Mensuration

Perimeter of Closed Figures

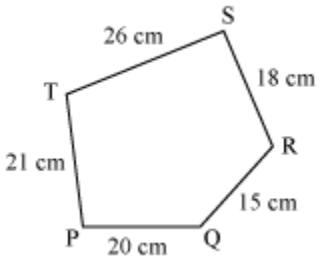
Perimeter is an important property of a closed figure as it gives an idea about the size of the figure.

Let us now discuss some more examples based on the perimeter of closed figures.

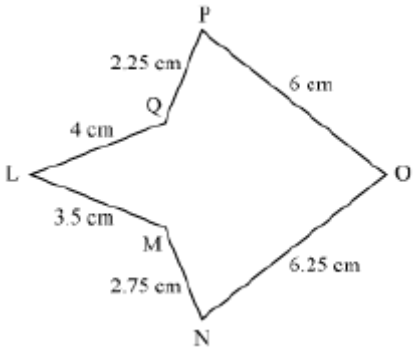
Example 1:

Find the perimeter of the following figures.

(a)



(b)



Solution:

(a) Perimeter of the given figure = PQ + QR + RS + ST + TP

= 20 cm + 15 cm + 18 cm + 26 cm + 21 cm

= 100 cm

(b) Perimeter of the given figure = LM + MN + NO + OP + PQ + QL

= 3.5 cm + 2.75 cm + 6.25 cm + 6.0 cm

+ 2.25 cm + 4.0 cm

= 24.75 cm

Example 2:

The lengths of three sides of a quadrilateral are 10.6 cm, 12.7 cm, and 9.2 cm. If the perimeter of the quadrilateral is 46.9 cm, then what is the length of the fourth side?

Solution:

We know that,

Perimeter of the quadrilateral = Sum of four sides of quadrilateral

⇒ Perimeter of the quadrilateral = Sum of three sides of quadrilateral + Fourth side

⇒ 46.9 cm = (10.6 cm + 12.7 cm + 9.2 cm) + Fourth side

⇒ 46.9 cm = 32.5 cm + Fourth side

Thus, fourth side = 46.9 cm – 32.5 cm

= 14.4 cm

Example 3:

What is the cost of fencing a pentagon-shaped land of sides 75 m, 47 m, 18 m, 39 m, and 31 m at the rate of Rs 8 per metre?

Solution:

To find the cost of fencing the land, we have to find out the perimeter of the pentagon-shaped land.

Perimeter of the pentagon-shaped land = Sum of five sides of the pentagon

= 75 m + 47 m + 18 m + 39 m + 31 m

= 210 m

Cost of fencing = Rs 8 per metre

∴Cost of fencing 210 m of the boundary = Rs (8 × 210) = Rs 1680

Thus, the cost of fencing the pentagon-shaped land is Rs 1680.

Rectangle is a quadrilateral with opposite sides equal. Let us try to find the general formula for perimeter of any rectangle with given length and breadth with the help of an example.

Let us discuss some more examples based on the perimeter of a rectangle.

Example 1:

Find the cost of fencing a rectangular park of length 217 m and breadth 183 m at the rate of Rs 12.50 per metre.

Solution:

Length of the rectangle = 217 m

Breadth of the rectangle = 183 m

To find the cost of fencing the rectangular park, we have to find out the perimeter of the rectangular park.

Now, perimeter of the rectangular park = 2 × (length + breadth)

= 2 × (217 m + 183 m)

= 2 × (400 m)

= 800 m

Cost of fencing = Rs 12.50 per metre

∴Cost of fencing 800 m = Rs (12.50 × 800) = Rs 10000

Thus, the cost of fencing the whole rectangular park is Rs 10000.

Example 2:

The lid of a rectangular box of size 60 cm by 20 cm is sealed all around with tape. Find the length of the tape required.

Solution:

Length of the rectangular box = 60 cm

Breadth of the rectangular box = 20 cm

The rectangular box is sealed all around with tape i.e., the tape covers the boundary of the rectangular box.

∴ Length of tape required = Perimeter of the rectangular box

= 2 × (Length + Breadth)

= 2 × (60 cm + 20 cm)

= 2 × (80 cm)

= 160 cm

Example 3:

A rectangular piece of land measures 0.75 km by 0.5 km. Each side is to be fenced with 6 rows of wires. Find the length of the wire required.

Solution:

Length of the rectangular land = 0.75 km

Breadth of the rectangular land = 0.5 km

It is given that each side of the land is to be fenced with 6 rows of wires.

Therefore, the total length of wire required for fencing is 6 times the perimeter of the land.

Perimeter of the rectangular land = 2 × (Length + Breadth)

= 2 × (0.75 km + 0.5 km)

= 2 × (1.25 km)

= 2.5 km

∴ Length of wire required = (6 × 2.5 km) = 15 km

Example 4:

Chulbul takes 10 rounds of a rectangular park, which is 65 m long and 35 m wide. Find the total distance covered by her.

Solution:

Length of the rectangular park = 65 m

Breadth of the rectangular park = 35 m

Total distance covered by Chulbul in one round is the perimeter of the rectangular park.

Perimeter of the rectangular park = 2 × (Length + Breadth)

= 2 × (65 m + 35 m)

= 2 × (100 m)

= 200 m

Therefore, total distance covered by Chulbul in 1 round = 200 m

∴ Distance covered by Chulbul in 10 rounds = (10 × 200 m) = 2000 m

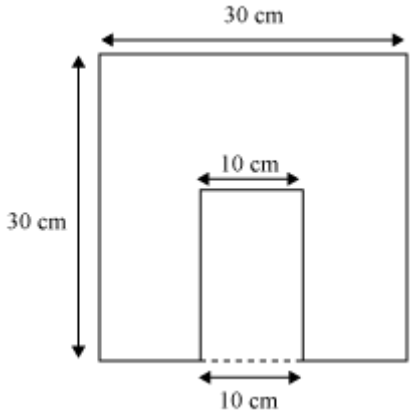
$$\text{However, } 1 \text{ m} = \left(\frac{1}{1000}\right) \text{ km}$$

$$\therefore 2000 \text{ m} = \left(2000 \times \frac{1}{1000}\right) \text{ km} = 2 \text{ km}$$

Thus, the distance covered by Chulbul is 2 km.

Example 5:

A rectangular portion is cut off from one side of a square sheet of paper as shown in the figure. If the length of the rectangular portion is 15 cm, then what will be the difference between the perimeters of the sheet of paper before and after cutting off the rectangular portion?



Solution:

Original perimeter of the sheet of paper = 4 × 30 cm = 120 cm

Length of the rectangular portion that is cut off = 15 cm

Breadth of the rectangular portion that is cut off = 10 cm

After cutting off the rectangular portion, three sides of the sheet of paper remain the same, but one side gets changed.

Thus, perimeter of the sheet of paper after the rectangular portion is cut off

= 30 cm + 30 cm + 30 cm + (30 − 10) cm + 15 cm + 10 cm + 15 cm

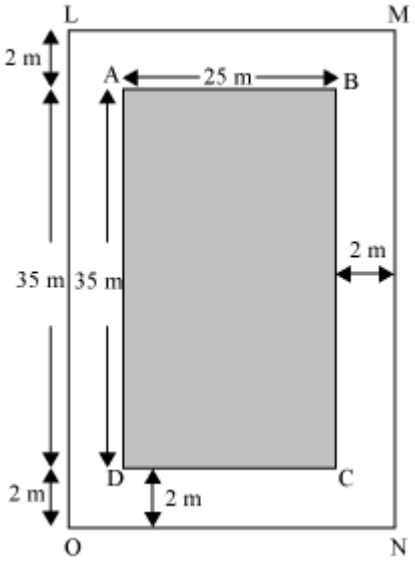
= 30 cm + 30 cm + 30 cm + 20 cm + 15 cm + 10 cm + 15 cm

= 150 cm

Thus, difference between the two perimeters = 150 cm − 120 cm = 30 cm

Example 6:

What is the difference between the perimeters of rectangles ABCD and LMNO?



Solution:

Length of rectangle ABCD = 35 m

Breadth of rectangle ABCD = 25 m

∴ Perimeter of rectangle ABCD = 2 (35 m + 25 m) = 2 × 60 m = 120 m

Length of rectangle LMNO = 2m + 35 m + 2 m = 39 m

Breadth of rectangle LMNO = 2 m + 25 m + 2 m = 29 m

∴ Perimeter of rectangle LMNO = 2 (39 m + 29 m) = 2 × 68 m = 136 m

∴ Difference between the perimeters of the two rectangles = 136 m − 120 m = 16 m

Example 7:

The perimeters of a square and a rectangle are equal. If the length of the rectangle is 9 cm and its breadth is 3 cm less than its length, then what is the length of each side of the square?

Solution:

Length (l) of the rectangle = 9 cm

Thus, breadth (b) of the rectangle = (9 − 3) cm = 6 cm

∴ Perimeter of the rectangle = 2 × (l + b) = 2 × (9 + 6) cm = 2 × 15 cm = 30 cm

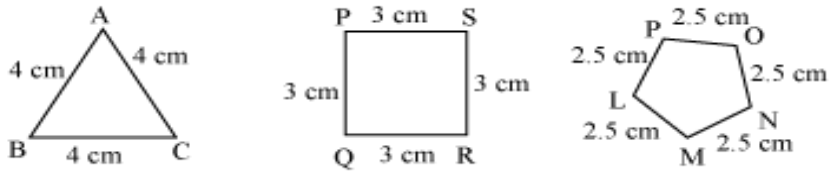
It is given that the perimeters of the square and the rectangle are equal.

Therefore, perimeter of the square = 4 × length of the square = 30 cm

Thus, length of each side of the square = $\frac{30 \text{ cm}}{4} = 7.5 \text{ cm}$

Perimeter of Regular Shapes

Look at the following figures.



What do we observe in these figures?

It can be observed that all the sides of the given figures are equal. These types of figures are known as **regular polygons.**

The first, second, and third figures are the figures of an equilateral triangle, a square, and a regular pentagon respectively.

How can we find the perimeter of a regular polygon?

Let us now discuss some examples based on the perimeter of a regular polygon to understand this concept better.

Example 1:

What is the perimeter of a regular hexagon with side 15 cm?

Solution:

Number of sides of a regular hexagon = 6

Length of each side of the regular hexagon = 15 cm

Perimeter of a regular hexagon = Number of sides of the hexagon × Length of a side of the hexagon

∴ Perimeter of the regular hexagon = (6 × 15) cm = 90 cm

Example 2:

A piece of string, which is 120 cm long, is used to form a regular pentagon. What will be the length of each side of this regular pentagon?

Solution:

Number of sides of a regular pentagon = 5

A piece of string 120 cm long is used to form a regular pentagon

i.e., perimeter of the regular pentagon = 120 cm

However, perimeter of a regular polygon = Number of sides of the polygon × Length of a side of the polygon

$$\therefore \text{Length of a side of a pentagon} = \frac{\text{Perimeter of the pentagon}}{\text{Number of sides of the pentagon}}$$

$$= \left(\frac{120}{5}\right) \text{cm}$$

$$= 24 \text{ cm}$$

Thus, the length of each side of the regular pentagon is 24 cm.

Example 3:

Find the cost of fencing a square park of side 375 m at the rate of Rs 16 per metre.

Solution:

Length of one side of square = 375 m

To find the cost of fencing around the square park, we have to find the perimeter of the square park.

Perimeter of the square park = 4 × Length of a side

= (4 × 375) m

= 1500 m

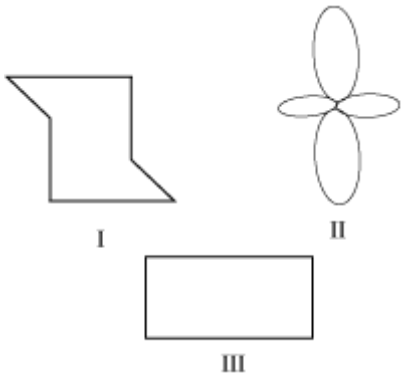
It is given that the cost of fencing 1 m of the park is Rs 16.

Therefore, cost of fencing 1500 m = Rs (1500 × 16) = Rs 24000

Thus, the cost of fencing around the square park is Rs 24000.

Estimation of Areas of Geometric Figures

Look at the following closed figures.



All of the above figures occupy some region or flat surface**. The amount of flat surface or region occupied by a closed figure is known as the area of the closed figure**.

Can we tell which one of the above three figures occupy a greater area?

We can answer this question, if we calculate the area of each figure. Now, **how can we do so?**

To calculate the area of each closed figure, we follow the below given steps.

Step 1: Firstly, we place the closed figure on a squared paper or a graph paper where every square measures 1 cm × 1 cm.

Step 2: Then we make an outline of the figure.

Step 3: Now we look at the squares enclosed by the figure. Some of them are completely enclosed, some half, some less than half and some more than half. Note down the number of squares of each category.

Step 4: Calculate the area of the closed figure by considering the following points.

(a) Takethe area of 1 full square as 1 square unit.

(b) Ignore portions of the area that are less than half a square.

(c) If some portion enclosed by the figure is more than half a square, then

count its area as one square unit.

(d) If exactly half of the square is counted, take its area as $\frac{1}{2}$ square unit.

Such a convention gives a fair estimate of the desired area.

Let us calculate the area of each figure using the above method and try to find out the figure whose area is more than the other two figures.

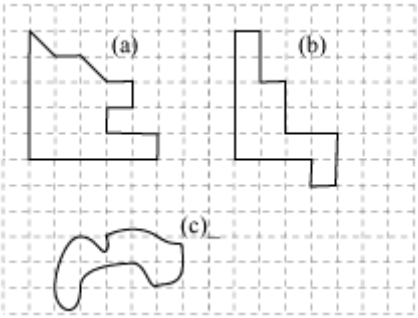
From the above calculations, we can say that figure III has more area than others.

Thus, figure III occupies more space than others.

Let us now discuss one more example based on the area of closed figures.

Example:

Find the areas of the following figures by counting the squares.



Solution:

(a) The number of completely-filled and half-filled squares for the given figure is 14 and 2 respectively.

Therefore, area covered by fully-filled squares = (14 × 1) square units = 14 square units

Area covered by half-filled squares = $\left(2 \times \frac{1}{2}\right)$ square units = 1 square unit

∴ Area of the given figure = (14 + 1) square units = 15 square units

(b) The number of completely-filled squares of the given figure is 11.

∴ Area of the given figure = 11 square units

Covered Area	Number	Area estimate (square units)
Completely-filled squares	1	1
Half-filled squares	0	0

More than half-filled squares	7	7
Less than half-filled squares	4	0

(c) Observation of the number of completely-filled squares, half-filled squares etc. can be represented by the following table.

∴ Estimated area of the given figure = (1 + 7) square units = 8 square units

Conversion of Units of Area

1 km^2	= 1 km × 1 km	= 1000 m × 1000 m = 10,00,000 m^2
1 hectare	= 1 hm × 1 hm	= 100 m × 100 m = 10,000 m^2
1 are	= 1 dcm × 1 dcm	= 10 m × 10 m = 100 m^2
1 m^2	= 1 m × 1 m	= 100 cm × 100 cm =10,000 cm^2
1 m^2	= 1 m × 1 m	= 1000 mm × 1000 mm =10,00,000 mm^2
1 cm^2	= 1 cm × 1 cm	= 10 mm × 10 mm = 100 mm^2

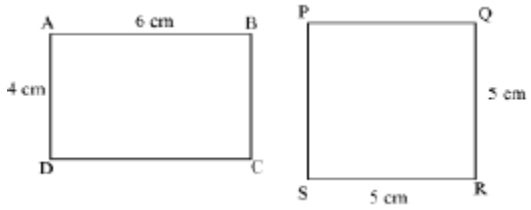
Area is represented in square millimetres or mm^2 for small surfaces and the area of slightly bigger surfaces is represented in cm^2 .

Areas of bigger and bigger surfaces are represented in m^2, ares, hectares and km^2.

Area of Rectangle and Square

We usually come across the situations in our life when we need to find the area of various types of things such as area of a piece of land, area of wall to be painted, area of cloth required etc. The most common shapes that we see in our life are square and rectangle and thus, it becomes necessary for us to learn how to find their area.

Look at the figures given below.



Here, the first figure, i.e. ABCD, is a rectangle of length 6 cm and breadth 4 cm whereas the second figure, i.e. PQRS, is a square of side 5 cm.

Can we find which of the two shapes has the greater area?

It is difficult to answer this question by merely looking at the figures. To find the area of a rectangle or a square, we have to know the formula for each of them.

If the measure of the diagonal of the square is known, then its area can be calculated using the following formula.

$$\text{Area of square} = \frac{(\text{Diagonal})^2}{2}$$

Now, let us consider a real life situation to understand the concept better.

The owner of a paddy field decides to construct a 3 m wide path outside the field along its boundary. What will be the cost of constructing the path at the rate of Rs 500 per m^2?

Now, let us discuss how to convert units of area.
As, we have already studied about the conversion of units for length and they are as follows:

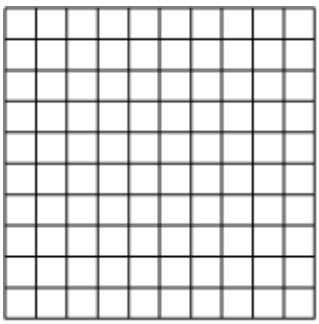
1 centimetre = 10 millimetres

1 metre = 100 centimetres

1 kilometre = 1000 metres

In the same manner, we can convert the unites of areas as well.

Let us consider a square of side 1 cm and divide that square into 100 small squares, each of side 1 mm.



It is evident from the figure that area of a square of side 1 cm will be equal to the areas of 100 small squares of side 1 mm.

$\Rightarrow 1\ cm^2 = 100 \times 1\ mm^2 \Rightarrow 1\ cm^2 = 100\ mm^2$⇒1 cm2=100×1 mm2⇒1 cm2=100 mm2⇒1 cm2=100 m m2

Similarly, we can say
that $1\ m^2 = 1\ m \times 1\ m = 100\ cm \times 100\ cm = 10000\ cm^2$1 m2=1 m×1 m=100 cm×100 cm=10000 cm21 m2=1 m×1 m=100 cm×1 00 cm=10000 cm2.

Now, to convert 1 km^2 into m^2, we will proceed as follows:

$1\ km^2 = 1\ km \times 1\ km = 1000\ m \times 1000\ m = 1000000\ m^2$1 km2=1 km×1 km=1000 m×1000 m=1000000 m21 km2=1 km×1 km=100 0 m×1000 m=1000000 m2.

It is quite significant to note that when we convert a unit of area to a smaller unit, the resulting number of units will be bigger.

The areas of land are usually measured in hectares where 1 hectare = Area of a square of side 100 m.

i.e. $1\ \text{hectare} = 100\ m \times 100\ m = 10000\ m^2$1 hectare=100 m×100 m=10000 m2.

Let us have a look at some more examples to be clear with the concept.

Example 1:

A paddy field is in the form of a square with length 100 m. Find the area of the field.

Solution:

It is given that the length of a side of the square paddy field is 100 m.

∴ Area of the paddy field = $(\text{Side})^2$

= $(100\ \text{m})^2$

= $10000\ \text{m}^2$

Example 2:

A square park is 300 m long diagonally. What is the area of the park?

Solution:

Length of diagonal of square park = 300 m

$$\text{Area of square park} = \frac{(\text{Diagonal})^2}{2}$$

$$\Rightarrow \text{Area of square park} = \frac{(300)^2}{2}\ \text{m}^2$$

$$\Rightarrow \text{Area of square park} = \frac{90000}{2}\ \text{m}^2$$

⇒ Area of square park = $45000\ \text{m}^2$

Example 3:

All the four walls of a room have to be painted. If all the four walls have the equal length of 4 m and equal breadth of 3 m, then find the total cost of painting the walls at the rate of Rs 7 per m^2.

Solution:

Length of a wall = 4 m

Breadth of a wall = 3 m

Now, area of one wall = length × breadth

= 4m × 3m

= 12 m^2

∴ Total area of the four walls = 4 ×12 m^2

= 48 m^2

Cost of painting the walls = Rs 7 per m^2.

∴ Total cost of painting the four walls = Rs (7 × 48)

= Rs 336

Example 4:

A 90 cm long wire is bent into a rectangle of length 30 cm and an 80 cm long wire is bent in the form of a square. Which encloses more area − the rectangle or the square, and by how much?

Solution:

Perimeter of the rectangle = Length of the wire = 90 cm

Length of the rectangle = 30 cm

Let the breadth of the rectangle be *b*.

∴ Perimeter of the rectangle = 90 cm = 2 (30 cm + *b*)

⇒ 90 cm = 2 × 30 cm + 2 × *b*

⇒ 90 cm = 60 cm + 2*b*

⇒ 2*b* = 90 cm − 60 cm = 30 cm

$$\Rightarrow b = \frac{30 \text{ cm}}{2} = 15 \text{ cm}$$

Thus, area enclosed by the rectangle = length × breadth = 30 cm × 15 cm = 450 cm^2

Perimeter of the square = Length of the wire = 80 cm

Let the length of the square be l.

∴ Perimeter of the square = 80 cm = $4l$

$$\Rightarrow l = \frac{80 \text{ cm}}{4} = 20 \text{ cm}$$

Thus, area enclosed by the square = side × side = 20 cm × 20 cm = 400 cm^2

Therefore, the rectangle encloses 450 cm^2 − 400 cm^2 = 50 cm^2 more area than the square.

Example 5:

From a rectangular sheet of paper of 30 cm length and 600 cm^2 area, the biggest possible square is cut out. What is the area of the sheet of paper left?

Solution:

Length of the rectangular sheet of paper = 30 cm

Let the width of the rectangular sheet of paper be w.

Area of the sheet of paper = 600 cm^2 = 30 cm × w

$$\Rightarrow w = \frac{600 \text{ cm}^2}{30 \text{ cm}} = 20 \text{ cm}$$

∴ Width of the sheet = 20 cm

The biggest possible square that can be cut off from this sheet has each side of length equal to the width of the square i.e., 20 cm.

Thus, area of the square that is cut off = 20 cm × 20 cm = 400 cm^2

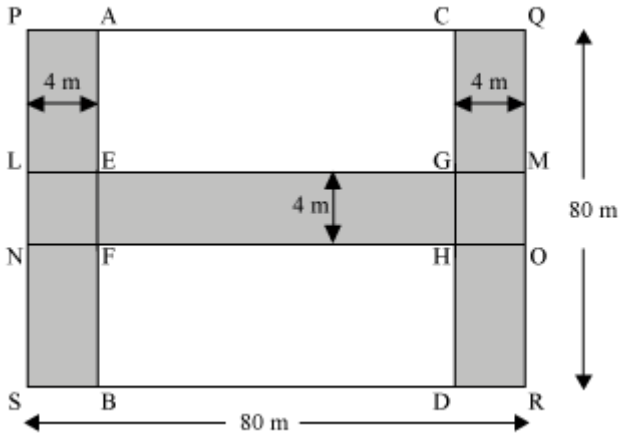
Thus, area of the sheet left = Original area of the sheet − Area of the square that is cut off

= 600 cm^2 − 400 cm^2

= 200 cm^2

Example 6:

Two jogging tracks, each of width 4 m, run along the two opposite sides inside a park. Another jogging track runs through the centre of the park and intersects the two tracks perpendicularly. What is the total area of the tracks?



Solution:

In the given figure, the shaded portion represents the jogging tracks.

Now, area of the roads

= area of PABS + area of CQRD + area of LMON − Area of LEFN − area of GMOH

= (80 m × 4 m) + (80 m × 4 m) + (80 m × 4 m) − (4 m × 4 m) − (4 m × 4 m)

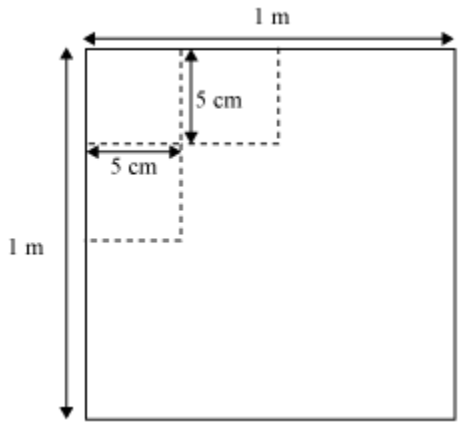
= 320 m^2 + 320 m^2 + 320 m^2 − 16 m^2 − 16 m^2

= 960 m^2 − 32 m^2

= 928 m^2

Example 7:

Raju was given a sheet of paper having all the sides of length 1 m. He was asked to draw squares with side 5 cm on the given sheet without leaving any space. How many squares can be drawn?



Solution:

Given, length of the sheet = 1 m = 100 cm

The length of all sides of the sheet is the same i.e., the sheet is in the form of a square.

Therefore, area of the sheet = $(\text{side})^2$

= $(100\ \text{cm})^2$

= $10000\ \text{cm}^2$

Now, side of each square to be drawn = 5 cm

∴ Area of one square = $(\text{Side})^2$

= $(5\ \text{cm})^2$

= $25\ \text{cm}^2$

Number of squares that can be drawn on the sheet

$$= \frac{\text{Area of the sheet}}{\text{Area of one square}}$$

$$= \frac{10000}{25} = 400$$

Thus, 400 squares can be drawn on the sheet.

Example 8:

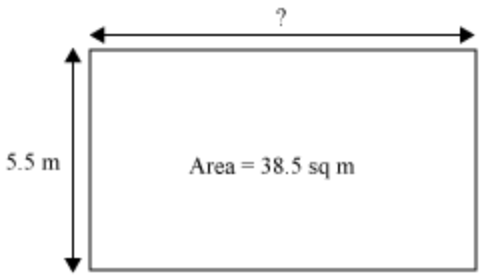
The area of a rectangular pond is 38.5 m^2. If the breadth of the pond is 5.5 m, then find the length of the pond.

Solution:

Given, area of the pond = 38.5 m^2

Breadth of the pond = 5.5 m

We have to find the length of the pond.



Since the pond is rectangular in shape,

Length × Breadth = Area of the pond

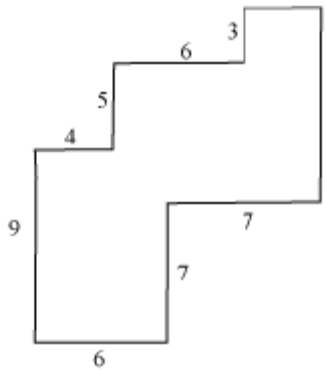
⇒ Length × 5.5 m = 38.5 m^2

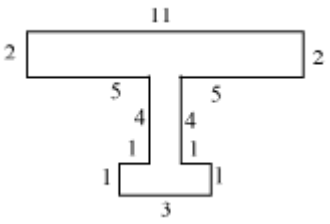
⇒ Length = $\left(\frac{38.5}{5.5}\right)$ m = 7 m

Thus, the length of the pond is 7 m.

Example 9:

By splitting the following figures into rectangles, find their areas. (The measures are given in centimetres)

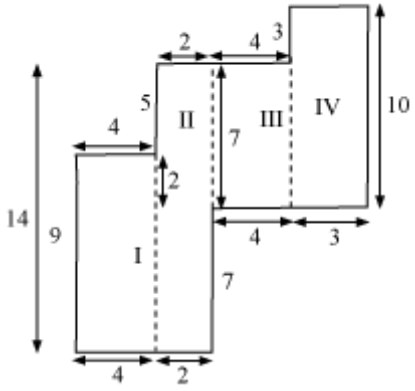




(a) **(b)**

Solution:

(a) By splitting the given figure into rectangles, we will obtain the following figure.



For rectangle I,

Length = 4 cm and breadth = 9 cm

∴ Area of rectangle I = Length × Breadth = (4 × 9) sq cm = 36 cm^2

For rectangle II,

Length = 14 cm and breadth = 2 cm

∴ Area of rectangle II = Length × Breadth = (14 × 2) sq cm = 28 cm^2

For rectangle III,

Length = 7 cm and breadth = 4 cm

∴ Area of rectangle III = Length × Breadth = (7 × 4) sq cm = 28 cm^2

For rectangle IV,

Length = 10 cm and breadth = 3 cm

∴ Area of rectangle IV = Length × Breadth = (10 × 3) sq cm = 30 cm^2

∴ Area of the given figure = Area of rectangle I + Area of rectangle II +

Area of rectangle III + Area of rectangle IV

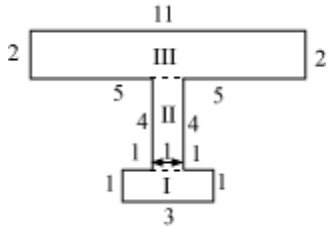
= 36 cm^2 + 28 cm^2 + 28 cm^2 + 30 cm^2

= (36 + 28 + 28 + 30) cm^2

= 122 cm^2

Thus, the area of the given figure is 122 cm^2.

(b) By splitting the given figure into rectangles, we will obtain the following figure.



For rectangle I,

Length = 3 cm and breadth = 1 cm

∴ Area of rectangle I = Length × Breadth = (3 × 1) cm^2 = 3 cm^2

For rectangle II,

Length = 4 cm and breadth = 1 cm

∴ Area of rectangle II = Length × Breadth = $(4 \times 1)\ cm^2 = 4\ cm^2$

For rectangle III,

Length = 11 cm and breadth = 2 cm

∴ Area of rectangle III = Length × Breadth = $(11 \times 2)\ cm^2 = 22\ cm^2$

∴ Area of the given figure = Area of rectangle I + Area of rectangle II +

Area of rectangle III

$= (3 + 4 + 22)\ cm^2$

$= 29\ cm^2$

Thus, the area of the given figure is $29\ cm^2$.