18. Binomial Theorem

Very Short Answer

1. Question

Write the number of terms in the expansion of $\left(2+\sqrt{3}x\right)^{10}+\left(2-\sqrt{3}x\right)^{10}$.

Answer

Given:

$$(x+a)^n=\sum_{k=0}^{n}\binom{n}{k}x^k a^{n-k}$$

$$\left(2+\sqrt{3}x\right)^{10}=\sum_{k=0}^{10}\binom{10}{k}2^k\left(\sqrt{3}x\right)^{10-k}$$

$$=\binom{10}{0}2^0\left(\sqrt{3}x\right)^{10}+\binom{10}{1}2^1\left(\sqrt{3}x\right)^{10-1}+\cdots+\binom{10}{9}2^9\left(\sqrt{3x}\right)^{10-9}+\binom{10}{10}2^{10}\left(\sqrt{3}x\right)^{10-10}\quad(1)$$

$$\left(2-\sqrt{3}x\right)^{10}=\sum_{k=0}^{10}\binom{10}{k}2^k\left(-\sqrt{3}x\right)^{10-k}$$

$$=\binom{10}{0}2^0\left(-\sqrt{3}x\right)^{10}+\binom{10}{1}2^1\left(-\sqrt{3}x\right)^{10-1}+\cdots+\binom{10}{9}2^9\left(-\sqrt{3x}\right)^{10-9}+\binom{10}{9}2^{10}\left(-\sqrt{3}x\right)^{10-10}\quad(2)$$

Add both equations;

$$\left(2+\sqrt{3}x\right)^{10}+\left(2-\sqrt{3}x\right)^{10}$$

$$=\binom{10}{0}2^0\left(\sqrt{3x}\right)^{10}+\binom{10}{1}2^1\left(\sqrt{3x}\right)^{10-1}+\cdots+\binom{10}{9}2^9\left(\sqrt{3x}\right)^{10-9}+\binom{10}{10}2^{10}\left(\sqrt{3x}\right)^{10-10}+$$

$$\binom{10}{0}2^0\left(-\sqrt{3}x\right)^{10}+\binom{10}{1}2^1\left(-\sqrt{3}x\right)^{10-1}+\cdots+\binom{10}{9}2^9\left(-\sqrt{3x}\right)^{10-9}+\binom{10}{9}2^{10}\left(-\sqrt{3}x\right)^{10-10}$$

The even terms; i.e. k=1,3,5,7 & 9 cancel each other

So, we are left with only terms with k=0,2,4,6,8 &10

So total number of terms = 6

2. Question

Write the sum of the coefficients in the expansion $\left(1-3x+x^2\right)^{111}$.

Answer

Given:

$$(x+a)^n=\sum_{k=0}^{n}\binom{n}{k}x^{n-k}a^k$$

$(1-3x+x^2)^{111}$

For sum of coefficients; put x=1

We have;

$(1-3+1)^{111}=(-1)^{111}$

= -1

3. Question

Write the number of terms in the expansion of $(1-3x+3x^2-x^3)^8$.

Answer

Given:

$(1-3x+3x^2-x^3)^8$

Highest power is $(x^3)^8 = x^{24}$

And lowest power is x^0

So the expansion contains all the terms ranging from 0 to 24

Therefore, total number of terms = 25

4. Question

Write the middle term in the expansion of $\left(\frac{2x^2}{3}+\frac{3}{2x^2}\right)^{10}$.

Answer

Given:

$$(x+a)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} a^k$$

$$\left(\frac{2x^2}{3}+\frac{3}{2x^2}\right)^{10} = \sum_{k=0}^{10} \binom{10}{k} \left(\frac{2x^2}{3}\right)^{10-k} \left(\frac{3}{2x^2}\right)^k$$

Total number of terms = n+1 = 11

So middle term = 6th term, i.e. k=5

$$= \binom{10}{5} \left(\frac{2x^2}{3}\right)^{10-5} \left(\frac{3}{2x^2}\right)^5$$

$$= \frac{10!}{5! \times 5!}$$

=252

5. Question

Which term is independent of x, in the expansion of $\left(x-\frac{1}{3x^2}\right)^9$?

Answer

Given:

$$(x+a)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} a^k$$

$$\left(x - \frac{1}{3x^2}\right)^9 = \sum_{k=0}^{9} \binom{9}{k} x^{9-k} \left(\frac{-1}{3x^2}\right)^k$$

$$= \sum_{k=0}^{9} \binom{9}{k} x^{9-k} \left(\frac{-1}{3}\right)^k x^{-2k}$$

$\Rightarrow x^{(9-k-2k)} = x^0$

$\Rightarrow 9-3k=0$

$\Rightarrow k=3$

So 4^{th} term is independent of x.

6. Question

If a and b denote respectively the coefficients of x^m and x^n in the expansion of $(1+x)^{m+n}$, then write the relation between a and b.

Answer

Given:

$$(x+a)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} a^k$$

$$(1+x)^{m+n} = \sum_{k=0}^{m+n} \binom{m+n}{k} 1^{m+n-k} x^k$$

Coefficient of x^m;k=m

$$a = \binom{m+n}{m} 1^{m+n-m}$$

$a = \frac{(m+n)!}{m! \times n!}$ .... (1)

Coefficient of x^n;k=n

$$b = \binom{m+n}{n} 1^{m+n-n}$$

$b = \frac{(m+n)!}{n! \times m!}$ .... (2)

Divide both equations;

We get;

a=b

7. Question

If a and b are coefficients of x^n in the expansion of $(1+x)^{2n}$ and $(1+x)^{2n-1}$ respectively, then write the relation between a and b.

Answer

Given:

$(x+a)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} a^k$

$(1+x)^{2n} = \sum_{k=0}^{2n} \binom{2n}{k} 1^{2n-k} x^k$

Coefficient of x^n;k=n

$a = \binom{2n}{n} 1^{2n-n}$ ..... (1)

$(1+x)^{2n-1} = \sum_{k=0}^{2n-1} \binom{2n-1}{k} 1^{2n-1-k} x^k$

Coefficient of x^n;k=n

$b = \binom{2n-1}{n} 1^{2n-1-n}$ ..... (2)

Divide both equations;

$\frac{a}{b} = \frac{2n!}{n! \times n!} \times \frac{n! \times (n-1)!}{(2n-1)!}$

$\frac{a}{b} = \frac{2n(2n-1)!}{n! \times (n-1)!} \times \frac{n! \times (n-1)!}{(2n-1)!}$

$\frac{a}{b} = 2n$

a=2b

8. Question

Write the middle term in the expansion of $\left(x + \frac{1}{x}\right)^{10}$.

Answer

Given:

$(x+a)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} a^k$

Total terms = n+1 =11

So middle term= 6th term ; i.e. k=5

$\left(x + \frac{1}{x}\right)^{10} = \sum_{k=0}^{10} \binom{10}{k} x^{10-k} \left(\frac{1}{x}\right)^k$

For k=5;

$= \binom{10}{5} x^{10-5} \left(\frac{1}{x}\right)^5$

$= {}^{10}C_5$

9. Question

If a and b denote the sum of the coefficients in the expansions of $\left(1-3x+10x^2\right)^n$ and $\left(1+x^2\right)^n$ respectively, then write the relation between a and b.

Answer

Given:

$(1-3x+10x^2)^n$

Sum of coefficients = a

a = $(1-3+10)^n$

$=(2^3)^n$

$=(2^n)^3$

$(1+x^2)^n$

Sum of coefficients = b

b= $(1+1)^n$

$=2^n$

Put value of b in a; we get:

$a=b^3$

10. Question

Write the coefficient of the middle term in the expansion of $(1 + x)^{2n}$

Answer

Given:

$$(x+a)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} a^k$$

$$(1 + x)^{2n} = \sum_{k=0}^{2n} \binom{2n}{k} 1^{2n-k} x^k$$

Total terms = 2n+1

Middle term = (2n+1)/2

i.e. (n+1)th term

so k=n

$$= \binom{2n}{n} 1^{2n-n} x^n$$

$= {}^{2n}C_n$

11. Question

Write the number of terms in the expansion of $\{(2x + y^3)^4\}^7$

Answer

Given:

$\{(2x + y^3)^4\}^7 = (2x + y^3)^{28}$

$$(x+a)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} a^k$$

$$(2x + y^3)^{28} = \sum_{k=0}^{28} \binom{28}{k} 2x^{28-k}(y^3)^k$$

So total number of terms = n+1

= 28+1

= 29

12. Question

Find the sum of coefficients of two middle terms in the binomial expansion of $(1 + x)^{2n-1}$

Answer

Given:

Total terms after expansion = 2n-1+1=2n

Middle term = 2n/2 = nth term

So two required middle terms are : nth & (n+1)th term

k= (n-1) & n for both terms respectively.

$$(x + a)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} a^k$$

$$(1 + x)^{2n-1} = \sum_{k=0}^{2n-1} \binom{2n-1}{k} 1^{2n-1-k} x^k$$

Coefficient of nth term;

$= {}^{2n-1}C_{n-1}$

Coefficient of (n+1)th term ;

$= {}^{2n-1}C_n$

Sum of coefficients = ${}^{2n-1}C_{n-1} + {}^{2n-1}C_n$

$= {}^{2n-1+1}C_n$

$= {}^{2n}C_n$

13. Question

Find the ratio of the coefficients of x^p and x^q in the expansion of $(1 + x)^{p+q}$.

Answer

Given:

$$(x + a)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} a^k$$

$$(1 + x)^{p+q} = \sum_{k=0}^{p+q} \binom{p+q}{k} 1^{p+q-k} x^k$$

For x^p; k=p

Coefficient = ${}^{p+q}C_p$ (1)

For x^q; k=q

Coefficient = ${}^{p+q}C_q$ (2)

Divide both equations;

$$\frac{p}{q} = \frac{(p+q)!}{p! \times q!} \times \frac{p! \times q!}{(p+q)!}$$

$$\frac{p}{q} = 1$$

14. Question

Write last two digits of the number 3^{400}.

Answer

Given:

$3^{400} = (3^2)^{200}$

$= 9^{200}$

$= (10-1)^{200}$

By binomial expansion, $(x+a)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} a^k$

$$(1-10)^{200} = \sum_{k=0}^{200} \binom{200}{k} 1^{200-k} (-10)^k$$

$= \binom{200}{0}(-10)^0 + \binom{200}{1}(-10)^1 + \binom{200}{2}(-10)^2 + \cdots + \binom{200}{200}(-10)^{200}$

$=1-2000+ 10^2$ {I}

=1+100(I-20)

So, the last two digits would be 01.

15. Question

Find the number of terms in the expansion of $(a + b + c)^n$.

Answer

Given:

$T_n = \frac{n!}{p! \times q! \times r!} a^p b^q c^r$;

Where p + q + r = n

Since number of ways in which we can divide n different things into r different things is : ${}^{n+r-1}C_{r-1}$

Here, n=n & r=3

So, ${}^{n+3-1}C_{3-1} = {}^{n+2}C_2$

$$= \frac{(n+2)!}{2! \times n!}$$

$$= \frac{(n+2)(n+1)n!}{2! \times n!}$$

$$= \frac{(n+1)(n+2)}{2}$$

so, the number of terms $= \frac{(n+1)(n+2)}{2}$

16. Question

If a and b are the coefficients of x^n in the expansions $(1+x)^{2n}$ and $(1+x)^{2n-1}$ respectively, find $\frac{a}{b}$.

Answer

Given:

$$(x+a)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} a^k$$

$$(1+x)^{2n} = \sum_{k=0}^{2n} \binom{2n}{k} 1^{2n-k} x^k$$

Coefficient of x^n;k=n

$a = \binom{2n}{n} 1^{2n-n}$ (1)

$$(1+x)^{2n-1} = \sum_{k=0}^{2n-1} \binom{2n-1}{k} 1^{2n-1-k} x^k$$

Coefficient of x^n;k=n

$b = \binom{2n-1}{n} 1^{2n-1-n}$ (2)

Divide both equations;

$$\frac{a}{b} = \frac{2n!}{n! \times n!} \times \frac{n! \times (n-1)!}{(2n-1)!}$$

$$\frac{a}{b} = \frac{2n(2n-1)!}{n! \times (n-1)!} \times \frac{n! \times (n-1)!}{(2n-1)!}$$

$$\frac{a}{b} = 2n$$

a=2b

17. Question

Write the total number of terms in the expansion of $(x+a)^{100} + (x-a)^{100}$.

Answer

Given:

$$(x+a)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} a^k$$

$$(x+a)^{100} = \sum_{k=0}^{100} \binom{100}{k} x^{100-k} a^k$$

$$= \binom{100}{0} x^{100} a^0 + \binom{100}{1} x^{100-1} a^1 + \cdots + \binom{100}{99} x^1 a^{99} + \binom{100}{100} x^0 a^{100}$$

$$(x-a)^n = \sum_{k=0}^{100} \binom{100}{k} x^{100-k}(-a)^k$$

$$= \binom{100}{0} x^{100-0}(-a)^0 + \binom{100}{1} x^{100-1}(-a)^1 + \cdots + \binom{100}{99} x^1(-a)^{99} + \binom{100}{100} x^0(-a)^{100}$$

$$(x+a)^{100} + (x-a)^{100}$$

$$= \binom{100}{0} x^{100}a^0 + \binom{100}{1} x^{100-1}a^1 + \cdots + \binom{100}{99} x^1a^{99} + \binom{100}{100} x^0a^{100} +$$

$$\binom{100}{0} x^{100-0}(-a)^0 + \binom{100}{1} x^{100-1}(-a)^1 + \cdots + \binom{100}{99} x^1(-a)^{99} + \binom{100}{100} x^0(-a)^{100}$$

$$= 2\left\{\binom{100}{0} x^{100}a^0 + \binom{100}{2} x^{100-2}a^2 + \cdots + \binom{100}{100} x^0a^{100}\right\}$$

So odd powers of x cancel each other, we are left with even powers of x or say odd terms of expansion.

So total number of terms are $T_1, T_3, \ldots T_{99}, T_{101}$

$$= \frac{1+101}{2}$$

=51

18. Question

If $(1 - x + x^2)^n = a_0 + a_1x + a_2x^2 + \ldots + a_{2n}x^{2n}$, find the value of $a_0 + a_2 + a_4 + \ldots + a_{2n}$.

Answer

$(1 - x + x^2)^n = a_0 + a_1x + a_2x^2 + \ldots + a_{2n}x^{2n}$

At x = 1

$(1 - 1 + 1^2)^n = a_0 + a_1(1) + a_2(1)^2 + \ldots + a_{2n}(1)^{2n}$

$a_0 + a_1 + a_2 + \ldots + a_{2n} = 1$ ...(1)

At x = -1

$(1 - (-1) + (-1)^2)^n = a_0 + a_1(-1) + a_2(-1)^2 + \ldots + a_{2n}(-1)^{2n}$

$a_0 - a_1 + a_2 - \ldots + a_{2n} = 3^n$ ...(2)

On adding eq.1 and eq.2

$(a_0 + a_1 + a_2 + \ldots + a_{2n}) + (a_0 - a_1 + a_2 - \ldots + a_{2n}) = 1 + 3^n$

$2(a_0 + a_2 + a_4 + \ldots + a_{2n}) = 1 + 3^n$

$a_0 + a_2 + a_4 + \ldots + a_{2n} = \frac{1+3^n}{2}$

MCQ

1. Question

Mark the correct alternative in the following :

If in the expansion of $(1+x)^{20}$, the coefficient of rth and (r +4) th terms are equal, then r is equal to

A.7

B. 8

C. 9

D. 10

Answer

Given:

$$(x+a)^n=\sum_{k=0}^{n}\binom{n}{k}x^k a^{n-k}$$

In rth term; k=r-1

& in (r+4)th term ; k=r+3

So, the terms are;

$\binom{20}{r-1}1^{21-r}x^{r-1}$ & $\binom{20}{r+3}1^{17-r}x^{r+3}$

Coefficients of both terms are equal:

$$\binom{20}{r-1}=\binom{20}{r+3}$$

$$\frac{20!}{(r-1)!(21-r)!}=\frac{20!}{(r+3)!(17-r)!}$$

$$\frac{1}{(r-1)!(21-r)(20-r)(19-r)(18-r)(17-r)!}=\frac{1}{(r+3)(r+2)(r+1)r(r-1)!(17-r)!}$$

$$\frac{1}{(21-r)(20-r)(19-r)(18-r)}=\frac{1}{(r+3)(r+2)(r+1)r}$$

]

$$(r+3)(r+2)(r+1)r=(21-r)(20-r)(19-r)(18-r)$$

So, r= (21-r);

(r+1)= (20-r);

(r+2)= (19-r);

(r+3)= (18-r)

We get;

r=9

2. Question

Mark the correct alternative in the following :

The term without x in the expansion of $\left(2x-\frac{1}{2x^2}\right)^{12}$ is

A.495

B. -495

C. -7920

D. 7920

Answer

Given:

$$(x+a)^n = \sum_{k=0}^{n} \binom{n}{k} x^k a^{n-k}$$

$$\left(2x - \frac{1}{2x^2}\right)^{12} = \sum_{k=0}^{12} \binom{12}{k} (2x)^{12-k} \left(\frac{-1}{2x^2}\right)^k$$

$$= \sum_{k=0}^{12} \binom{12}{k} 2^{12-k} (x)^{12-k} \left(\frac{-1}{2}\right)^k x^{-2k}$$

The term without x is where :

$$x^{12-k-2k} = x^0$$

12-3k=0

k=4

for k=4; the term is :

$$= \binom{12}{4} (2x)^{12-4} \left(\frac{-1}{2x^2}\right)^4$$

$$= \frac{12!}{4! \times 8!} \times 2^8 \times x^8 \times \left(\frac{-1}{2}\right)^4 \times x^{-8}$$

$$= 7920$$

3. Question

Mark the correct alternative in the following :

If rth term in the expansion of $\left(2x^2 - \frac{1}{x}\right)^{12}$ is without x, then r is equal to.

A.8

B. 7

C. 9

D. 10

Answer

Given:

$$(x+a)^n = \sum_{k=0}^{n} \binom{n}{k} x^k a^{n-k}$$

$$\left(2x^2 - \frac{1}{x}\right)^{12} = \sum_{k=0}^{12} \binom{12}{k} (2x^2)^{12-k} \left(\frac{-1}{x}\right)^k$$

$$= \sum_{k=0}^{12} \binom{12}{k} 2^k (-1)^{12-k} x^{2(12-k)} x^{-k}$$

For term without x:

$x^{2(12-k)-k} = x^0$

24-2k-k=0

24-3k=0

k=8

for k= 8;

term = 8+1=9th term

4. Question

Mark the correct alternative in the following :

If in the expansion of $(a+b)^n$ and $(a+b)^{n+3}$, the ratio of the coefficients of second and third terms, and third and fourth terms respectively are equal, then n is

A.3

B. 4

C. 5

D. 6

Answer

Given:

$$(x+a)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} x^k$$

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

$T_2 = \binom{n}{1} a^{n-1} b^1$; $T_3 = \binom{n}{2} a^{n-2} b^2$

$$\frac{T1}{T3} = \frac{\binom{n}{1} a^{n-1} b^1}{\binom{n}{2} a^{n-2} b^2}$$

$$= \frac{n! \times 2! \times (n-2)! \times a^n a^{-1} b}{n! \times 1! \times (n-1)! \times a^n a^{-2} b^2}$$

$$= \frac{2 \times (n-2)! \times a}{(n-1)(n-2)! \times b}$$

$= \frac{2a}{(n-1)b}$ (1)

$$(a+b)^{n+3} = \sum_{k=0}^{n+3} \binom{n+3}{k} a^{n+3-k} b^k$$

$T_3 = \binom{n+3}{2} a^{n+3-2} b^2$; $T_4 = \binom{n+3}{3} a^{n+3-3} b^3$

$$\frac{T3}{T4} = \frac{\binom{n+3}{2} a^{n+1} b^2}{\binom{n+3}{3} a^n b^3}$$

$$= \frac{n! \times 3! \times n! \times a^n a^1 b^2}{n! \times 2! \times (n+1)! \times a^n b^3}$$

$$= \frac{3! \times n! \times a}{2! \times (n+1) n! \times b}$$

$$= \frac{3a}{(n+1)b} \quad (2)$$

Equating both equations:

$$\frac{2a}{(n-1)b} = \frac{3a}{(n+1)b}$$

2(n+1)=3(n-1)

2n+2 = 3n-3

n=5

5. Question

Mark the correct alternative in the following :

If A and B are the sums of odd and even terms respectively in the expansion of $(x+a)^n$, then $(x+a)^{2n} - (x-a)^{2n}$ is equal to

A.4 (A+B)

B. 4 (A – B)

C. AB

D. 4 AB

Answer

Given:

$$(x+a)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} a^k$$

$$= \binom{n}{0} x^n a^0 + \binom{n}{1} x^{n-1} a^1 + \cdots + \binom{n}{n-1} x^1 a^{n-1} + \binom{n}{n} x^0 a^n$$

$$(x-a)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} (-a)^k$$

$$= \binom{n}{0} x^{n-0} (-a)^0 + \binom{n}{1} x^{n-1} (-a)^1 + \cdots + \binom{n}{n-1} x^1 (-a)^{99} + \binom{n}{n} x^0 (-a)^n$$

So, $(x+a)^n + (x-a)^n = \binom{n}{0} x^n a^0 + \binom{n}{1} x^{n-1} a^1 + \cdots + \binom{n}{n-1} x^1 a^{n-1} + \binom{n}{n} x^0 a^n +$

$$\binom{n}{0} x^{n-0} (-a)^0 + \binom{n}{1} x^{n-1} (-a)^1 + \cdots + \binom{n}{n-1} x^1 (-a)^{n-1} + \binom{n}{n} x^0 (-a)^n$$

$$= 2\left\{\binom{n}{0} x^n a^0 + \binom{n}{2} x^{n-2} a^2 + \cdots + \binom{n}{n} x^0 a^n\right\}$$

= 2A

So, $(x+a)^n-(x-a)^n = \binom{n}{0}x^n a^0+\binom{n}{1}x^{n-1}a^1+\cdots+\binom{n}{n-1}x^1a^{n-1}+\binom{n}{n}x^0a^n -$

$\binom{n}{0}x^{n-0}(-a)^0+\binom{n}{1}x^{n-1}(-a)^1+\cdots+\binom{n}{n-1}x^1(-a)^{n-1}+\binom{n}{n}x^0(-a)^n$

$=2\left\{\binom{n}{1}x^{n-1}a^1+\binom{n}{1}x^{n-3}a^3+\cdots+\binom{n}{n-1}x^1a^{n-1}\right\}$

=2B

$(x+a)^{2n}-(x-a)^{2n}=[(x+a)^n]^2-[(x-a)^n]^2$

$=\{(x+a)^n+(x-a)^n\}\times\{(x+a)^n-(x-a)^n\}$

=2A×2B

=4AB

6. Question

Mark the correct alternative in the following :

The number of irrational terms in the expansion of $\left(4^{1/5}+7^{1/10}\right)^{45}$ is

A. 40

B. 5

C. 41

D. None of these

Answer

Given:

$(x+a)^n=\sum_{k=0}^{n}\binom{n}{k}x^k a^{n-k}$

$\left(4^{1/5}+7^{1/10}\right)^{45}=\sum_{k=0}^{45}\binom{45}{k}\left(4^{1/5}\right)^k\left(7^{1/10}\right)^{45-k}$

Total number of terms in expansion =n+1

=45+1

=46

irrational terms = total terms – rational terms

For rational terms; the power of each term should be integer.

Therefore, k must be divisible by 5 and (45-k) by 10.

i.e. the terms having power as multiples of 5.

i.e. 0,5,10,15,20,25,30,35,40 & 45

for k= 5,15,25,35 & 45;

(45-k) do not give an integral power, so these powers have to be rejected.

Now, we have k= 0,10,20,30 & 40 which give us rational terms.

Hence, irrational terms = 46-5 = 41

7. Question

Mark the correct alternative in the following :

The coefficient of x^{-17} in the expansion of $\left(x^4 - \frac{1}{x^3}\right)^{15}$ is

A.1365

B. -1365

C. 3003

D. -3003

Answer

Given:

$$(x+a)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} a^k$$

$$\left(x^4 - \frac{1}{x^3}\right)^{15} = \sum_{k=0}^{15} \binom{15}{k} (x^4)^{15-k} \left(\frac{-1}{x^3}\right)^k$$

$$= \sum_{k=0}^{15} \binom{15}{k} x^{4(15-k)} (-1)^k x^{-3k}$$

$$x^{4(15-k)-3k} = x^{-17}$$

60-4k-3k = -17

-7k = -77

k= 11

$$= \binom{15}{11} x^{4(15-11)} (-1)^{11} x^{-3\times 11}$$

$$= \binom{15}{11} x^{16} (-1)^{11} x^{-33}$$

$$= -\frac{15!}{11! \times 4!} x^{-17}$$

Coefficient = -1365

8. Question

Mark the correct alternative in the following :

In the expansion of $\left(x^2 - \frac{1}{3x}\right)^9$, the term without x is equal to

A. $\frac{28}{81}$

B. $\frac{-28}{243}$

C. $\frac{28}{243}$

D. None of these

Answer

Given:

$$(x+a)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} a^k$$

$$\left(x^2 - \frac{1}{3x}\right)^9 = \sum_{k=0}^{9} \binom{9}{k} (x^2)^{9-k} \left(\frac{-1}{3x}\right)^k$$

$$= \sum_{k=0}^{9} \binom{9}{k} x^{2(9-k)} \left(\frac{-1}{3}\right)^k x^{-k}$$

$\Rightarrow x^{2(9-k)-k} = x^0$

$\Rightarrow 18-2k-k = 0$

$\Rightarrow 18-3k = 0$

$\Rightarrow k = 6$

$$= \frac{9!}{6! \times 3!} \left(\frac{-1}{3}\right)^6$$

$$= \frac{28}{243}$$

9. Question

Mark the correct alternative in the following :

If in the expansion of $(1+x)^{15}$, the coefficients of $(2r+3)^{th}$ and $(r-1)^{th}$ terms are equal, then the value of r is

A.5

B. 6

C. 4

D. 3

Answer

Given:

$$(x+a)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} a^k$$

$$(1+x)^{15} = \sum_{k=0}^{15} \binom{15}{k} 1^{15-k} x^k$$

For (2r+3)th term; k=(2r+2)

$$\binom{15}{2r+2} 1^{15-2r-2} x^{2r+2}$$

For (r-1)th term; k=r-2

$$\binom{15}{r-2} 1^{15-r+2} x^{r-2}$$

Coefficients of both terms are equal;

$$\binom{15}{2r+2} = \binom{15}{r-2}$$

$$\Rightarrow \frac{15!}{(2r+2)!\,(13-2r)!} = \frac{15!}{(r-2)!\,(17-r)!}$$

$$\Rightarrow \frac{1}{(2r+2)(2r+1)(2r)!\,(13-2r)!} = \frac{1}{(r-2)(r-1)r!\,(17-r)!}$$

$$\Rightarrow \frac{1}{2(2r+2)(2r+1)(13-2r)!} = \frac{1}{(r-2)(r-1)(17-r)!}$$

10. Question

Mark the correct alternative in the following :

The middle term in the expansion of $\left(\frac{2x^2}{3} + \frac{3}{2x^2}\right)^{10}$ is

A.251

B. 252

C. 250

D. None of these

Answer

Given:

n= 10

Total number of terms on expansion = n+1 = 11

So middle term is 6^{th} term; i.e. k=5

$$= \frac{10!}{5! \times 5!}\left(\frac{2x^2}{3}\right)^5\left(\frac{3}{2x^2}\right)^5$$

= 252

11. Question

Mark the correct alternative in the following :

If in the expansion of $\left(x^4 - \frac{1}{3}\right)^{15}$, x^{-17} occurs in r^{th} term, then

A.r = 10

B. r = 11

C. r = 12

D. r = 13

Answer

Given:

$$(x+a)^n = \sum_{k=0}^{n}\binom{n}{k}x^{n-k}a^k$$

$$\left(x^4 - \frac{1}{x^3}\right)^{15} = \sum_{k=0}^{15} \binom{15}{k} (x^4)^{15-k} \left(\frac{-1}{x^3}\right)^k$$

$$= \sum_{k=0}^{15} \binom{15}{k} x^{4(15-k)} (-1)^k x^{-3k}$$

$\Rightarrow x^{4(15-k)-3k} = x^{-17}$

$\Rightarrow 60-4k-3k = -17$

$\Rightarrow -7k = -77$

$\Rightarrow k = 11$

So, the term is 12^{th} term.

12. Question

Mark the correct alternative in the following :

In the expansion of $\left(x - \frac{1}{3x^2}\right)^9$, the term independent of x is

A. T_3

B. T_4

C. T_5

D. None of these

Answer

Given:

$$(x + a)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} a^k$$

$$\left(x - \frac{1}{3x^2}\right)^9 = \sum_{k=0}^{9} \binom{9}{k} x^{9-k} \left(\frac{-1}{3x^2}\right)^k$$

$$= \sum_{k=0}^{9} \binom{9}{k} x^{9-k} \left(\frac{-1}{3}\right)^k x^{-2k}$$

$\Rightarrow x^{9-k-2k} = x^0$

$\Rightarrow 9-3k = 0$

$\Rightarrow k = 3$

So, the term is 4^{th} term.

13. Question

Mark the correct alternative in the following :

If in the expansion of $(1 + y)^n$, the coefficients of 5^{th}, 6^{th} and 7^{th} terms are in A.P., then n is equal to

A. 7, 11

B. 7, 14

C. 8, 16

D. None of these

Answer

Given:

$$(x+a)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} a^k$$

$$(1+y)^n = \sum_{k=0}^{n} \binom{n}{k} 1^{n-k} y^k$$

$T_5 = \binom{n}{4} 1^{n-4} y^4$; $T_6 = \binom{n}{5} 1^{n-5} y^5$ & $T_7 = \binom{n}{6} 1^{n-6} y^6$

Since T_5 , T_6 & T_7 are in AP

Then; $2(T_6) = T_5 + T_7$

i.e. $\binom{n}{4} 1^{n-4} y^4 + \binom{n}{6} 1^{n-6} y^6 = 2 \times \binom{n}{5} 1^{n-5} y^5$

$$\frac{n!}{4!\,(n-4)!} + \frac{n!}{6!\,(n-6)!} = 2 \times \frac{n!}{5!\,(n-5)!}$$

$$\frac{1}{4!\,(n-4)(n-5)(n-6)!} + \frac{1}{6!\,(n-6)!} = 2 \times \frac{1}{5!\,(n-5)(n-6)!}$$

$$\frac{1}{(n-4)(n-5)} + \frac{1}{30} - \frac{2}{5(n-5)} = 0$$

$$\frac{30 + (n-4)(n-5) - 12(n-4)}{30(n-4)(n-5)} = 0$$

$\Rightarrow 30+(n-4)(n-5)-12(n-4)=0$

$\Rightarrow 30+n^2-9n+20-12n+48=0$

$\Rightarrow n^2-21n+98=0$

$\Rightarrow (n-7)(n-14)=0$

$\Rightarrow n=7,14$

14. Question

Mark the correct alternative in the following :

In the expansion of $\left(\frac{1}{2}x^{1/3} + x^{-1/5}\right)^8$, the term independent of x is

A. T_5

B. T_6

C. T_7

D. T_8

Answer

Given:

$$(x+a)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} a^k$$

$$\left(\frac{1}{2}x^{\frac{1}{3}} + x^{-\frac{1}{5}}\right)^8 = \sum_{k=0}^{8} \binom{8}{k} \left(\frac{1}{2}x^{1/3}\right)^{8-k} \left(x^{-1/5}\right)^k$$

$$= \sum_{k=0}^{8} \binom{8}{k} \left(\frac{1}{2}\right)^{8-k} x^{\frac{(8-k)}{3}} x^{\frac{-k}{5}}$$

$$\Rightarrow x^{\frac{(8-k)}{3} - \frac{k}{5}} = x^0$$

$$\Rightarrow \frac{(8-k)}{3} - \frac{k}{5} = 0$$

$$\Rightarrow \frac{5(8-k) - 3k}{15} = 0$$

$\Rightarrow$40-5k-3k = 0

$\Rightarrow$40-8k =0

$\Rightarrow$ k = 5

So, the term is 6^{th} term.

15. Question

Mark the correct alternative in the following :

If the sum of odd numbered terms and the sum of even numbered terms in the expansion of $(x+a)^n$ are A and B respectively, then the value of $(x^2 - a^2)^n$ is.

A. $A^2 - B^2$

B. $A^2 + B^2$

C. 4 AB

D. None of these

Answer

Given:

$$(x+a)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} a^k$$

$$= \binom{n}{0} x^n a^0 + \binom{n}{1} x^{n-1} a^1 + \cdots + \binom{n}{n-1} x^1 a^{n-1} + \binom{n}{n} x^0 a^n$$

= A+B

$$(x-a)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} (-a)^k$$

$$= \binom{n}{0} x^{n-0} (-a)^0 + \binom{n}{1} x^{n-1} (-a)^1 + \cdots + \binom{n}{n-1} x^1 (-a)^{99} + \binom{n}{n} x^0 (-a)^n$$

= A-B

$(x^2-a^2)^n = [(x+a)(x-a)]^n$

$=(x+a)^n (x-a)^n$

$= (A+B)(A-B)$

$=A^2-B^2$

16. Question

Mark the correct alternative in the following :

If the coefficient of x in $\left(x^2+\frac{\lambda}{x}\right)^5$ is 270, then λ =

A.3

B. 4

C. 5

D. None of these

Answer

Given:

$$(x+a)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} a^k$$

$$\left(x^2+\frac{\lambda}{x}\right)^5 = \sum_{k=0}^{5} \binom{5}{k} (x^2)^{5-k} \left(\frac{\lambda}{x}\right)^k$$

$$= \sum_{k=0}^{5} \binom{5}{k} x^{2(5-k)} \lambda^k x^{-k}$$

$\Rightarrow x^{2(5-k)-k} = x^1$

$\Rightarrow 10-2k-k=1$

$\Rightarrow 9-3k=0$

$\Rightarrow k=3$

for k=3;

$$\Rightarrow \binom{5}{3} x^{2(5-3)} \lambda^3 x^{-3}$$

$$\Rightarrow \frac{5!}{3! \times 2!} \lambda^3 = 270$$

$\Rightarrow \lambda^3=27$

$\Rightarrow \lambda=3$

17. Question

Mark the correct alternative in the following :

The coefficient of x^4 in $\left(\frac{x}{2}-\frac{3}{2}\right)^{10}$ is.

A. $\frac{405}{256}$

B. $\frac{504}{259}$

C. $\frac{450}{263}$

D. None of these

Answer

Given:

$$\Rightarrow (x+a)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} a^k$$

$$\Rightarrow \left(\frac{x}{2} - \frac{3}{2}\right)^{10} = \sum_{k=0}^{10} \binom{10}{k} \left(\frac{X}{2}\right)^{10-k} \left(\frac{-3}{2}\right)^k$$

$\Rightarrow x^{10-k} = x^4$

$\Rightarrow$ 10-k=4

$\Rightarrow$ k=6

for k=6;

$$= \binom{10}{6} \left(\frac{X}{2}\right)^{10-6} \left(\frac{-3}{2}\right)^6$$

$$= \frac{10!}{6! \times 4!} \times \frac{(-3)^6}{2^{10}} x^4$$

So, the coefficient of $x^4 = 105 \times \frac{729}{512}$

18. Question

Mark the correct alternative in the following :

The total number of terms in the expansion of $(x+a)^{100} + (x-a)^{100}$ after simplification is

A.202

B. 51

C. 50

D. None of these

Answer

Given:

$$(x+a)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} a^k$$

$$(x+a)^{2n} = \sum_{k=0}^{2n} \binom{2n}{k} x^{2n-k} a^k$$

$= \binom{2n}{0}x^{2n}a^0 + \binom{2n}{1}x^{2n-1}a^1 + \cdots + \binom{2n}{99}x^1a^{99} + \binom{2n}{2n}x^0a^{2n}$

$$(x-a)^n = \sum_{k=0}^{2n}\binom{2n}{k}x^{2n-k}(-a)^k$$

$= \binom{2n}{0}x^{2n-0}(-a)^0 + \binom{2n}{1}x^{2n-1}(-a)^1 + \cdots + \binom{2n}{99}x^1(-a)^{99} + \binom{2n}{2n}x^0(-a)^{2n}$

$(x+a)^{100} + (x-a)^{100}$

$= \binom{2n}{0}x^{2n}a^0 + \binom{2n}{1}x^{2n-1}a^1 + \cdots + \binom{2n}{99}x^1a^{99} + \binom{2n}{2n}x^0a^{2n} +$

$\binom{2n}{0}x^{2n-0}(-a)^0 + \binom{2n}{1}x^{2n-1}(-a)^1 + \cdots + \binom{2n}{99}x^1(-a)^{99} + \binom{2n}{2n}x^0(-a)^{2n}$

$= 2\left\{\binom{2n}{0}x^{2n}a^0 + \binom{2n}{2}x^{2n-2}a^2 + \cdots + \binom{2n}{2n}x^0a^{2n}\right\}$

So odd powers of x cancel each other, we are left with even powers of x or say odd terms of expansion.

So total number of terms are $T_1, T_3, \ldots T_{99}, T_{101}$

$= \frac{1 + 101}{2}$

=51

19. Question

Mark the correct alternative in the following :

If T_2 / T_3 in the expansion of $(a+b)^n$ and T_3 / T_4 in the expansion of $(a+b)^{n+3}$ are equal, then n =

A.3

B. 4

C. 5

D. 6

Answer

Given:

$$(x+a)^n = \sum_{k=0}^{n}\binom{n}{k}a^{n-k}x^k$$

$$(a+b)^n = \sum_{k=0}^{n}\binom{n}{k}a^{n-k}b^k$$

$T_2 = \binom{n}{1}a^{n-1}b^1$; $T_3 = \binom{n}{2}a^{n-2}b^2$

$$\frac{T1}{T3} = \frac{\binom{n}{1}a^{n-1}b^1}{\binom{n}{2}a^{n-2}b^2}$$

$$= \frac{n! \times 2! \times (n-2)! \times a^n a^{-1} b}{n! \times 1! \times (n-1)! \times a^n a^{-2} b^2}$$

$$= \frac{2 \times (n-2)! \times a}{(n-1)(n-2)! \times b}$$

$= \frac{2a}{(n-1)b}$ (1)

$$(a+b)^{n+3} = \sum_{k=0}^{n+3} \binom{n+3}{k} a^{n+3-k} b^k$$

$T_3 = \binom{n+3}{2} a^{n+3-2} b^2$; $T_4 = \binom{n+3}{3} a^{n+3-3} b^3$

$$\frac{T3}{T4} = \frac{\binom{n+3}{2} a^{n+1} b^2}{\binom{n+3}{3} a^n b^3}$$

$$= \frac{n! \times 3! \times n! \times a^n a^1 b^2}{n! \times 2! \times (n+1)! \times a^n b^3}$$

$$= \frac{3! \times n! \times a}{2! \times (n+1)n! \times b}$$

$= \frac{3a}{(n+1)b}$ (2)

Equating both equations:

$$\frac{2a}{(n-1)b} = \frac{3a}{(n+1)b}$$

$\Rightarrow 2(n+1)=3(n-1)$

$\Rightarrow 2n+2 = 3n-3$

$\Rightarrow n=5$

20. Question

Mark the correct alternative in the following :

The coefficient of $\frac{1}{x}$ in the expansion of $(1+x)^n \left(1+\frac{1}{x}\right)^n$ is.

A. $\frac{n!}{\{(n-1)!(n+1)!\}}$

B. $\frac{(2n)!}{[(n-1)!(n+1)!]}$

C. $\frac{(2n)!}{(2n-1)!(2n+1)!}$

D. None of these

Answer

Given:

$$(x+a)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} a^k$$

$(1+x)^n\left(1+\frac{1}{x}\right)^n = \frac{(1+x)^n(1+x)^n}{x^n}$

$= \frac{(1+x)^{2n}}{x^n}$

$= \frac{1}{x^n}\sum_{k=0}^{2n}\binom{2n}{k}1^{2n-k}x^k$

For x^{-1};

$\Rightarrow \frac{x^k}{x^n} = x^{-1}$

$\Rightarrow x^{k-n} = x^{-1}$

⇒k-n=-1

⇒k=n-1

So, coefficient $= \frac{2n!}{(n-1)!(n+1)!}$

21. Question

Mark the correct alternative in the following :

If the sum of the binomial coefficients of the expansion $\left(2x+\frac{1}{x}\right)^n$ is equal to 256, then the term independent of x is

A.1120

B. 1020

C. 512

D. None of these

Answer

Given:

Sum of binomial coefficients = 2^n

=256

⇒$2^n=2^8$

⇒ n=8

so total terms = n+1

=9

Middle term = 5^{th} term; i.e. k=4

So, term independent of $x = \binom{8}{4}(2x)^{8-4}\left(\frac{1}{x}\right)^4$

$= \frac{8!}{4!\times 4!}2^4$

= 1120

22. Question

Mark the correct alternative in the following :

If the fifth term of the expansion $\left(a^{2/3}+a^{-1}\right)^{n}$ does not contain ‘a’. Then n is equal to

A.2

B. 5

C. 10

D. None of these

Answer

Given:

$$(x+a)^n=\sum_{k=0}^{n}\binom{n}{k}x^{n-k}a^k$$

$$\left(a^{\frac{2}{3}}+a^{-1}\right)^n=\sum_{k=0}^{n}\binom{n}{k}\left(a^{\frac{2}{3}}\right)^{n-k}(a^{-1})^k$$

$$=\sum_{k=0}^{n}\binom{n}{k}a^{\frac{2(n-k)}{3}-k}$$

Term 5 ; i.e. k=4:

$$a^{\frac{2(n-k)}{3}-k}=a^0$$

$$\frac{2(n-4)-3(4)}{3}=0$$

⇒2n-8-12=0

⇒ n=10

23. Question

Mark the correct alternative in the following :

The coefficient of x^{-3} in the expansion of $\left(x-\frac{m}{x}\right)^{11}$ is

A. $-924m^7$

B. $-792m^5$

C. $-792m^6$

D. $-330m^7$

Answer

Given:

$$(x+a)^n=\sum_{k=0}^{n}\binom{n}{k}x^{n-k}a^k$$

$$\left(x-\frac{m}{x}\right)^{11}=\sum_{k=0}^{11}\binom{11}{k}x^{11-k}\left(\frac{-m}{x}\right)^{k}$$

$$=\sum_{k=0}^{11}\binom{11}{k}x^{11-k}(-m)^{k}x^{-k}$$

$\Rightarrow x^{11-k-k}=x^{-3}$

⇒11-2k=-3

⇒14-2k=0

⇒ k =7

for k=7; coefficient is:

$$=\frac{11!}{7!\times 4!}(-m)^{7}$$

$=-330m^7$

24. Question

Mark the correct alternative in the following :

The coefficient of the term independent of x in the expansion of $\left(ax+\frac{b}{x}\right)^{14}$ is

A. $14!a^7b^7$

B. $\frac{14!}{7!}a^7b^7$

C. $\frac{14!}{(7!)^2}a^7b^7$

D. $\frac{14!}{(7!)^3}a^7b^7$

Answer

Given:

$$(x+a)^n=\sum_{k=0}^{n}\binom{n}{k}x^{n-k}a^k$$

$$\left(ax+\frac{b}{x}\right)^{14}=\sum_{k=0}^{14}\binom{14}{k}(ax)^{14-k}\left(\frac{b}{x}\right)^{k}$$

$$=\sum_{k=0}^{14}\binom{14}{k}a^{14-k}b^{k}x^{14-k}x^{-k}$$

$x^{14-k-k}=x^0$

14-2k=0

k = 7

So, the coefficient is:

$$= \frac{14!}{7! \times 7!} a^7 b^7$$

25. Question

Mark the correct alternative in the following :

The coefficient of x^5 in the expansion of $(1+x)^{21} + (1+x)^{22} + \ldots + (1+x)^{30}$ is.

A. ${}^{51}C_5$

B. ${}^{9}C_5$

C. ${}^{31}C_6 - {}^{21}C_6$

D. ${}^{30}C_5 + {}^{20}C_5$

Answer

Given:

$$(x+a)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} a^k$$

$(1+x)^{21}+(1+x)^{22}+\ldots+(1+x)^{30}$

Coefficient of x^5in any expansion = $\binom{n}{5} 1^{n-5} x^5$; i.e. ${}^nC_5$

So, coefficient of x^5 in above expansion = ${}^{21}C_5$ + ${}^{22}C_5$ + ${}^{23}C_5$ +...+ ${}^{30}C_5$

26. Question

Mark the correct alternative in the following :

The coefficient of $x^8 y^{10}$ in the expansion $(x+y)^{18}$ is.

A. ${}^{18}C_8$

B. ${}^{18}P_{10}$

C. 2^{18}

D. None of these

Answer

Given:

$$(x+y)^{18} = \sum_{k=0}^{18} \binom{18}{k} x^{18-k} y^k$$

For x^8 y^{10}; k=10

So coefficient is ${}^{18}C_{10}$

Also ${}^{18}C_{10}$ = ${}^{18}C_8$

So coefficient = $^{18}C_8$

27. Question

Mark the correct alternative in the following :

If the coefficients of the $(n+1)^{th}$ term and the $(n+3)^{th}$ term in the expansion of $(1+x)^{20}$ are equal, then the value of n is

A.10

B. 8

C. 9

D. None of these

Answer

Given:

$$(x+a)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} a^k$$

$$(1+x)^{20} = \sum_{k=0}^{20} \binom{20}{k} 1^{20-k} x^k$$

For nth term ; k=n-1

So for (n+1)th term ; k= n

& for (n+3)th term ; k =n+2

Coefficients for the above terms are equal;

$$\frac{20!}{n!\,(20-n)!} = \frac{20!}{(n+2)!\,(18-n)!}$$

$$\frac{1}{n! \times (20-n)(19-n)(18-n)!} = \frac{1}{(n+2)(n+1)n! \times (18-n)!}$$

(20-n)(19-n) = (n+2)(n+1)

$380-39n+n^2 = n^2+3n+2$

42n-378=0

n=9

28. Question

Mark the correct alternative in the following :

If the coefficients of 2^{nd}, 3^{rd} and 4^{th} terms in the expansion of $(1+x)^n, n \in N$ are in A.P., then n =

A.7

B. 14

C. 2

D. None of these

Answer

Given:

$$(x+a)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} a^k$$

$$(1+x)^n = \sum_{k=0}^{n} \binom{n}{k} 1^{n-k} x^k$$

$T_2 = \binom{n}{1} 1^{n-1} x^1$; $T_3 = \binom{n}{2} 1^{n-2} x^2$ & $T_4 = \binom{n}{3} 1^{n-3} x^3$

Since T_2 , T_3 & T_4 are in AP

Then; $2(T_3) = T_2 + T_4$

i.e. $\binom{n}{1} 1^{n-1} x^1 + \binom{n}{3} 1^{n-3} x^3 = 2 \times \binom{n}{2} 1^{n-2} x^2$

$$\frac{n!}{1!\,(n-1)!} + \frac{n!}{3!\,(n-3)!} = 2 \times \frac{n!}{2!\,(n-2)!}$$

$$\frac{1}{1!\,(n-1)(n-2)(n-3)!} + \frac{1}{3!\,(n-3)!} = 2 \times \frac{1}{2!\,(n-2)(n-3)!}$$

$$\frac{1}{(n-1)(n-2)} + \frac{1}{6} = \frac{1}{(n-2)}$$

$$\frac{1}{(n-1)(n-2)} + \frac{1}{6} - \frac{1}{(n-2)} = 0$$

$$\frac{6}{6(n-1)(n-2)} + \frac{(n-1)(n-2)}{6(n-1)(n-2)} - \frac{6(n-1)}{6(n-1)(n-2)} = 0$$

$(n-1)(n-2)-6(n-1)+6=0$

$n^2-3n+2-6n+6+6=0$

$n^2-9n+14=0$

$(n-2)(n-7)=0$

$n= 2,7$

$n=2$ rejected for term 3^{rd}

So $n=7$

29. Question

Mark the correct alternative in the following :

The middle term in the expansion of $\left(\frac{2x}{3} - \frac{3}{2x^2}\right)^{2n}$ is.

A. ${}^{2n}C_n$

B. $(-1)^n\ {}^{2n}C_n\ x^{-n}$

C. ${}^{2n}C_n\ x^{-n}$

D. None of these

Answer

Given:

$$(x+a)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} a^k$$

$$\left(\frac{2x}{3} - \frac{3}{2x^2}\right)^{2n} = \sum_{k=0}^{2n} \binom{2n}{k} \left(\frac{2x}{3}\right)^{2n-k} \left(\frac{-3}{2x^2}\right)^k$$

For middle term,

$$T_n = \binom{2n}{n} \left(\frac{2x}{3}\right)^{2n-k=\frac{2n}{2}=nn} \left(\frac{-3}{2x^2}\right)^n$$

$$= \binom{2n}{n} \left(\frac{2}{3}\right)^n \left(\frac{3}{2}\right)^n (-1)^n x^n x^{-2n}$$

$$= \binom{2n}{n} (-1)^n x^{-n}$$

$=(-1)^n\ {}^{2n}C_n\ x^{-n}$

30. Question

Mark the correct alternative in the following :

If r^{th} term is the middle term in the expansion of $\left(x^2 - \frac{1}{2x}\right)^{20}$, then $(r+3)^{th}$ term is

A. ${}^{20}C_{14}\left(\frac{x}{2^{14}}\right)$

B. ${}^{20}C_{12}x^2 2^{-12}$

C. $-{}^{20}C_7 x.2^{-13}$

D. None of these

Answer

Given:

$$(x+a)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} a^k$$

$$\left(x^2 - \frac{1}{2x}\right)^{20} = \sum_{k=0}^{20} \binom{20}{k} (x^2)^{20-k} \left(\frac{-1}{2x}\right)^k$$

Total terms = n+1 = 21

Mid term = 21/2 = 11^{th} term

For k= 10,it is rth term.

So (r+3)th term = 11^{th} term

k=13

$$T_{14} = \binom{20}{13} (x^2)^{20-13} \left(\frac{-1}{2x}\right)^{13}$$

$= \binom{20}{13}(x^2)^7\left(\frac{-1}{2}\right)^{13}x^{-13}$

$= \binom{20}{13}\left(\frac{-1}{2}\right)^{13}x^{-13}x^{14}$

= $-{}^{20}C_{13}$ x. 2^{-13}

= $-{}^{20}C_{7}$ x. 2^{-13}

31. Question

Mark the correct alternative in the following :

The number of terms with integral coefficients in the expansion of $\left(17^{1/3}+35^{1/2}x\right)^{600}$ is

A.2n

B. 50

C. 150

D. 101

Answer

Given:

$$(x+a)^n = \sum_{k=0}^{n}\binom{n}{k}x^{n-k}a^k$$

$$\left(17^{\frac{1}{3}}+35^{\frac{1}{2}}x\right)^{600} = \sum_{k=0}^{600}\binom{600}{k}\left(17^{1/3}\right)^{600-k}\left(35^{1/2}x\right)^{k}$$

For integral coefficients; (600-k) should be divisible by 3 and k should be disable bye 2.

It indicates that k should be multiple of 6.

So, the values of k would be = 6,12,18…,594,600

32. Question

Mark the correct alternative in the following :

Constant term in the expansion of $\left(x-\frac{1}{x}\right)^{10}$ is

A.152

B. -152

C. -252

D. 252

Answer

Given:

$$(x+a)^n = \sum_{k=0}^{n}\binom{n}{k}x^{n-k}a^k$$

$$\left(x-\frac{1}{x}\right)^{10}=\sum_{k=0}^{10}\binom{10}{k}x^{10-k}\left(\frac{-1}{x}\right)^{k}$$

$$=\sum_{k=0}^{10}\binom{10}{k}x^{10-k}(-1)^{k}x^{-k}$$

For constant term,

$x^{10-k-k}=x^{0}$

10-2k = 0

k = 5

Term = $\binom{10}{5}x^{10-5}(-1)^{5}x^{-5}$

= -252

33. Question

Mark the correct alternative in the following :

If the coefficients of x^2 and x^3 in the expansion of $(3+ax)^9$ are the same, then the value of a is.

A. $-\frac{7}{9}$

B. $-\frac{9}{7}$

C. $\frac{7}{9}$

D. $\frac{9}{7}$

Answer

Given:

$$(x+a)^{n}=\sum_{k=0}^{n}\binom{n}{k}x^{n-k}a^{k}$$

$$(3+ax)^{9}=\sum_{k=0}^{9}\binom{9}{k}3^{9-k}(ax)^{k}$$

Coefficient of x^2 ; k=2

$$=\binom{9}{2}3^{9-2}a^{2}$$

$=\binom{9}{2}3^{7}a^{2}$ (1)

Coefficient of x^3 ; k=3

$$=\binom{9}{3}3^{9-3}a^{3}$$

$=\binom{9}{3}3^{6}a^{3}$ (2)

Equate both equations;

$$\binom{9}{2}3^7a^2 = \binom{9}{3}3^6a^3$$

$$\frac{9!}{2! \times 7!} \times 3 = \frac{9!}{3! \times 6!}a$$

$$\frac{1}{7} \times 3 = \frac{1}{3}a$$

$$\frac{9}{7} = a$$