I.E.

CHAPTER-1 [BEA]

- $S = F + V + P$
- $x = \dfrac{F+P}{s-v}$
- $x_{BEP} = \dfrac{F}{s-v}$
- Contribution margin $= (s-v)\,x$

  Contribution $= s-v$
- Profit vol. ratio $= \left(\frac{P}{V}\right)_{ratio} = \dfrac{\Delta P}{\Delta S}$
- MOS = margin of sales $= S_x - S_{BEP}$

CHAPTER-2 [INVENTORY]

Characteristics of Inventory

- Dependent or Independent Demand inventory
- Inventory review system
  - Q-System
    - Fixed order Sys
    - ROL system
    - 2 bin system
    - OC is high
    - Continous monitoring
    - Single inventory
  - P-System
    - Fixed Period System
    - Periodic review system
    - Perishable
    - SS ↑
    - HC ↑
    - Periodic monitoring
    - multiple item inventory
- Deterministic 4 types or Probabilistic Inventory models 2 types

Deterministic Inventory models

1. Economic order Quantity (EOQ)

or

Harris willson model

or

infinite rate of replenishment model

$$TAC = DC + \frac{DC_o}{Q} + \frac{Q}{2} C_h$$

$$TIC = \frac{DC_o}{Q} + \frac{Q}{2} C_h$$

$$Q^* = \sqrt{\frac{2DC_o}{C_h}} \quad ; \quad TIC^* = \sqrt{2DC_oC_h}$$

* $C_h \longrightarrow$ rs/unit-year [always taken as %age of C]

$C_o \longrightarrow$ rs/order

$D \longrightarrow$ unit/yr

$\Rightarrow \quad Q^* = \sqrt{Q_1 Q_2}$ when $TIC(Q_1) = TIC(Q_2)$

2. EOQ with Price Brake

if C_h is varying, start checking EOQ from smallest C_h value. Check TAC at EOQ & Discounts after EOQ.

3. Production or Build up model

$$TAC = DC + \frac{DC_o}{Q} + \frac{Q}{2} C_h \left(\frac{P-d}{P}\right)$$

$$TIC = \frac{DC_o}{Q} + \frac{Q}{2} C_h \left(\frac{P-d}{P}\right)$$

$$Q^* = \sqrt{\frac{2DC_o}{C_h}} \times \sqrt{\frac{P}{P-d}}$$

$\Rightarrow$ Production factor >1

$$TIC^* = \sqrt{2DC_oC_h} \times \sqrt{\frac{P-d}{P}}$$

4. Shortage or Stockout or Backorder

$$TAC = DC + \frac{DC_o}{Q} + \frac{(Q-S)^2 C_h}{2Q} + \frac{S^2 C_b}{2Q}$$

$$TIC = \frac{DC_o}{Q} + \frac{(Q-S)^2 C_h}{2Q} + \frac{S^2 C_b}{2Q}$$

$$Q^* = \sqrt{\frac{2DC_o}{C_h}} \times \sqrt{\frac{C_b + C_h}{C_b}}$$ → cost factor > 1

here, C_b = cost of backorder

$$TIC^* = \sqrt{2DC_oC_h} \times \sqrt{\frac{C_b}{C_b + C_h}}$$

$$S^* = Q^* \times \left(\frac{C_h}{C_b + C_h}\right)$$ = optimum stock out units

$$m^* = Q^* \times \left(\frac{C_b}{C_b + C_h}\right)$$ = max. inventory

5. Production & Shortage [Never asked in paper]

$$Q^* = \sqrt{\frac{2DC_o}{C_h}} \times \sqrt{\frac{P}{P-d}} \times \sqrt{\frac{C_b + C_h}{C_b}}$$

$$TIC^* = \sqrt{2DC_oC_h} \times \sqrt{\frac{P-d}{P}} \times \sqrt{\frac{C_b}{C_b + C_h}}$$

Probabilistic Inventory models

$ROL = LT \times d + SS$

$Q_{max} = Q + SS$; $Q_{avg} = \frac{Q}{2} + SS$; $Q_{min} = SS$

1. Demand Profit or Static inventory model

P = Selling Price − Cost Price + C_b

l = Cost Price − (Salvage or Scrape) + C_h

- $P(S-1) < \frac{P}{P+l} \leq P(S)$

here $P(S-1)$ & $P(S)$ are commulative Probability

- $P(S-1) < \frac{C_b}{C_b + C_h} \leq P(S)$ [when SP = CP = Scrap value]

2. Service level model

Service level = $\left(\frac{\text{no. of units supplied without delay}}{\text{Total no. of units demanded}}\right)_{LT}$

ROL = LT × d + SS

$\bar{X}$ = LT × d

SS = $Z\sigma$

σ = Standard deviation of demand variation during lead time

$$\sigma = \sqrt{\frac{(x_1-\bar{x})^2 + (x_2-\bar{x})^2 - - - (x_n-\bar{x})^2}{n}}$$

Z	Service level
0.50	50%
.84	80%
1.28	90%
1.645	95%
2.33	99%

- if σ_i is given for different part of a cycle

then $\sigma_{overall} = \sqrt{\sigma_1^2 + \sigma_2^2 - - - \sigma_n^2}$

- If partial data is given

SS = max. demand during lead time − Average demand during lead time

SS = $(LT \times d)_{max.} - (LT \times d)_{avg.}$

- Demand during lead time is assumed to be normal distribution

- ABC = Always better control

A ⇒ 50 − 60% usage value & 10 − 20% items

B ⇒ 30 − 40% usage value & 30 − 40% item

C ⇒ 10 − 20% usage value & 50 − 60% item

CHAPTER-3 [SEQUENCING]

- Job flow time of a job = time from starting point upto completion of that job
- Makespan time = time from starting to last job finished
  = job flow time of last job.
- Tardiness = amount of time by which a job is delayed beyond its due date
- Avg no. of job in system = $\frac{\text{Sum of All job flow time}}{\text{make span time}}$
- Avg job flow time / job = $\frac{\text{Sum of All job flow time}}{\text{no. of job}}$
- N job on 1 m/c
  - → Shortest Processing time (SPT)
  - → Earliest due date (EDD)
  - → Critical ratio ($CR = \frac{\text{Due date}}{\text{Processing time}}$)
  - → Slack time remaining (Due date − processing time)
- N job on 2 m/c

Jobs	A	B
1	12	8
2	7	11
3	10	9

SQ ⇒ 2 3 1

Jobs	A	B
1	10	8
2	9	11
3	8	12

SQ = 3 2 1

Jobs	A	B
1	7	13
2	9	8
3	7	10

SQ ⇒ 3 1 2

Jobs	A	B
1	10	9
2	8	6
3	11	6

SQ ⇒ 1 3 2

Jobs	A	B
1	8	11
2	12	10
3	8	11

SQ ⇒ 1 3 2 or 3 1 2

Jobs	A	B
1	10	12
2	9	9
3	11	10

SQ ⇒ 2 1 3 and 1 3 2

- N jobs on 3 m/c

A B C

↓ X ↓ Y

$X_i = A_i + B_i$

$Y_i = B_i + C_i$

when sequence of m/cing is A → B → C

CHAPTER-4 [PERT CPM]

- PERT ⇒ Program (Project) Evaluation & Review technique
  - ⇒ event oriented
  - ⇒ 3 time estimation
  - ⇒ Research & development projects
- CPM ⇒ Critical Path method
  - ⇒ activity oriented
  - ⇒ Single time based
  - ⇒ Construction Projects

- PERT

$$t_E = \frac{t_o + 4t_m + t_p}{6} = \frac{a + 4m + b}{6}$$

$$\sigma = \left(\frac{b-a}{6}\right) \quad ; \text{variance} = \sigma^2 = \left(\frac{b-a}{6}\right)^2$$

- Probability of completing Project within scheduled time(T_S)

Z	Probability
0.0	50%
.84	80%
1.28	90%
1.645	95%
2.33	99%

$$Z = \frac{T_S - T_E}{\sigma}$$

PERT ⇒ Activity → β distribution
Project → normal distribution

- σ along overall critical activity

$$\sigma = \sqrt{\text{Sum of variance } (\sigma^2) \text{ along Critical path}}$$

- Head slack = $S_j = L_j - E_j$
- Tail slack = $S_i = L_i - E_i$
- CPM
  - Total float = $TF = L_j - (E_i + t_E^{ij})$
  - Free float = $FF = E_j - (E_i + t_E^{ij})$
  - Independent float = $IF = E_j - (L_i + t_E^{ij})$

- Crashing ⇒ done to minimise cost (in general)
  ⇒ Sometimes time is needed to minimise

Total cost = Direct cost + Indirect cost

Direct cost → Sum of all normal costs of all activities
⇓
DC ↑ when T ↓

Indirect cost → Fixed + Variable

Fixed [assume 0 if not given]

Variable → Cost of project perday is given
⇓
V.C. ↓ when T ↓

- CHAPTER-5 [Forecasting]

Forecasting

new product (long 4-5 yrs) → Qualitative or Subjective

Short (1-3 month)
medium (3 month - 2 yrs) → Quantitative or objective

Qualitative or Subjective → Judgemental
- opinion Survey
- market trial
- market research
- Delphi technique

Quantitative or objective → Time Series
- Past avg
- moving avg
- weighted movig avg
- Exponential smoothing

Quantitative or objective → Causal or Econometric
- Correlation
- Regression

- weighted moving average
  weight is given as $\frac{n}{\Sigma n}, \frac{n-1}{\Sigma n}, \frac{n-2}{\Sigma n} \text{ ---- } \frac{1}{\Sigma n}$

- Exponential smoothing
- $F_t = F_{t-1} + \alpha(D_{t-1} - F_{t-1})$
- $F_t = \alpha D_{t-1} + \alpha(1-\alpha) D_{t-2} + \alpha(1-\alpha)^2 D_{t-3}$ ----
  ⇓
  not used for 4-5 observation
- $\alpha = \frac{2}{n+1}$

- $F_t = F_{t-1} + \alpha e_{t-1}$ $[\because e_{t-1} = D_{t-1} - F_{t-1}]$
- $0 \leftarrow \alpha \rightarrow 1$

  Stability Responsiveness
- Forecast Error
- Mean absolute deviation (MAD)

$$MAD = \frac{\sum_{i=1}^{n} |D_i - F_i|}{n}$$

- Mean forecast error (MFE) or Bias

$$Bias = \frac{\sum_{i=1}^{n} (D_i - F_i)}{n}$$

$\Rightarrow$ Running Sum forecast error $= RSFE = \sum_{i=1}^{n} (D_i - F_i)$

$\Rightarrow$ $Bias = \frac{RSFE}{n}$

- Mean square error (MSE)

$$MSE = \frac{\sum_{i=1}^{n} (D_i - F_i)^2}{n} \qquad \Rightarrow \sigma = \sqrt{MSE}$$

- Mean absolute %age Error

$$MAPE = \frac{\sum_{i=1}^{n} \left| \frac{D_i - F_i}{D_i} \right| \times 100}{n}$$

$\Rightarrow$ ± 15 to $\pm 20\%$ are aceptable.

- Tracking Signal $= TS = \frac{RSFE}{MAD}$ $[\because \pm 5$ are acceptable limit$]$
- Preference order $\Rightarrow$ MSE > MAD > Bias > MAPE > TS
- Corelation [causal or Econometry]

$y = r x$

here, $r = \frac{S_{xy}}{\sqrt{S_{xx} \cdot S_{yy}}}$

here $S_{xy} = \sum (x-\bar{x}).(y-\bar{y})$

$S_{xx} = \sum (x-\bar{x})^2$

$S_{yy} = \sum (y-\bar{y})^2$

- Linear regression [Causual or econometric]

$y = a + bx$

$b = \dfrac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2}$; $a = \dfrac{\sum y - b\sum x}{n}$

⇒ Least square method [when $\sum x = 0$]

$b = \dfrac{\sum xy}{\sum x^2}$; $a = \dfrac{\sum y}{n}$

CHAPTER - 6 [Line Balancing]

- Balance delay ⇒ $BD\% = \left(\dfrac{nT_c - TWC}{nT_c}\right) \times 100$
- line efficiency ⇒ $\eta_e = \dfrac{TWC}{nT_c} \times 100 = 1 - BD\%$
- Smoothness Index ⇒ $SI = \sqrt{\sum_{i=1}^{n} \left[(T_{si})_{max} - T_i\right]^2}$
- minimum no. of work station $= \dfrac{TWC}{T_c}$
- Largest candidate rule
  - ⇒ List all elements in decreasing order of their tasktime
  - ⇒ To assign a task in a work station
    1) Precedance should be checked
    2) Work station time should not exceed.
  - ⇒ Strike off the task which is assigned. workstation
  - ⇒ Continue down ward after assigning a task.
  - ⇒ Start from top again when workstation is closed

CHAPTER - 7 [Queueing theory]

- Arrival Pattern ⇒ Poission's distribution
- Service Pattern ⇒ exponential distribution

- Inter arrival rate => exponential distribution
- Inter service rate => Poisson's distribution
- Arrival rate = λ

  Service rate = μ
- Representation of Queueing model

  $(a/b/c) : (d/e/f)$

  a = Probability distribution of arrival pattern

  b = Probability distribution of service pattern

  c = no. of server in System

  d = Service rule

  e = Size or capacity of System

  f = Size or capacity of calling population
- Customer's Attitude

  - Patient
  - In-Patient
    - [Keep on changing Queue] Jockey
    - [Doesnot join Queue] Balking
    - [Join Queue for short time] Reneging
    - Cheater
- System utilisation = $\rho = \frac{\lambda}{\mu}$
- Probability that system is idle or Probability of zero customer in System

  $$P_0 = 1 - \rho$$
- Probability of having exactly 'n' customer in the System => $P_n = \rho^n P_0$
- Probability of atleast 'n' customer in system

  $$P(\text{Cust.} \geq n) = \rho^n$$

- Average no. of customer in the system

$$L_s = \frac{\rho}{1-\rho}$$

- Average no. of customer in the queue

$$L_q = \frac{\rho^2}{1-\rho}$$

- Little's Law

$\Rightarrow$ $W_s - W_q = \frac{1}{\mu}$

$\Rightarrow$ $W_s = \frac{L_s}{\lambda}$

$\rightarrow$ $W_q = \frac{L_q}{\lambda}$

- Average length of non-empty Queue or average length of Queue containing atleast 1 customer

$$L_q' = \frac{1}{1-\rho}$$

- Probability of 'n' arrival in system during Period 'T'

$$P(n,T) = \frac{e^{-\lambda T} (\lambda T)^n}{n!}$$

λ = cust / hr

T = hr

- Probability that more than 'T' time period is required to service a customer

$$P(\text{time} > T) = e^{-\mu T}$$

- Probability that waiting time in Queue is greater than 'T'

$$P(W_q > T) = \rho \, e^{-T/W_s}$$

- Probability that waiting time in system is greater than 'T'

$$P(W_s > T) = e^{-T/W_s}$$

CHAPTER-8 [LINEAR PROGRAMMING]

⇒ Graphical method

Special cases

1) Infinite of multi optimum solu^n

- when slope of objective function becomes equal to one of the binding constraints.

2) No solution or infeasiblility

- when there is no feasible region

3) Unbounded solu^n

- In maximisation problem, graph beomes open & highest value becomes inifinity.

⇒ Simplex method

- number of alternate salu^n $= {}^nC_m = \frac{n!}{m!\,(n-m)!}$

  n = no. of variables

  m = no. of constraints

- Big-M method

  ⇒ when sign of resource inequalities beome $\boxed{= \text{ or } \geq}$

  ⇒ An artificial variable 'A' is introduced to get initial working solu^n.

  ⇒ Coefficient of A in objective func^n is 'M'

  +M → minimisation

  −M → maximisation

  [∵ M = greater than any finite value]

Special cases

1) Infinite or multioptimum solu^n

- when a variable becomes zero in Δj raw [in bottom) other than basic variables.

2) Unbounded solu^n

- If in case, the value of replacement ratio column $Q_i = \frac{b_i}{a_{ij}}$ is either −ve or ∞ [i.e. no minimum +ve value available]

3) No solu^n or infeasibility
- when Artificial variable 'A' still remains in final solution.

4) Degeneracy solution
- when one or more basic variable becomes equal to zero [in b_i column]

⇒ Duality

Primal		Dual
max.	⟷	mini
n	⟷	m
m	⟷	n
b_i	⟷	c_j
c_j	⟷	b_i
$\leq$	⟷	$\geq$.
$=$	⟷	$\leq$ & $\geq$

⇒ TRANSPORTATION

- For a balanced problem Σ demand $= \Sigma$ supply
  $\Sigma b_j = \Sigma a_i$
- if initial allocations are $m+n-1$ in a $m \times n$ problem ⇒ basic feasible solu^n
- if these $m+n-1$ allocations are at independent position i.e. no loop is formed ⇒ non-degenerate basic feasible solu^n. & optimility can be performed

Initial solu^n
1) North west corner rule
2) Raw minima method
3) Column minima method
4) Least cost method
5) Vogel's Approximation method (VAM)
   or
   unit cost penalty

optimality
1) Stepping stone method
2) Modified Distribution method [MODI method]
   or
   U-V method

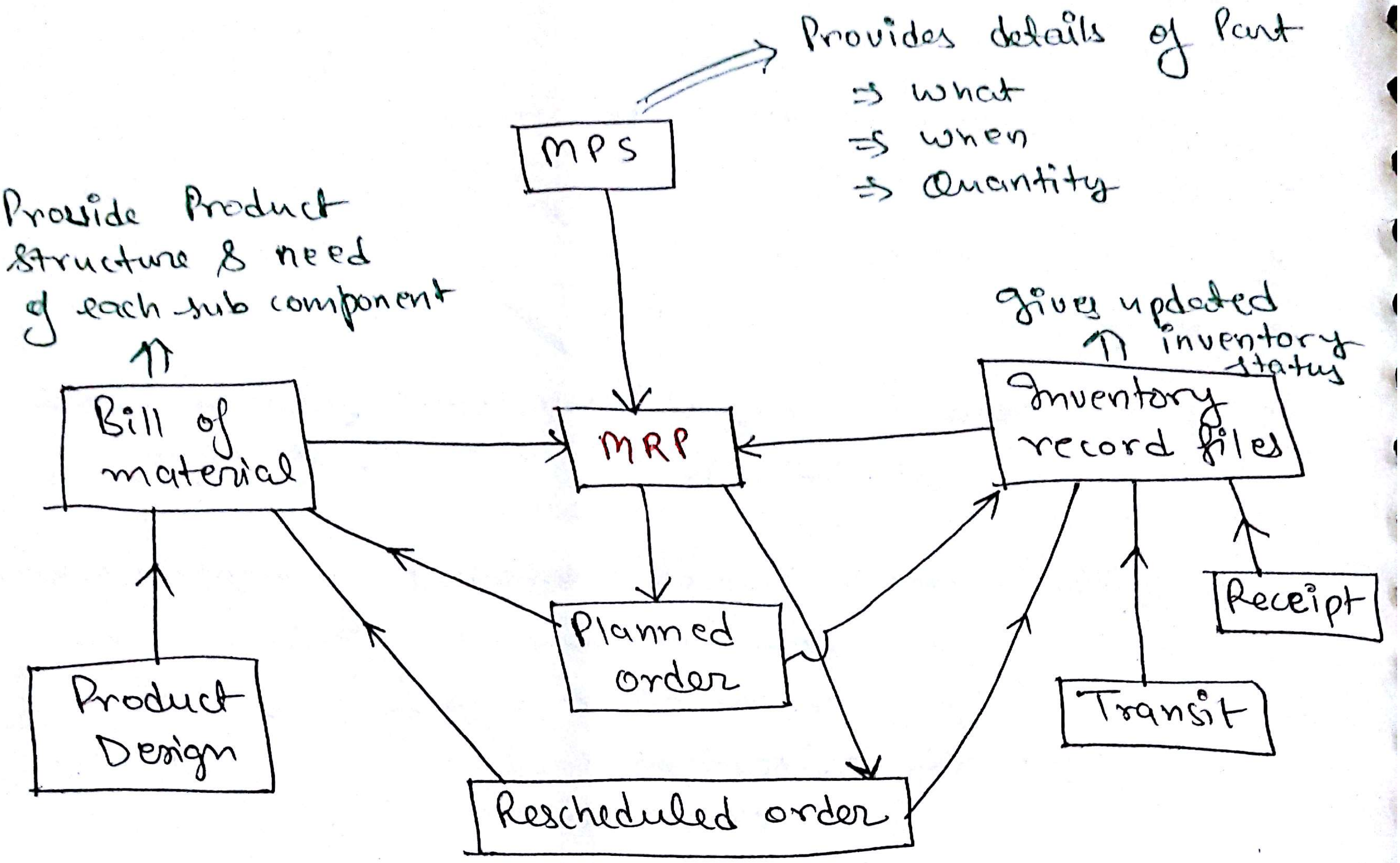
- when total allocations are less than $m+n-1$ then soln is called degenerate soln.

⇒ Assignment

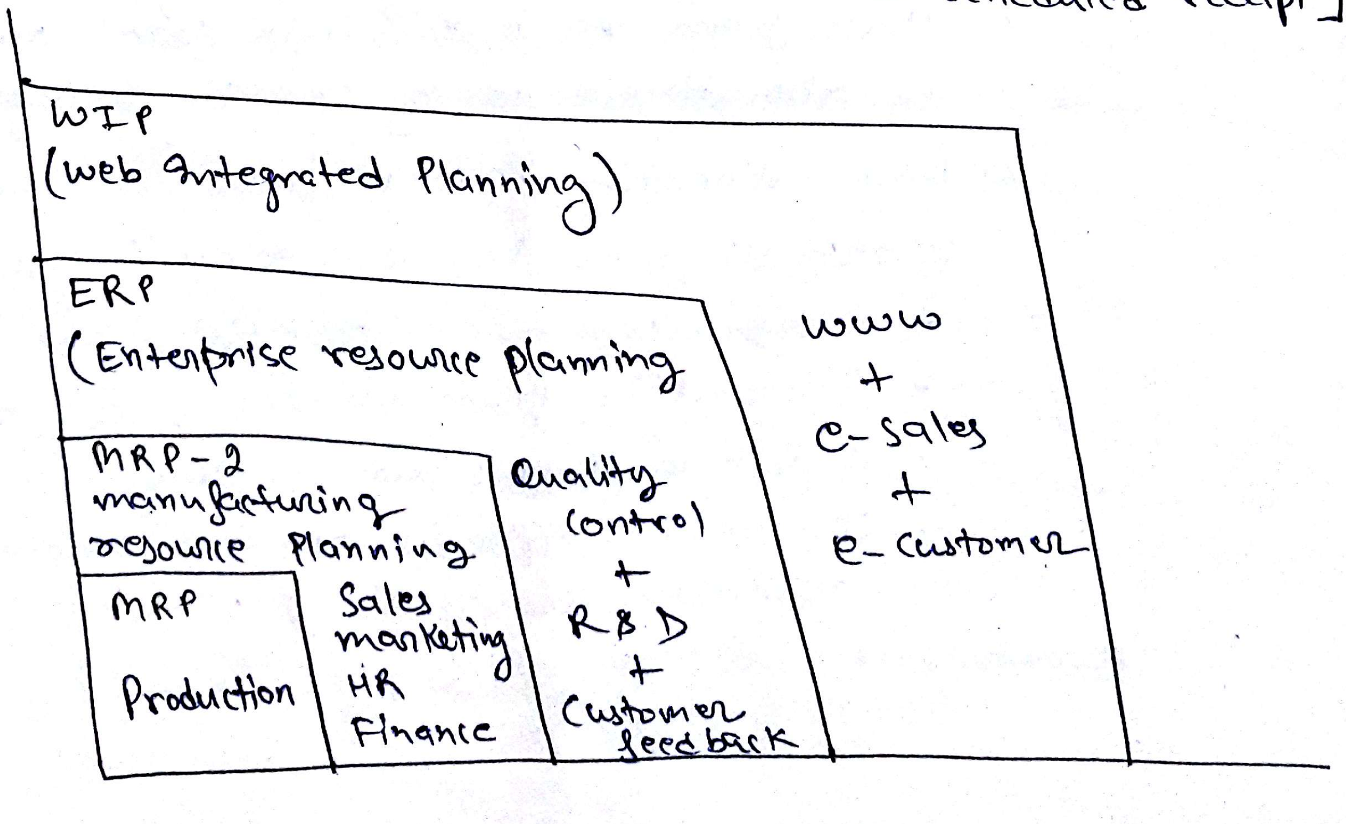
- Square matrix [$m \times n$; $m = n$]
- $x_{ij} = 0$ or 1
- no. of variables $= n^2$
- no of eqn $= 2n-1$

Steps
1) Raw ⇒ Subtract smallest element in raw from each element in that raw.
Do for all rows.
2) Column ⇒ Subtract smallest element in column from each element in that column.
Do for all columns.
3) make allocations
   - 1st check raw
   - 2nd check column
   - 3rd check raw and so on.
4) when single zero not found in any raw or column check for 2 and allocate the leftmost zero while checking raws.
5) start from raw & single zero once an allocation done by checking 2 zeros.
6) when allocations are less than 'n', then cover all zeros by less than 'n' lines
   - Select minimum uncovered value
   - Subtract it from all uncovered values
   - add it to intersection of lines
   - The new matrix is obtained, now new allocations are done.

CHAPTER - 9 [Material requirement Planning]



- MRP makes available raw things at appropriate time so that production can be carried out smoothly.
- net requirement = Gross Requirement − [Inventory on hand + Scheduled receipt]
-



MRP	JIT (Just in time)
• Push system	• Pull system
• Keep safety stock	• no safety stock
• Suited for batch or job production	• suited for mass production

Kanban System

- Japanese word means card or sign.

- P- Kanban : indicate to Produce more
  C- Kanban : indicate to deliver part to next work station

- no. of cards or container or Kanban

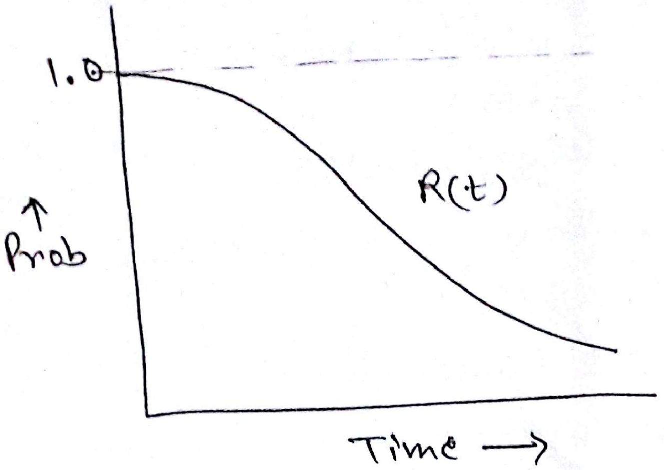
$$N = \frac{D.T.}{C} \times (1+X)$$

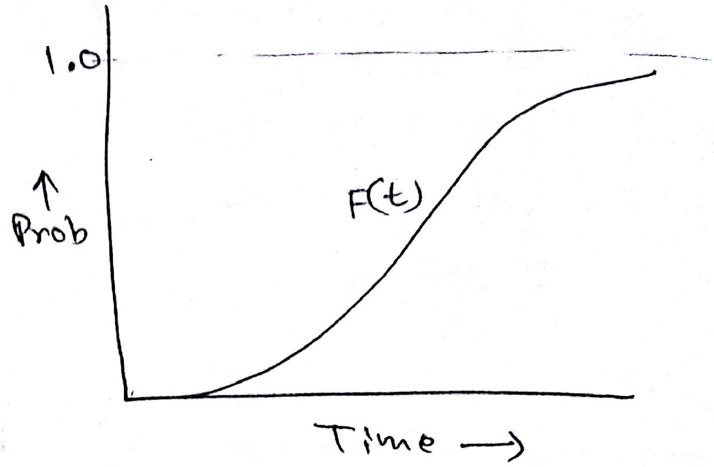
; D = Demand rate
T = avg. waiting time for Production or conveyance
X = Safety factor
C = container capacity

CHAPTER-10 [Reliability & Failure Analysis]
(only ESE)

- Reliability is ability of Product to perform its funcn properly for a Specified period of time under given circumstances.
- Failure is the variation of properties of product from prescribed conditions
- Failure rate $(\lambda) = \frac{\text{no. of failure}}{\text{Time}}$
- Mean time b/w failure = MTBF = $\frac{1}{\lambda}$
- mean time to failure = MTTF = $\frac{t_1 + t_2 + \cdots t_n}{n}$
- Reliability function = $R(t)$ = Probability $\{T \geq t]$
- Failure function = $F(t)$ = Probability $[T < t]$

  here T = failure time





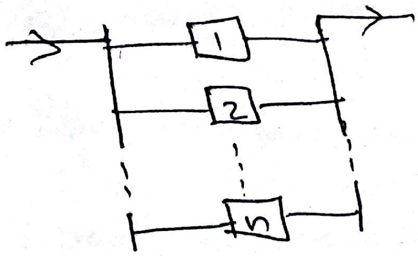
- $F(t) = 1 - R(t)$
- Probability density function $f(t)$
- $R(t) = \int_t^{\infty} f(t)\, dt$ $\quad \left[\therefore \int_{-\infty}^{\infty} f(t)\, dt = \text{Area of graph} = 1 \right.$
- $F(t) = \int_{-\infty}^{t} f(t)\, dt$
- $f(t) = \frac{dF(t)}{dt} = -\frac{dR(t)}{dt}$
- $\lambda(t) = \frac{f(t)}{R(t)}$
- $MTTF = \int_0^{\infty} R(t)\, dt$

⇒ Components connected in series

$R_{sys} = R_1 . R_2 . R_3 . --- R_n$

$\Rightarrow R_{sys} \leq \text{mini}\,(R_i)$

Components connected in parallel



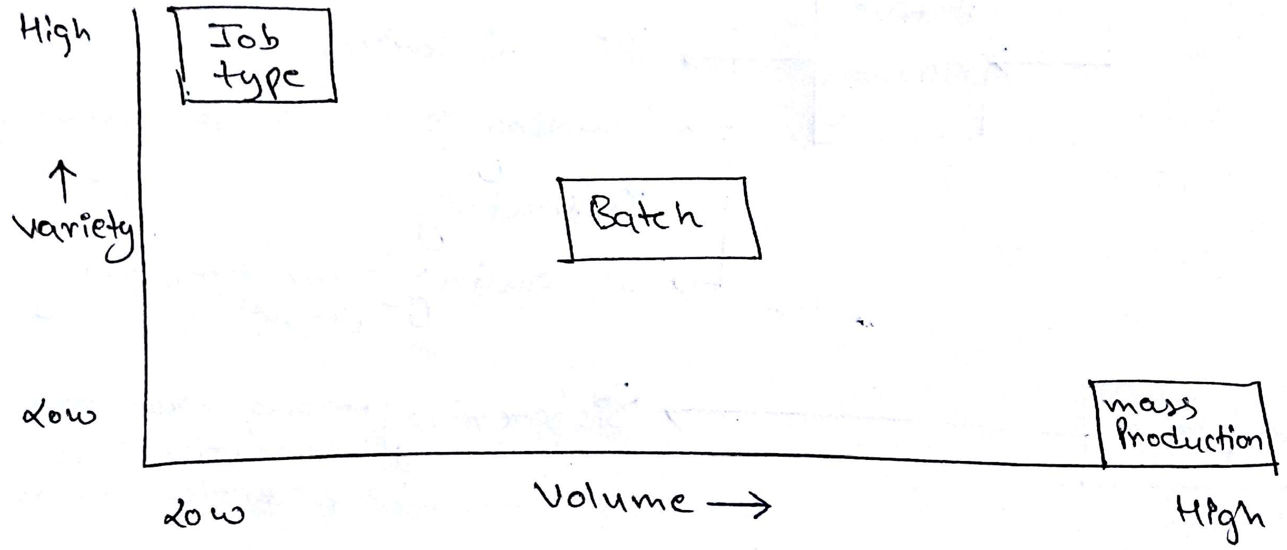
$R_{sys} = 1 - [(1-R_1)(1-R_2) -- (1-R_n)]$

$\Rightarrow R_{sys} \geq \text{max.}\,(R_i)$

CHAPTER – 11 [Plant Layout & PPC]

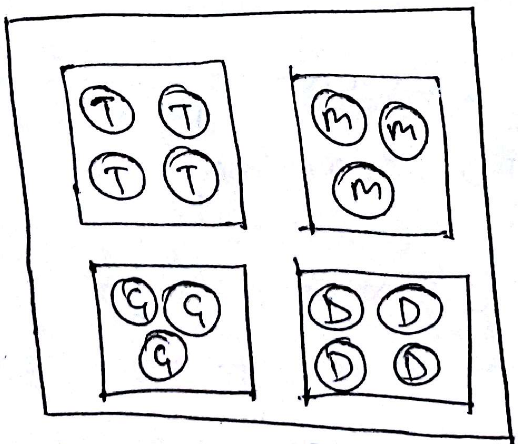
Types of Production

- **Continous Production**
  - **mass Production** – operations on single m/c. ex:- nails, Screws, nut bolt.
  - **Flow Production** – operations on more than 1 m/c. ex:- TV, A/c, motor cycle.
  - **Process Production** – continous demand products. ex: Petrol, Electricity
- **Intermittent Production**
  - **Batch Production** – Similar to Prod. & Build up model (P > d)
  - **Job Production** – ex: space vehicle, custom clothing etc.



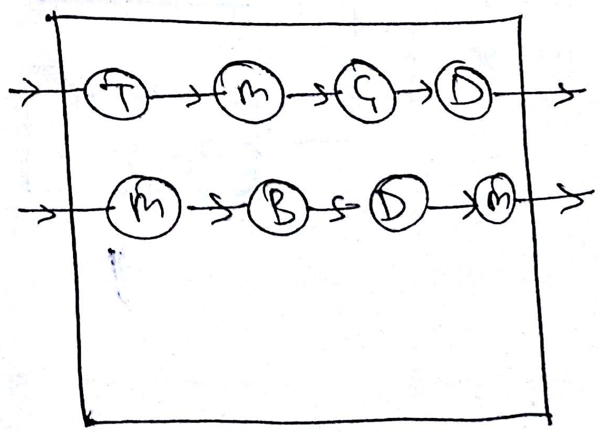
⇒ Types of Plant Layout

1.



Functional or Process Layout

2.



Product or line Layout

3. Combined, hybrid or mixed type Layout
4. Fixed position layout
   ex:- Ship building, aircraft manufacturing etc.

⇒ Production Planning & control

- Planning Phase
  - Prior Planning
    - Forecasting
    - order writting [Placing order for various item in written
    - Product design
  - Active Planning
    - Routing [Sequencing]
    - Tool control
    - Material control
    - Loading [arranging m/c & labour]
    - Balancing
    - Scheduling [setting time for prodation run]
- Action phase
  - Dispatching [handing Action responsibili to production manager. here production starts
- Control phase
  - Progress reporting
    - Data collection
    - Data interpretition
  - corrective action
    - Expediting [corrections to achiev planned path
    - Replanning [Production fails, needed to start again]