CBSE Board
Class XII Mathematics
Sample Paper 2

Time: 3 hrs **Total Marks: 100**

General Instructions:

1. **All** the questions are **compulsory**.
2. The question paper consists of **37** questions divided into **three parts** A, B, and C.
3. **Part A** comprises of **20** questions of **1 mark** each. **Part B** comprises of **11** questions of **4 marks** each. **Part C** comprises of **6** questions of **6 marks** each.
4. There is no overall choice. However, an internal choice has been provided in **three questions of 4 marks** each, **four questions of 6 marks** each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculator is **not** permitted.

Section A

Q 1 – Q 20 are multiple choice type questions. Select the correct option.

1. If A and B are two events associated to a random experiment such that $P(A \cap B) = \frac{7}{10}$ and $P(B) = 17/20$, then $P(A/B) =$

A. $\frac{14}{17}$

B. $\frac{17}{14}$

C. $\frac{4}{7}$

D. $\frac{7}{4}$

2. A parallelepiped is formed by planes drawn through the points (2, 3, 5) and (5, 9, 7) parallel to the coordinate planes. The length of a diagonal of the parallelepiped is

A. 40

B. $\sqrt{38}$

C. $\sqrt{155}$

D. 7

3. The value of $\tan^{-1}\left(\sin\left(-\frac{\pi}{2}\right)\right)$ is equal to

A. $\frac{\pi}{4}$

B. −1

C. 1

D. $-\frac{\pi}{4}$

4. The distance of a point P(a, b, c) from the x-axis is

A. $\sqrt{b^2 + c^2}$

B. $\sqrt{a^2 + c^2}$

C. $\sqrt{a^2 + b^2}$

D. none of these

5. If the function $f(x) = 2x^2 - kx + 5$ is increasing on [1, 2], then k lies in the interval:

A. [0, 2]

B. $(-\infty, 4)$

C. $(4, \infty)$

D. (−4, 4)

6. Find the number of all possible matrices of order 3 × 3 with each entry 0 or 1.

A. 64

B. 128

C. 256

D. 512

7. If x < 0, y < 0 such that xy = 1, then $\tan^{-1}x + \tan^{-1}y$ equals

A. 0

B. $\frac{\pi}{2}$

C. $-\frac{\pi}{2}$

D. $\frac{3\pi}{2}$

8. If $\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} - 5\hat{k}$, then find the position vector of their mid-point.

A. $3\hat{i} + \hat{j} - 4\hat{k}$

B. $3\hat{i} - \hat{j} + 4\hat{k}$

C. $\hat{i} + 3\hat{j} + 4\hat{k}$

D. $-\hat{i} - \hat{j} - \hat{k}$

9. Which of the following is the integrating factor of the differential equation

$\cos^2 x \frac{dy}{dx} + y = \tan x$?

A. tan x

B. sec x tan x

C. $e^{\tan x}$

D. $e^{\sec x}$

10. The area bounded by the curve $x^2 = 4y$ and $y = 4$ in the first quadrant is

A. $\frac{32}{3}$ sq. units

B. $\frac{4}{3}$ sq. units

C. $\frac{16}{3}$ sq. units

D. $\frac{2}{3}$ sq. units

11. If $y = 10^{10^x}$ then $\frac{dy}{dx}$ is equal to

A. $10^{10^x} \cdot 10^x (\log 10)^2$

B. $10^x (\log 10)^2$

C. $10^{10^x} \cdot 10^x (\log 10)$

D. $10^{10^x} (\log 10)^2$

12. Two vectors a and b have the same magnitude such that the angle between them is $60°$ and their scalar product is $\frac{9}{2}$. Then their magnitude is

A. $\sqrt{3}$

B. 3

C. 1

D. $3\sqrt{3}$

13. The function $f(x) = \frac{-x}{2} + \sin x$ defined on $\left[-\frac{\pi}{3}, \frac{\pi}{3}\right]$ is increasing on

A. $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

B. $\left[-\frac{\pi}{6}, \frac{\pi}{6}\right]$

C. $\left[-\frac{\pi}{3}, \frac{\pi}{3}\right]$

D. $\left[0, \frac{\pi}{2}\right]$

14. The number of binary operations that can be defined on a set of 2 elements is

A. 8

B. 4

C. 1

D. 16

15. If $f(x) = (x + 1)^{\cot x}$ be continuous at x = 0, then f(0) is equal to

A. 1

B. ∞

C. e

D. log e

16. Find the general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}$.

A. $y = \frac{k}{x}$

B. y = kx

C. k = 0

D. $y = e^x k$

17. Let $\begin{vmatrix} x & 2 & x \\ x^2 & x & 6 \\ x & x & 6 \end{vmatrix} = ax^4 + bx^3 + cx^2 + dx + e$. Then, the value of 5a + 4b + 3c +2d + e is equal to:

A. 0

B. 11

C. −11

D. 1

18. Find $\int \left(\frac{1 - \cos 2x}{1 + \cos 2x}\right) dx$.

A. −2 tan x log(cos x) + c

B. tan x − x + c

C. tan x + c

D. x + c

19. Let R = {(a, a^3): a is a prime number less than 5} be a relation. Then the range of R is

A. {2, 3}

B. {8, 27}

C. R

D. {4, 9}

20. Evaluate $\int_{1}^{\sqrt{3}} \frac{1}{1+x^2} dx$.

A. $-\frac{\pi}{6}$

B. $\frac{\pi}{6}$

C. $-\frac{\pi}{12}$

D. $\frac{\pi}{12}$

Section B

21. Find the family of curves passing through the point (x, y) for which the slope of the tangent is equal to the sum of y-coordinate and exponential raise to the power of x-coordinate.

OR

Form the differential equation of the family of curves y = A cos 2x + B sin 2x, where A and B are constants.

22. Evaluate: $\int \frac{\sin x}{(1-\cos x)(2-\cos x)} dx$

OR

Evaluate $\int \frac{2x^2 - x + 4}{x^3 + 4x} dx$.

23. Show that the function, $f(x) = \begin{cases} \frac{\sin x}{x} + \cos x & , x \neq 0 \\ 2 & , x = 0 \end{cases}$

is continuous at x = 0.

OR

If sin y = x sin(a + y), prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$.

24. Let f: N → N be defined by $f(n) = \begin{cases} \frac{n+1}{2} & \text{if n is odd} \\ \frac{n}{2} & \text{if n is even} \end{cases}$

Find whether the function f is bijective or not.

25. The probability of a shooter hitting a target is $\frac{3}{4}$. How many minimum numbers of times must he fire so that the probability of hitting the target at least once is more than 0.99?

26. If $\tan^{-1}\left(\frac{2x-4}{2x-3}\right) + \tan^{-1}\left(\frac{2x+4}{2x+3}\right) = \frac{\pi}{4}$, then find the value of x.

27. Without expanding the determinant prove that $\begin{vmatrix} 3x+y & 2x & x \\ 4x+3y & 3x & 3x \\ 5x+6y & 4x & 6x \end{vmatrix} = x^3$.

28. If $x = \frac{a}{1+t^3}$ and $y = \frac{at}{1+t^3}$, then find $\frac{dy}{dx}$.

29. Find the value of $\int_0^4 (x-3)(x-1)\,dx$.

30. If the vectors $\vec{a} \neq 0, \vec{b}$ and $\vec{c}$ have magnitude 1, 1 and 4 respectively such that $|\vec{b} \times \vec{c}| = \sqrt{15}$ and $\vec{c} - 2\vec{b} = \lambda \vec{a}$. Then find the value(s) of λ.

31. A line passes through the point (–1, 3, –4) and it is also perpendicular to the plane x + 2y – 5z + 9 = 0. Find the equation of the line.

Section C

32. Prove that the curves $y^2 = 4x$ and $x^2 = 4y$ divide the area of the square bounded by x = 0, x = 4, y = 4, and y = 0 into three equal parts.

OR

Find the area of the region $\{(x, y): 0 \le y \le x^2 + 1, 0 \le y \le x + 1, 0 \le x \le 2\}$.

33. Find the volume of the largest cylinder which can be inscribed in a sphere of radius r.

OR

Find the equations of the normals to the curve $3x^2 - y^2 = 8$, parallel to the line x + 3y = 4.

34. Find the inverse of $A = \begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$ by elementary row transformation.

OR

Given two matrices, $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, verify that BA = 6I, use the result to solve the system x – y = 3, 2x + 3y + 4z = 17, y + 2z = 7

35. A variable plane which remains at a constant distance 3k from the origin cuts the coordinate axes at A, B, C. Show that the locus of the centroid of ΔABC is $x^{-2} + y^{-2} + z^{-2} = k^{-2}$.

36. A manufacturing company makes two models A and B of a product. Each piece of Model A requires 9 labour hours for fabricating and 1 labour hour for finishing. Each piece of Model B requires 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available are 180 and 30 respectively. The company makes a profit of Rs. 8000 on each piece of model A and Rs. 12000 on each piece of Model B. How many pieces of Model A and Model B should be manufactured per week to realise a maximum profit? What is the maximum profit per week?

37. A bag contains 25 balls of which 10 are purple and the remaining are pink. A ball is drawn at random, its colour is noted and it is replaced. 6 balls are drawn in this way, find the probability that

i. All balls were purple
ii. Not more than 2 were pink
iii. An equal number of purple and pink balls were drawn.
iv. Atleast one ball was pink

OR

A doctor is to visit a patient. Form past experience, it is known that the probabilities that he will come by train, bus, scooter or by other means of transport are respectively $\frac{3}{10}, \frac{1}{5}, \frac{1}{10}$ and $\frac{2}{5}$. The probabilities that he will be late are $\frac{1}{4}, \frac{1}{3}$ and $\frac{1}{12}$ if he comes by train, bus and scooter respectively. But if he comes by other means of transport, then he will not be late. When he arrives, he is late. What is the probability that the doctor came by train?

CBSE Board
Class XII Mathematics
Sample Paper 2 – Solution

Section A

1. Correct option: A
Explanation:-

$$P(A \cap B) = \frac{7}{10},\ P(B) = \frac{17}{20}$$

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A/B) = \frac{\frac{7}{10}}{\frac{17}{20}} = \frac{14}{17}$$

2. Correct option: D
Explanation:-

Edges of parallelepiped are $5-2,\ 9-3,\ 7-5 \Rightarrow 3,\ 6,\ 2$

Length of the diagonal $= \sqrt{9+36+4}$

Length of the diagonal $= 7$

3. Correct option: D
Explanation:-

$$\tan^{-1}\left(\sin\left(-\frac{\pi}{2}\right)\right) = \tan^{-1}\left(-\sin\frac{\pi}{2}\right) = \tan^{-1}(-1)$$

As tan(−x) = −tan x

$$\therefore \tan^{-1}(-1) = \tan^{-1}\left(-\tan\frac{\pi}{4}\right) = \tan^{-1}\left[\tan\left(-\frac{\pi}{4}\right)\right] = -\frac{\pi}{4}$$

Hence, $\tan^{-1}\left(\sin\left(-\frac{\pi}{2}\right)\right) = -\frac{\pi}{4}$

4. Correct option: A
Explanation:-

Coordinate of a point on x − axis $(a,0,0)$

The distance of the point $P(a,b,c)$ from x − axis

$$= \sqrt{(a-a)^2 + b^2 + c^2}$$

$$= \sqrt{b^2 + c^2}$$

5. **Correct option: B**

Explanation:-

$f(x) = 2x^2 - kx + 5$

$f'(x) = 4x - k$

$f(x)$ is increasing

$4x - k > 0$ on $[1,2]$

$k < 4x$

Minimum value of k is 4.

$k < 4$

$k \in (-\infty, 4)$

6. **Correct option: D**

Explanation:-

Matrix of order 3 × 3 has 9 elements.

Now the entries have to be either 0 or 1 so that each of the 9 places can be filled with 2 choices 0 or 1.

So $2^9 = 512$ matrices are possible.

7. **Correct option: C**

Explanation:-

Given that xy = 1

Consider,

$\tan^{-1}x + \tan^{-1}y$

$= \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

$= \tan^{-1}(-\infty)$ .....($\because$ $x < 0, y < 0$)

$= -\frac{\pi}{2}$

8. **Correct option: A**

Explanation:-

$\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$;

$\vec{b} = \hat{i} + 3\hat{j} - 5\hat{k}$

$\Rightarrow \vec{a} + \vec{b} = 6\hat{i} + 2\hat{j} - 8\hat{k}$

$\frac{\vec{a} + \vec{b}}{2} = 3\hat{i} + \hat{j} - 4\hat{k}$

9. Correct option: C

Explanation:-

Given differential equation is $\cos^2 x \frac{dy}{dx} + y = \tan x$

$$\Rightarrow \frac{dy}{dx} + \frac{1}{\cos^2 x} y = \frac{\tan x}{\cos^2 x}$$

It is a linear differential equation with $P(x) = \frac{1}{\cos^2 x}$ and $Q(x) = \frac{\tan x}{\cos^2 x}$

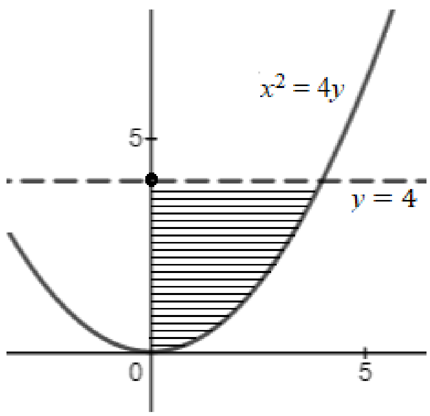
Integrating factor is $e^{\int P(x)dx} = e^{\int \frac{1}{\cos^2 x}dx} = e^{\int \sec^2 x dx} = e^{\tan x}$

Hence, the integrating factor is $e^{\tan x}$.

10. Correct option: A

Explanation:-

Graph of the curve $x^2 = 4y$ and the line $y = 4$ is given by



Area is bounded between the lines y = 0 and y = 4

So, the required area is

$$A = \int_0^4 2\sqrt{y}\,dy = \left[2 \times \frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4 = \frac{4}{3}\left[y^{\frac{3}{2}}\right]_0^4 = \frac{4}{3}[8] = \frac{32}{3}$$

Hence, the area is $\frac{32}{3}$ sq. units.

11. Correct option: A

Explanation:-

Given: $y = 10^{10^x}$

Taking log on both the sides, we get

$\log_{10} y = 10^x (\log 10)$

Differentiating w.r.t x, we get

$$\frac{1}{y}\frac{dy}{dx} = (\log 10)\frac{d}{dx}(10^x) \dots \text{(i)}$$

Let $m = 10^x$

Therefore, $\log_{10} m = x \log 10$

$$\frac{1}{m}\frac{dm}{dx} = \log 10$$

$$\Rightarrow \frac{dm}{dx} = m \log 10$$

$$\Rightarrow \frac{d}{dx}\left(10^x\right) = 10^x \log 10$$

From (i), we get

$$\frac{dy}{dx} = 10^{10^x}\left(\log 10\right)10^x\left(\log 10\right)$$

$$\Rightarrow \frac{dy}{dx} = 10^{10^x} \cdot 10^x \left(\log 10\right)^2$$

12. Correct option: B

Explanation:-

Vectors $\vec{a}$ and $\vec{b}$ have the same magnitude

$$\Rightarrow \left|\vec{a}\right| = \left|\vec{b}\right| \text{..........(i)}$$

Let θ be the angle between the two vectors $\Rightarrow \theta = 60° \text{........(ii)}$

Also, $\vec{a} \cdot \vec{b} = \frac{9}{2} \text{.........(iii)}$

$$\Rightarrow \left|\vec{a}\right| \left|\vec{b}\right| \cos 60° = \frac{9}{2}$$

$$\Rightarrow \left|\vec{a}\right| \left|\vec{a}\right| \cos 60° = \frac{9}{2} \text{ ..... From (i)}$$

$$\Rightarrow \left|\vec{a}\right|^2 \times \frac{1}{2} = \frac{9}{2}$$

$$\Rightarrow \left|\vec{a}\right|^2 = 9$$

$$\Rightarrow \left|\vec{a}\right| = 3$$

$$\Rightarrow \left|\vec{a}\right| = \left|\vec{b}\right| = 3 \text{ .... From (i)}$$

13. Correct option: C

Explanation:-

$$f(x) = \frac{-x}{2} + \sin x$$

$$\Rightarrow f'(x) = \frac{-1}{2} + \cos x$$

$\because \cos x > \frac{1}{2}$ for $x \in \left[-\frac{\pi}{3}, \frac{\pi}{3}\right]$

$\Rightarrow \frac{-1}{2} + \cos x > 0$

$\Rightarrow f'(x) > 0$

Hence, f is increasing on $\left[-\frac{\pi}{3}, \frac{\pi}{3}\right]$.

14. Correct option: D

Explanation:-

Total number of binary operations on a set containing n elements is $(n)^{n^2}$

So, for n = 2

The no. of binary operations defined on a set of 2 elements $= (2)^{2^2} = 2^4 = 16$

15. Correct option: C

Explanation:-

f(0) = $\lim_{x \to 0} f(x)$

$= \lim_{x \to 0} (x + 1)^{\cot x}$

$= \lim_{x \to 0} \left[(1 + x)^{\frac{1}{x}}\right]^{x \cot x}$

$= \lim_{x \to 0} [e]^{x \cot x}$

$= \lim_{x \to 0} [e]^{\frac{x}{\tan x}}$

$= [e]^{\lim_{x \to 0} \frac{1}{\frac{\tan x}{x}}}$

$= e$

16. Correct option: B

Explanation:-

Given differential equation is $\frac{dy}{dx} = \frac{y}{x}$

$\Rightarrow \frac{dy}{y} = \frac{dx}{x}$

Integrating on both sides,

$\Rightarrow \int \frac{dy}{y} = \int \frac{dx}{x}$

$\Rightarrow \log|y| = \log|x| + \log k$

$\Rightarrow \log\left(\frac{y}{x}\right) = \log k$

$\Rightarrow y = kx$

17. Correct option: C

Explanation:-

$$\begin{vmatrix} x & 2 & x \\ x^2 & x & 6 \\ x & x & 6 \end{vmatrix}$$

$= x\left(6x - 6x\right) - 2\left(6x^2 - 6x\right) + x\left(x^3 - x^2\right)$

$= 0 - 12x^2 + 12x + x^4 - x^3$

$= x^4 - x^3 - 12x^2 + 12x$

Comparing with RHS $ax^4 + bx^3 + cx^2 + dx + e$, we have

$a = 1, b = -1, c = -12, d = 12, e = 0$

$\Rightarrow 5a + 4b + 3c + 2d + e = 5 - 4 - 36 + 24 = -11$

18. Correct option: B

Explanation:-

Let I = $\int\left(\frac{1 - \cos 2x}{1 + \cos 2x}\right)dx$

$I = \int\left(\frac{1 - \cos 2x}{1 + \cos 2x}\right)dx$

$= \int\left(\frac{2\sin^2 x}{2\cos^2 x}\right)dx$

$= \int \tan^2 x\,dx = \int\left(\sec^2 x - 1\right)dx$

$= \int \sec^2 x\,dx - \int dx$

$= \tan x - x + c$

19. Correct option: B

Explanation:-

R = {(2, 8), (3, 27)}

$\therefore$ The range set of R is {8, 27}.

20. Correct option: D

Explanation:-

Let $I = \int_1^{\sqrt{3}} \frac{1}{1 + x^2}dx$

$\therefore I = \left[\tan^{-1} x\right]_1^{\sqrt{3}}$

$I = \tan^{-1}\left(\sqrt{3}\right) - \tan^{-1}\left(1\right)$

$I = \frac{\pi}{3} - \frac{\pi}{4}$

$I = \frac{\pi}{12}$

Section B

21. We know that the slope of the tangent is given by $\frac{dy}{dx}$

According to the question, $\frac{dy}{dx} = y + e^x$

Or, $\frac{dy}{dx} - y = e^x$ …… (i)

This is a linear differential equation of the form

$\frac{dy}{dx} + Py = Q$

where, P = -1 and Q = e^x

Therefore,

$\text{I.F.} = e^{\int P dx} = e^{-\int 1 dx} = e^{-x}$

Solution of (i) is given by

$ye^{-x} = \int e^x e^{-x} dx + c$

$\Rightarrow ye^x = x + c$

This is the required family of curves.

OR

Given: y = A cos 2x + B sin 2x

Differentiating w.r.t. x, we get

$\frac{dy}{dx} = A(-\sin 2x) \times 2 + B(\cos 2x) \times 2$

$\frac{dy}{dx} = -2A \sin 2x + 2B \cos 2x$

Again differentiating w. r. t. x, we get

$\frac{d^2y}{dx^2} = -2A \cos 2x \cdot 2 + 2B(-\sin 2x) \cdot 2$

$\frac{d^2y}{dx^2} = -4(A \cos 2x + B \sin 2x)$

$\frac{d^2y}{dx^2} = -4y \Rightarrow \frac{d^2y}{dx^2} + 4y = 0.$

22. Let I = $\int \frac{\sin x}{(1-\cos x)(2-\cos x)} dx$

Here substitute -cosx = t ⇒ sinx dx = dt

$\int \frac{\sin x}{(1 - \cos x)(2 - \cos x)} dx = \int \frac{dt}{(1 + t)(2 + t)}$

Let $\frac{1}{(1+t)(2+t)} = \frac{A}{(1+t)} + \frac{B}{(2+t)}$

1 = A(2 + t) + B(1 + t)

Solving the equation we get

B = -1

A = 1

$$\int \frac{dt}{(1+t)(2+t)} = \int \frac{dt}{1+t} - \int \frac{dt}{2+t}$$

$$= \log|1+t| - \log|2+t| + C$$

$$= \log\left|\frac{1+t}{2+t}\right| + C$$

And so

$$\int \frac{\sin x}{(1-\cos x)(2-\cos x)} dx = \log\left|\frac{1-\cos x}{2-\cos x}\right| + C$$

OR

Let $I = \int \frac{2x^2 - x + 4}{x^3 + 4x} dx$

Now, $\frac{2x^2 - x + 4}{x^3 + 4x} = \frac{2x^2 - x + 4}{(x^2+4)x}$

Let $\frac{2x^2 - x + 4}{(x^2+4)x} = \frac{A}{x} + \frac{Bx + C}{x^2 + 4}$ ... (By partial fractions)

$$\Rightarrow 2x^2 - x + 4 = A(x^2 + 4) + (Bx + C)x$$

$$\Rightarrow 2x^2 - x + 4 = (A + B)x^2 + Cx + 4A$$

Equating the corresponding coefficients, we get

A = 1, B = 1 and C = –1

Substituting the values of A, B and C we have

$$I = \int \left(\frac{1}{x} + \frac{x-1}{x^2+4}\right) dx$$

$$= \int \frac{dx}{x} + \int \left(\frac{x-1}{x^2-4}\right) dx$$

$$= \int \frac{dx}{x} + \frac{1}{2}\int \frac{2x}{x^2-4} dx - \int \frac{1}{x^2 - 2^2} dx$$

$$= \log x + \frac{1}{2}\log(x^2 - 4) - \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C$$

23. To prove the continuity of f(x) at x = 0, we need to prove that

$$\lim_{x \to 0} f(x) = f(0)$$

Consider,

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} \left(\frac{\sin x}{x} + \cos x \right)$$

$$= \lim_{x \to 0} \frac{\sin x}{x} + \lim_{x \to 0} \cos x$$

$$= 1 + \cos 0 \ \ldots \left(\because \lim_{x \to 0} \frac{\sin x}{x} = 1 \right)$$

$$= 1 + 1$$

$$= 2$$

Therefore, $\lim_{x \to 0} f(x) = 2$

Also, f(0) = 2

$\therefore \lim_{x \to 0} f(x) = f(0)$

Hence, f(x) is continuous at x = 0.

OR

Given: sin y = x sin(a + y) … (i)

To Prove: $\frac{dy}{dx} = \frac{\sin^2(a + y)}{\sin a}$

$$\frac{\sin y}{\sin(a + y)} = x \ \ldots \text{From (i)}$$

$$\frac{\sin(a + y - a)}{\sin(a + y)} = x$$

$$\frac{\sin(a + y)\cos a - \cos(a + y)\sin a}{\sin(a + y)} = x$$

$$\cos a - \cot(a + y)\sin a = x$$

$$\operatorname{cosec}^2(a + y)\sin a . \frac{dy}{dx} = 1$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\sin a \operatorname{cosec}^2(a + y)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sin^2(a + y)}{\sin a}$$

24. f : N → N is defined as

$$f(n) = \begin{cases} \frac{n + 1}{2} & \text{if } n \text{ is odd} \\ \frac{n}{2} & \text{if } n \text{ is even} \end{cases}$$

Let $f(n_1) = f(n_2)$

Case 1 : n_1, n_2 are odd

Let $f(n_1) = f(n_2)$

$\Rightarrow \frac{n_1 + 1}{2} = \frac{n_2 + 1}{2}$

$\Rightarrow n_1 = n_2$

Case 2: n_1, n_2 are even

$f(n_1) = f(n_2) \Rightarrow \frac{n_1}{2} = \frac{n_2}{2} \Rightarrow n_1 = n_2$

Case3: n_1 is odd and n_2 is even

$f(n_1) = f(n_2) \Rightarrow \frac{n_1 + 1}{2} = \frac{n_2}{2}$

$\Rightarrow n_1 + 1 = n_2$

$\Rightarrow n_1 \neq n_2$

Hence,

$f(n_1) = f(n_2)$ does not imply $n_1 = n_2 \ \forall \ n_1, n_2 \in N$

$\therefore$ f is not one – one

Function f is onto and hence, f is surjective.

Hence, f is not bijective.

25. Let the shooter fire n times. Then n fires are Bernoulli's trials

Let p = probability of hitting the target = $\frac{3}{4}$

q = probability of not hitting the target = $\frac{1}{4}$

$\Rightarrow P(X = r) = {}^nC_r \, q^{n-r} p^r$

$$= {}^nC_r \left(\frac{1}{4}\right)^{n-r} \left(\frac{3}{4}\right)^r = {}^nC_r \frac{3^r}{4^n}$$

$\Rightarrow$ P(hitting the target at least once) > 0.99

$P(X \geq 1) > 0.99$

$1 - P(X = 0) > 0.99$

$1 - {}^nC_0 \frac{1}{4^n} > 0.99$

${}^nC_0 \frac{1}{4^n} < 0.01$

$\frac{1}{4^n} < 0.01$

$4^n > \frac{1}{0.01} = 100$

The minimum value of n is 4

Thus the shooter must fire at least 4 times.

26. Given: $\tan^{-1}\left(\frac{2x-4}{2x-3}\right)+\tan^{-1}\left(\frac{2x+4}{2x+3}\right)=\frac{\pi}{4}$

$$\Rightarrow \tan^{-1}\left[\frac{\frac{2x-4}{2x-3}+\frac{2x+4}{2x+3}}{1-\left(\frac{2x-4}{2x-3}\right)\left(\frac{2x+4}{2x+3}\right)}\right]=\frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left[\frac{(2x-4)(2x+3)+(2x+4)(2x-3)}{(2x-3)(2x+3)-(2x-4)(2x+4)}\right]=\frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left[\frac{4x^2-2x-12+4x^2+2x-12}{4x^2-9-4x^2+16}\right]=\frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left[\frac{8x^2-24}{-7}\right]=\frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left[\frac{24-8x^2}{7}\right]=\frac{\pi}{4}$$

$$\Rightarrow \frac{24-8x^2}{7}=\tan\frac{\pi}{4}$$

$$\Rightarrow 24-8x^2=7$$

$$\Rightarrow 8x^2=17$$

$$\Rightarrow x=\pm\sqrt{\frac{17}{8}}$$

27. To prove that $\begin{vmatrix}3x+y & 2x & x\\ 4x+3y & 3x & 3x\\ 5x+6y & 4x & 6x\end{vmatrix}=x^3$

Consider, LHS

$$=\begin{vmatrix}3x+y & 2x & x\\ 4x+3y & 3x & 3x\\ 5x+6y & 4x & 6x\end{vmatrix}=\begin{vmatrix}3x & 2x & x\\ 4x & 3x & 3x\\ 5x & 4x & 6x\end{vmatrix}+\begin{vmatrix}y & 2x & x\\ 3y & 3x & 3x\\ 6y & 4x & 6x\end{vmatrix}$$

$$=x^3\begin{vmatrix}3 & 2 & 1\\ 4 & 3 & 3\\ 5 & 4 & 6\end{vmatrix}+x^2y\begin{vmatrix}1 & 2 & 1\\ 3 & 3 & 3\\ 6 & 4 & 6\end{vmatrix}$$

$$=x^3\begin{vmatrix}3 & 2 & 1\\ 4 & 3 & 3\\ 5 & 4 & 6\end{vmatrix}+x^2y\times 0 \quad \ldots[\because C_1 \text{ and } C_3 \text{ are identical}]$$

$$=x^3\begin{vmatrix}3 & 2 & 1\\ 4 & 3 & 3\\ 5 & 4 & 6\end{vmatrix}$$

Applying $C_1 \to C_1 - C_2$

$$= x^3 \begin{vmatrix} 1 & 2 & 1 \\ 1 & 3 & 3 \\ 1 & 4 & 6 \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_2$

$$= x^3 \begin{vmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & 1 & 3 \end{vmatrix}$$

$= x^3 \times (3 - 2)$ … [Expanding along C_1]

$= x^3$

= R.H.S.

Hence, $\begin{vmatrix} 3x + y & 2x & x \\ 4x + 3y & 3x & 3x \\ 5x + 6y & 4x & 6x \end{vmatrix} = x^3.$

28. Given: $x = \frac{a}{1 + t^3}$ and $y = \frac{at}{1 + t^3}$

Differentiating x w.r.t t, we get

$$\frac{dx}{dt} = -a\left(1 + t^3\right)^{-2} \frac{d}{dt}\left(1 + t^3\right)$$

$$\therefore \frac{dx}{dt} = -\frac{3t^2 a}{\left(1 + t^3\right)^2} \text{ … (i)}$$

Differentiating y w.r.t t, we get

$$\frac{dy}{dt} = \frac{\left(1 + t^3\right)a - at\left(3t^2\right)}{\left(1 + t^3\right)^2}$$

$$\frac{dy}{dt} = \frac{\left(1 + t^3 - 3t^3\right)a}{\left(1 + t^3\right)^2}$$

$$\therefore \frac{dy}{dt} = \frac{\left(1 - 2t^3\right)a}{\left(1 + t^3\right)^2} \text{ … (ii)}$$

From (i) and (ii), we get

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{3t^2 a}{\left(1 + t^3\right)^2}}{\frac{\left(1 - 2t^3\right)a}{\left(1 + t^3\right)^2}} = \frac{3t^2 a}{\left(1 - 2t^3\right)a}$$

$$\therefore \frac{dy}{dx} = \frac{3t^2}{1 - 2t^3}$$

29. Let $I = \int_0^4 |x^2 - 4x + 3|dx$

Now, $x^2 - 4x + 3 = (x - 3)(x - 1)$

$|x^2 - 4x + 3| = x^2 - 4x + 3$ if $x^2 - 4x + 3 > 0$

i.e. $(x - 3)(x - 1) > 0$

i.e. $x - 3 > 0$ & $x - 1 > 0$ OR $x - 3 < 0$ & $x - 1 < 0$

i.e. $x > 3$ & $x > 1$ OR $x < 3$ & $x < 1$

i.e. $x > 3$ OR $x < 1$

Therefore, $|x^2 - 4x + 3| = x^2 - 4x + 3$ for $x \in (-\infty, 1) \cup (3, \infty)$ ... (i)

$|x^2 - 4x + 3| = -(x^2 - 4x + 3)$ if $x^2 - 4x + 3 < 0$

i.e. $(x - 3)(x - 1) < 0$

i.e. $x - 3 > 0$ & $x - 1 < 0$ OR $x - 3 < 0$ & $x - 1 > 0$

i.e. $x > 3$ & $x < 1$ OR $x < 3$ & $x > 1$

i.e. $1 < x < 3$ i.e. $x \in (1, 3)$

Therefore, $|x^2 - 4x + 3| = -(x^2 - 4x + 3)$ for $x \in (1, 3)$ ... (ii)

$$I = \int_0^1 (x^2 - 4x + 3)dx - \int_1^3 (x^2 - 4x + 3)dx + \int_3^4 (x^2 - 4x + 3)dx$$

$$= \left[\frac{x^3}{3} - 2x^2 + 3x\right]_0^1 - \left[\frac{x^3}{3} - 2x^2 + 3x\right]_1^3 + \left[\frac{x^3}{3} - 2x^2 + 3x\right]_3^4$$

$$= \left[\frac{1}{3} - 2 + 3\right] - \left[9 - 18 + 9 - \frac{1}{3} + 2 - 3\right] + \left[\frac{64}{3} - 32 + 12 - 9 + 18 - 9\right]$$

$$= \left[\frac{1}{3} + 1\right] - \left[-\frac{1}{3} - 1\right] + \left[\frac{64}{3} - 20\right]$$

$$= \frac{1}{3} + \frac{1}{3} + \frac{64}{3} + 1 + 1 - 20$$

$$= \frac{66}{3} - 18$$

$$= 22 - 18$$

$$= 4$$

30. $a \neq 0$, b and c are three vectors such that $|\vec{a}| = |\vec{b}| = 1$ and $|\vec{c}| = 4$

As $|\vec{b} \times \vec{c}| = \sqrt{15}$

$$\Rightarrow |\vec{b}||\vec{c}|\sin\theta = \sqrt{15} \ \ldots (\theta \text{ is the angle between } \vec{b} \text{ and } \vec{c})$$

$$\Rightarrow 1 \times 4\sin\theta = \sqrt{15}$$

$$\Rightarrow \sin\theta = \frac{\sqrt{15}}{4}$$

$$\Rightarrow \cos\theta = \sqrt{1 - \sin^2\theta} = \frac{1}{4} \ \ldots (i)$$

Also, $\vec{c} - 2\vec{b} = \lambda\vec{a}$

$\Rightarrow \left|\vec{c} - 2\vec{b}\right|^2 = \left|\lambda\vec{a}\right|^2$

$\Rightarrow \left|\vec{c}\right|^2 + 4\left|\vec{b}\right|^2 - 4\vec{b}\cdot\vec{c} = \lambda^2\left|\vec{a}\right|^2$

$\Rightarrow \left|\vec{c}\right|^2 + 4\left|\vec{b}\right|^2 - 4\left|\vec{c}\right|\left|\vec{b}\right|\cos\theta = \lambda^2\left|\vec{a}\right|^2$

$\Rightarrow 4^2 + 4(1)^2 - 4(4)(1)\times\frac{1}{4} = \lambda^2(1)^2$ ... From (i)

$\Rightarrow \lambda^2 = 16$

$\Rightarrow \lambda = \pm 4$

Hence, the values of λ are -4 and 4.

31. Let l, m and n be the direction ratios of the given line.

Since the line passes through the point (–1, 3, –4), so the equation will be of the form

$\frac{x-(-1)}{l} = \frac{y-3}{m} = \frac{z-(-4)}{n}$

i.e. $\frac{x+1}{l} = \frac{y-3}{m} = \frac{z+4}{n}$ ... (i)

As this line is perpendicular to the plane x + 2y – 5z + 9 = 0

So, the direction ratios of the line will be proportional to the direction ratios of the given line.

$\therefore \frac{l}{1} = \frac{m}{2} = \frac{n}{-5} = \lambda$ ... (As 1, 2 & –5 are direction ratios)

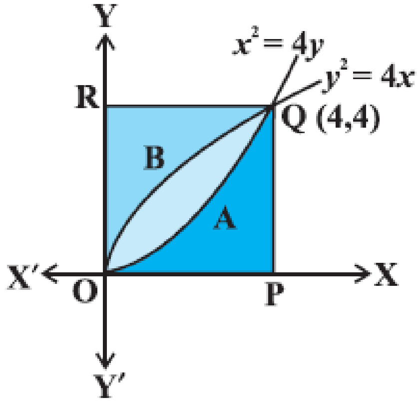
$\therefore l = \lambda,\ m = 2\lambda$ and $n = -5\lambda$

Putting these values in (i), we get

$\frac{x+1}{1} = \frac{y-3}{2} = \frac{z+4}{-5}$ which is the equation of the line

Section C

32. The point of intersection of the parabolas $y^2 = 4x$ and $x^2 = 4y$ are (0, 0) and (4, 4)



Now, the area of the region OAQBO bounded by curves $y^2 = 4x$ and $x^2 = 4y$

$$\int_0^4 \left(2\sqrt{x} - \frac{x^2}{4}\right) dx = \left[2\frac{x^{3/2}}{\frac{3}{2}} - \frac{x^3}{12}\right]_0^4 = \frac{32}{3} - \frac{16}{3} = \frac{16}{3} \text{sq.units} \text{ .... (i)}$$

Again, the area of the region OPQAO bounded by the curves $x^2 = 4y$, $x = 0$, $x = 4$ and the x-axis.

$$\int_0^4 \frac{x^2}{4} dx = \left[\frac{x^3}{12}\right]_0^4 = \left(\frac{64}{12}\right) = \frac{16}{3} \text{sq.units} \text{ .... (ii)}$$

Similarly, the area of the region OBQRO bounded by the curve $y^2 = 4x$ and the y-axis, $y = 0$ and $y = 4$

$$\int_0^4 \frac{y^2}{4} dy = \left[\frac{y^3}{12}\right]_0^4 = \frac{16}{3} \text{sq.units} \text{ .... (iii)}$$

From (i) (ii), and (iii), it is concluded that the area of the region OAQBO = area of the region OPQAO = area of the region OBQRO.

i.e., the parabolas $y^2 = 4x$ and $x^2 = 4y$ divide the area of the square bounded by $x = 0$, $x = 4$, $y = 4$ and $y = 0$ in three equal parts.

OR

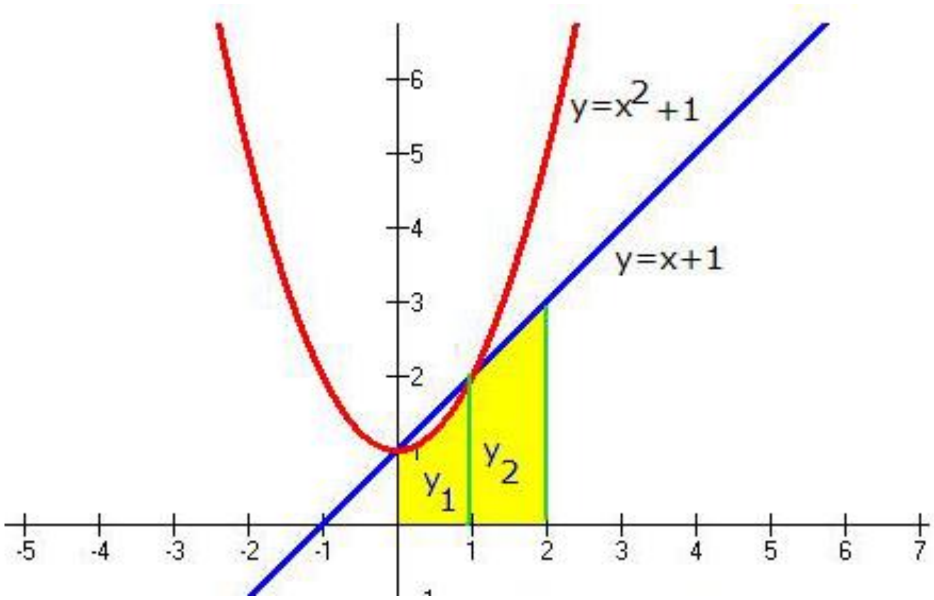
Points of intersection of $y = x^2 + 1$, $y = x + 1$

$x^2 + 1 = x + 1$

$\Rightarrow x(x - 1) = 0$

$\Rightarrow x = 0, 1$

So points of intersection are P(0, 1) and Q(1, 2). The graph is represented as



Required area is given by

$$A = \int_0^1 y_1 \, dx + \int_1^2 y_2 \, dx,$$

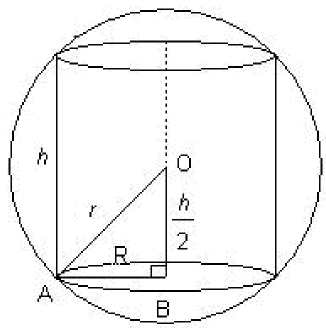
where y_1 and y_2 represent the y co-ordinate of the parabola and straight line respectively.

$$\therefore A = \int_0^1 \left(x^2 + 1\right) dx + \int_1^2 (x + 1)\, dx$$

$$= \left[\left(\frac{x^3}{3} + x\right)\right]_0^1 + \left(\frac{x^2}{2} + x\right)_1^2$$

$$= \left[\left(\frac{1}{3} + 1\right) - 0\right] + \left[(2 + 2) - \left(\frac{1}{2} + 1\right)\right] = \frac{23}{6} \text{ sq. units}$$

33. Radius of the sphere is r. Let *h* and *R* be the height and radius of the cylinder inscribed in the sphere.



Volume of cylinder $(V) = \pi R^2 h$ ..... (1)

In right ΔOBA,

$$AB^2 + OB^2 = OA^2$$

$$R^2 + \frac{h^2}{4} = r^2$$

$$\text{So}, R^2 = r^2 - \frac{h^2}{4}$$

Putting the value of R^2 in equation (1), we get

$$V = \pi\left(r^2 - \frac{h^2}{4}\right) \cdot h$$

$$V = \pi\left(r^2 h - \frac{h^3}{4}\right) \text{ .... (3)}$$

$$\therefore \frac{dV}{dh} = \pi\left(r^2 - \frac{3h^2}{4}\right) \text{ .... (4)}$$

For stationary point, $\frac{dV}{dh} = 0$

$$\pi\left(r^2 - \frac{3h^2}{4}\right) = 0$$

$$r^2 = \frac{3h^2}{4} \Rightarrow h^2 = \frac{4r^2}{3} \Rightarrow h = \frac{2r}{\sqrt{3}}$$

Now, $\frac{d^2V}{dh^2} = \pi\left(-\frac{6}{4}h\right)$

$\therefore \left[\frac{d^2V}{dh^2}\right]_{\left(at\ h=\frac{2r}{\sqrt{3}}\right)} = \pi\left(-\frac{3}{2}\cdot\frac{2r}{\sqrt{3}}\right) < 0$

$\therefore$ Volume is maximum at $h = \frac{2r}{\sqrt{3}}$

Maximum volume is

$$= \pi\left(r^2 . \frac{2r}{\sqrt{3}} - \frac{1}{4} . \frac{8r^3}{3\sqrt{3}}\right)$$

$$= \pi\left(\frac{2r^3}{\sqrt{3}} - \frac{2r^3}{3\sqrt{3}}\right)$$

$$= \pi\left(\frac{6r^3 - 2r^3}{3\sqrt{3}}\right)$$

$$= \frac{4\pi r^3}{3\sqrt{3}} \text{cu. unit}$$

OR

The given line is x + 3y = 4 i.e. $y = -\frac{1}{3}x + \frac{4}{3}$

$\therefore$ Slope of the given line is $-\frac{1}{3}$

$\Rightarrow$ Slope of the required normal = $-\frac{1}{3}$ ... (i) (As required normal is parallel to the given line)

Let the point of contact be (x_1, y_1).

Now, the given curve is $3x^2 - y^2 = 8$

$\Rightarrow 6x - 2y\frac{dy}{dx} = 0$ ... (Diff w.r.t. x)

$\Rightarrow \frac{dy}{dx} = \frac{3x}{y}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(x_1, y_1)} = \frac{3x_1}{y_1}$

$\therefore$ Slope of the normal = $\frac{-1}{\left(\frac{dy}{dx}\right)_{(x_1, y_1)}} = \frac{-y_1}{3x_1}$ .... (ii)

Thus from (i) and (ii), we have

$\frac{-y_1}{3x_1} = -\frac{1}{3}$

$\Rightarrow x_1 = y_1$ .... (iii)

Also, since (x_1, y_1) lies on the given curve, we have

$3x_1^2 - y_1^2 = 8$

$\Rightarrow 3x_1^2 - x_1^2 = 8$ ... From (iii)

$\Rightarrow 2x_1^2 = 8$ or $\Rightarrow x_1^2 = 4$ or $\Rightarrow x_1 = \pm 2$

$\Rightarrow y_1 = \pm 2$ ... From (iii)

Thus, the points of contact are (2, 2) and (−2, −2).

The equation of the required normal at (2, 2) is $\frac{y-2}{x-2} = \frac{-1}{3}$ i.e. $x + 3y - 8 = 0$.

The equation of the required normal at (−2, −2) is $\frac{y+2}{x+2} = \frac{-1}{3}$ i.e. $x + 3y + 8 = 0$.

34. Given:

$$A = \begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$$

$$A = AI$$

$$\begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix} = A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_2 \to R_2 - \frac{2}{3}R_1$$

$$\begin{bmatrix} 3 & 0 & -1 \\ 0 & 3 & \frac{2}{3} \\ 0 & 4 & 1 \end{bmatrix} = A \begin{bmatrix} 1 & 0 & 0 \\ -\frac{2}{3} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_1 \to \frac{R_1}{3}$$

$$\begin{bmatrix} 1 & 0 & -\frac{1}{3} \\ 0 & 3 & \frac{2}{3} \\ 0 & 4 & 1 \end{bmatrix} = A \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ -\frac{2}{3} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_3 \to R_3 - \frac{4}{3}R_2$$

$$\begin{bmatrix} 1 & 0 & -\frac{1}{3} \\ 0 & 3 & \frac{2}{3} \\ 0 & 0 & \frac{1}{9} \end{bmatrix} = A \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ -\frac{2}{3} & 1 & 0 \\ \frac{8}{9} & -\frac{4}{3} & 1 \end{bmatrix}$$

$R_3 \to 9R_3$

$$\begin{bmatrix} 1 & 0 & -\frac{1}{3} \\ 0 & 3 & \frac{2}{3} \\ 0 & 0 & 1 \end{bmatrix} = A \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ -\frac{2}{3} & 1 & 0 \\ 8 & -12 & 9 \end{bmatrix}$$

$R_2 \to R_2 + 2R_1$

$$\begin{bmatrix} 1 & 0 & -\frac{1}{3} \\ 2 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} = A \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & 1 & 0 \\ 8 & -12 & 9 \end{bmatrix}$$

$R_1 \to R_1 + \frac{1}{3}R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} = A \begin{bmatrix} 3 & -4 & 3 \\ 0 & 1 & 0 \\ 8 & -12 & 9 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} = A \begin{bmatrix} 3 & -4 & 3 \\ -6 & 9 & -6 \\ 8 & -12 & 9 \end{bmatrix}$$

$R_2 \to \frac{R_2}{3}$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = A \begin{bmatrix} 3 & -4 & 3 \\ -2 & 3 & -2 \\ 8 & -12 & 9 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} 3 & -4 & 3 \\ -2 & 3 & -2 \\ 8 & -12 & 9 \end{bmatrix}$$

OR

Given: $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ and $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$

$$BA = \begin{bmatrix} 2\times1+2\times2-4\times0 & 2\times(-1)+2\times3-4\times1 & 2\times0+2\times4-4\times2 \\ -4+4 & 4+6-4 & 8-8 \\ 2-2 & -2-3+5 & -4+10 \end{bmatrix}$$

$$BA = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} = 6I$$

System of equations x – y = 3, 2x + 3y + 4z = 17, y + 2z = 7, can be written as

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, C = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$$AX = C$$

$BA = 6I \quad \Rightarrow \quad B = 6I\,A^{-1} \quad \Rightarrow A^{-1} = \frac{1}{6}B$

$$A^{-1} = \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$$

$$X = \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$$X = \frac{1}{6}\begin{bmatrix} 6 + 34 - 28 \\ -12 + 34 - 28 \\ 6 - 17 + 35 \end{bmatrix}$$

$$X = \begin{bmatrix} \frac{12}{6} \\ \frac{-6}{6} \\ \frac{24}{6} \end{bmatrix}$$

$$X = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

x = 2, y = −1, z = 4

35. Let the equation of the variable plane be

$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ ….. (i)

This plane cuts the coordinate axes at A, B, C i.e. x-axis, y-axis and z-axis at the points A(a, 0, 0), B(0, b, 0) and C(0, 0, c) respectively.

Let (p, q, r) be the coordinates of the centroid of ΔABC.

Then, $p = \frac{a + 0 + 0}{3}, q = \frac{0 + b + 0}{3}, r = \frac{0 + 0 + c}{3}$

$\Rightarrow p = \frac{a}{3},\ q = \frac{b}{3},\ r = \frac{c}{3}$

$\Rightarrow a = 3p,\ b = 3q,\ c = 3r$ ….. (ii)

Therefore, 3k = length of the perpendicular from (0, 0, 0) to the plane (i)

$$\Rightarrow 3k = \frac{\left|\frac{0}{a}+\frac{0}{b}+\frac{0}{c}-1\right|}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}} = \frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}$$

$$\Rightarrow \sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}} = \frac{1}{3k}$$

$$\Rightarrow \frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2} = \frac{1}{9k^2}$$

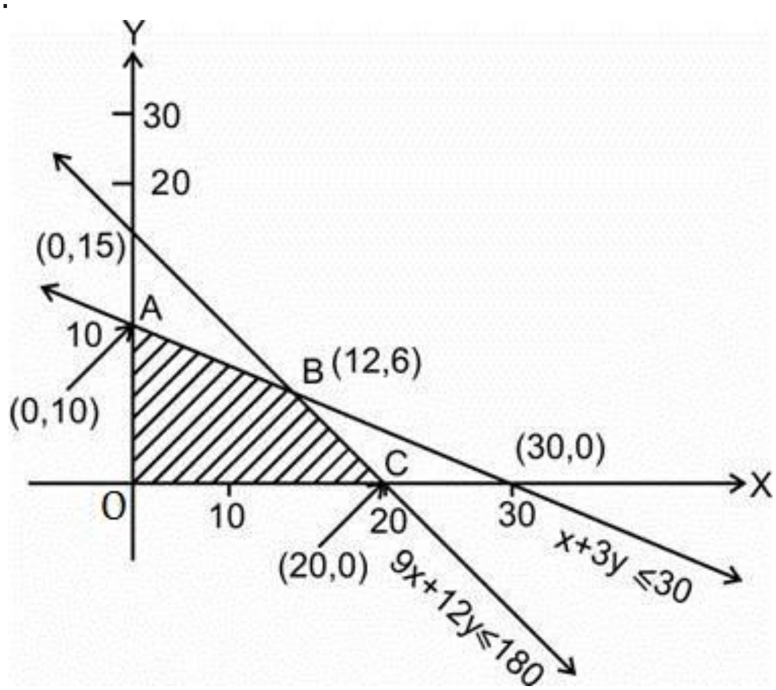
$$\Rightarrow \frac{1}{9p^2}+\frac{1}{9q^2}+\frac{1}{9r^2} = \frac{1}{9k^2}$$

$$\Rightarrow \frac{1}{p^2}+\frac{1}{q^2}+\frac{1}{r^2} = \frac{1}{k^2}$$

$$\Rightarrow p^{-2}+q^{-2}+r^{-2} = k^{-2}$$

Hence, the required locus is $x^{-2}+y^{-2}+z^{-2}=k^{-2}$.

36. Suppose x is the number of pieces of Model A and y is the number of pieces of Model B.
Then, total profit (in Rs.) = $8000x + 12000y$
Let Z = $8000x + 12000y$
Mathematical statement for the given problem is as follows:
Maximise Z = $8000\,x + 12000\,y$ ... (1)
subject to the constraints,
$9x + 12y \le 180$ (Fabrication constraint) i.e. $3x + 4y \le 60$ .... (2)
$x + 3y \le 30$ (Finishing constraint) .... (3)
$x \ge 0,\ y \ge 0$ .... (4)
The feasible region (shaded) OABC determined by the linear inequalities (2) to (4) is shown below.



Corner Point	$Z = 8000x + 12000y$
A(0, 10)	120000
B(12, 6)	168000 ⟵ Maximum
C(20, 0)	160000

The company should produce 12 pieces of Model A and 6 pieces of Model B to realise maximum profit and the maximum profit will be Rs. 1, 68, 000.

37. Let Success: Getting a purple ball on a draw

Let E_1 be the event that a red ball is transferred from bag A to bag B

Let E_2 be the event that a black ball is transferred from bag A to bag B

$\therefore E_1$ and E_2 are mutually exclusive and exhaustive.

$P(E_1) = 3/7$; $P(E_2) = 4/7$

Let E be the event that a red ball is drawn from bag

$$P(E|E_1) = \frac{4+1}{(4+1)+5} = \frac{5}{10} = \frac{1}{2}$$

$$P(E|E_2) = \frac{3+1}{(5+1)+4} = \frac{4}{10} = \frac{2}{5}$$

(a)

$$\therefore \text{ Required probability } = P(E_2|E) = \frac{P(E|E_2)P(E_2)}{P(E|E_1)P(E_1) + P(E|E_2)P(E_2)}$$

$$= \frac{\frac{4}{10} \times \frac{4}{7}}{\frac{1}{2} \times \frac{3}{7} + \frac{4}{10} \times \frac{4}{7}} = \frac{\frac{16}{70}}{\frac{3}{14} + \frac{16}{70}} = \frac{\frac{16}{70}}{\frac{31}{70}} = \frac{16}{31}$$

(b)

$$\therefore \text{ Required probability } = P(E_1|E) = \frac{P(E|E_1)P(E_1)}{P(E|E_1)P(E_1) + P(E|E_2)P(E_2)}$$

$$= \frac{\frac{1}{2} \times \frac{3}{7}}{\frac{1}{2} \times \frac{3}{7} + \frac{4}{10} \times \frac{4}{7}} = \frac{\frac{3}{14}}{\frac{3}{14} + \frac{16}{70}} = \frac{\frac{3}{14}}{\frac{31}{70}} = \frac{15}{31}$$

OR

The events A, E_1, E_2, E_3, and E_4 are given by

A = event when doctor visits patients late

E_1 = doctor comes by train

E_2 = doctor comes by bus

E_3 = doctor comes by scooter

E_4 = doctor comes by other means of transport

So, $P(E_1) = \frac{3}{10}$, $P(E_2) = \frac{1}{5}$, $P(E_3) = \frac{1}{10}$, $P(E_4) = \frac{2}{5}$

$P(A/E_1)$ = Probability that the doctor arrives late, given that he is comes by train.

$= \frac{1}{4}$

Similarly $P(A/E_2) = \frac{1}{3}$, $P(A/E_3) = \frac{1}{12}$, $P(A/E_4) = 0$

Required probability of the doctor arriving late by train by using Baye's theorem, $P(E_1/A)$

$$= \frac{P(E_1)P\left(A/E_1\right)}{P(E_1)P\left(A/E_1\right) + P(E_2)P\left(A/E_2\right) + P(E_3)P\left(A/E_3\right) + P(E_4)P\left(A/E_4\right)}$$

$$= \frac{\frac{3}{10} \times \frac{1}{4}}{\frac{3}{10} \times \frac{1}{4} + \frac{1}{5} \times \frac{1}{3} + \frac{1}{10} \times \frac{1}{12} + \frac{2}{5} \times 0}$$

$$= \frac{3}{40} \times \frac{120}{18} = \frac{1}{2}$$

Hence the required probability is $\frac{1}{2}$.