Linear Equation

Introduction

We know that a mathematical statement of equality which involves one or more than one variables is called an equation.

An equation in which variables are of one degree is called linear equation. If there Is only one variable, then the equation is said to be linear equation in one variable.

Linear Equation

The general form of linear equation in one variable is $ax+b=0$, where a and b are constant.

The general form of linear equation in two variable is $ax+by+c=0$, where a, b, c are constants, for example $4x+5=5$ is a linear equation in one variable.

Solutions of the Linear Equations

The real number which satisfies the given equation is called the solution of the equation, for example 6 is the root of the equation $3x+4=5x-8$, because when we put x = 6 then L.H.S = 22, R.H.S = 22. That is why ,to find the solution of an equation means that you have to find the value of variable for which L.H.S = R.H.S The following are the methods to solve a linear equation:

Trial and Error Method

In this method we just guess the roots of the equation

The value of variable for which L.H.S = R.H.S is the solution of the equation.

v+3e.g. $\frac{y+3}{3}+8=11$, we guess different values of y, suppose 3, 6, 9 etc.

If we put y = 3 then L.H.S. = 10 and R.H.S = 11. Therefore, L.H.S $\neq$ R.H.S. hence y = 3 is not a solution. Put y = 6 then L.H.S. = 11 and R.H.S. = 11. Therefore, L.H.S. = R.H.S. hence y = 6 is the solution of given equation

Systematic Method

- Add or subtract same number from both sides of the equation
- Multiply or divide both sides of the equation by the same non-zero number.

Illustrative EXAMPLE

Solve $19x+2=40$

Solution:

$19x+2=40$

$\Rightarrow 19x+2-2=40-2$ (subtracting 2 from both sides)

$\Rightarrow 19x+0=38 \Rightarrow 19x=38$

$\Rightarrow \frac{19x}{19}=\frac{38}{19}(dividing\ both\ sides\ by\ 19)$

$\Rightarrow x=2$. Therefore, $x=2$ is the solution of given equation.

Transposition Method

In this method the following steps are involved:

Step 1: Identify the unknown quantity

Step 2: Transfer all the term containing variable on the left hand side and constant term on the right hand side.

Step 3: Simplify left hand side and right hand side in such a way that each side containing only one term.

Step 4: Solve the equation obtained above by systematic method.

Illustrative EXAMPLE

Solve $3(y+3)-2\,(y-1)=5(y-5)$

Solution:

$3(y+3)-2(y-1)=5(y-5)$

$\Rightarrow 3y+9-2y+2=5y-25$ (Expanding the bracket)

$\Rightarrow y+11=5y-25$

$\Rightarrow y-5y=-25-11$ (Transposing 5y on LHS and 11 on RHS)

$\Rightarrow 4y=-36\Rightarrow \frac{-4y}{-4}=\frac{-36}{-4}=9$

Therefore, y = 9 is the solution of given equation.

Illustrative EXAMPLE

The solution of the equation $2(3x-7)+4(3x+2)=6(5x+9)+3$ **is:**

(a) A natural number (b) A rational number

(c) An integer (d) A fraction

(e) None of these

Answer: (b)

Explanation

We have: $2(3x-7)+4(3x+2)=6(5x+9)+3$

$\Rightarrow 6x-14+12x+8=30x+54+3$ [Group like term each side]

$\Rightarrow 6x+12x-30x=54+3+14-8$

$\Rightarrow 18x-30x=63\Rightarrow -12x=63$

$\therefore x=\frac{-63}{12}=\frac{-21}{4}$ is the solution of the equation which is a rational number.

Illustrative EXAMPLE

The Solution of the equation $\frac{x}{2}+\frac{1}{2}=\frac{x}{3}-\frac{1}{3}$ **represents:**

(a) An integer which is less than -4

(b) An integer which is less than 0

(c) An integer which is between 0 and -10

(d) An integer which is equal to -5

(e) All of the above

Answer: (e)

Explanation

$\frac{x}{2}+\frac{1}{2}=\frac{x}{3}-\frac{1}{3}\Rightarrow \frac{x}{3}-\frac{x}{3}=-\frac{1}{3}-\frac{1}{2}\Rightarrow \frac{3x-2x}{6}=\frac{-2-3}{6}$ [Group like term each side and solve]

$\Rightarrow \frac{x}{6}=\frac{-5}{6}\Rightarrow x=\frac{-5}{6}\times 6=-5\Rightarrow =-5$ is the solution or root of the given equation.

Illustrative EXAMPLE

Find the value of x which satisfies the equation $\frac{x^2+5}{2-x^2}=\frac{-3}{2}$.

(a) $x=5$ (b) $x=\pm5$
(c) $x=\pm4$ (d) $x=9$
(e) None of these

Answer: (c)
Explanation

Putting $x^2=y$ in the given equation we get $\frac{y+5}{2-y}=\frac{-3}{2}$.

By cross multiplication, $2(y+5)=-3(2-y)$
Or, $2y+10=-6+3y\Rightarrow 2y-3y=-6-10$
Or, $-y=-16, y=16$ or *or*, $x=\pm4$

Commonly Asked QUESTIONS

Find the solution of the equation $\frac{2x-1}{3x+5}=5.$

(a) $x=-3$ (b) $x=-5$
(c) $x=-2$ (d) $x=-9$
(e) None of these

Answer: (c)

Explanation

We have $\frac{2x-1}{3x+5}=5$

$\frac{2x-1}{3x+5}\times(3x+5)=5x(3x+5)$ (Multiplying both sides by $(3x+5)$

$\Rightarrow 2x-1=15x+25$

$\Rightarrow 2x-1=15x+25$ [Transposing 15x, to L.H.S. & (-1) to R.H.S.]

$\Rightarrow -13=26$, or $x=\frac{26}{-13}=-2$

Which one of the following options is the solution of the equation $\frac{4x-3}{2x+3}=\frac{5}{7}$?

(a) 2 (b) 4
(c) 10 (d) 9
(e) None of these

Answer: (a)

Explanation

$\frac{4x-3}{2x+3}=\frac{5}{7}$ Multiplying both sides of the above equation by $(2x+3)$

$\frac{4x-3}{2x+3}\times(2x+3)=\frac{5}{7}\times(2x+3)$

$\Rightarrow 4x-3=\frac{5}{7}(2x+3)\Rightarrow 4x-3$

$=\frac{10x+15}{7}\Rightarrow 7(4x-3)=10x+15$

$\Rightarrow 28x-21=10x+15\Rightarrow 28x-10x$

$=15+21\Rightarrow 18x=36\Rightarrow x=\frac{36}{18}=2$

If $\frac{(z+2)}{4}+\frac{z-3}{5}=\frac{5z-4}{6}$, then value of z which satisfies the given equation is:

(a) Any natural number
(b) A natural number lies between 7 and 9
(c) A natural number less than 8
(d) It is not a natural number
(e) None of these

Answer: (b)

If $\frac{z+6}{4}+\frac{z-3}{5}=\frac{5z-4}{6}$ then the value of z is

(a) 10
(b) -10
(c) Cannot be determined
(d) 5
(e) None of these

Answer :(a)

If $\frac{x-3}{x+4}=\frac{x+1}{x-2}$, then which of the following value satisfies it?

(a) 1
(b) $\frac{2}{19}$
(c) $\frac{2}{12}$
(d) $\frac{2}{10}$

Answer: (d)

Application of Linear Equation

When you are solving the word problem you should follow the following steps:

Step 1: Read the problem carefully and specify the given and required parameters.
Step 2: Represent the unknown quantity by variables like x, y w....etc
Step 3: Convert the mathematical statements into mathematical problem.
Step 4: Use the conditions to form an equation.

Step 5: Solve the equation for the unknown and check whether the solution satisfies the equating or not.

Illustrative EXAMPLE

The sum of three consecutive multiples of 8 is 888. Which one of following options is the group of those numbers?

(a) 504, 342 and 342 (b) 234, 567 and 604
(c) 234, 564 and 905 (d) 288, 296 and 304
(e) None of these

Answer: (d)
Explanation

Let the first multiple of 8 be 8x then the next two multiples of 8 will be $8(x+1)\ \&\ 8(x+2)$
It is given that the sum of these three consecutive multiples is 888.
$\therefore 8x+8(x+1)+8(x+2) = 888$
$\Rightarrow 8x+8x+8+8x+16 = 888 \Rightarrow 24x+24 = 888$
$\Rightarrow 24x = 888-24 \Rightarrow 24x = 864 \Rightarrow x = \frac{864}{24} = 36$ Therefore, three consecutive multiples of 8 are, $8\times 36, 8\times 37\ \&\ 8\times 38$. i.e., 288, 296 and 304

Illustrative EXAMPLE

The denominator of a rational number is greater than its numerator by 6. If enumerator is increased by 5 and the denominator is decreased by 3 then the number obtained is $\frac{5}{4}$, find the rational number.

(a) $\frac{5}{11}$ (b) $\frac{11}{5}$
(c) $\frac{12}{3}$ (d) $\frac{9}{8}$
(e) None of these

Answer: (a)

Explanation
Let the numerator of the rational number be x.
Then the denominator of the rational number will be $x+6$ It is given that the numerator and denominator of the number are increased and decreased by 5 and 3 respectively then the number obtained is $\frac{5}{4}$
$\therefore$ Numerator of the new rational number $= x+5$
Denominator of the new rational number $= (x+6)-3 = x+3$
$\therefore$ New rational number $= \frac{x+5}{x+3}$
But the new rational number is given as
$\therefore \frac{x+5}{x+3} = \frac{5}{4} \Rightarrow 4(x+5) = 5(x+3)$ (By cross multiplication)
$\Rightarrow 4x+20 = 5x+15$
$4x-5x = 15-20$ [transposing 5x to L.H.S. and 20 to R.H.S.]
$\Rightarrow -x = -5 \Rightarrow$ or $x = 5$

Numerator of the rational number $=5$
Denominator of the rational number $=5+6=11$
$\therefore$ The required rational number $=\frac{5}{11}$

Illustrative EXAMPLE

A steamer goes downstream from one port to another in 6 hours. It covers the same distance up stream in 7 hours. If the speed of the stream is 2 km/hours then find the speed of the steamer in still water.

(a) 20km/h (b) 30 Km/h
(c) 26 Km/h (d) 48Km/h
(e) None of these

Answer: (c)

Explanation

Let the speed of the steamer in still water be $x\ Km/h$
It is given that while going down stream the steamer takes 6 hours to cover the distance between two ports.
$\therefore$ Speed of the steamer down stream $=(x+2)$ Km/h.
Distance covered in $1\ h=(x+2)Km$
Distance covered in $6h=6(x+2)\ Km$
$\therefore$ Distance between 2 $ports=6(x+2)\ Km$ ..... (i)
It is given that while going up stream, the steamer takes 7 hours to cover the distance.
Speed of the steamer up stream $=(x-2)\ Km/h$
Distance covered in $1h=(x-2)\ Km$
Distance covered in $7h=7(x-2)\ Km$
$\therefore$ Distance covered in this case $=7(x-2)km$ .....(ii)
The distance between two ports is same
$\therefore$ From (i) & (ii) we get
$6(x+2)=7(x-2)$
$6x+12=7x-14$
$\Rightarrow 6x-7x=-14-12$ [Transposing $7x$ to L.H.S. & 12 to R.H.S.]
$\Rightarrow -x=-26\Rightarrow x=26$
$\therefore$ The speed of the streamer in still water $=26Km/hrs$

Commonly Asked QUESTIONS

The present ages of Peter & Jony are in the ratio of 4 : 3, four years late their ages will be in the ratio of 6 : 5. What is their present ages?

(a) 8 years and 9 years (b) 6 years and 9 years
(c) 8 years and 6 years (d) 5 years and 9 years
(e) None of these

Answer: (c)

Explanation

Since the ratio of the present ages of Peter & Jony is given as $4:3$.
Let the present age of Peter is $3x$ years, and present age of Jony is $4x$ years
After four years
Peter's age $=(4x+4)$ years
Jony's age $=(3x+4)$ years
According to the given condition
$(4x+4):(3x+4)=6:5$
$\frac{4x+4}{3x+4}=\frac{6}{5}\Rightarrow 5(4x+4)=$
$6(3x+4)\Rightarrow 20x+20=18x+24$
$\Rightarrow 20x-18x=24-20$ [Transposing $18x$ to L.H.S. & 20 to R.H.S.]
$\Rightarrow 2x=4$, or $x=2$
$\therefore$ Present age of Peter $=(4x2)$ years, i.e. 8 years.
$\therefore$ Present age of Jony $=(3x2)$ years, i.e. 6 years.

The sum of the digits of a two digit numbers is 10. The number obtain by interchanging the digits exceeds the original number by 54, find original number.

(a) 29 (b) 28
(c) 55 (d) 95
(e) None of these

Answer: (b)

Explanation

Since the required number is a two digit number so, we have to find its units digit & tens digit.
Let the digit at ones place be x.
It is given that the sum of the digit of the number is 10.
$\therefore$ The digit at the tens place $=10-x$
Thus the original number $=10x(10-x)+x$
$=100$ -$10x+x$
$=100-9x$

On interchanging the digits of the given number the digit at the ones place becomes $(10-x)$ & the digit at the tens place becomes x.
$\therefore$ New number $=10x+(10-x)=9x+10$

It is given that the new number exceeds the original number by 54. .
i.e. New number-original number $=54$
$(9x+10)-(100-9x)=54$
$\Rightarrow 9x+10-100+9x=54$ Or, $18x-90=54$
$\Rightarrow 18x=54+90$ or, $18x=144$ or, $x=\frac{144}{18}=8$
The digit at the ones place $=8$
The digit at the tens place $=(10-8)=2$
Original number $=28$

A Monkey climbing up a pole ascends 10 meters and slep down 2 metres in alternate minutes. If the pole is 57 metres high, how long will take him to reach the top of pole?

(a) 14 minutes, 6 seconds (b) 16 minutes, 4 seconds
(c) 20 minutes, 30 seconds (d) 10 minutes, 18 seconds
(e) None of these

Answer: (a)

Two trains of equal length are running on parallel tracks in the same direction at 46 km per hour. The faster train passes the slower train in 36 /seconds, the length of each train is:

(a) 46m (b) 33m
(c) 53m (d) cannot be determined
(e) None of these

Answer: (d)

The denominator of a number is greater than its numerator by 8. If the numerator increased by one the number obtained is $\frac{2}{3}$. The number is

(a) $\frac{3}{11}$ (b) $\frac{13}{21}$
(c) $\frac{11}{19}$ (d) $\frac{14}{22}$
(e) None of these

SUMMARY

- An equation is a statement which contains one or more than one variables.
- An equation in which the highest power of variable is one is called linear equation.
- The value of variable which satisfies the given linear equation is called solution.
- Generally there are three methods to solve a linear equation.
  (a) Trial and error method.
  (b) Systematic method.
  (c) Transposition method.

You Must KNOW

- Do you know a linear equations appear with great regularity because so many measurable quantities are proportional to other quantities as in related linearly.
- Linear equations are helpful first approximations of computationally prohibitive nonlinear phenomena.

Self Evaluation TEST

Duration 10 Minutes

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1. **Find the solution of the equation** $\frac{7y-5}{4y+2}=\frac{8}{7}$.

(a) 33 (b) 35
(c) 3 (d) 4
(e) None of these

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2. **Which one of the following options is the solution of :** $\frac{7-x}{5x+1}=3$

(a) $\frac{1}{10}$ (b) $\frac{2}{15}$
(c) $\frac{5}{5}$ (d) $\frac{1}{4}$
(e) None of these

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3. **Two trains start simultaneously from two stations 300 Km apart & move towards each other. The speed of one train is more than the other by 20 Km/ hour. If the distance between the two trains after two hours is 20 km. then find the respective speed of the two trains.**

(a) 50 Km/h, 30 Km/h (b) 60 Km/h, 80 Km/h
(c) 170 Km/h, 60 Km/h (d) 190 Km/h, 80 Km/h
(e) None of these

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4. **The perimeter of a rectangle is 100 m. If the length is decreased by 2 m & the breadth is increased by 3 m then area increased by 44 m2. Find the length and breadth of the rectangle.**

(a) 30m, 20m (b) 40 m, 30 m
(c) 50m, 40m (d) 100 m, 90 m
(e) None of these

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5. **What is the speed of boat in still water?**
Statement 1: It takes 1 hour to cover the distance between two points P and Qin downstream.
Statement 2: It takes 3 hours to cover the distance between P and Q in up stream

(a) Statement 1 is sufficient to answer the question.
(b) Statement 2 is sufficient to answer the question.
(c) Both the statement 1 and 2 are sufficient to answer the question.
(d) Both the statement 1 and 2 are not sufficient to answer the question.
(e) None of these

6. **A number is 7 less than the other and its square is 77 less than the square of the greater number. The smaller number is:**

(a) 9 (b) 2
(c) 4 (d) 5
(e) None of these

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7. **Peter is three times as old as Jasmine. The sum of their ages five years from now will twice Peters present age. The age of Peter is:**

(a) 13 years (b) 14 years
(c) 15 years (d) 12 years
(e) None of these

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8. **The ratio of two smaller sides of a right angled triangle is 4: 3. A rectangle is formed on the largest side of the triangle in such a way that largest side will be the length of the rectangle. The breadth of rectangle is four fifth of its length. Find the length of shortest side of triangle if the perimeter of rectangle is 1.8 m.**

(a) 60cm (b) 40cm
(c) 15cm (d) 30cm
(e) None of these

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9. **A boat goes downstream in a river and covers a distance between two coastal town A and B in five hours and it covers the same distance along upstream in six hours. If the speed of the flow of the river is 2 km/h then the speed of boat is:**

(a) 20 km/h (b) 22 km/h
(c) 24 km/h (d) 21 km/h
(e) None of these

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10. **Find the dimension of rectangle when its length is 20 m more than its width. If its width is reduced by 20 m and length is increased by 100 m then the perimeter will be twice the perimeter of original one.**

(a) 30m, 50m (b) 40 m, 60 m
(c) 20m, 40m (d) 10 m, 30 m
(e) None of these

Answers – Self Evaluation Test

1.	A	**2.**	A	**3.**	C	**4.**	C	**5.**	B	**6.**	D	**7.**	C	**8.**	A	**9.**	B	**10.**	A

Self Evaluation Test

SOLUTIONS

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1. $\frac{7y-5}{4y+2}=-\frac{8}{7}\Rightarrow 49y-35=32y+16$

$\Rightarrow 17y=51\Rightarrow y=3$

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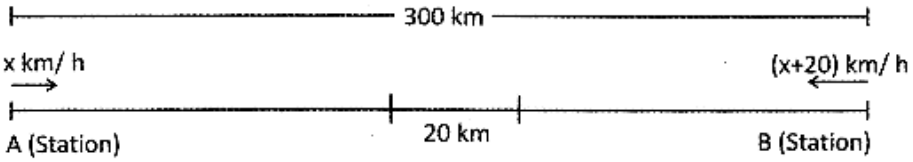
2. $7-\times 3(5x-1)\Rightarrow x=\frac{1}{4}$

3.



Let the speed of slower train be x and the speed of faster train be x + 20.

When train move opposite to each other then,

$=x+x+20$

$=2x+20$

According to question

$(2x+20)\times 2=280\Rightarrow 4x+40=280-40\Rightarrow 4x=240$

$\Rightarrow x=60km/h\Rightarrow x+20=(60+20)=80kh/m$

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4. Let the length be x and breadth be y, then according to question

$2(x+y)=100\Rightarrow x+y=50\Rightarrow y=50-x$

$(x-2)\ (53-x)=x(50-x)+44$

$\Rightarrow -\cancel{x}^{2}+53x+2x-106=50x-\cancel{x}^{2}+44$

$\Rightarrow 5x=106+44\Rightarrow 5x=150$

$\Rightarrow x=30m\Rightarrow y=50-x=20m$

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6. Let the smaller number be x

Then the larger number $=x+7$

$(x+7)^2=x^2+77\Rightarrow x^2+14x+49=x^2+77$

$\Rightarrow 14x=77-49$

$\Rightarrow x=\frac{\cancel{28}^{2}}{\cancel{14}}\Rightarrow x=2$

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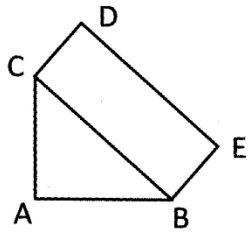
7. Let the age of Jasmine x, then the age of Peter be $3x$

According to question

$(x+5)+(3x+5)=(6x) \Rightarrow x=5$ years

8.



Let the small sides are $4x$ and $3x$.

From the figure

$QC^2 = AB^2 + AC^2$ (Pythagoras theorem)

$\Rightarrow BC^2 = (3x)^2 + 4x^2$

$\Rightarrow BC^2 = 9x^2 + 16x^2$

$\Rightarrow BC^2 = 25x^2 \Rightarrow BC = 5x$

According to question,

Length of rectangle $= 5x$

Breadth of rectangle $= \Rightarrow \frac{4}{5} \times 5x = 4x$

Perimeter of rectangle $= 2(5x+4x)$

$= 18x$

$\Rightarrow 18x = 180cm$

$\Rightarrow x = 10cm$

The length of shortest side $= 3 \times 10 = 30\ cm$

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9. Let the speed of boat be x km/h in still water and the speed of river be km/ h

The speed of boat in upstream $= (x-2)$ km/h

The speed of boat in downstream $= (x+2)$ km/h

Let the distance between the town A and B be d.

According to question,

$d = (x+2)\ x\ 5 \ldots\ldots\ldots (1)$

and $d = (x-2)x6 \ldots\ldots\ldots\ldots\ldots (2)$

$\Rightarrow (x+2)5 = (x-2) \times 6$

$\Rightarrow x - 5x = 10 + 12$

$x = 22\ km/h$

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10. Let the breadth of rectangle be x.

Then length will be $x+20$

According to question

$2\left(x+100+x-20\right)=2\left[2\left(x+x+20\right)\right]$

$\Rightarrow 2(2x+80)=4(2x+20)$

$\Rightarrow 4x+160=8x+80$

$\Rightarrow 4x=80$

$\Rightarrow x=20$

breadth $=$ $20\ m$

length $=20+20=40\ m$
