9. Outcomes of an experiment are **equally likely** if each has the same chance of occurring.
10. **Probability of an event** $= \dfrac{\text{Number of outcomes that make an event}}{\text{Total number of outcomes of the experiment}}$, when the outcomes are equally likely.
11. One or more outcomes of an experiment make an **event**.
12. Chances and probability are related to real life.

CHAPTER

6

Squares and Square Roots

6.1 Introduction

You know that the area of a square = side × side (where 'side' means 'the length of a side'). Study the following table.

Side of a square (in cm)	Area of the square (in cm^2)
1	$1 \times 1 = 1 = 1^2$
2	$2 \times 2 = 4 = 2^2$
3	$3 \times 3 = 9 = 3^2$
5	$5 \times 5 = 25 = 5^2$
8	$8 \times 8 = 64 = 8^2$
a	$a \times a = a^2$

What is special about the numbers 4, 9, 25, 64 and other such numbers?

Since, 4 can be expressed as $2 \times 2 = 2^2$, 9 can be expressed as $3 \times 3 = 3^2$, all such numbers can be expressed as the product of the number with itself.

Such numbers like 1, 4, 9, 16, 25, ... are known as **square numbers**.

In general, if a natural number m can be expressed as n^2, where n is also a natural number, then m is a **square number**. Is 32 a square number?

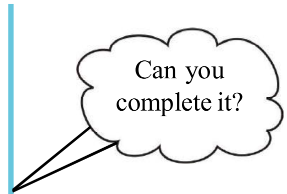
We know that $5^2 = 25$ and $6^2 = 36$. If 32 is a square number, it must be the square of a natural number between 5 and 6. But there is no natural number between 5 and 6.

Therefore 32 is not a square number.

Consider the following numbers and their squares.

Number	Square
1	$1 \times 1 = 1$
2	$2 \times 2 = 4$

3	3 × 3 = 9
4	4 × 4 = 16
5	5 × 5 = 25
6	-----------
7	-----------
8	-----------
9	-----------
10	-----------



From the above table, can we enlist the square numbers between 1 and 100? Are there any natural square numbers upto 100 left out?

You will find that the rest of the numbers are not square numbers.

The numbers 1, 4, 9, 16 ... are square numbers. These numbers are also called **perfect squares**.

TRY THESE

1. Find the perfect square numbers between (i) 30 and 40 (ii) 50 and 60

6.2 Properties of Square Numbers

Following table shows the squares of numbers from 1 to 20.

Number	Square	Number	Square
1	1	11	121
2	4	12	144
3	9	13	169
4	16	14	196
5	25	15	225
6	36	16	256
7	49	17	289
8	64	18	324
9	81	19	361
10	100	20	400

Study the square numbers in the above table. What are the ending digits (that is, digits in the one's place) of the square numbers? All these numbers end with 0, 1, 4, 5, 6 or 9 at unit's place. None of these end with 2, 3, 7 or 8 at unit's place.

Can we say that if a number ends in 0, 1, 4, 5, 6 or 9, then it must be a square number? Think about it.

TRY THESE

1. Can we say whether the following numbers are perfect squares? How do we know?
   (i) 1057 (ii) 23453 (iii) 7928 (iv) 222222
   (v) 1069 (vi) 2061

Write five numbers which you can decide by looking at their one's digit that they are not square numbers.

2. Write five numbers which you cannot decide just by looking at their unit's digit (or one's place) whether they are square numbers or not.

- Study the following table of some numbers and their squares and observe the one's place in both.

Table 1

Number	Square	Number	Square	Number	Square
1	1	11	121	21	441
2	4	12	144	22	484
3	9	13	169	23	529
4	16	14	196	24	576
5	25	15	225	25	625
6	36	16	256	30	900
7	49	17	289	35	1225
8	64	18	324	40	1600
9	81	19	361	45	2025
10	100	20	400	50	2500

The following square numbers end with digit 1.

Square	Number
1	1
81	9
121	11
361	19
441	21

TRY THESE

Which of 123^2, 77^2, 82^2, 161^2, 109^2 would end with digit 1?

Write the next two square numbers which end in 1 and their corresponding numbers.

You will see that if a number has 1 or 9 in the unit's place, then it's square ends in 1.

- Let us consider square numbers ending in 6.

Square	Number
16	4
36	6
196	14
256	16

TRY THESE

Which of the following numbers would have digit 6 at unit place.

(i) 19^2 (ii) 24^2 (iii) 26^2

(iv) 36^2 (v) 34^2

We can see that *when a square number ends in 6, the number whose square it is, will have either 4 or 6 in unit's place*.

Can you find more such rules by observing the numbers and their squares (Table 1)?

TRY THESE

What will be the "one's digit" in the square of the following numbers?

(i) 1234 (ii) 26387 (iii) 52698 (iv) 99880
(v) 21222 (vi) 9106

- Consider the following numbers and their squares.

We have one zero

$10^2 = 100$

$20^2 = 400$

$80^2 = 6400$

But we have two zeros

We have two zeros

$100^2 = 10000$

$200^2 = 40000$

$700^2 = 490000$

$900^2 = 810000$

But we have four zeros

If a number contains 3 zeros at the end, how many zeros will its square have ?
What do you notice about the number of zeros at the end of the number and the number of zeros at the end of its square?
Can we say that square numbers can only have even number of zeros at the end?

- See Table 1 with numbers and their squares.
What can you say about the squares of even numbers and squares of odd numbers?

TRY THESE

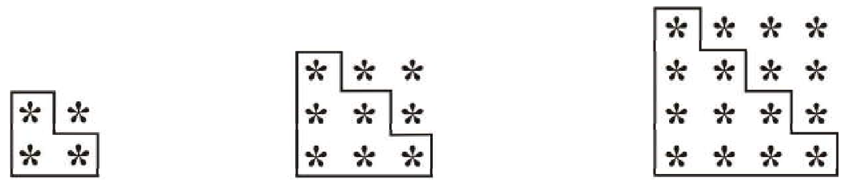
1. The square of which of the following numbers would be an odd number/an even number? Why?
(i) 727 (ii) 158 (iii) 269 (iv) 1980
2. What will be the number of zeros in the square of the following numbers?
(i) 60 (ii) 400

6.3 Some More Interesting Patterns

1. **Adding triangular numbers.**
Do you remember triangular numbers (numbers whose dot patterns can be arranged as triangles)?

*	* * *	* * * * * *	* * * * * * * * * *	* * * * * * * * * * * * * * *
1	3	6	10	15

If we combine two consecutive triangular numbers, we get a square number, like



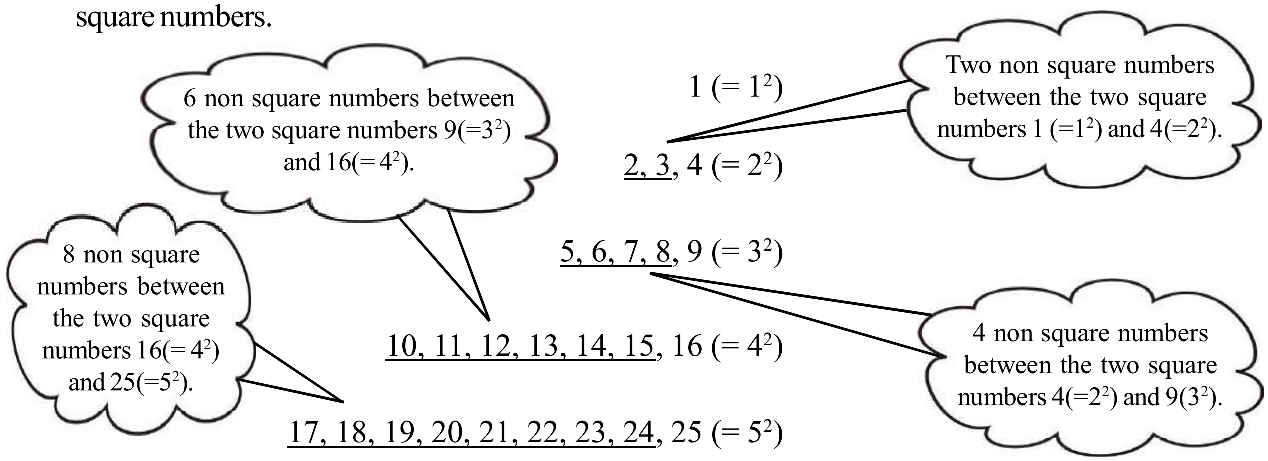
$1 + 3 = 4$
$= 2^2$

$3 + 6 = 9$
$= 3^2$

$6 + 10 = 16$
$= 4^2$

2. Numbers between square numbers

Let us now see if we can find some interesting pattern between two consecutive square numbers.



Between $1^2(=1)$ and $2^2(= 4)$ there are two (i.e., 2×1) non square numbers 2, 3.

Between $2^2(= 4)$ and $3^2(= 9)$ there are four (i.e., 2×2) non square numbers 5, 6, 7, 8.

Now, $3^2 = 9$, $4^2 = 16$

Therefore, $4^2 - 3^2 = 16 - 9 = 7$

Between $9(=3^2)$ and $16(= 4^2)$ the numbers are 10, 11, 12, 13, 14, 15 that is, six non-square numbers which is 1 less than the difference of two squares.

We have $4^2 = 16$ and $5^2 = 25$

Therefore, $5^2 - 4^2 = 9$

Between $16(=4^2)$ and $25(=5^2)$ the numbers are 17, 18, ... , 24 that is, eight non square numbers which is 1 less than the difference of two squares.

Consider 7^2 and 6^2. Can you say how many numbers are there between 6^2 and 7^2? If we think of any natural number n and $(n + 1)$, then,

$$(n + 1)^2 - n^2 = (n^2 + 2n + 1) - n^2 = 2n + 1.$$

We find that between n^2 and $(n + 1)^2$ there are $2n$ numbers which is 1 less than the difference of two squares.

Thus, in general we can say that *there are 2n non perfect square numbers between the squares of the numbers n and* $(n + 1)$. Check for $n = 5$, $n = 6$ etc., and verify.

TRY THESE

1. How many natural numbers lie between 9^2 and 10^2 ? Between 11^2 and 12^2?
2. How many non square numbers lie between the following pairs of numbers
   (i) 100^2 and 101^2 (ii) 90^2 and 91^2 (iii) 1000^2 and 1001^2

3. Adding odd numbers

Consider the following

1 [one odd number]	$= 1 = 1^2$
1 + 3 [sum of first two odd numbers]	$= 4 = 2^2$
1 + 3 + 5 [sum of first three odd numbers]	$= 9 = 3^2$
1 + 3 + 5 + 7 [...]	$= 16 = 4^2$
1 + 3 + 5 + 7 + 9 [...]	$= 25 = 5^2$
1 + 3 + 5 + 7 + 9 + 11 [...]	$= 36 = 6^2$

So we can say that the *sum of first n odd natural numbers is* n^2.

Looking at it in a different way, we can say: 'If the number is a square number, it has to be the sum of successive **odd** numbers starting from 1.

Consider those numbers which are not perfect squares, say 2, 3, 5, 6, ... Can you express these numbers as a sum of successive odd natural numbers beginning from 1?

You will find that these numbers cannot be expressed in this form.

Consider the number 25. Successively subtract 1, 3, 5, 7, 9, ... from it

(i) $25 - 1 = 24$ (ii) $24 - 3 = 21$ (iii) $21 - 5 = 16$ (iv) $16 - 7 = 9$
(v) $9 - 9 = 0$

This means, $25 = 1 + 3 + 5 + 7 + 9$. Also, 25 is a perfect square.

Now consider another number 38, and again do as above.

(i) $38 - 1 = 37$ (ii) $37 - 3 = 34$ (iii) $34 - 5 = 29$ (iv) $29 - 7 = 22$
(v) $22 - 9 = 13$ (vi) $13 - 11 = 2$ (vii) $2 - 13 = -11$

This shows that we are not able to express 38 as the sum of consecutive odd numbers starting with 1. Also, 38 is not a perfect square.

So we can also say that *if a natural number cannot be expressed as a sum of successive odd natural numbers starting with 1, then it is not a perfect square*.

We can use this result to find whether a number is a perfect square or not.

TRY THESE

Find whether each of the following numbers is a perfect square or not?

(i) 121 (ii) 55 (iii) 81
(iv) 49 (v) 69

4. A sum of consecutive natural numbers

Consider the following

$3^2 = 9 = 4 + 5$

$5^2 = 25 = 12 + 13$

$7^2 = 49 = 24 + 25$

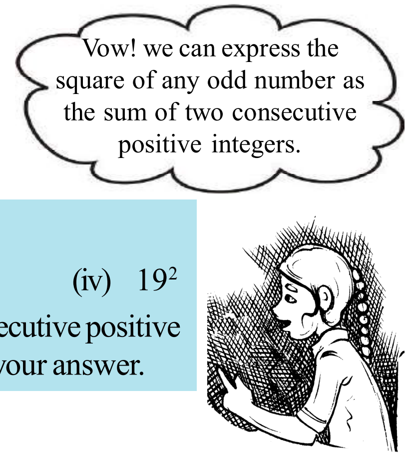
First Number $= \dfrac{3^2 - 1}{2}$

Second Number $= \dfrac{3^2 + 1}{2}$

$9^2 = 81 = 40 + 41$

$11^2 = 121 = 60 + 61$

$15^2 = 225 = 112 + 113$



TRY THESE

1. Express the following as the sum of two consecutive integers.
   (i) 21^2 (ii) 13^2 (iii) 11^2 (iv) 19^2
2. Do you think the reverse is also true, i.e., is the sum of any two consecutive positive integers is perfect square of a number? Give example to support your answer.

5. Product of two consecutive even or odd natural numbers

$11 \times 13 = 143 = 12^2 - 1$

Also $11 \times 13 = (12 - 1) \times (12 + 1)$

Therefore, $11 \times 13 = (12 - 1) \times (12 + 1) = 12^2 - 1$

Similarly, $13 \times 15 = (14 - 1) \times (14 + 1) = 14^2 - 1$

$29 \times 31 = (30 - 1) \times (30 + 1) = 30^2 - 1$

$44 \times 46 = (45 - 1) \times (45 + 1) = 45^2 - 1$

So in general we can say that $(a + 1) \times (a - 1) = a^2 - 1$.

6. Some more patterns in square numbers

Observe the squares of numbers; 1, 11, 111 ... etc. They give a beautiful pattern:

$1^2 =$ 1

$11^2 =$ 1 2 1

$111^2 =$ 1 2 3 2 1

$1111^2 =$ 1 2 3 4 3 2 1

$11111^2 =$ 1 2 3 4 5 4 3 2 1

$11111111^2 =$ 1 2 3 4 5 6 7 8 7 6 5 4 3 2 1

Another interesting pattern.

$7^2 = 49$

$67^2 = 4489$

$667^2 = 444889$

$6667^2 = 44448889$

$66667^2 = 4444488889$

$666667^2 = 444444888889$

The fun is in being able to find out why this happens. May be it would be interesting for you to explore and think about such questions even if the answers come some years later.

TRY THESE

Write the square, making use of the above pattern.

(i) 111111^2 (ii) 1111111^2

TRY THESE

Can you find the square of the following numbers using the above pattern?

(i) 6666667^2 (ii) 66666667^2

EXERCISE 6.1

1. What will be the unit digit of the squares of the following numbers?

(i) 81 (ii) 272 (iii) 799 (iv) 3853
(v) 1234 (vi) 26387 (vii) 52698 (viii) 99880
(ix) 12796 (x) 55555

2. The following numbers are obviously not perfect squares. Give reason.

(i) 1057 (ii) 23453 (iii) 7928 (iv) 222222
(v) 64000 (vi) 89722 (vii) 222000 (viii) 505050

3. The squares of which of the following would be odd numbers?

(i) 431 (ii) 2826 (iii) 7779 (iv) 82004

4. Observe the following pattern and find the missing digits.

$$11^2 = 121$$
$$101^2 = 10201$$
$$1001^2 = 1002001$$
$$100001^2 = 1 \text{ .... } 2 \text{ .... } 1$$
$$10000001^2 = \text{...........................}$$

5. Observe the following pattern and supply the missing numbers.

$$11^2 = 1\ 2\ 1$$
$$101^2 = 1\ 0\ 2\ 0\ 1$$
$$10101^2 = 102030201$$
$$1010101^2 = \text{...........................}$$
$$\text{............}^2 = 10203040504030201$$

6. Using the given pattern, find the missing numbers.

$1^2 + 2^2 + 2^2 = 3^2$
$2^2 + 3^2 + 6^2 = 7^2$
$3^2 + 4^2 + 12^2 = 13^2$
$4^2 + 5^2 + \_^2 = 21^2$
$5^2 + \_^2 + 30^2 = 31^2$
$6^2 + 7^2 + \_^2 = \_\_^2$

To find pattern
Third number is related to first and second number. How?
Fourth number is related to third number. How?

7. Without adding, find the sum.

(i) 1 + 3 + 5 + 7 + 9
(ii) 1 + 3 + 5 + 7 + 9 + I1 + 13 + 15 + 17 +19
(iii) 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23

8. (i) Express 49 as the sum of 7 odd numbers.
(ii) Express 121 as the sum of 11 odd numbers.

9. How many numbers lie between squares of the following numbers?

(i) 12 and 13 (ii) 25 and 26 (iii) 99 and 100

6.4 Finding the Square of a Number

Squares of small numbers like 3, 4, 5, 6, 7, ... etc. are easy to find. But can we find the square of 23 so quickly?

The answer is not so easy and we may need to multiply 23 by 23.

There is a way to find this without having to multiply 23×23.

We know $23 = 20 + 3$

Therefore $23^2 = (20 + 3)^2 = 20(20 + 3) + 3(20 + 3)$

$= 20^2 + 20 \times 3 + 3 \times 20 + 3^2$

$= 400 + 60 + 60 + 9 = 529$

Example 1: Find the square of the following numbers without actual multiplication.

(i) 39 (ii) 42

Solution: (i) $39^2 = (30 + 9)^2 = 30(30 + 9) + 9(30 + 9)$

$= 30^2 + 30 \times 9 + 9 \times 30 + 9^2$

$= 900 + 270 + 270 + 81 = 1521$

(ii) $42^2 = (40 + 2)^2 = 40(40 + 2) + 2(40 + 2)$

$= 40^2 + 40 \times 2 + 2 \times 40 + 2^2$

$= 1600 + 80 + 80 + 4 = 1764$

6.4.1 Other patterns in squares

Consider the following pattern:

$25^2 = 625 = (2 \times 3)$ hundreds + 25

$35^2 = 1225 = (3 \times 4)$ hundreds + 25

$75^2 = 5625 = (7 \times 8)$ hundreds + 25

$125^2 = 15625 = (12 \times 13)$ hundreds + 25

Now can you find the square of 95?

Consider a number with unit digit 5, i.e., $a5$

$(a5)^2 = (10a + 5)^2$

$= 10a(10a + 5) + 5(10a + 5)$

$= 100a^2 + 50a + 50a + 25$

$= 100a(a + 1) + 25$

$= a(a + 1)$ hundred + 25

TRY THESE

Find the squares of the following numbers containing 5 in unit's place.

(i) 15 (ii) 95 (iii) 105 (iv) 205

6.4.2 Pythagorean triplets

Consider the following

$$3^2 + 4^2 = 9 + 16 = 25 = 5^2$$

The collection of numbers 3, 4 and 5 is known as **Pythagorean triplet**. 6, 8, 10 is also a Pythagorean triplet, since

$$6^2 + 8^2 = 36 + 64 = 100 = 10^2$$

Again, observe that

$5^2 + 12^2 = 25 + 144 = 169 = 13^2$. The numbers 5, 12, 13 form another such triplet.

Can you find more such triplets?

For any natural number $m > 1$, we have $(2m)^2 + (m^2 - 1)^2 = (m^2 + 1)^2$. So, $2m$, $m^2 - 1$ and $m^2 + 1$ forms a Pythagorean triplet.

Try to find some more Pythagorean triplets using this form.

Example 2: Write a Pythagorean triplet whose smallest member is 8.

Solution: We can get Pythagorean triplets by using general form $2m$, $m^2 - 1$, $m^2 + 1$.

Let us first take $m^2 - 1 = 8$

So, $m^2 = 8 + 1 = 9$

which gives $m = 3$

Therefore, $2m = 6$ and $m^2 + 1 = 10$

The triplet is thus 6, 8, 10. But 8 is not the smallest member of this.

So, let us try $2m = 8$

then $m = 4$

We get $m^2 - 1 = 16 - 1 = 15$

and $m^2 + 1 = 16 + 1 = 17$

The triplet is 8, 15, 17 with 8 as the smallest member.

Example 3: Find a Pythagorean triplet in which one member is 12.

Solution: If we take $m^2 - 1 = 12$

Then, $m^2 = 12 + 1 = 13$

Then the value of m will not be an integer.

So, we try to take $m^2 + 1 = 12$. Again $m^2 = 11$ will not give an integer value for m.

So, let us take $2m = 12$

then $m = 6$

Thus, $m^2 - 1 = 36 - 1 = 35$ and $m^2 + 1 = 36 + 1 = 37$

Therefore, the required triplet is 12, 35, 37.

Note: All Pythagorean triplets may not be obtained using this form. For example another triplet 5, 12, 13 also has 12 as a member.

EXERCISE 6.2

1. Find the square of the following numbers.

(i) 32 (ii) 35 (iii) 86 (iv) 93

(v) 71 (vi) 46

2. Write a Pythagorean triplet whose one member is.

(i) 6 (ii) 14 (iii) 16 (iv) 18

6.5 Square Roots

Study the following situations.

(a) Area of a square is 144 cm^2. What could be the side of the square?

We know that the area of a square = side^2

If we assume the length of the side to be 'a', then $144 = a^2$

To find the length of side it is necessary to find a number whose square is 144.

(b) What is the length of a diagonal of a square of side 8 cm (Fig 6.1)?

Can we use Pythagoras theorem to solve this ?

We have, $AB^2 + BC^2 = AC^2$

i.e., $8^2 + 8^2 = AC^2$

or $64 + 64 = AC^2$

or $128 = AC^2$

Again to get AC we need to think of a number whose square is 128.

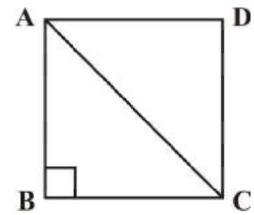


Fig 6.1

(c) In a right triangle the length of the hypotenuse and a side are respectively 5 cm and 3 cm (Fig 6.2).

Can you find the third side?

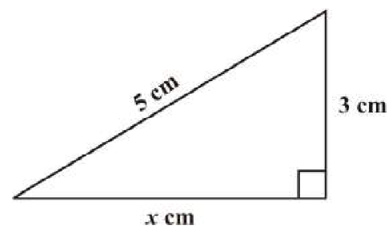


Fig 6.2

Let x cm be the length of the third side.

Using Pythagoras theorem $5^2 = x^2 + 3^2$

$25 - 9 = x^2$

$16 = x^2$

Again, to find x we need a number whose square is 16.

In all the above cases, we need to find a number whose square is known. Finding the number with the known square is known as finding the square root.

6.5.1 Finding square roots

The inverse (opposite) operation of addition is subtraction and the inverse operation of multiplication is division. Similarly, finding the square root is the inverse operation of squaring.

We have, $1^2 = 1$, therefore square root of 1 is 1

$2^2 = 4$, therefore square root of 4 is 2

$3^2 = 9$, therefore square root of 9 is 3

Since $9^2 = 81$,
and $(-9)^2 = 81$
We say that square roots of 81 are 9 and –9.

TRY THESE

(i) $11^2 = 121$. What is the square root of 121?

(ii) $14^2 = 196$. What is the square root of 196?

THINK, DISCUSS AND WRITE

$(-1)^2 = 1$. Is –1, a square root of 1? $(-2)^2 = 4$. Is –2, a square root of 4?

$(-9)^2 = 81$. Is –9 a square root of 81?

From the above, you may say that there are two integral square roots of a perfect square number. In this chapter, we shall take up only positive square root of a natural number. Positive square root of a number is denoted by the symbol $\sqrt{\ }$.

For example: $\sqrt{4} = 2$ (not –2); $\sqrt{9} = 3$ (not –3) etc.

Statement	Inference
$1^2 = 1$	$\sqrt{1} = 1$
$2^2 = 4$	$\sqrt{4} = 2$
$3^2 = 9$	$\sqrt{9} = 3$
$4^2 = 16$	$\sqrt{16} = 4$
$5^2 = 25$	$\sqrt{25} = 5$

Statement	Inference
$6^2 = 36$	$\sqrt{36} = 6$
$7^2 = 49$	$\sqrt{49} = 7$
$8^2 = 64$	$\sqrt{64} = 8$
$9^2 = 81$	$\sqrt{81} = 9$
$10^2 = 100$	$\sqrt{100} = 10$

6.5.2 Finding square root through repeated subtraction

Do you remember that the sum of the first n odd natural numbers is n^2? That is, every square number can be expressed as a sum of successive odd natural numbers starting from 1.

Consider $\sqrt{81}$. Then,

(i) $81 - 1 = 80$ (ii) $80 - 3 = 77$ (iii) $77 - 5 = 72$ (iv) $72 - 7 = 65$
(v) $65 - 9 = 56$ (vi) $56 - 11 = 45$ (vii) $45 - 13 = 32$ (viii) $32 - 15 = 17$
(ix) $17 - 17 = 0$

From 81 we have subtracted successive odd numbers starting from 1 and obtained 0 at 9th step.

Therefore $\sqrt{81} = 9$.

Can you find the square root of 729 using this method? Yes, but it will be time consuming. Let us try to find it in a simpler way.

TRY THESE

By repeated subtraction of odd numbers starting from 1, find whether the following numbers are perfect squares or not? If the number is a perfect square then find its square root.

(i) 121
(ii) 55
(iii) 36
(iv) 49
(v) 90

6.5.3 Finding square root through prime factorisation

Consider the prime factorisation of the following numbers and their squares.

Prime factorisation of a Number	Prime factorisation of its Square
$6 = 2 \times 3$	$36 = 2 \times 2 \times 3 \times 3$
$8 = 2 \times 2 \times 2$	$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$
$12 = 2 \times 2 \times 3$	$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$
$15 = 3 \times 5$	$225 = 3 \times 3 \times 5 \times 5$

How many times does 2 occur in the prime factorisation of 6? Once. How many times does 2 occur in the prime factorisation of 36? Twice. Similarly, observe the occurrence of 3 in 6 and 36 of 2 in 8 and 64 etc.

You will find that each prime factor in the prime factorisation of the square of a number, occurs twice the number of times it occurs in the prime factorisation of the number itself. Let us use this to find the square root of a given square number, say 324.

2	324
2	162
3	81
3	27
3	9
	3

We know that the prime factorisation of 324 is

$$324 = 2 \times 2 \times 3 \times 3 \times 3 \times 3$$

By pairing the prime factors, we get

$$324 = \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{3 \times 3} = 2^2 \times 3^2 \times 3^2 = (2 \times 3 \times 3)^2$$

So, $\sqrt{324} = 2 \times 3 \times 3 = 18$

Similarly can you find the square root of 256? Prime factorisation of 256 is

$$256 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

2	256
2	128
2	64
2	32
2	16
2	8
2	4
	2

By pairing the prime factors we get,

$$256 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} = (2 \times 2 \times 2 \times 2)^2$$

Therefore, $\sqrt{256} = 2 \times 2 \times 2 \times 2 = 16$

Is 48 a perfect square?

We know $48 = \underline{2 \times 2} \times \underline{2 \times 2} \times 3$

Since all the factors are not in pairs so 48 is not a perfect square.

Suppose we want to find the smallest multiple of 48 that is a perfect square, how should we proceed? Making pairs of the prime factors of 48 we see that 3 is the only factor that does not have a pair. So we need to multiply by 3 to complete the pair.

Hence $48 \times 3 = 144$ is a perfect square.

Can you tell by which number should we divide 48 to get a perfect square?

The factor 3 is not in pair, so if we divide 48 by 3 we get $48 \div 3 = 16 = \underline{2 \times 2} \times \underline{2 \times 2}$ and this number 16 is a perfect square too.

Example 4: Find the square root of 6400.

Solution: Write $6400 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{5 \times 5}$

2	6400
2	3200
2	1600
2	800
2	400
2	200
2	100
2	50
5	25
	5

Therefore $\sqrt{6400} = 2 \times 2 \times 2 \times 2 \times 5 = 80$

Example 5: Is 90 a perfect square?

2	90
3	45
3	15
	5

Solution: We have $90 = 2 \times 3 \times 3 \times 5$

The prime factors 2 and 5 do not occur in pairs. Therefore, 90 is not a perfect square. That 90 is not a perfect square can also be seen from the fact that it has only one zero.

Example 6: Is 2352 a perfect square? If not, find the smallest multiple of 2352 which is a perfect square. Find the square root of the new number.

2	2352
2	1176
2	588
2	294
3	147
7	49
	7

Solution: We have $2352 = \underline{2 \times 2} \times \underline{2 \times 2} \times 3 \times \underline{7 \times 7}$

As the prime factor 3 has no pair, 2352 is not a perfect square.

If 3 gets a pair then the number will become perfect square. So, we multiply 2352 by 3 to get,

$$2352 \times 3 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{7 \times 7}$$

Now each prime factor is in a pair. Therefore, $2352 \times 3 = 7056$ is a perfect square. Thus the required smallest multiple of 2352 is 7056 which is a perfect square.

And, $\sqrt{7056} = 2 \times 2 \times 3 \times 7 = 84$

Example 7: Find the smallest number by which 9408 must be divided so that the quotient is a perfect square. Find the square root of the quotient.

Solution: We have, $9408 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times 3 \times \underline{7 \times 7}$

If we divide 9408 by the factor 3, then

$9408 \div 3 = 3136 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{7 \times 7}$ which is a perfect square. (Why?)

Therefore, the required smallest number is 3.

And, $\sqrt{3136} = 2 \times 2 \times 2 \times 7 = 56.$

2	6, 9, 15
3	3, 9, 15
3	1, 3, 5
5	1, 1, 5
	1, 1, 1

Example 8: Find the smallest square number which is divisible by each of the numbers 6, 9 and 15.

Solution: This has to be done in two steps. First find the smallest common multiple and then find the square number needed. The least number divisible by each one of 6, 9 and 15 is their LCM. The LCM of 6, 9 and 15 is $2 \times 3 \times 3 \times 5 = 90$.

Prime factorisation of 90 is $90 = 2 \times \underline{3 \times 3} \times 5$.

We see that prime factors 2 and 5 are not in pairs. Therefore 90 is not a perfect square.

In order to get a perfect square, each factor of 90 must be paired. So we need to make pairs of 2 and 5. Therefore, 90 should be multiplied by 2×5, i.e., 10.

Hence, the required square number is $90 \times 10 = 900$.

EXERCISE 6.3

1. What could be the possible 'one's' digits of the square root of each of the following numbers?
   (i) 9801 (ii) 99856 (iii) 998001 (iv) 657666025
2. Without doing any calculation, find the numbers which are surely not perfect squares.
   (i) 153 (ii) 257 (iii) 408 (iv) 441
3. Find the square roots of 100 and 169 by the method of repeated subtraction.
4. Find the square roots of the following numbers by the Prime Factorisation Method.
   (i) 729 (ii) 400 (iii) 1764 (iv) 4096
   (v) 7744 (vi) 9604 (vii) 5929 (viii) 9216
   (ix) 529 (x) 8100
5. For each of the following numbers, find the smallest whole number by which it should be multiplied so as to get a perfect square number. Also find the square root of the square number so obtained.
   (i) 252 (ii) 180 (iii) 1008 (iv) 2028
   (v) 1458 (vi) 768
6. For each of the following numbers, find the smallest whole number by which it should be divided so as to get a perfect square. Also find the square root of the square number so obtained.
   (i) 252 (ii) 2925 (iii) 396 (iv) 2645
   (v) 2800 (vi) 1620
7. The students of Class VIII of a school donated ₹ 2401 in all, for Prime Minister's National Relief Fund. Each student donated as many rupees as the number of students in the class. Find the number of students in the class.

8. 2025 plants are to be planted in a garden in such a way that each row contains as many plants as the number of rows. Find the number of rows and the number of plants in each row.
9. Find the smallest square number that is divisible by each of the numbers 4, 9 and 10.
10. Find the smallest square number that is divisible by each of the numbers 8, 15 and 20.

6.5.4 Finding square root by division method

When the numbers are large, even the method of finding square root by prime factorisation becomes lengthy and difficult. To overcome this problem we use Long Division Method.

For this we need to determine the number of digits in the square root.
See the following table:

Number	Square	
10	100	which is the smallest 3-digit perfect square
31	961	which is the greatest 3-digit perfect square
32	1024	which is the smallest 4-digit perfect square
99	9801	which is the greatest 4-digit perfect square

So, what can we say about the number of digits in the square root if a perfect square is a 3-digit or a 4-digit number? We can say that, if a perfect square is a 3-digit or a 4-digit number, then its square root will have 2-digits.

Can you tell the number of digits in the square root of a 5-digit or a 6-digit perfect square?

The smallest 3-digit perfect square number is 100 which is the square of 10 and the greatest 3-digit perfect square number is 961 which is the square of 31. The smallest 4-digit square number is 1024 which is the square of 32 and the greatest 4-digit number is 9801 which is the square of 99.

THINK, DISCUSS AND WRITE

Can we say that if a perfect square is of n-digits, then its square root will have $\frac{n}{2}$ digits if n is even or $\frac{(n+1)}{2}$ if n is odd?

The use of the number of digits in square root of a number is useful in the following method:

- Consider the following steps to find the square root of 529.
  Can you estimate the number of digits in the square root of this number?

Step 1 Place a bar over every pair of digits starting from the digit at one's place. If the number of digits in it is odd, then the left-most single digit too will have a bar.

Thus we have, $\overline{5}\,\overline{29}$.

Step 2 Find the largest number whose square is less than or equal to the number under the extreme left bar ($2^2 < 5 < 3^2$). Take this number as the divisor and the quotient with the number under the extreme left bar as the dividend (here 5). Divide and get the remainder (1 in this case).

$$\begin{array}{r|l} & 2 \\ \hline 2 & \overline{5}\,\overline{29} \\ & -4 \\ \hline & 1 \end{array}$$

$$\begin{array}{r|l} & \;\;2 \\ \hline 2 & \overline{5}\,\overline{29} \\ & -4 \\ \hline & 1\,29 \end{array}$$

Step 3 Bring down the number under the next bar (i.e., 29 in this case) to the right of the remainder. So the new dividend is 129.

Step 4 Double the quotient and enter it with a blank on its right.

$$\begin{array}{r|l} & \;\;2 \\ \hline 2 & \overline{5}\,\overline{29} \\ & -4 \\ \hline 4\_ & 129 \end{array}$$

Step 5 Guess a largest possible digit to fill the blank which will also become the new digit in the quotient, such that when the new divisor is multiplied to the new quotient the product is less than or equal to the dividend.

In this case $42 \times 2 = 84$.

As $43 \times 3 = 129$ so we choose the new digit as 3. Get the remainder.

$$\begin{array}{r|l} & \;\;23 \\ \hline 2 & \overline{5}\,\overline{29} \\ & -4 \\ \hline 43 & 1\,29 \\ & -129 \\ \hline & \;\;\;0 \end{array}$$

Step 6 Since the remainder is 0 and no digits are left in the given number, therefore, $\sqrt{529} = 23$.

- Now consider $\sqrt{4096}$

Step 1 Place a bar over every pair of digits starting from the one's digit. ($\overline{40}\;\overline{96}$).

$$\begin{array}{r|l} & 6 \\ \hline 6 & \overline{40}\,\overline{96} \\ & -36 \\ \hline & \;\;4 \end{array}$$

Step 2 Find the largest number whose square is less than or equal to the number under the left-most bar ($6^2 < 40 < 7^2$). Take this number as the divisor and the number under the left-most bar as the dividend. Divide and get the remainder i.e., 4 in this case.

$$\begin{array}{r|l} & 6 \\ \hline 6 & \overline{40}\,\overline{96} \\ & -36 \\ \hline & 496 \end{array}$$

Step 3 Bring down the number under the next bar (i.e., 96) to the right of the remainder. The new dividend is 496.

$$\begin{array}{r|l} & 6 \\ \hline 6 & \overline{40}\,\overline{96} \\ & -36 \\ \hline 12\_ & 496 \end{array}$$

Step 4 Double the quotient and enter it with a blank on its right.

Step 5 Guess a largest possible digit to fill the blank which also becomes the new digit in the quotient such that when the new digit is multiplied to the new quotient the product is less than or equal to the dividend. In this case we see that $124 \times 4 = 496$.

So the new digit in the quotient is 4. Get the remainder.

$$\begin{array}{r|l} & 64 \\ \hline 6 & \overline{40}\,\overline{96} \\ & -36 \\ \hline 12\underline{4} & 496 \\ & -496 \\ \hline & \;\;\;\;0 \end{array}$$

Step 6 Since the remainder is 0 and no bar left, therefore, $\sqrt{4096} = 64$.

Estimating the number

We use bars to find the number of digits in the square root of a perfect square number.

$$\sqrt{\overline{5}\,\overline{29}} = 23 \qquad \text{and} \qquad \sqrt{\overline{40}\,\overline{96}} = 64$$

In both the numbers 529 and 4096 there are two bars and the number of digits in their square root is 2. Can you tell the number of digits in the square root of 14400?

By placing bars we get $\overline{1}\,\overline{44}\,\overline{00}$. Since there are 3 bars, the square root will be of 3 digit.

TRY THESE

Without calculating square roots, find the number of digits in the square root of the following numbers.

(i) 25600 (ii) 100000000 (iii) 36864

Example 9: Find the square root of : (i) 729 (ii) 1296

Solution:

(i)
$$\begin{array}{r|l} & 27 \\ \hline 2 & \overline{7}\,\overline{29} \\ & -4 \\ \hline 47 & 329 \\ & 329 \\ \hline & 0 \end{array}$$

Therefore $\sqrt{729} = 27$

(ii)
$$\begin{array}{r|l} & 36 \\ \hline 3 & \overline{12}\,\overline{96} \\ & -9 \\ \hline 66 & 396 \\ & 396 \\ \hline & 0 \end{array}$$

Therefore $\sqrt{1296} = 36$

Example 10: Find the least number that must be subtracted from 5607 so as to get a perfect square. Also find the square root of the perfect square.

$$\begin{array}{r|l} & 74 \\ \hline 7 & \overline{56}\,\overline{07} \\ & -49 \\ \hline 14\underline{4} & 707 \\ & -576 \\ \hline & 131 \end{array}$$

Solution: Let us try to find $\sqrt{5607}$ by long division method. We get the remainder 131. It shows that 74^2 is less than 5607 by 131.

This means if we subtract the remainder from the number, we get a perfect square.

Therefore, the required perfect square is $5607 - 131 = 5476$. And, $\sqrt{5476} = 74$.

Example 11: Find the greatest 4-digit number which is a perfect square.

$$\begin{array}{r|l} & 99 \\ \hline 9 & \overline{99}\,\overline{99} \\ & -81 \\ \hline 18\underline{9} & 1899 \\ & -1701 \\ \hline & 198 \end{array}$$

Solution: Greatest number of 4-digits = 9999. We find $\sqrt{9999}$ by long division method. The remainder is 198. This shows 99^2 is less than 9999 by 198.

This means if we subtract the remainder from the number, we get a perfect square.

Therefore, the required perfect square is $9999 - 198 = 9801$.

And, $\sqrt{9801} = 99$

Example 12: Find the least number that must be added to 1300 so as to get a perfect square. Also find the square root of the perfect square.

$$\begin{array}{r|l} & 36 \\ \hline 3 & \overline{13}\,\overline{00} \\ & -9 \\ \hline 6\underline{6} & 400 \\ & -396 \\ \hline & 4 \end{array}$$

Solution: We find $\sqrt{1300}$ by long division method. The remainder is 4.

This shows that $36^2 < 1300$.

Next perfect square number is $37^2 = 1369$.

Hence, the number to be added is $37^2 - 1300 = 1369 - 1300 = 69$.

6.6 Square Roots of Decimals

Consider $\sqrt{17.64}$

Step 1 To find the square root of a decimal number we put bars on the integral part (i.e., 17) of the number in the usual manner. And place bars on the decimal part

$$\begin{array}{r|l} & \;4 \\ \hline 4 & \overline{17}.\overline{64} \\ & -16 \\ \hline & \;1 \end{array}$$

(i.e., 64) on every pair of digits beginning with the first decimal place. Proceed as usual. We get $\overline{17}.\overline{64}$.

Step 2 Now proceed in a similar manner. The left most bar is on 17 and $4^2 < 17 < 5^2$. Take this number as the divisor and the number under the left-most bar as the dividend, i.e., 17. Divide and get the remainder.

$$\begin{array}{r|l} & \;4 \\ \hline 4 & \overline{17}.\overline{64} \\ & -16 \\ \hline 8\_ & \;1\;64 \end{array}$$

Step 3 The remainder is 1. Write the number under the next bar (i.e., 64) to the right of this remainder, to get 164.

$$\begin{array}{r|l} & \;4. \\ \hline 4 & 17.64 \\ & -16 \\ \hline 82 & \;164 \end{array}$$

Step 4 Double the divisor and enter it with a blank on its right.

Since 64 is the decimal part so put a decimal point in the quotient.

Step 5 We know $82 \times 2 = 164$, therefore, the new digit is 2. Divide and get the remainder.

$$\begin{array}{r|l} & \;4.2 \\ \hline 4 & 17.64 \\ & -16 \\ \hline 82 & \;164 \\ & -164 \\ \hline & \;0 \end{array}$$

Step 6 Since the remainder is 0 and no bar left, therefore $\sqrt{17.64} = 4.2$.

Example 13: Find the square root of 12.25.

Solution:

$$\begin{array}{r|l} & \;3.5 \\ \hline 3 & \overline{12}.\overline{25} \\ & -9 \\ \hline 65 & \;325 \\ & \;325 \\ \hline & \;0 \end{array}$$

Therefore, $\sqrt{12.25} = 3.5$

Which way to move

Consider a number 176.341. Put bars on both integral part and decimal part. In what way is putting bars on decimal part different from integral part? Notice for 176 we start from the unit's place close to the decimal and move towards left. The first bar is over 76 and the second bar over 1. For .341, we start from the decimal and move towards right. First bar is over 34 and for the second bar we put 0 after 1 and make $.\overline{34}\overline{10}$.

$$\begin{array}{r|l} & \;48 \\ \hline 4 & \overline{23}\,\overline{04} \\ & -16 \\ \hline 88 & \;704 \\ & \;704 \\ \hline & \;0 \end{array}$$

Example 14: Area of a square plot is 2304 m^2. Find the side of the square plot.

Solution: Area of square plot = 2304 m^2

Therefore, side of the square plot = $\sqrt{2304}$ m

We find that, $\sqrt{2304} = 48$

Thus, the side of the square plot is 48 m.

Example 15: There are 2401 students in a school. P.T. teacher wants them to stand in rows and columns such that the number of rows is equal to the number of columns. Find the number of rows.

Solution: Let the number of rows be x

So, the number of columns = x

Therefore, number of students = $x \times x = x^2$

Thus, $x^2 = 2401$ gives $x = \sqrt{2401} = 49$

The number of rows = 49.

$$\begin{array}{r|l} & \quad 49 \\ \hline 4 & \overline{24}\,\overline{01} \\ & -16 \\ \hline 89 & \quad 801 \\ & \quad 801 \\ \hline & \quad\; 0 \end{array}$$

6.7 Estimating Square Root

Consider the following situations:

1. Deveshi has a square piece of cloth of area 125 cm^2. She wants to know whether she can make a handkerchief of side 15 cm. If that is not possible she wants to know what is the maximum length of the side of a handkerchief that can be made from this piece.
2. Meena and Shobha played a game. One told a number and other gave its square root. Meena started first. She said 25 and Shobha answered quickly as 5. Then Shobha said 81 and Meena answered 9. It went on, till at one point Meena gave the number 250. And Shobha could not answer. Then Meena asked Shobha if she could atleast tell a number whose square is closer to 250.

In all such cases we need to *estimate* the square root.

We know that $100 < 250 < 400$ and $\sqrt{100} = 10$ and $\sqrt{400} = 20$.

So $10 < \sqrt{250} < 20$

But still we are not very close to the square number.

We know that $15^2 = 225$ and $16^2 = 256$

Therefore, $15 < \sqrt{250} < 16$ and 256 is much closer to 250 than 225.

So, $\sqrt{250}$ is approximately 16.

TRY THESE

Estimate the value of the following to the nearest whole number.

(i) $\sqrt{80}$ (ii) $\sqrt{1000}$ (iii) $\sqrt{350}$ (iv) $\sqrt{500}$

EXERCISE 6.4

1. Find the square root of each of the following numbers by Division method.

(i) 2304	(ii) 4489	(iii) 3481	(iv) 529
(v) 3249	(vi) 1369	(vii) 5776	(viii) 7921
(ix) 576	(x) 1024	(xi) 3136	(xii) 900

2. Find the number of digits in the square root of each of the following numbers (without any calculation).

(i) 64	(ii) 144	(iii) 4489	(iv) 27225
(v) 390625			

3. Find the square root of the following decimal numbers.
   (i) 2.56 (ii) 7.29 (iii) 51.84 (iv) 42.25
   (v) 31.36
4. Find the least number which must be subtracted from each of the following numbers so as to get a perfect square. Also find the square root of the perfect square so obtained.
   (i) 402 (ii) 1989 (iii) 3250 (iv) 825
   (v) 4000
5. Find the least number which must be added to each of the following numbers so as to get a perfect square. Also find the square root of the perfect square so obtained.
   (i) 525 (ii) 1750 (iii) 252 (iv) 1825
   (v) 6412
6. Find the length of the side of a square whose area is 441 m^2.
7. In a right triangle ABC, $\angle B = 90°$.
   (a) If AB = 6 cm, BC = 8 cm, find AC (b) If AC = 13 cm, BC = 5 cm, find AB
8. A gardener has 1000 plants. He wants to plant these in such a way that the number of rows and the number of columns remain same. Find the minimum number of plants he needs more for this.
9. There are 500 children in a school. For a P.T. drill they have to stand in such a manner that the number of rows is equal to number of columns. How many children would be left out in this arrangement.

WHAT HAVE WE DISCUSSED?

1. If a natural number m can be expressed as n^2, where n is also a natural number, then m is a **square number**.
2. All square numbers end with 0, 1, 4, 5, 6 or 9 at unit's place.
3. Square numbers can only have even number of zeros at the end.
4. **Square root** is the inverse operation of square.
5. There are two integral square roots of a perfect square number.

   Positive square root of a number is denoted by the symbol $\sqrt{\ }$.

   For example, $3^2 = 9$ gives $\sqrt{9} = 3$

CHAPTER 7

Cubes and Cube Roots

7.1 Introduction

This is a story about one of India's great mathematical geniuses, S. Ramanujan. Once another famous mathematician Prof. G.H. Hardy came to visit him in a taxi whose number was 1729. While talking to Ramanujan, Hardy described this number "a dull number". Ramanujan quickly pointed out that 1729 was indeed interesting. He said it is the smallest number that can be expressed as a sum of two cubes in two different ways:

$$1729 = 1728 + 1 = 12^3 + 1^3$$

$$1729 = 1000 + 729 = 10^3 + 9^3$$

1729 has since been known as the Hardy – Ramanujan Number, even though this feature of 1729 was known more than 300 years before Ramanujan.

Hardy – Ramanujan Number

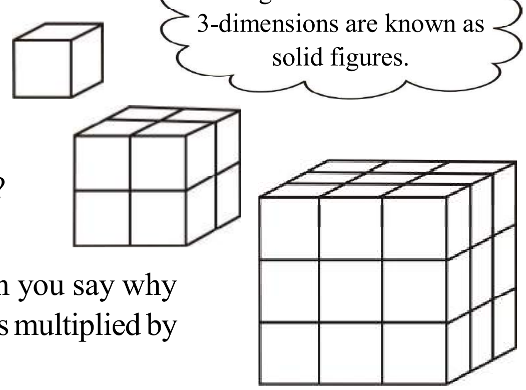
1729 is the smallest Hardy–Ramanujan Number. There are an infinitely many such numbers. Few are 4104 (2, 16; 9, 15), 13832 (18, 20; 2, 24), Check it with the numbers given in the brackets.

How did Ramanujan know this? Well, he loved numbers. All through his life, he experimented with numbers. He probably found numbers that were expressed as the sum of two squares and sum of two cubes also.

There are many other interesting patterns of cubes. Let us learn about cubes, cube roots and many other interesting facts related to them.

7.2 Cubes

Figures which have 3-dimensions are known as solid figures.



You know that the word 'cube' is used in geometry. A cube is a solid figure which has all its sides equal. How many cubes of side 1 cm will make a cube of side 2 cm?

How many cubes of side 1 cm will make a cube of side 3 cm?

Consider the numbers 1, 8, 27, ...

These are called **perfect cubes or cube numbers**. Can you say why they are named so? Each of them is obtained when a number is multiplied by itself three times.