JCECE
ENGINEERING ENTRANCE EXAM
SOLVED PAPER 2007

Physics

1. A simple pendulum is made of a body which is a hollow sphere containing mercury suspended by means of a wire. If a little mercury is drained off, the period of pendulum will
(a) remain unchanged
(b) increase
(c) decrease
(d) become erratic

2. Two balloons are filled, one with pure He gas and the other by air, respectively. If the pressure and temperature of these balloons are same then the number of molecules per unit volume is
(a) more in the He filled balloon
(b) same in both balloons
(c) more in air filled balloon
(d) in the ratio of 1 : 4

3. Which of the following logic gates is an universal gate ?
(a) OR (b) NOT
(c) AND (d) NAND

4. We consider a thermodynamic system. If ΔU represents the increase in its internal energy and W the work done by the system, which of the following statements is true ?
(a) $\Delta U = -W$ in an adiabatic process
(b) $\Delta U = W$ in an isothermal process
(c) $\Delta U = -W$ in an isothermal process
(d) $\Delta U = W$ in an adiabatic process

5. Aerofils are so designed that the speed of air
(a) on top side is more than on lower side
(b) on top side is less than on lower side
(c) is same on both sides
(d) is turbulent

6. According to Bernoulli's equation
$$\frac{P}{\rho g} + h + \frac{1}{2}\frac{v^2}{g} = \text{constant}$$
The terms A, B and C are generally called respectively :
(a) gravitational head, pressure head and velocity head
(b) gravity, gravitational head and velocity head
(c) pressure head, gravitational head and velocity head
(d) gravity, pressure and velocity head

7. The time of reverberation of a room A is one second. What will be the time (in seconds) of reverberation of a room, having all the dimensions double of those of room A?
(a) 2 (b) 4
(c) $\frac{1}{2}$ (d) 1

8. For sky wave propagation of 10 MHz signal, what should be the minimum electron density in ionosphere ?
(a) $\sim 1.2 \times 10^{12}$ m^{-3}
(b) $\sim 10^{6}$ m^{-3}
(c) $\sim 10^{14}$ m^{-3}
(d) $\sim 10^{22}$ m^{-3}

9. If the mass of moon is $\frac{M}{81}$, where M is the mass of earth, find the distance of the point from the moon, where gravitational field due to earth and moon cancel each other. Given that distance between earth and moon is $60R$, where R is the radius of earth.
(a) $6R$ (b) $8R$
(c) $2R$ (d) $4R$

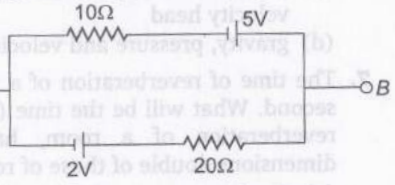
10. 27 small drops each having charge q and radius r coalesce to from big drop. How many times charge and capacitance will become?
(a) 3, 27 (b) 27, 3
(c) 27, 27 (d) 3, 3

11. The magnetic moment has dimensions of
(a) [LA] (b) $[L^2A]$
(c) $[LT^{-1}A]$ (d) $[L^2T^{-1}A]$

12. Which of the following circular rods, (given radius r and length l) each made of the same material and whose ends are maintained at the same temperature will conduct most heat?
(a) $r = 2r_0; l = 2l_0$ (b) $r = 2r_0; l = l_0$
(c) $r = r_0; l = l_0$ (d) $r = r_0; l = 2l_0$

13. For a satellite moving in an orbit around the earth, the ratio of kinetic energy to potential energy is
(a) 2 (b) $\frac{1}{2}$
(c) $\frac{1}{\sqrt{2}}$ (d) $\sqrt{2}$

14. The current in the given circuit is



(a) 0.3 A (b) 0.1 A
(c) 0.4 A (d) 0.2 A

15. Interference was observed in an interference chamber when air was present. Now, the chamber is evacuated and if the same light is used, a careful observation will show
(a) no interference
(b) interference with bright band
(c) interference with dark bands
(d) interference in which breadth of the fringe will be slightly increased

16. When a charged particle moving with velocity $\vec{v}$ is subjected to a magnetic field of induction $\vec{B}$, the force on it is non-zero. This implies that
(a) angle between $\vec{v}$ and $\vec{B}$ is necessarily 90°
(b) angle between $\vec{v}$ and $\vec{B}$ can have any value other than 90°
(c) angle between $\vec{v}$ and $\vec{B}$ can have any value other than zero and 180°
(d) angle between $\vec{v}$ and $\vec{B}$ is either zero or 180°

17. A particle moves along a straight line OX. At a time t (in seconds) the distance x (in metres) of the particle from O is given by
$$x = 40 + 12t - t^3$$
How long would the particle travel before coming to rest?
(a) 24 m (b) 40 m
(c) 56 m (d) 16 m

18. Which one of the following statements is true?
(a) Both light and sound waves in air are transverse
(b) The sound waves in air are longitudinal while the light waves are transverse
(c) Both light and sound waves in air are longitudinal
(d) Both light and sound waves can travel in vacuum

19. When power is drawn from the secondary coil of the transformer, the dynamic resistance
(a) increases
(b) decreases
(c) remains unchanged
(d) changes erratically

20. A reference frame attached to the earth
(a) is an inertial frame by definition
(b) cannot be an inertial frame because earth is revolving round the sun
(c) is an inertial frame because Newton's laws are applicable
(d) is an inertial frame because the earth is rotating about its own axis

21. Pressure of an ideal gas is increased by keeping temperature constant. What is the effect on kinetic energy of molecules?
(a) Increase
(b) Decrease
(c) No change
(d) Can't be determined

22. What maximum frequency can be reflected from ionosphere?
(a) 5 MHz
(b) 6 GHz
(c) 5 kHz
(d) 500 MHz

23. A telescope has focal length of objective and eyepiece as 200 cm and 5 cm respectively. What is magnification of telescope?
(a) 40 (b) 80
(c) 50 (d) 0.01

24. A small disc of radius 2 cm is cut from a disc of radius 6 cm. If the distance between their centres is 3.2 cm, what is the shift in the centre of mass of the disc?

(a) 0.4 cm (b) 2.4 cm
(c) 1.8 cm (d) 1.2 cm

25. Which of the following is not a process involved in fabrication of IC?

(a) Polymerization (b) Diffusion
(c) Photolithography (d) Metallisation

26. Permanent magnet has properties retentivity and coercivity respectively

(a) high-high (b) low-low
(c) low-high (d) high-low

27. Hydrogen bomb is based upon

(a) fission
(b) fusion
(c) chemical reaction
(d) transmutation

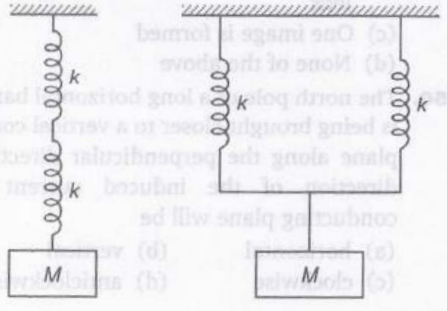
28. What is not true for equipotential surface for uniform electric field?

(a) Equipotential surface is flat
(b) Equipotential surface is spherical
(c) Electric lines are perpendicular to equipotential surface
(d) Work done is zero

29. A closed organ pipe of length 20 cm is sounded with tuning fork in resonance. What is the frequency of tuning fork? (v = 332 m/s)

(a) 300 Hz (b) 350 Hz
(c) 375 Hz (d) 415 Hz

30. Two identical springs of spring constant k are connected in series and parallel as shown in figure. A mass M is suspended from them. The ratio of their frequencies of vertical oscillation will be



(a) 2 : 1 (b) 1 : 2
(c) 4 : 1 (d) 1 : 4

31. The area of the acceleration-displacement curve of a body gives

(a) impulse
(b) change in momentum per unit mass
(c) change in KE per unit mass
(d) total change in energy

32. The speed of earth's rotation about its axis is ω. Its speed is increased to x times to make the effective acceleration due to gravity equal to zero at the equator. Then x is

(a) 1 (b) 8.5
(c) 17 (d) 34

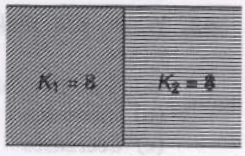
33. Hard X-rays for the study of fractures in bones should have a minimum wavelength of 10^{-11} m. The accelerating voltage for electrons in X-ray machine should be

(a) < 124 kV
(b) > 124 kV
(c) between 60 kV and 70 kV
(d) = 100 kV

34. In photoelectric effect, the electrons are ejected from metals if the incident light has a certain minimum

(a) wavelength
(b) frequency
(c) amplitude
(d) angle of incidence

35. A capacitor having capacitance 1 μF with air, is filled with two dielectrics as shown. How many times capacitance will increase?



(a) 12 (b) 6
(c) 8/3 (d) 3

36. A leaf which contains only green pigments, is illuminated by a laser light of wavelength 0.6328 μm. It would appear to be

(a) brown
(b) black
(c) red
(d) green

37. The operation of a nuclear reactor is said to be critical, if the multiplication factor (k) has a value

(a) 1 (b) 1.5
(c) 2.1 (d) 2.5

38. A galvanometer having a resistance of 8 Ω is shunted by a wire of resistance 2 Ω. If the total current is 1 A, the part of it passing through the shunt will be
(a) 0.25 A (b) 0.8 A
(c) 0.2 A (d) 0.5 A

39. Flash light equipped with a new set of batteries, produces bright white light. As the batteries wear out
(a) the light intensity gets reduced with no change in its colour
(b) light colour changes first to yellow and then red with no change in intensity
(c) it stops working suddenly while giving white light
(d) colour changes to red and also intensity gets reduced

40. If alpha, beta and gamma rays carry same momentum, which has the longest wavelength?
(a) Alpha rays
(b) Beta rays
(c) Gamma rays
(d) None, all have same wavelength

41. An amplifier has a voltage gain $A_v = 1000$. The voltage gain in dB is
(a) 30 dB (b) 60 dB
(c) 3 dB (d) 20 dB

42. When you make ice cubes, the entropy of water
(a) does not change
(b) increases
(c) decreases
(d) may either increase or decrease depending on the process used

43. Half-lives of two radioactive substances *A* and *B* are respectively 20 min and 40 min. Initially the samples of *A* and *B* have equal number of nuclei. After 80 min the ratio of remaining number of *A* and *B* nuclei is
(a) 1 : 16 (b) 4 : 1
(c) 1 : 4 (d) 1 : 1

44. If equation of a sound wave is $y = 0.0015 \sin (62.8x + 314t)$ then its wavelength will be
(a) 0.1 unit (b) 2 unit
(c) 0.3 unit (d) 0.2 unit

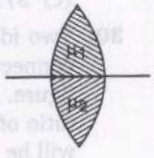
45. A projectile is thrown in the upward direction making an angle of 60° with the horizontal direction with a velocity of 147 ms^{-1}. Then the time after which its inclination with the horizontal is 45°, is
(a) 15 s (b) 10.98 s
(c) 5.49 s (d) 2.745 s

46. A motor cycle is going on an overbridge of radius *R*. The driver maintains a constant speed. As the motor cycle is ascending on the overbridge, the normal force on it
(a) increases
(b) decreases
(c) remains the same
(d) fluctuates erratically

47. If we throw a body upwards with velocity of 4 m/s, at what height does its kinetic energy reduce to half of the initial value? (Take $g = 10\ ms^{-2}$)
(a) 4 m (b) 2 m
(c) 1 m (d) 0.4 m

48. Two glass plates are separated by water. If surface tension of water is 75 dynes/cm and area of each plate wetted by water is 8 cm^2 and the distance between the plates is 0.12 mm, then the force applied to separate the two plates is
(a) 10^2 dyne (b) 10^4 dyne
(c) 10^5 dyne (d) 10^6 dyne

49. Which of the following is true for rays coming from infinity?



(a) Two images are formed
(b) Continuous image is formed between focal points of upper and lower lens
(c) One image is formed
(d) None of the above

50. The north pole of a long horizontal bar magnet is being brought closer to a vertical conducting plane along the perpendicular direction. The direction of the induced current in the conducting plane will be
(a) horizontal (b) vertical
(c) clockwise (d) anticlockwise

Chemistry

1. Calcium carbide reacts with H_2O to produce
(a) ethane (b) methane
(c) ethylene (d) acetylene

2. Propyne and propene can be distinguished by
(a) dil. $KMnO_4$
(b) conc. H_2SO_4
(c) $AgNO_3$ in ammonia
(d) Br_2 in CCl_4

3. Oxidation number of Fe in Fe_2O_3 is
(a) $\frac{1}{2}$ (b) $\frac{2}{6}$
(c) $\frac{8}{3}$ (d) $\frac{3}{2}$

4. Bauxite is an ore of
(a) Al (b) Ca
(c) Cu (d) Ni

5. The best source of vitamin A is
(a) beans (b) pulses
(c) orange (d) carrot

6. Plaster of Paris is
(a) $CaSO_4 \cdot \frac{1}{2} H_2O$
(b) $CaSO_4 \cdot 2H_2O$
(c) $CaSO_4 \cdot H_2O$
(d) $CaSO_4 \cdot 4H_2O$

7. Penicillin is
(a) analgesic (b) antipyretic
(c) antimalarials (d) antibiotic

8. The most stable compound is
(a) LiF (b) LiCl
(c) LiBr (d) LiI

9. Heavy water is
(a) $CaSO_4$
(b) water contain $CaSO_4$, $MgSO_4$
(c) D_2O
(d) water contain $CaCO_3$

10. In the extraction of copper from its sulphide ore, the metal is formed by reduction of Cu_2O with
(a) FeS (b) CO
(c) Cu_2S (d) SO_2

11. BaO_2 and ozone reacts to produce
(a) Ba (b) Ba_2O_3
(c) BaO (d) $Ba(OH)_3$

12. Heisenberg uncertainty principle can be explained as
(a) $\Delta x \geq \frac{\Delta p \times h}{4\pi}$ (b) $\Delta x \times \Delta p \geq \frac{h}{4\pi}$
(c) $\Delta x \times \Delta p \geq \frac{h}{\pi}$ (d) $\Delta p \geq \frac{\pi h}{\Delta x}$

13. For decolourisation of 1 mole of $KMnO_4$, the moles of H_2O_2 required is
(a) $\frac{1}{2}$ (b) $\frac{3}{2}$
(c) $\frac{5}{2}$ (d) $\frac{7}{2}$

14. Bleaching powder is obtained by treating Cl_2 with
(a) $Ca(OH)_2$ (b) CaO
(c) $CaCO_3$ (d) $CaCl_2$

15. Magnalium contains
(a) Mg + Al (b) Mg + Cu
(c) Mg + Fe (d) Mg + Mn

16. The de-Broglie wavelenght of a particle with mass 1 kg and velocity 100 m/s is
(a) 6.6×10^{-33} m (b) 6.6×10^{-36} m
(c) 3.3×10^{33} m (d) 3.3×10^{-36} m

17. The volume of a gas measured at 27°C and 1 atm pressure is 10 L. To reduce the volume to 2 L at 1 atm. pressure, the temperature required is
(a) 60 K (b) 75 K
(c) 150 K (d) 225 K

18. Isotonic solutions have
(a) same vapour pressure
(b) same osmotic pressure
(c) same boiling point
(d) same temperature

19. Epsom salt is
(a) $BaSO_4 \cdot 2H_2O$ (b) $CaSO_4 \cdot H_2O$
(c) $MgSO_4 \cdot 2H_2O$ (d) $MgSO_4 \cdot 7H_2O$

20. Paracetamol is an
(a) analgesic
(b) antipyretic
(c) both (a) and (b)
(d) none of the above

21. The number of moles of oxygen obtained by the electrolytic decomposition of 108 g water is
(a) 2.5 (b) 3
(c) 5 (d) 7.5

22. The change in entropy for the fusion of 1 mole of ice is [mp of ice = 273 K, molar enthalpy of fusion for ice = 6.0 kJ mol^{-1}]
(a) 11.73 JK^{-1} mol^{-1}
(b) 18.84 JK^{-1} mol^{-1}
(c) 21.97 JK^{-1} mol^{-1}
(d) 24.47 JK^{-1} mol^{-1}

23. Which does not give a precipitate with $AgNO_3$ solution ?
(a) $[Co(NH_3)_6]Cl_3$ (b) $[Co(NH_3)_5Cl]Cl_2$
(c) $[Co(NH_3)_4Cl_2]Cl$ (d) $[Co(NH_3)_3Cl_3]$

24. Total volume of atoms present in a face centred cubic unit cell of a metal is (r is atomic radius)
(a) $\frac{16}{3}\pi r^3$ (b) $\frac{20}{3}\pi r^3$
(c) $\frac{24}{3}\pi r^3$ (d) $\frac{12}{3}\pi r^3$

25. Pure silicon doped with phosphorus is a
(a) metallic conductor
(b) insulator
(c) n-type semiconductor
(d) p-type semiconductor

26. Which of the following is a biodegradable polymer ?
(a) Cellulose (b) Polythene
(c) Polyvinyl chloride (d) Nylon 6

27. What is X in the following nuclear reaction ?
$${}_7N^{14} + {}_1H^1 \longrightarrow {}_8O^{15} + X$$
(a) ${}_0n^1$ (b) ${}_{-1}e^0$ (c) ${}_{+1}e^0$ (d) γ

28. Solubility product of $PbCl_2$ at 298 K is 1×10^{-6}. At this temperature solubility of $PbCl_2$ in mol/L is
(a) $(1 \times 10^{-6})^{1/2}$ (b) $(1 \times 10^{-6})^{1/3}$
(c) $(0.25 \times 10^{-6})^{1/3}$ (d) $(0.25 \times 10^{-6})^{1/2}$

29. The pH of a 0.001 M solution of HCl is
(a) 0 (b) 3 (c) 5 (d) 10

30. Gold number is associated with
(a) amount of gold (b) protective colloids
(c) purple of cassius (d) electrophoresis

31. Noble gases are used in discharge tubes to give different colours. Reddish-orange glow is due to
(a) Ar (b) Ne (c) Xe (d) Kr

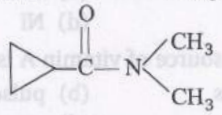
32. The set representing the correct order for first ionisation potential is
(a) K > Na > Li (b) Be > Mg > Ca
(c) B > C > N (d) Ge > Si > C

33. Dry ice is
(a) solid CO_2
(b) solid camphor
(c) solid SO_2
(d) solid NO_2

34. Methanol and ethanol are miscible in water due to
(a) covalent character
(b) hydrogen bonding character
(c) oxygen bonding character
(d) none of the above

35. Which of the following does not have optical isomer ?
(a) $[Co(NH_3)_3Cl_3]$
(b) $[Co(en)_3]Cl_2$
(c) $[Co(en)_2Cl_2]Cl$
(d) $[Co(en)(NH_3)_2Cl_2]Cl$

36. IUPAC name of the following compound



(a) N, N-dimethylcyclopropanecarboxamide
(b) N-methylcyclopropanamide
(c) cyclopropionamide
(d) none of the above

37. The product of following reaction is
$$2CH_3MgBr + CH_2O \longrightarrow$$
(a) CH_3OH (b) C_2H_5OH
(c) CH_4 (d) C_2H_6

38. Freon used as refrigerant is
(a) $CF_2 = CF_2$ (b) CH_2F_2
(c) CCl_2F_2 (d) CF_4

39. Lucas reagent is
(a) anhy. $ZnCl_2$ and NH_3
(b) anhy. $ZnCl_2$ and $CaCl_2$
(c) anhy. $ZnCl_2$ and conc. HCl
(d) anhy. $ZnCl_2$ and HCl gas

40. The enzyme which can catalyse the conversion of glucose to ethanol is
(a) zymase (b) invertase
(c) maltase (d) diastase

41. When dihydroxy acetone reacts with HIO_4, the product is/are
(a) HCHO
(b) HCOOH
(c) HCHO and HCOOH
(d) HCHO and CO_2

42. Which of the following does not reduce Fehling's solution?
(a) Benzaldehyde (b) Formic acid
(c) Glucose (d) Fructose

43. Sodium formate on heating gives
(a) oxalic acid and H_2
(b) sodium oxalate and H_2
(c) sodium oxalate
(d) CO_2 and caustic soda

44. Reaction of ethyl formate with excess of CH_3MgI followed by hydrolysis gives
(a) *n*-propyl alcohol
(b) isopropyl alcohol
(c) acetaldehyde
(d) acetone

45. Hydrolysis of phenyl isocyanide forms
(a) benzoic acid (b) formic acid
(c) acetic acid (d) none of these

46. Styrene can be purified by
(a) simple distillation
(b) fractional distillation
(c) steam distillation
(d) vacuum distillation

47. Which of the following is not reducing sugar ?
(a) Glucose (b) Fructose
(c) Lactose (d) Sucrose

48. The monomer of teflon is :
(a) $CHF = CH_2$ (b) $CF_2 = CF_2$
(c) $CHCl = CHCl$ (d) $CHF = CHCl$

49. The hybridisation state of carbon in fullerene is
(a) sp (b) sp^2
(c) sp^3 (d) sp^3d

50. A fruity smell is produced by the reaction of C_2H_5OH with
(a) CH_3COCH_3 (b) CH_3COOH
(c) PCl_5 (d) CH_3CHO

Mathematics

1. If $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$, then x is equal to
(a) 0, $2a$ (b) a, $2a$
(c) 0, $3a$ (d) none of these

2. If the resultant of two forces $2p$ and $\sqrt{2}p$ is $\sqrt{10}p$, then the angle between them will be
(a) $\frac{\pi}{2}$ (b) π
(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

3. When $y = \sin^{-1}\left(\frac{2t}{1+t^2}\right)$, $x = \tan^{-1}\left(\frac{2t}{1-t^2}\right)$ where $-1 < t < 1$, then $\frac{dy}{dx}$ is equal to
(a) 0 (b) 1
(c) -1 (d) ∞

4. $\lim_{x\to\infty} \sqrt{\frac{x - \sin x}{x + \cos^2 x}}$ is equal to
(a) -1 (b) 0
(c) $\frac{1}{2}$ (d) 1

5. The least number of times a fair coin must be tossed so that the probability of getting at least one head in at least 0.8 is :
(a) 3 (b) 6
(c) 5 (d) none of these

6. If $\frac{dy}{dx} + y = 2e^{2x}$, then y is equal to
(a) $ce^{-x} + \frac{2}{3}e^{2x}$ (b) $(1+x)e^{-x} + \frac{2}{3}e^{2x} + c$
(c) $ce^{-x} + \frac{2}{3}e^{2x} + c$ (d) $e^{-x} + \frac{2}{3}e^{2x} + c$

7. A particle is projected under gravity ($g = 9.81$ m/s^2) with a velocity of 29.43 m/s at an elevation of 30°. The time of flight in seconds to a height of 9.81 m are
(a) 1, 2 (b) 5, 1.5
(c) 1.5, 2 (d) 2, 3

8. $\lim_{x\to 1} \frac{x^8 - 2x + 1}{x^4 - 2x + 1}$ equals
(a) 3 (b) 0
(c) -3 (d) 1

9. $\lim_{x\to\frac{\pi}{6}} \frac{\sin 2x}{\sin x}$ is equal to
(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$
(c) 2 (d) $\frac{1}{2}$

10. The position vectors of three points are $\hat{\mathbf{i}}$, $\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$ and $\hat{\mathbf{i}} + 3\hat{\mathbf{k}}$. The area of the triangle formed by these three points, is
(a) 3 sq unit (b) $3\sqrt{3}$ sq unit
(c) 4 sq unit (d) 5 sq unit

11. $\int_0^{\pi/3} \frac{\cos x + \sin x}{\sqrt{1+\sin 2x}}\,dx$ is equal to

(a) $\frac{4\pi}{3}$ (b) $\frac{2\pi}{3}$

(c) π (d) $\frac{\pi}{3}$

12. If x, y are any two elements of a Boolean lattice, then $(x+y)\cdot(x+y')\cdot(x'+y)$ is equal to

(a) $x+y$ (b) $x\cdot y$

(c) $x'+y$ (d) $x\cdot y'$

13. Area of quadrilateral whose vertices are (2, 3) (3, 4), (4, 5) and (5, 6) is equal to

(a) 0 (b) 4

(c) 6 (d) none of these

14. In a triangle vertex angles are A , B , C and side BC are given. The area of ΔABC is

(a) $\frac{s(s-a)(s-b)(s-c)}{2}$

(b) $\frac{b^2 \sin C \sin A}{\sin B}$

(c) $ab \sin C$

(d) $\frac{1}{2}\frac{a^2 \sin B \sin C}{\sin A}$

15. $f(x) = \sin|x|$. Then $f(x)$ is not differentiable at

(a) $x = 0$ only

(b) all x

(c) multiples of π

(d) multiples of $\frac{\pi}{2}$

16. Which of the following is a logical statement ?

(a) Are you going to chennai ?

(b) Shyam is a handsome boy.

(c) Alas ! I were a king.

(d) Sum of two irrational number is always irrational.

17. For a given integer k, in the interval $\left[2\pi k - \frac{\pi}{2}, 2\pi k + \frac{\pi}{2}\right]$ the graph of $\sin x$ is

(a) increasing from –1 to 1

(b) decreasing from –1 to 0

(c) decreasing from 0 to 1

(d) none of the above

18. Coefficient of the term independent of x in the expansion $\left(x + \frac{1}{x^2}\right)^6$ is equal to

(a) 10 (b) 15

(c) 16 (d) none of these

19. A particle starts from rest with uniform acceleration and acquires a velocity of 40 m/s in 10 s. The displacement of the particle at the end of 10 s is

(a) 4 m (b) 200 m

(c) 20 m (d) none of these

20. If $P(A) = P(B) = P(C) = \frac{1}{4}$,

$P(AB) = P(CB) = 0$ and $P(A \cap C) = \frac{1}{8}$,

then $P(A+B)$ is equal to

(a) $\frac{5}{8}$ (b) $\frac{37}{64}$

(c) $\frac{3}{4}$ (d) $\frac{1}{2}$

21. Angles A, B and C of a triangle are in AP with common difference 15 degree, then angle A is equal to

(a) 45° (b) 60°

(c) 75° (d) 30°

22. Length of major axis of ellipse $9x^2 + 7y^2 = 63$ is

(a) 3 (b) 9

(c) 6 (d) $2\sqrt{7}$

23. Area between curves $y = x^2$, $x = y^2$ is

(a) $\frac{1}{9}$ sq unit (b) $\frac{1}{3}$ sq unit

(c) $\frac{1}{\sqrt{3}}$ sq unit (d) $\frac{2}{3}$ sq unit

24. $\int \log 2x\,dx$ is equal to

(a) $x \log 2x - \frac{x^2}{2} + c$ (b) $x \log 2x - \frac{x}{2} + c$

(c) $x^2 \log 2x - \frac{x}{2} + c$ (d) $x \log 2x - x + c$

25. $f(x) = \frac{1}{x^2}$, is

(a) symmetric about x-axis

(b) symmetric about y-axis

(c) symmetric about x and y axes

(d) none of the above

26. 1 , 2 , 3 , 4 are four numbers. How many numbers can be made using all four numbers

(a) 1 (b) 3

(c) 2 (d) 64

27. $f(x) = \begin{cases} \frac{\sin 3x}{\sin x}, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous, if k is

(a) 3 (b) 0

(c) –3 (d) –1

28. If $y = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$, then $y'(0)$ is

(a) 1/2 (b) 0
(c) 1 (d) does not exist

29. The slope of the tangent at the point (h, h) to the circle $x^2 + y^2 = a^2$ is

(a) 0 (b) 1
(c) -1 (d) will depend on h

30. If $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, I is the unit matrix of order 2 and a, b are arbitary constants, then $(aI + bA)^2$ is equal to

(a) $a^2I - abA$ (b) $a^2I + 2abA$
(c) $a^2I + b^2A$ (d) none of these

31. The value of $\tan 1° \tan 2° \tan 3° \ldots \tan 89°$ is equal to

(a) -1 (b) 2
(c) $\frac{\pi}{2}$ (d) 1

32. Let α and β be the roots of the equation $x^2 + x + 1 = 0$, then the equation whose roots are α^{19}, β^7, is

(a) $x^2 - x - 1 = 0$ (b) $x^2 - x + 1 = 0$
(c) $x^2 + x - 1 = 0$ (d) $x^2 + x + 1 = 0$

33. If sum of two unit vectors is also an unit vector, then the angle between the two vectors is

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$
(c) $\frac{\pi}{4}$ (d) $\frac{2\pi}{3}$

34. Distance from the point (1, 6, 3) to the line $\vec{r} = (\hat{j} + 2\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$ is

(a) $\sqrt{13}$ (b) 13
(c) $2\sqrt{3}$ (d) none of these

35. The value of $\int_{-\pi}^{\pi} (1 - x^2) \sin x \cos^2 x \, dx$ is

(a) 0 (b) $\pi - \frac{\pi^3}{3}$
(c) $2\pi - \pi^3$ (d) $\frac{7}{2} - 2\pi^3$

36. The equation of circle with origin as centre and passing through the vertices of an equilateral triangle whose median is of length $3a$, is

(a) $x^2 + y^2 = 9a^2$
(b) $x^2 + y^2 = 16a^2$
(c) $x^2 + y^2 = 4a^2$
(d) $x^2 + y^2 = a^2$

37. The angle between the pair of tangents drawn to the ellipse $3x^2 + 2y^2 = 5$ from the point (1, 2) is

(a) $\tan^{-1}\left(\frac{12}{5}\right)$ (b) $\tan^{-1}(6\sqrt{5})$
(c) $\tan^{-1}\left(\frac{12}{\sqrt{5}}\right)$ (d) $\tan^{-1}(12\sqrt{5})$

38. The values of x and y such that y satisfy the equation ($x, y \in$ real numbers) $x^2 - xy + y^2 - 4x - 4y + 16 = 0$ is

(a) 4, 4 (b) 3, 3
(c) 2, 2 (d) none of these

39. The complex number $z = x + iy$ which satisfy the equation $\left|\frac{z - 5i}{z + 5i}\right| = 1$ lies on

(a) the axis of x
(b) the straight line $y = 5$
(c) the circle passing through the origin
(d) none of the above

40. $\left(1 + \cos\frac{\pi}{8}\right)\left(1 + \cos\frac{3\pi}{8}\right)\left(1 + \cos\frac{5\pi}{8}\right)\left(1 + \cos\frac{7\pi}{8}\right)$ is equal to

(a) $\frac{1}{2}$ (b) $\cos\frac{\pi}{8}$
(c) $\frac{1}{8}$ (d) $\frac{1+\sqrt{2}}{2\sqrt{2}}$

41. If $a \le 0$, then one of the roots of $x^2 - 2a|x - a| - 3a^2 = 0$ is

(a) $(-1 + \sqrt{6})a$ (b) $(\sqrt{6} - 1)a$
(c) a (d) none of these

42. If the radius of the circumcircle of an isosceles triangle PQR is equal to $PQ = PR$, then the angle P is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$
(c) $\frac{\pi}{2}$ (d) $\frac{2\pi}{3}$

43. If $z_r = \cos\left(\frac{\pi}{2^r}\right) + i \sin\left(\frac{\pi}{2^r}\right)$, $r = 1, 2, \ldots,$ then $z_1 z_2 z_3 \ldots$ is equal to

(a) 1 (b) $-i$
(c) i (d) -1

44. The condition for which $a^2x^4 + bx^3 + cx^2 + dx + f^2$ may be a perfect square, is

(a) $4a^2c - b^2 = 8a^3f$
(b) $4a^2c = 8a^3f$
(c) $2a^2c = a^3f$
(d) none of these

45. If $\tan(\cot x) = \cot(\tan x)$, then $\sin 2x$ is equal to

(a) $\frac{2}{(2n+1)\pi}$ (b) $\frac{4}{(2n+1)\pi}$

(c) $\frac{2}{n(n+1)\pi}$ (d) $\frac{4}{n(n+1)\pi}$

46. Let a, b, c be real, if $ax^2 + bx + c = 0$ has two real roots α and β, where $\alpha < -2$ and $\beta > 2$, then

(a) $4 - \frac{2b}{a} + \frac{c}{a} < 0$ (b) $4 + \frac{2b}{a} - \frac{c}{a} < 0$

(c) $4 - \frac{2b}{a} + \frac{c}{a} = 0$ (d) $4 + \frac{2b}{a} + \frac{c}{a} = 0$

47. The maximum value of $x^3 - 3x$ in the interval $[0, 2]$ is

(a) -2 (b) 0

(c) 2 (d) 1

48. y intercept of line passes through (2, 2) and is perpendicular to the line $3x + y = 3$, is

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$

(c) 1 (d) $\frac{4}{3}$

49. If α, β, γ be the angles which a line makes with the co-ordinates axes, then

(a) $\sin^2\alpha + \cos^2\beta + \sin^2\gamma = 1$

(b) $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 1$

(c) $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

(d) $\cos^2\alpha + \sin^2\beta + \sin^2\gamma = 1$

50. The value of x satisfying $\sin^{-1}x + \sin^{-1}(1-x) = \cos^{-1}x$ are

(a) 0 (b) 1, –1

(c) $0, \frac{1}{2}$ (d) none of these

ANSWERS

PHYSICS

1.	(b)	2.	(b)	3.	(d)	4.	(a)	5.	(a)	6.	(c)	7.	(a)	8.	(a)
9.	(a)	10.	(b)	11.	(b)	12.	(b)	13.	(b)	14.	(b)	15.	(d)	16.	(c)
17.	(c)	18.	(b)	19.	(a)	20.	(b)	21.	(c)	22.	(a)	23.	(a)	24.	(a)
25.	(a)	26.	(a)	27.	(b)	28.	(b)	29.	(d)	30.	(b)	31.	(c)	32.	(c)
33.	(a)	34.	(b)	35.	(b)	36.	(b)	37.	(a)	38.	(b)	39.	(d)	40.	(d)
41.	(a)	42.	(c)	43.	(c)	44.	(a)	45.	(c)	46.	(a)	47.	(d)	48.	(c)
49.	(a)	50.	(c)												

CHEMISTRY

1.	(d)	2.	(c)	3.	(c)	4.	(a)	5.	(d)	6.	(a)	7.	(d)	8.	(a)
9.	(c)	10.	(a)	11.	(c)	12.	(b)	13.	(c)	14.	(a)	15.	(a)	16.	(b)
17.	(a)	18.	(b)	19.	(d)	20.	(c)	21.	(b)	22.	(c)	23.	(d)	24.	(a)
25.	(c)	26.	(a)	27.	(d)	28.	(c)	29.	(b)	30.	(b)	31.	(b)	32.	(b)
33.	(a)	34.	(b)	35.	(a)	36.	(a)	37.	(b)	38.	(c)	39.	(c)	40.	(a)
41.	(d)	42.	(d)	43.	(b)	44.	(c)	45.	(b)	46.	(d)	47.	(d)	48.	(b)
49.	(b)	50.	(b)												

MATHEMATICS

1.	(c)	2.	(d)	3.	(b)	4.	(d)	5.	(a)	6.	(a)	7.	(a)	8.	(a)
9.	(a)	10.	(a)	11.	(d)	12.	(b)	13.	(a)	14.	(d)	15.	(a)	16.	(d)
17.	(a)	18.	(b)	19.	(b)	20.	(d)	21.	(b)	22.	(c)	23.	(b)	24.	(d)
25.	(b)	26.	(d)	27.	(a)	28.	(a)	29.	(c)	30.	(b)	31.	(d)	32.	(d)
33.	(d)	34.	(a)	35.	(a)	36.	(c)	37.	(c)	38.	(a)	39.	(a)	40.	(c)
41.	(b)	42.	(b)	43.	(d)	44.	(a)	45.	(b)	46.	(a)	47.	(c)	48.	(d)
49.	(c)	50.	(c)												

HINTS & SOLUTIONS

Physics

1. Length of simple pendulum =Distance between point of suspension and centre of gravity of system

On draining off a little mercury, the centre of gravity gets lowered, that is, the length of simple pendulum increases. The time period of simple pendulum is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$\Rightarrow \quad T \propto \sqrt{l}$$

As length of simple pendulum increases, the time period increases.

2. Ideal gas equation can be written as

$$PV = nRT \qquad \ldots(i)$$

In this equation,

n = number of moles of the gas

P = pressure of the gas

V = volume of the gas

R = universal gas constant

and T = temperature of the gas

From Eq. (i), we have

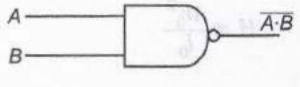
$$\frac{n}{V} = \frac{P}{RT} = \text{constant}$$

So, at constant pressure and temperature, both balloons will contain equal number of molecules per unit volume.

Note : This result is nothing but Avogadro's law.

3. The NAND gate and the NOR gate can be said to be universal gates since combinations of them can be used to accomplish any of the basic operations and can thus, produce an inverter, an OR gate or an AND gate. The non-inverting gates do not have the versatility since they can't produce an invert.

Note : The basic symbol and truth table of NAND gate are shown



A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

4. An isothermal process is a constant temperature process. In this process,

$$T = \text{constant or } \Delta T = 0.$$

$$\therefore \quad \Delta U = nC_V \Delta T = 0$$

An adiabatic process is defined as one with no heat transfer into or out of a system. Therefore,

$$Q = 0$$

From first law of thermodynamics

$$W = -\Delta U$$

or $$\Delta U = -W$$

5. The aerofils are so designed that

$$P_{\text{upper side}} < P_{\text{lower side}}$$

so that the aerofils get a lifting force in upward direction.

According to Bernoulli's theorem, where the pressure is large, the velocity will be minimum or *vice-versa*.

Thus, $$v_{\text{upper side}} > v_{\text{lower side}}$$

6. According to Bernoulli's theorem, in case of steady flow of incompressible and non-viscous liquid through a tube of non-uniform cross-section, the sum of the pressure, the potential energy per unit volume and the kinetic energy per unit volume is same at every point in the tube, *i. e.*,

$$P + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}$$

Dividing this expression by ρg, we have

$$\frac{P}{\rho g} + \frac{v^2}{2g} + h = \text{constant}$$

In this expression $\frac{P}{\rho g}$ is called the 'pressure head', $\frac{v^2}{2g}$ the 'velocity head' and h the 'gravitational head'.

7. Sabine formula for reverberation time is

$$T = \frac{0.16\ \text{V}}{\Sigma\, aS}$$

where V is volume of hall in m^3.

$\Sigma aS = a_1S_1 + a_2S_2 + \ldots$

= total absorption of the hall (room)

Here, S_1, S_2, S_3 ..... are surface areas of the absorbers and a_1, a_2, a_3 ..... are their respective absorption coefficients.

$$\therefore \quad \frac{T'}{T} = \frac{V'}{S'} \times \frac{S}{V} = \frac{(2)^3}{(2)^2} = \frac{8}{4} = 2$$

Hence, $T' = 2T = 2 \times 1 = 2$ s

8. The critical frequency of sky wave undergoing reflection from a layer of atmosphere is

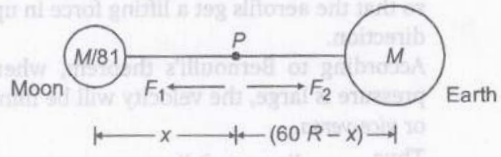
$$f_c = \sqrt[2]{N_{max}}$$

where N is electron density per m^3.

$$\therefore \quad N_{max} = \frac{f_c^2}{81} = \frac{(10 \times 10^6)^2}{81}$$
$$= 1.2 \times 10^{12} \text{ m}^{-3}$$

9. Key Idea : *Where gravitational field due to earth and moon cancel each other, there the gravitational force is equal.*

From Newton's law of gravitation the force of attraction between any two material particles is given by



$$F = \frac{Gm_1m_2}{r^2}$$

where m_1, m_2 are masses and r is the distance between the two.

Since gravitational fields cancel each other the force of attraction is same and opposite.

i.e., $F_1 = F_2$

$$\frac{G\left(\frac{M}{81}\right)m}{x^2} = \frac{GM \times m}{(60\,R - x)^2}$$

$$\Rightarrow \quad \frac{1}{81x^2} = \frac{1}{(60\,R - x)^2}$$

Taking square root of the above expression, we have

$$\frac{1}{9x} = \frac{1}{60R - x}$$

$$\Rightarrow \quad 9x = 60R - x$$

$$\Rightarrow \quad x = 6R$$

Hence, distance of that point from moon is $6R$.

10. Key Idea : *In coalescing into a single drop charge remains conserved. Also volume before and after coalescing remains same.*

Let R and r be the radii of bigger and each smaller drop respectively. In coalescing into a single drop, charge remains conserved.

Hence, charge on bigger drop
= 27 × charge on smaller drop

ie, $q' = 27q$

Now, before and after coalescing, volume remains same. That is,

$$\frac{4}{3}\pi R^3 = 27 \times \frac{4}{3}\pi r^3$$

$$\therefore \quad R = 3r$$

Hence, capacitance of bigger drop

$$C' = 4\pi\varepsilon_0 R$$
$$= 4\pi\varepsilon_0 (3r)$$
$$= 3(4\pi\varepsilon_0 r)$$
$$= 3C$$

11. Magnetic moment of a current carrying coil is defined as the product of current in the coil with the area of coil in vector form. That is,

$$\vec{\mathbf{M}} = I\,\vec{\mathbf{A}}$$

Thus, dimensions of $M = [\text{A}]\,[\text{L}^2]$
$= [\text{L}^2\text{A}]$

12. Heat conduction through a rod is given by

$$H = \frac{\Delta Q}{\Delta t} = KA\left(\frac{T_1 - T_2}{l}\right)$$

$$\Rightarrow \quad H \propto \frac{r^2}{l} \qquad \ldots(\text{i})$$

(a) When $r = 2r_0$, $l = 2l_0$

$$H \propto \frac{(2r_0)^2}{2l_0}$$

$$\Rightarrow \quad H \propto \frac{2r_0^2}{l_0}$$

(b) When $r = 2r_0$; $l = l_0$

$$H \propto \frac{(2r_0)^2}{l_0}$$

$$\Rightarrow \quad H \propto \frac{4r_0^2}{l_0}$$

(c) When $r = r_0; l = l_0$

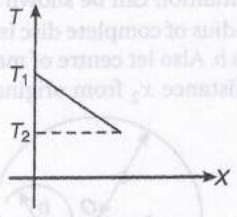
$$H \propto \frac{r_0^2}{l_0}$$

(d) When $r = r_0; l = 2l_0$

$$H \propto \frac{r_0^2}{2l_0}$$

It is obvious that heat conduction will be more in case (b).

Note : It is fact that the temperature of whole rod does not become equal when heat is being continuously supplied due to the reason that temperature difference in the rod for the heat flow is same as we require a potential difference across a resistance for the current flow through it.



13. Key Idea : *Kinetic energy of satellite is half of its potential energy.*

Potential energy of satellite

$$U = -\frac{GM_e m}{R_e}$$

where R_e is radius of earth, M_e the mass of earth, m the mass of satellite and G the gravitational constant.

$$|U| = \frac{GM_e m}{R_e}$$

Kinetic energy of satellite

$$K = \frac{1}{2}\frac{GM_e m}{R_e}$$

Thus, $$\frac{K}{|U|} = \frac{1}{2}\frac{GM_e m}{R_e} \times \frac{R_e}{GM_e m}$$

$$= \frac{1}{2}$$

14. Key Idea : *Emf's of cells in the given circuit are in opposite directions, hence resultant emf is difference of the two emf's.*

We know that when current flowing is same then resistors are connected in series, hence resultant resistance is

$$R' = R_1 + R_2 = 10\Omega + 20\Omega$$
$$= 30\Omega$$

Also since cells are connected in opposite direction, the resultant emf is

$$E = E_1 - E_2$$
$$= 5V - 2V = 3V$$

From ohm's law $E = IR$

$$\Rightarrow I = \frac{E}{R} = \frac{3}{30} = 0.1\ A$$

15. When chamber is evacuated, the refractive index (μ) decreases. Therefore, wavelength $\left(\lambda \propto \frac{1}{\mu}\right)$ increases.

Now, fringe width is given by

$$\beta = \frac{\lambda D}{d}$$

$$\Rightarrow \beta \propto \lambda$$

Therefore, as wavelength increases, the fringe width will increase.

16. When a charged particle q is moving in a uniform magnetic field $\vec{B}$ with velocity $\vec{v}$ such that angle between $\vec{v}$ and $\vec{B}$ be θ, then due to interaction between the magnetic field produced due to moving charge and magnetic force applied, the charge q experiences a force which is given by

$$F = qvB \sin\theta$$

If $\theta = 0°$ or $180°$, then $\sin\theta = 0$

$$\therefore \quad F = qvB \sin\theta = 0$$

Since, force on charged particle is non-zero, so angle between $\vec{v}$ and $\vec{B}$ can have any value other than zero and 180°.

Note : Force experienced by the charged particle is Lorentz force.

17. Key Idea : *Speed is rate of change of distance.*

Distance travelled by the particle is

$$x = 40 + 12t - t^3$$

We know that, speed is rate of change of distance *i.e.*, $v = \frac{dx}{dt}$

$$\therefore \quad v = \frac{d}{dt}(40 + 12t - t^3)$$
$$= 0 + 12 - 3t^2$$

But final velocity $v = 0$

$$\therefore \quad 12 - 3t^2 = 0$$

or $$t^2 = \frac{12}{3} = 4$$

or $$t = 2s$$

Hence, distance travelled by the particle before coming to rest is given by

$$x = 40 + 12(2) - (2)^3$$
$$= 40 + 24 - 8 = 64 - 8$$
$$= 56 \text{ m}$$

18. In a longitudinal wave, the particles of the medium oscillate about their mean or equilibrium position along the direction of propagation of the wave itself. Sound waves are longitudinal in nature. In transverse wave, the particles of the medium oscillate about their mean or equilibrium position at right angles to the direction of propagation of wave itself. Light waves being electromagnetic are transverse waves.

19. When the secondary coil circuit is open, the magnetic flux in the core is produced by the primary current only. When the secondary circuit is closed, the currents in the secondary coil also produce magnetic flux in the core but in opposite direction. This decreases the core flux and hence reduces the back emf produced in the primary coil. The source emf is now in excess of the back emf, more current is drawn in the primary coil. Hence, power factor is no longer zero. The power factor has increased or the phase difference is no longer 90°, *i.e.*, phase difference has decreased. Thus, dynamic resistance has increased.

20. The frame of reference which are at rest or in uniform motion are called inertial frames while frames which are accelerated with respect to each other are non-inertial frames. Spinning or rotating frames are accelerated frames, hence these are non-inertial frames.

21. Kinetic energy of ideal gas depends only on its temperature. Hence, it remains constant whether its pressure is increased on decreased.

22. The ionosphere extends from a height of 80 km to 300 km. The refractive index of ionosphere is less than its free space value. That is, it behaves as a rarer medium. As we go deep into the ionosphere, the refractive index keeps on decreasing. The bending of beam (away from the normal) will continue till it reaches critical angle after which it will be reflected back. The different points on earth receive signals reflected from different depths of the ionosphere. There is a critical frequency f_c (5 to 100 MHz) beyond which the waves cross the ionosphere and do not return back to earth.

23. In relaxed eye state of telescope or when final image is formed at infinity, the magnification of telescope is given by

$$|M| = \frac{f_o}{f_e}$$

where f_0 = focal length of objective = 200 cm

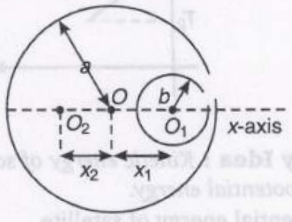
f_e = focal length of eyepiece = 5 cm

Hence, $$|M| = \frac{200}{5} = 40$$

Note : The aperture of the objective lens is kept large so that more and more rays coming from the heavenly body may enter the telescope and a bright image is formed by the objective lens. The aperture of the eye lens is kept comparatively small so that all the rays may enter the eye.

24. The situation can be shown as :

Let radius of complete disc is a and that of small disc is b. Also let centre of mass now shifts to O_2 at a distance x_2 from original centre.



The position of new centre of mass is given by

$$X_{CM} = \frac{-\sigma \cdot \pi b^2 \cdot x_1}{\sigma \cdot \pi a^2 - \sigma \cdot \pi b^2}$$

Here, $a = 6$ cm, $b = 2$ cm, $x_1 = 3.2$ cm

Hence, $$X_{CM} = \frac{-\sigma \times \pi (2)^2 \times 3.2}{\sigma \times \pi \times (6)^2 - \sigma \times \pi \times (2)^2}$$
$$= \frac{12.8\pi}{32\pi}$$
$$= -0.4 \text{ cm}$$

25. The different processes involved in the fabrication of an IC are :

(i) **Epitaxial growth** of n or p-type layer, whenever desired. This involves cracking of silane.

(ii) **Oxidation** which gives a layer of insulating SiO_2 and can be used to separate one region of the silicon chip from the other.

(iii) **Photolithography** is a process in which different regions of silicon chip are photographically selected and etched so that different components can be fabricated in different regions.

(iv) **Diffusion** of different impurities to obtain different device structure.

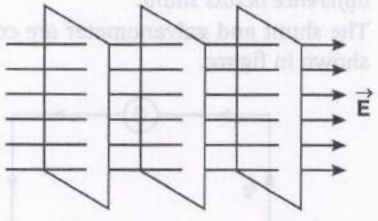
(v) **Metallisation** involves deposition of metal films which inter-connect different components on a chip to obtain the circuit.

Hence, option (a) is not involved in fabrication of IC.

26. The materials for a permanent magnet should have high retentivity (so that the magnet is strong) and high coercivity (so that the magnetism is not wiped out by stray magnetic fields). As the material in this case is never put to cyclic changes of magnetisation, hence hysteresis is immaterial.

27. The working of hydrogen bomb is based on nuclear fusion. But for the process of nuclear fusion, a very high temperature ($\approx 10^7$ K) and a very high pressure is required, hence to construct a hydrogen bomb, first atom bomb is constructed. The atom bomb is then covered from all sides by an enclosure of a compound of heavy hydrogen (deuteron) such as lithium hydride. First the atom bomb is exploded which produces such a high temperature and pressure that deuteron nuclei get fused and a huge amount of energy is released.

28. An equipotential surface is a surface with a constant value of potential at all points on the surface. For a uniform electric field, say, along the x-axis, the equipotential surfaces are planes

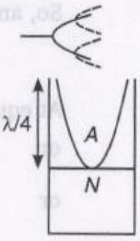


normal to the x-axis, *i e*, plane parallel to the y-z plane. Equipotential surfaces for a dipole and its electric field lines are shown in figure.

As said above that on equipotential surface, potential at all points is constant, this means that on equipotential surface work done in moving a test charge from one point to other point is zero.

29. **Key Idea :** *When length of air column is* $\frac{\lambda}{4}$, *then first resonance occurs.*

If we adjust the length of air-column in closed organ pipe as such its any natural frequency equals to the frequency of tuning fork, then the amplitude of forced vibrations of air-column increases very much. This is the state of resonance. At first resonance



$$l = \frac{\lambda}{4}$$

So, frequency of tuning fork

$$f = \frac{v}{\lambda} = \frac{v}{4l}$$

Given, $l = 20$ cm $= 0.2$ m, $v = 332$ m/s

Hence, $f = \frac{332}{4 \times 0.2} = 415$ Hz

30. **Key Idea :** *In first set up, the springs are joined in series and in second, the springs are joined in parallel.*

When springs are connected in series the effective spring constant is

$$\frac{1}{k} = \frac{1}{k} + \frac{1}{k} = \frac{2}{k}$$

$$\Rightarrow \quad k' = \frac{k}{2}$$

Hence, frequency $n' = \frac{1}{2\pi}\sqrt{\frac{k'}{m}}$

$$= \frac{1}{2\pi}\sqrt{\left(\frac{k}{2m}\right)} \qquad \ldots(i)$$

When springs are connected in parallel, the effective force constant is

$$k'' = k + k = 2k$$

Therefore, frequency is

$$n'' = \frac{1}{2\pi}\sqrt{\frac{2k}{m}} \qquad \ldots(ii)$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{n'}{n''} = \frac{\sqrt{(k/2m)}}{\sqrt{(2k/m)}} = \frac{1}{2}$$

31. Area of acceleration-displacement curve gives change in KE per unit mass.

$$\frac{1}{2}m(v^2 - u^2) = Fs = \frac{mdv}{dt} \times s$$

$$\therefore \quad \frac{\text{Change in KE}}{\text{Mass}} = \frac{dv}{dt} \times s$$

32. Earth rotates about its own axis in 24 h.

So, $T = 24 \times 60 \times 60$ s

So, angular speed of earth about its own axis

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{24 \times 60 \times 60} \text{ rad/s}$$

At equator, $g' = g - R_e\omega'^2$

or $0 = g - R_e\omega'^2$

or $\omega' = \sqrt{\frac{g}{R_e}} = \frac{2\pi}{84.6 \times 60}$

$\therefore \quad \frac{\omega'}{\omega} = \frac{24 \times 60 \times 60}{84.6 \times 60} \approx 17$

$\Rightarrow \quad \omega' = 17\ \omega$

So, value of x is 17.

33. From conservation of energy the kinetic energy of electron equals the maximum photon energy (we neglect the work function ϕ because it is normally so small compared to eV_0).

$\therefore \quad eV_0 = h\nu_{max}$

or $\quad eV_0 = \frac{hc}{\lambda_{min}}$

$\therefore \quad V_0 = \frac{hc}{e\lambda_{min}}$

or $\quad V_0 = \frac{12400 \times 10^{-10}}{10^{-11}}$

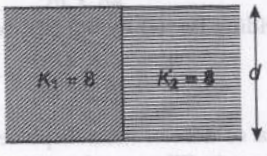
$= 124$ kV

Hence, accelerating voltage for electrons in X-ray machine should be less than 124 kV.

34. In photoelectric effect for a given photosensitive material, there exists a certain minimum cut-off frequency, called the threshold frequency, below which no emission of photoelectrons takes place no matter how intense the light is.

35. **Key Idea :** *After filling with dielectrics the two capacitors will be in parallel order.*

As shown, the two capacitors are connected in parallel. Initially the capacitance of capacitor



$$C = \frac{\varepsilon_0 A}{d}$$

where A is area of each plate and d is the separation between the plates.

After filling with dielectrics, we have two capacitors of capacitance.

$$C_1 = \frac{K_1\varepsilon_0(A/2)}{d} = \frac{8}{2}\frac{\varepsilon_0 A}{d} = \frac{4\varepsilon_0 A}{d} = 4C$$

and $C_2 = \frac{K_2\varepsilon_0(A/2)}{d} = \frac{4}{2}\frac{\varepsilon_0 A}{d} = \frac{2\varepsilon_0 A}{d} = 2C$

Hence, their equivalent capacitance

$$C_{eq} = C_1 + C_2$$
$$= 4C + 2C = 6C$$

i.e., new capacitance will be six times of the original.

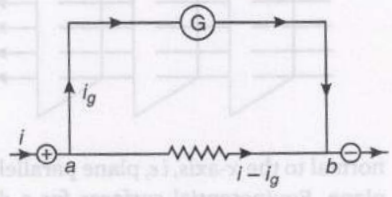
36. If an object reflects the colour of light incident on it, it will appear with that colour but if object absorbs the colour of light, it will appear to be black. Since, the given wavelength does not belong to green, so it will be absorbed by the leaf and hence, it would appear to be black.

37. The multiplication factor (k) is an important reactor parameter and is the ratio of number of neutrons present at the beginning of a particular generation to the number present at the beginning of the next generation. It is a measure of the growth rate of the neutrons in the reactor. For $k = 1$, the operation of the reactor is said to be critical.

Note : If k becomes greater than one, the reaction rate and the reactor power increase exponentially. Unless the factor k is brought down very close to unity, the reactor will become supercritical and can even explode.

38. **Key Idea :** *Potential difference across galvanometer should be equal to potential difference across shunt.*

The shunt and galvanometer are connected as shown in figure.



Let total current through the parallel combination is i, the current through the galvanometer is i_g and the current through the shunt is $i - i_g$.

The potential difference V_{ab} $(= V_a - V_b)$ is the same for both paths, so

$$i_g G = (i - i_g)\, S$$

or $\quad i_g\,(G + S) = i\,S$

or $\frac{i_g}{i} = \frac{S}{S+G}$

The fraction of current passing through shunt

$$= \frac{i - i_g}{i} = 1 - \frac{i_g}{i}$$

$$= 1 - \frac{S}{S+G} = \frac{G}{S+G}$$

$$= \frac{8}{2+8} = \frac{8}{10}$$

$$= 0.8 \text{ A}$$

39. As batteries wear out, temperature of filament of flash light attains lesser value, therefore intensity of radiation reduces. Also dominating wavelength (λ_m) in spectrum, which is the red colour, increases.

40. On the basis of dual nature of light, Lious de-Broglie suggested that the dual nature is not only of light, but each moving material particle has the dual nature. He assumed a wave to be associated with each moving material particle which is called the matter wave. The wavelength of this wave is determined by the momentum of the particle. If p is the momentum of the particle, the wavelength of the wave associated with it is

$$\lambda = \frac{h}{p}$$

where h is Planck's constant.

Since, it is given that, alpha, beta and gamma rays carry same momentum, so they will have same wavelength.

41. Voltage gain, $A_v = 1000$

In dB, voltage gain

$$A = 10 \log_{10} 1000 \text{ dB}$$

$$= (10 \times 3) \log_{10} 10 \text{ dB}$$

$$= 30 \text{ dB} \qquad (\because \log_{10} 10 = 1)$$

42. The entropy function gives us a numerical measure of the irreversibility of a given process, *ie*, it is a measure of disorder of a system. During formation of ice cubes orderedness increases, *ie*, disorderness decreases, hence entropy decreases.

43. **Key Idea :** *Total number of nuclei remained after n half lives is* $N = N_0 \left(\frac{1}{2}\right)^n$.

Total time given = 80 min

Number of half-lives of A, $n_A = \frac{80 \text{ min}}{20 \text{ min}} = 4$

Number of half-lives of B, $n_B = \frac{80 \text{ min}}{40 \text{ min}} = 2$

Number of nuclei remains undecayed

$$N = N_0 \left(\frac{1}{2}\right)^n$$

where N_0 is initial number of nuclei.

$$\therefore \quad \frac{N_A}{N_B} = \frac{\left(\frac{1}{2}\right)^{n_A}}{\left(\frac{1}{2}\right)^{n_B}}$$

or $$\frac{N_A}{N_B} = \frac{\left(\frac{1}{2}\right)^4}{\left(\frac{1}{2}\right)^2} = \frac{\left(\frac{1}{16}\right)}{\left(\frac{1}{4}\right)}$$

or $$\frac{N_A}{N_B} = \frac{1}{4}$$

44. **Key Idea :** *The standard equation of sound wave is*

$$y = a \sin 2\pi \left(\frac{x}{\lambda} + \frac{t}{T}\right)$$

where a is amplitude, λ is wavelength and T is time period.

Given, equation of wave is

$$y = 0.0015 \sin (62.8x + 314t) \qquad \ldots(i)$$

Comparing given Eq. (i) with standard wave equation, we get

$$\frac{2\pi}{\lambda} = 62.8$$

$$\Rightarrow \quad \lambda = \frac{2\pi}{62.8} = \frac{2 \times 3.14}{62.8}$$

$$= 0.1 \text{ unit}$$

45. **Key Idea :** *At the two points of the trajectory during projection, the horizontal component of the velocity is the same.*

Horizonal component of velocity at angle 60°

= Horizontal component of velocity at 45°

i. e., $u \cos 60° = v \sin 45°$

or $$147 \times \frac{1}{2} = v \times \frac{1}{\sqrt{2}}$$

or $$v = \frac{147}{\sqrt{2}} \text{ m/s}$$

Vertical component of $u = u \sin 60°$

$$= \frac{147\sqrt{3}}{2} \text{ m}$$

Vertical component of $v = v \sin 45°$

$$= \frac{147}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$$

$$= \frac{147}{2} \text{ m}$$

but $v_y = u_y + at$

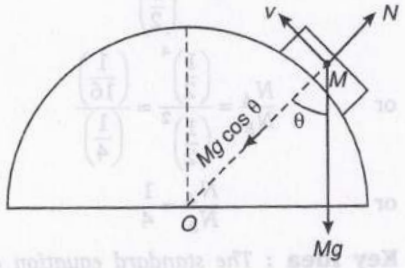
$\therefore \quad \frac{147}{2} = \frac{147\sqrt{3}}{2} - 9.8\,t$

or $9.8\,t = \frac{147}{2}(\sqrt{3} - 1)$

$\therefore \quad t = 5.49\ s$

46. Reaction of normal force on the motor cyclist

$$Mg\cos\theta - N = \frac{mv^2}{R}$$



or $N = Mg\cos\theta - \frac{Mv^2}{R}$

As the motor cyclist moves upward on the bridge, θ decreases and therefore, N increases and becomes maximum at the highest point.

47. Key Idea : *At a given height the half of the kinetic energy of the body is equal to its potential energy.*

Initial kinetic energy of the body

$$= \frac{1}{2}mv^2$$

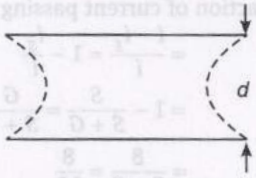
$$= \frac{1}{2}m(4)^2 = 8m$$

Let at height h, the kinetic energy reduces to half, *ie*, it becomes 4 m. It is also equal to potential energy. Hence,

$$mgh = 4\,m$$

or $h = \frac{4}{g} = \frac{4}{10} = 0.4$ m

48. The shape of water layer between the two plates is shown in the figure.



Thickness d of the film = 0.12 mm

= 0.012 cm

Radius R of the cylindrical face $= \frac{d}{2}$

Pressure difference across the surface

$$= \frac{T}{R} = \frac{2T}{d}$$

Area of each plate wetted by water = A

Force F required to separate the two plates is given by

F = pressure difference × area

$$= \frac{2T}{d}A$$

Putting the given values, we get

$$F = \frac{2 \times 75 \times 8}{0.012} = 10^5 \text{ dynes}$$

49. Since, lens is made of two layers of different refractive indices, for a given wavelength of light it will have two focal lengths or will form two images at two different points as there are μ's as

$$\frac{1}{f} \propto (\mu - 1)$$

50. The induced emf will oppose the motion of the magnet. Applying the right hand rule, the direction of induced current will be clockwise.

Chemistry

1. Calcium carbide reacts with H_2O to produce acetylene.

$$CaC_2 + 2H_2O \longrightarrow C_2H_2 + Ca(OH)_2$$

2. Propyne and propene can be distinguished by $AgNO_3$ in ammonia. Propyne gives a white precipitate with ammoniacal silver nitrate while propene has no effect on ammoniacal $AgNO_3$.

3. Let the oxidation number of Fe in Fe_3O_4 is x.

Hence, $3x + 4(-2) = 0$

$3x + (-8) = 0$

$3x = 8$

$x = \frac{8}{3}$

4. Bauxite ($Al_2O_3 \cdot 2H_2O$) is an ore of aluminium.

5. The best source of vitamin A is carrot. Other sources are butter, milk, spinach, cheese, etc. Its deficiency causes night blindness.

6. Calcium sulphate hemihydrate $\left(CaSO_4 \cdot \frac{1}{2}H_2O\right)$ is called plaster of Paris.

7. Penicillin is a antibiotic. It is narrow spectrum antibiotic.

8. The electronegativity of halogen is as :

$$F > Cl > Br > I$$

Among alkali metals the electropositive character increases as :

$$Li < Na < K < Rb < Cs$$

Hence, LiF is the most stable compound.

9. D_2O is called heavy water. It is used as moderator in nuclear reactor.

10. The copper metal is mainly extracted from its sulphide ore, copper pyrites ($CuFeS_2$). The Cu_2O obtained can be reduced to copper by treating with FeS.

$$Cu_2O + FeS \longrightarrow FeO + Cu_2S$$

$$2Cu_2S \xrightarrow{3O_2} 2Cu_2O + 2SO_2$$

$$2Cu_2O + Cu_2S \longrightarrow 6Cu + SO_2$$

11. BaO_2 and ozone reacts to produce BaO and oxygen.

$$BaO_2 + O_3 \longrightarrow BaO + 2O_2$$

12. According to Heisenberg uncertainty principle, the product of uncertainty in momentum (Δp) and in position (Δx) remains constant.

$$\Delta x \times \Delta p \geq \frac{h}{4\pi}$$

13. The reaction between H_2O_2 and acidified $KMnO_4$ takes place as follows :

$$2KMnO_4 + 3H_2SO_4 + 5H_2O_2 \longrightarrow K_2SO_4 + 2MnSO_4 + 8H_2O + 5O_2$$

In this ways we can see that 5 moles of H_2O_2 are required to reduce 2 moles of $KMnO_4$. Therefore, $\frac{5}{2}$ moles of H_2O_2 are required per mole of $KMnO_4$.

14. Bleaching powder is obtained by treating Cl_2 with lime water.

$$Ca(OH)_2 + Cl_2 \longrightarrow CaOCl_2 + H_2O$$

15. Magnalium is an alloy of Al and Mg. It is used to make light instruments.

16. $\because \quad \lambda = \frac{h}{mv} = \frac{6.6 \times 10^{-34}}{1 \times 100} = 6.6 \times 10^{-36}$ m

17. Here $V_1 = 10$ L, $V_2 = 2$ L

$P_1 = 1$ atm, $P_2 = 1$ atm

$T_1 = 300$ K, $T_2 = ?$

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$\frac{1 \times 10}{300} = \frac{1 \times 2}{T_2}$$

$$T_2 = \frac{300 \times 2}{10} = 60 \text{ K}$$

18. Isotonic solutions have same osmotic pressure.

19. $MgSO_4 \cdot 7H_2O$ is called Epsom salt.

20. Paracetamol is used as an antipyretic as well as analgesic.

21. $2H_2O \xrightarrow{\text{Electrolysis}} 2H_2 + O_2$

2 mole 1 mole

$2 \times 18 = 36$ g

$\because$ 36 g of H_2O produce 1 mole of oxygen.

$\therefore$ 108 g of water will produce oxygen.

$$= \frac{108}{36} = 3 \text{ mol}$$

22. Entropy change of fusion

$$\Delta S_f = \frac{\Delta H_f}{T}$$

$\Delta H_f = 60 \times 10^3$ J mol^{-1}, $T = 273$ K

$$\Delta S_f = \frac{60 \times 10^3}{273}$$

$$= 21.97 \text{ J K}^{-1} \text{ mol}^{-1}$$

23. $[Co(NH_3)_3Cl_3]$ does not give a precipitate with $AgNO_3$ solution because in it all the chlorine atoms are attached by secondary valencies.

24. In fcc unic cell $z = 4$

Hence, occupied volume by sphere $= 4 \times \frac{4}{3}\pi r^3$

$$= \frac{16}{3}\pi r^3$$

25. Excess of one valence electron in P atom (5 valence electron) over Si (4 valence electron) will form *n*-type semiconductor.

26. Cellulose is the biodegradable polymer. Different enzymes found in bacteria acts on it and degrade it. Synthetic polymers/like PE, PVC, nylon-6 etc.) are not acted upon by bacteria, hence are non biodegradable.

27. $_7N^{14} + {}_1H^1 \longrightarrow {}_8O^{15} + \gamma$

28. $PbCl_2 \rightleftharpoons \underset{s}{Pb^{2+}} + \underset{2s}{2Cl^-}$

$$K_{sp} = (s)(2s)^2$$

$$1 \times 10^{-6} = 4s^3$$

$$s = \left(\frac{1}{4} \times 10^{-6}\right)^{1/3}$$

$$= (0.25 \times 10^{-6})^{1/3}$$

29. $[H^+] = 0.001$

$$pH = -\log [H^+]$$

$$= -\log [10^{-3}]$$

$$= 3$$

30. The protective power of a colloid is expressed as gold number.

31. Noble gases are used in discharge tubes to give different colours. Reddish orange glow is due to Ne.

32. The IP decreases in a group on moving downward because atomic radius increases hence the correct order is :

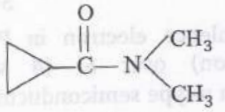
$$Be > Mg > Ca$$

33. Solid CO_2 is called ice because it changes to vapour state directly without changing to liquid state.

34. Methanol and ethanol are miscible in water due to hydrogen bonding character.

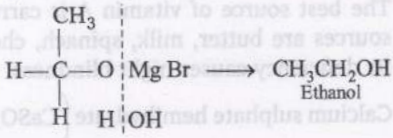
35. Octahedral complexes of type $[Ma_2b_2c_2]$ and $[Mabcdef]$, $[M(AA)_3]$, $[M(AA)_2a_2]$, $[M(AA)a_2b_2]$, $[M(AA)_2ab]$ and $[M(AA)_3]$ show optical isomerism due to asymmetric molecules, octahedral complex of type $[Ma_4b_2]$ and $[Ma_3b_3]$ do not show optical isomerism due to presence of plane of symmetry. Hence $[Co(NH_3)_3Cl_3]$ will not show optical isomerism.

36.



N,N-dimethyl cyclopropane carboxamide.

37. $CH_3MgBr + HCHO \longrightarrow$



$$\xrightarrow{} CH_3CH_2OH$$

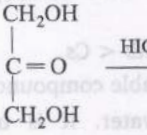
Ethanol

38. Freon is CCl_2F_2. It is used as refrigerant.

39. Anhy. $ZnCl_2$ and conc. HCl is called Lucas reagent. It is used to distinguish the primary, secondary and tertiary alcohol.

40. $C_6H_{12}O_6 \xrightarrow{\text{Zymase}} 2C_2H_5OH + CO_2$

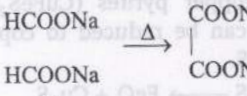
41.



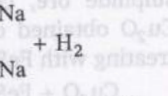
$$\xrightarrow{HIO_4} 2HCHO + CO_2$$

42. Fructose does not reduce Fehling's solution because it contains ketonic group.

43.



$$\xrightarrow{\Delta} + H_2$$



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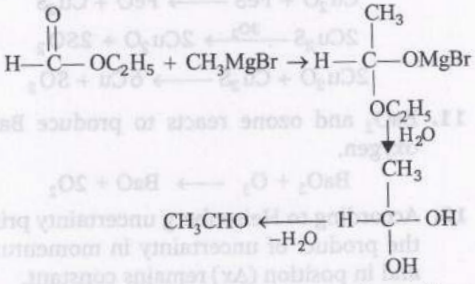


45. Hydrolysis of phenyl isocyanide forms formic acid.

46. Styrene can be purified by vaccum distillation.

47. Sucrose is not a reducing sugar.

48. Tetrafluoroethene is the monomer of teflon.

$$nCF_2 \equiv CF_2 \xrightarrow{\text{Polymerisation}} \underset{\text{Teflon}}{+CF_2 - CF_2+_n}$$

49. The hybridisation state of carbon in fullerene is sp^2.

50. C_2H_5OH reacts with CH_3COOH and gives ethylethanoate. Which is an ester and it has fruity smell.

$$C_2H_5OH + CH_3COOH \xrightarrow{\text{conc. } H_2SO_4}$$

$$\underset{\substack{\text{Ester}\\\text{(Fruity smell)}}}{CH_3COOC_2H_5} + H_2O$$

Mathematics

1. Given $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$

$$\Rightarrow \begin{vmatrix} 3a-x & a-x & a-x \\ 3a-x & a+x & a-x \\ 3a-x & a-x & a+x \end{vmatrix} = 0$$

$$(C \to C_1 + C_2 + C_3)$$

$$\Rightarrow (3a-x)\begin{vmatrix} 1 & a-x & a-x \\ 1 & a+x & a-x \\ 1 & a-x & a+x \end{vmatrix} = 0$$

$$\Rightarrow (3a-x)\begin{vmatrix} 1 & a-x & a-x \\ 0 & 2x & 0 \\ 0 & 0 & 2x \end{vmatrix} = 0$$

$$\begin{bmatrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{bmatrix}$$

$$\Rightarrow (3a-x)(4x^2) = 0$$

$$\Rightarrow x = 3a, 0$$

2. **Key Idea :** *If P and Q are two forces and Q be the angle between them, then*
Resultant, $R^2 = P^2 + Q^2 + 2PQ\cos\theta$.

Using the relation

$$R^2 = P^2 + Q^2 + 2PQ\cos\theta$$

$$\therefore (\sqrt{10}p)^2 = (2p)^2 + (\sqrt{2}p)^2 + 2(2p)(\sqrt{2}p)\cos\theta$$

$$\Rightarrow \cos\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{\pi}{4}$$

3. Given, $y = \sin^{-1}\left(\frac{2t}{1+t^2}\right)$, $x = \tan^{-1}\left(\frac{2t}{1-t^2}\right)$

$$-1 < t < 1$$

Let $t = \tan(\phi/2)$

$$\therefore \frac{2t}{1+t^2} = \frac{2\tan(\phi/2)}{1+\tan^2(\phi/2)}$$

$$= \sin 2(\phi/2) = \sin\phi$$

$$\Rightarrow \frac{2t}{1+t^2} = \sin\phi \quad \ldots. (i)$$

Also, $\frac{2t}{1-t^2} = \frac{2\tan(\phi/2)}{1-\tan^2(\phi/2)}$

$$= \tan 2(\phi/2) = \tan\phi \quad \ldots(ii)$$

From (i) and (ii),

$y = \sin^{-1}\sin\phi$ and $x = \tan^{-1}\tan(\phi)$

$$\Rightarrow y = \phi, x = \phi \Rightarrow \frac{dy}{d\phi} = 1, \frac{dx}{d\phi} = 1$$

$$\therefore \frac{dy}{dx} = 1$$

Alternate Method :

Given,

$$y = \sin^{-1}\left(\frac{2t}{1+t^2}\right), x = \tan^{-1}\left(\frac{2t}{1-t^2}\right)$$

$$\Rightarrow \frac{dy}{dt} = \frac{1}{\sqrt{1-\left(\frac{2t}{1+t^2}\right)^2}} \times \frac{d}{dt}\left(\frac{2t}{1+t^2}\right)$$

$$= \frac{1+t^2}{1-t^2}\left[\frac{(1+t^2)\times 2 - 2t\,(2t)}{(1+t^2)^2}\right] = \frac{2}{1+t^2} \ldots (i)$$

and

$$\frac{dx}{dt} = \frac{1}{1+\left(\frac{2t}{1-t^2}\right)^2}\frac{d}{dt}\left(\frac{2t}{1-t^2}\right)$$

$$= \frac{1}{1+\left(\frac{2t}{1-t^2}\right)^2}\frac{(1-t^2)\times 2 + 2t\,(2t)}{(1-t^2)^2}$$

$$= \frac{(1-t^2)^2}{(1+t^2)^2}\left[\frac{2(1+t^2)}{(1-t^2)^2}\right] = \frac{2}{1+t^2} \quad \ldots (ii)$$

$$\therefore \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{2}{1+t^2} \times \frac{1+t^2}{2} = 1$$

[from (i) and (ii)]

4. **Key Idea :**

$$\lim_{x\to\infty}\frac{\sin x}{x} = 0, \lim_{x\to\infty}\frac{\cos x}{x} = 0$$

Let $A = \lim_{x\to\infty}\sqrt{\frac{x-\sin x}{x+\cos^2 x}}$

$$\Rightarrow A = \lim_{x\to\infty}\sqrt{\frac{1-\frac{\sin x}{x}}{1+\frac{\cos^2 x}{x}}}$$

$$\Rightarrow A = \sqrt{\frac{1-0}{1+0}} = 1$$

5. Suppose the coin is tossed n times. Let x be the number of heads obtained. Then x follows a binomial distribution with parameters n and $p = \frac{1}{2}$, we have

$P(x \geq 1) \geq 0.8 \Rightarrow 1 - P(x = 0) \geq 0.8$

$\Rightarrow \quad P(x = 0) \leq 1 - 0.8 = 0.2$

$\Rightarrow \quad {}^nC_0 \left(\frac{1}{2}\right)^n \leq 0.2 = \frac{2}{10} = \frac{1}{5}$

$\Rightarrow \quad \left(\frac{1}{2}\right)^n \leq \frac{1}{5} \Rightarrow 2^n \geq 5$

$\therefore$ Least value of n is 3.

6. Given, $\frac{dy}{dx} + y = 2e^{2x}$

$\text{IF} = e^{\int 1\,dx} = e^x$

$\therefore$ Required solution is

$y(\text{IF}) = \int 2e^{2x} \cdot e^x\,dx$

$\Rightarrow \quad y\,e^x = 2\int e^{2x}\,e^x\,dx$

$= 2\int e^{3x}dx$

$= \frac{2}{3}e^{3x} + c$

$\Rightarrow \quad y = \frac{2}{3}e^{3x}\,e^{-x} + ce^{-x}$

$\Rightarrow \quad y = \frac{2}{3}e^{2x} + ce^{-x}$

7. For the vertical motion,

$9.81 = 29.43 \sin 30° - \frac{1}{2} \times 9.81 \times t^2$

[$\because$ Vertical displacement $= (u \sin \alpha)t - \frac{1}{2}gt^2$]

$\Rightarrow \quad 1 = 3 \sin 30° t - \frac{t^2}{2}$

$\Rightarrow \quad t^2 - 3t + 2 = 0$

$\Rightarrow \quad t = 1, 2$

8. $\lim_{x \to 1} \frac{x^8 - 2x + 1}{x^4 - 2x + 1} \left(\frac{0}{0} \text{ form}\right)$

$\Rightarrow \lim_{x \to 1} \frac{8x^7 - 2}{4x^3 - 2} = \frac{8 - 2}{4 - 2} = 3$

(using L' Hospital's rule)

Alternate Method :

Using factorisation method

$\therefore \quad \lim_{x \to 1} \frac{x^8 - 2x + 1}{x^4 - 2x + 1}$

$\lim_{x \to 1} \frac{(x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x - 1)(x - 1)}{(x^3 + x^2 + x - 1)(x - 1)}$

$= \lim_{x \to 1} \frac{(x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x - 1)}{x^3 + x^2 + x - 1}$

$= \frac{7 - 1}{3 - 1} = \frac{6}{2} = 3$

9. $\lim_{x \to \frac{\pi}{6}} \frac{\sin 2x}{\sin x} = \lim_{x \to \frac{\pi}{6}} \frac{2 \sin x \cos x}{\sin x}$

$= 2 \lim_{x \to \frac{\pi}{6}} \cos x$

$= 2\frac{\sqrt{3}}{2} = \sqrt{3}$

Alternate Method :

$\lim_{x \to \frac{\pi}{6}} \frac{\sin 2x}{\sin x} = \frac{\sin 2\frac{\pi}{6}}{\sin \frac{\pi}{6}} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$

10. Key Idea : *(i) The area of a triangle with adjacent sides* $\vec{a}$ *and* $\vec{b}$ *is* $\frac{1}{2}|\vec{a} \times \vec{b}|$.

(ii) The area of a triangle ABC is $\frac{1}{2}|\vec{AB} \times \vec{AC}|$

or $\frac{1}{2}|\vec{BC} \times \vec{BA}|$ or $\frac{1}{2}|\vec{CB} \times \vec{CA}|$

Let $\vec{OA} = \hat{i}, \vec{OB} = \hat{i} + 2\hat{j}, \vec{OC} = \hat{i} + 3\hat{k}$

$\therefore \quad \vec{AB} = \vec{OB} - \vec{OA} = (\hat{i} + 2\hat{j}) - (\hat{i}) = 2\hat{j}$

and $\vec{AC} = \vec{OC} - \vec{OA} = (\hat{i} + 3\hat{k}) - (\hat{i}) = 3\hat{k}$

$\therefore$ Area of $\Delta ABC = \frac{1}{2}|\vec{AB} \times \vec{AC}|$

$= \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{vmatrix} = \frac{1}{2}[\hat{i}(6) - \hat{j}(0) + \hat{k}(0)]$

$= \frac{1}{2}\sqrt{36 + 0 + 0} = \frac{6}{2} = 3$ sq units

11. Let $I = \int_0^{\pi/3} \frac{\cos x + \sin x}{\sqrt{1 + 2 \sin x \cos x}}\,dx$

$= \int_0^{\pi/3} \frac{\cos x + \sin x}{\sqrt{\cos^2 x + \sin^2 x + 2 \sin x \cos x}}\,dx$

$= \int_0^{\pi/3} \frac{\cos x + \sin x}{\sqrt{(\cos x + \sin x)^2}}\,dx$

$= \int_0^{\pi/3} \frac{\cos x + \sin x}{\cos x + \sin x}\,dx = \int_0^{\pi/3} 1\,dx = \frac{\pi}{3}$

Alternate Method :

$I = \int_0^{\pi/3} \frac{\cos x + \sin x}{\sqrt{1 + 2 \sin x \cos x}}\,dx$

$= \int_0^{\pi/3} \frac{\cos x + \sin x}{\sqrt{2 - (\sin x - \cos x)^2}}\,dx$

Let $\sin x - \cos x = t$

$\Rightarrow (\cos x + \sin x)\,dx = dt$

$\therefore \quad I = \left[\sin^{-1}\dfrac{(\sin x - \cos x)}{\sqrt{2}}\right]_0^{\pi/3}$

$= \left[\sin^{-1}\sin\left(x - \dfrac{\pi}{4}\right)\right]_0^{\pi/3}$

$= \left(x - \dfrac{\pi}{4}\right)_0^{\pi/3} = \dfrac{\pi}{3} - \dfrac{\pi}{4} - \left(0 - \dfrac{\pi}{4}\right)$

$= \dfrac{\pi}{3}$

12. $(x + y)\cdot(x + y')\cdot(x' + y)$

$= \{(x + y)\cdot(x + y')\}\cdot(x' + y)$

(By using associative law of multiplication)

$= \{x + (y\cdot y')\}\cdot(x' + y)$

$= (x + 0)\cdot(x' + y) = x\cdot(x' + y)$

$= (x\cdot x') + (x\cdot y)$

$= 0 + (x\cdot y) = x\cdot y$

13. Key Idea : *For cyclic quadrilateral*

$$\text{Area} = \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

$\therefore$ Let points are

$A(2, 3), B(3, 4), C(4, 5), D(5, 6)$

$\therefore \quad AB = \sqrt{(3-2)^2 + (4-3)^2} = \sqrt{2}$

Similarly, $BC = \sqrt{2}, CD = \sqrt{2}$

and $DA = 3\sqrt{2}$

$\therefore \quad a = \sqrt{2} = b = c,\ d = 3\sqrt{2}$

Now, $s = \dfrac{a + b + c + d}{2}$

$= \dfrac{\sqrt{2} + \sqrt{2} + \sqrt{2} + 3\sqrt{2}}{2} = \dfrac{6\sqrt{2}}{2} = 3\sqrt{2}$

$\therefore$ Area $= \sqrt{(s-a)(s-b)(s-c)(s-d)}$

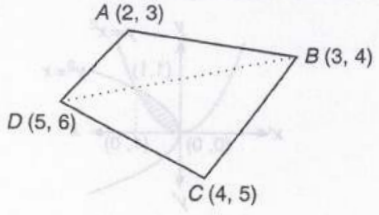
$= \sqrt{(3\sqrt{2} - \sqrt{2})(3\sqrt{2} - \sqrt{2})}$

$\sqrt{(3\sqrt{2} - \sqrt{2})(3\sqrt{2} - 3\sqrt{2})}$

$= 0$

Alternate Method :

Area of quadrilateral = Area of ΔABD + Area of ΔBDC ... (i)



Now, Area of $\Delta ABD = \dfrac{1}{2}\begin{vmatrix} 2 & 3 & 1 \\ 3 & 4 & 1 \\ 5 & 6 & 1 \end{vmatrix}$

$= \dfrac{1}{2}[2(4-6) - 3(3-5) + 1(18-20)]$

$= \dfrac{1}{2}[-4 + 6 - 2] = 0$ ...(ii)

Similarly,

Area of $\Delta BDC = \dfrac{1}{2}\begin{vmatrix} 5 & 6 & 1 \\ 4 & 5 & 1 \\ 3 & 4 & 1 \end{vmatrix}$

$= \dfrac{1}{2}[5(5-4) - 6(4-3) + 1(16-15)]$

$= \dfrac{1}{2}[5 - 6 + 1]$

$= 0$ ...(iii)

From (i), (ii) , (iii)

Area of quadrilateral $= 0 + 0 = 0$

Note: Notice that all the given points lies on the straight line $y = x + 1$.

$\therefore$ Points are collinear $\Rightarrow$ Area = 0.

14. Key Idea : *We know that*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$$

$Area = \dfrac{1}{2}ac\sin B = \dfrac{1}{2}ab\sin C = \dfrac{1}{2}bc\sin A$.

Given, the vertex angles are A, B, C and side BC.

$\therefore \quad \text{Area} = \dfrac{1}{2}a(k\sin C)\sin B$

$= \dfrac{1}{2}\dfrac{a^2\,k\sin C\sin B}{a} = \dfrac{1}{2}\dfrac{a^2\,k\sin C\sin B}{k\sin A}$

$= \dfrac{1}{2}a^2\dfrac{\sin B\sin C}{\sin A}$

15. Key Idea : *Any function $f(x)$ is derivable at $x = a$, if*

$$\lim_{h\to 0^+}\frac{f(a+h) - f(a)}{h} = \lim_{h\to 0^-}\frac{f(a-h) - f(a)}{-h}.$$

Given, $f(x) = \sin|x|$

$\Rightarrow f(x) = \begin{cases} \sin x & x > 0 \\ 0 & x = 0 \\ -\sin x & x < 0 \end{cases}$

$\text{RHD} = \lim\limits_{h\to 0}\dfrac{\sin|(0+h)| - \sin(0)}{h}$

$= \lim\limits_{h\to 0}\dfrac{\sin h}{h} = 1$

$$\text{LHD} = \lim_{h\to 0}\frac{\sin|(0-h)| - \sin(0)}{-h}$$

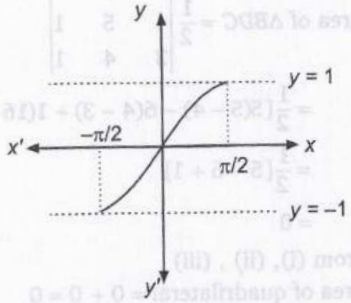
$$= \frac{-\sin h}{h} = -1$$

$\therefore$ LHD $\neq$ RHD at $x = 0$

$\therefore f(x)$ is not derivable at $x = 0$.

16. The statement given in option (d) is a logical statement as we can decide logically whether or not it is true.

17.



Since, $2\pi k - \frac{\pi}{2} \le \sin x \le 2\pi k + \frac{\pi}{2}$

For $k = 0$

$$-\pi/2 < \sin x < \pi/2$$

which increases from -1 to 1.

Similarly, for other values of k it is increases from -1 to 1.

18. Key Idea : *The general term in the expansion of* $(x + a)^n$ *is* ${}^nC_r\, x^{n-r}\, a^r = T_{r+1}$.

Given, $\left(x + \frac{1}{x^2}\right)^6$

$\therefore$ General term is

$$T_{r+1} = {}^6C_r\, x^{6-r}\left(\frac{1}{x^2}\right)^r$$

$$\Rightarrow \quad T_{r+1} = {}^6C_r\, x^{6-r}\, x^{-2r}$$

$$\Rightarrow \quad T_{r+1} = {}^6C_r\, x^{6-3r}$$

Now, for term independent of x

$$\Rightarrow \quad 3r = 6 \Rightarrow r = 2$$

$$\therefore \quad T_3 = {}^6C_2\, x^0 = {}^6C_2 = \frac{6\times 5}{2!} = 15$$

$$\Rightarrow \quad T_3 = 15$$

19. If the acceleration is f, then

$$f = \frac{40-0}{10}\ \text{m/s}^2 \qquad (\because v = u + ft)$$

and displacement in 10 s

$$= \left(0\times 10 + \frac{1}{2}\times 4\times 10^2\right)\text{m} \qquad \left(\because s = ut + \frac{1}{2}ft^2\right)$$

$$= 200\ \text{m}$$

20. Given $P(A) = P(B) = P(C) = \frac{1}{4}$

$$P(AB) = P(CB) = 0$$

and $\quad P(A \cap C) = \frac{1}{8}$

Now, $\quad P(A + B) = P(A \cup B)$

$$= P(A) + P(B) - P(A \cap B) = \frac{1}{4} + \frac{1}{4} - 0 = \frac{1}{2}$$

21. Given angles A, B, C of ΔABC are in AP with d (common difference) $= 15°$ ...(i)

$$\Rightarrow \quad B = A + 15° \qquad \text{...(ii)}$$

and $\quad C = A + 30°$ ...(iii)

Also, $A + B + C = 180°$

$(\because ABC$ forms a$\Delta)$

$$\Rightarrow \quad A + A + 15° + A + 30° = 180°$$

$$\Rightarrow \quad 3A + 45° = 180° \Rightarrow \angle A = 45°$$

$$\therefore \quad \angle B = 45° + 15° = 60°$$

22. Key Idea : *For the ellipse* $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

the length of major axis $= 2a$, *if* $a > b$ *and major axis is* $2b$, *if* $a < b$.

Given equation of ellipse $9x^2 + 7y^2 = 63$

$$\Rightarrow \quad \frac{x^2}{7} + \frac{y^2}{9} = 1$$

$$\therefore \quad a^2 = 7,\ b^2 = 9$$

Since, $\quad a < b$

$\Rightarrow$ Length of major axis $= 2b = 6$

23. Given curves $y = x^2$, $x = y^2$.

$\therefore$ Point of intersections

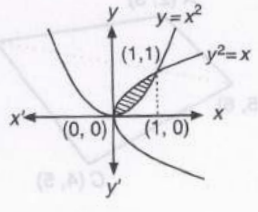
$$x = (x^2)^2 \Rightarrow x^4 - x = 0$$

$$\Rightarrow \quad x(x^3 - 1) = 0$$

$$\therefore \quad x = 0, 1$$

For $x = 0, y = 0$; $x = 1, y = 1$.

Area of shaded region



$$= \int_0^1 x^{1/2}\,dx - \int_0^1 x^2\,dx$$

$$= \left[\frac{2x^{3/2}}{3}\right]_0^1 - \left[\frac{1}{3}x^3\right]_0^1$$

$$= \frac{2}{3} - \frac{1}{3} = \frac{1}{3} \text{ sq unit}$$

$\therefore$ Required area $= \frac{1}{3}$ sq unit.

24. Let $I = \int \log 2x\,dx$

or $I = \int 1 \cdot \log 2x\,dx$

$$\Rightarrow I = \log 2x \int 1 \cdot dx - \int \left\{\left(\frac{d}{dx}\log 2x\right)\int 1\,dx\right\}dx$$

$$\Rightarrow I = x \log 2x - \int \frac{1}{2x} \cdot 2 \cdot x\ dx$$

$$\Rightarrow I = x \log 2x - x + c$$

25. Given $y = f(x) = \frac{1}{x^2}$

(i) $f(-x) = \frac{1}{(-x)^2} = \frac{1}{x^2} = f(x)$

$\therefore$ Curve is symmetrical about y-axis.

26. Given four numbers 1, 2, 3 and 4.

One digit number that can be formed $= {}^4P_1 = 4$

Two digit numbers $= {}^4P_2 = 12$

Three digit numbers $= {}^4P_3 = 24$

and four digit numbers $= {}^4P_4 = 24$

$\therefore$ Total number of numbers that can be formed

$$= 4 + 12 + 24 + 24 = 64$$

27. Key Idea : *For function $f(x)$ to be continuous at $x = a$*

$$= \lim_{x \to a} f(x) = \lim_{x \to a} f(x) = \lim_{x \to a^+} f(x).$$

Given, $f(x) = \begin{cases} \frac{\sin 3x}{\sin x}, & x \neq 0 \\ k\ , & x = 0 \end{cases}$

$\therefore$ For continuity

$$\lim_{x \to 0} f(x) = k$$

$$\Rightarrow \lim_{x \to 0} \frac{\sin 3x}{\sin x} = k$$

$$\Rightarrow \lim_{x \to 0} \frac{\sin 3x}{3x} \cdot \frac{3x}{\sin x} = k$$

$$\Rightarrow 3 = k$$

$\therefore$ Value of $k = 3$.

28. Given, $y = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$

Putting $x = \tan\theta$, we get

$$\frac{\sqrt{1+x^2}-1}{x} = \frac{\sec\theta - 1}{\tan\theta} = \frac{1-\cos\theta}{\sin\theta} = \tan\frac{\theta}{2}$$

$$\therefore \quad y = \frac{1}{2}\tan^{-1} x$$

$$\Rightarrow \quad y' = \frac{1}{2(1+x^2)}$$

$$\Rightarrow \quad y'(0) = \frac{1}{2(1+0^2)} = \frac{1}{2}$$

29. Key Idea : *Equation of tangent at (x_1, y_1) to the circle $x^2 + y^2 = a^2$ is $xx_1 + yy_1 = a^2$.*

Given circle $x^2 + y^2 = a^2$ and point (h, h).

$\therefore$ Equation of tangent at (h, h) is

$$xh + yh = a^2 \Rightarrow x + y = \frac{a^2}{h}$$

$$\therefore \quad y = -x + \frac{a^2}{h}$$

$\therefore$ Slope of tangent is -1.

Alternate Method :

Given curve

$$x^2 + y^2 = a^2$$

On differentiating w.r.t. x we get

$$2x + 2y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{-2x}{2y}$$

$\therefore$ Slope of tangent at (h, h)

is $\left(\frac{dy}{dx}\right)_{(h,h)} = \frac{-2h}{2h} = -1$

30. Given, $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$(aI + bA)^2 = (aI + bA)(aI + bA)$$

$$= a^2 I^2 + aI(bA) + bA(aI) + (bA)^2$$

$$= a^2 I^2 + 2abIA + (b^2 A^2)$$

Now, $I^2 = I$ and $IA = A$

$$\therefore \quad (aI + bA)^2 = a^2 I + 2abA + b^2(A^2)$$

Now, $A^2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$$\Rightarrow \quad A^2 = 0$$

$$\therefore \quad (aI + bA)^2 = a^2 I + 2abA$$

31. Given $\tan 1° \tan 2° \tan 3° \ldots. \tan 89°$

Now, $\tan 89° = \text{of } \tan(90-1) = \cot 1°$

$\tan 88° = \cot 2°$ and so on.

$\therefore \quad \tan 1° \tan 2° \ldots \tan 89°$

$= (\tan 1° \cot 1°)(\tan 2° \cot 2°) \ldots. \tan 45°$

$1 \cdot 1 = 1$

32. Given α, β are the roots of $x^2 + x + 1 = 0$

$\Rightarrow \quad x^2 - (\omega + \omega^2)x + 1 = 0$

$\Rightarrow \quad (x - \omega)(x - \omega^2) = 0$

$\therefore \quad \alpha = \omega, \beta = \omega^2$

Now, $\alpha^{19} = \omega^{19} = (\omega^3)^6 \omega = \omega$

and $\beta^7 = (\omega^2)^7 = (\omega^3)^4 \omega^2 = \omega^2$

$\therefore$ Equation whose roots are

$\alpha^{19} = \omega$ and $\beta^7 = \omega^2$ is

$x^2 - (\omega + \omega^2)x + \omega \cdot \omega^2 = 1$

$\therefore \quad x^2 + x + 1 = 0$

33. Let $\vec{a}$ and $\vec{b}$ be two unit vectors

$\therefore \quad |\vec{a}| = |\vec{b}| = 1$

Now, $|\vec{a} + \vec{b}|^2 = 1$ (given)

$\Rightarrow \quad |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b} = 1$

$\Rightarrow \quad 2\vec{a} \cdot \vec{b} = 1 - 1 - 1 = -1$

$\Rightarrow \quad \vec{a} \cdot \vec{b} = \frac{-1}{2}$

$\therefore \quad |\vec{a}| \cdot |\vec{b}| \cos\theta = -\frac{1}{2}$

$\Rightarrow \quad \theta = \frac{2\pi}{3}$

34. **Key Idea :** *The perpendicular distance from* (α, β, γ) *to the line*

$$\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n} \quad \ldots(i)$$

is given by

$$\sqrt{(\alpha - x_1)^2 + (\beta - y_1)^2 + (\gamma - z_1)^2 - [l(\alpha - x_1) + m(\beta - y_1) + n(\gamma - z_1)]^2} \quad \ldots(ii)$$

Given line $\vec{r} = (\hat{j} + 2\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$ and point $(1, 6, 3)$.

$\therefore$ Line in cartesian co-ordinates is

$$\frac{x - 0}{1} = \frac{y - 1}{2} = \frac{z - 2}{3} \quad \ldots(iii)$$

$\therefore \quad l = \frac{1}{\sqrt{14}}, m = \frac{2}{\sqrt{14}}, n = \frac{3}{\sqrt{14}}$ are direction ratio's of line.

Also, by comparing (i) and (iii), we get

$x_1 = 0, y_1 = 1, z_1 = 2$

and $\alpha = 1, \beta = 6, \gamma = 3$

$\therefore$ From (ii)

Required distance

$$\sqrt{(1-0)^2 + (6-1)^2 + (3-2)^2) - \left[\frac{1}{\sqrt{14}}\{1 + 2\times 5) + 3(1)^2\}\right]^2}$$

$$= \sqrt{27 - 14} = \sqrt{13}$$

35. **Key Idea :** *If* $f(-x) = -f(x)$, *then*

$$\int_{-a}^{a} f(x)\,dx = 0$$

Let $I = \int_{-\pi}^{\pi} (1 - x^2) \sin x \cos^2 x \, dx$

Let $f(x) = (1 - x^2) \sin x \cos^2 x$

$\Rightarrow f(-x) = (1 - (-x)^2)(\sin(-x)) \cos^2(-x)$

$\Rightarrow \quad = -(1 - x^2) \sin x \cos^2 x$

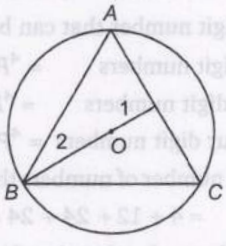
$\Rightarrow \quad f(-x) = -f(x)$

$\Rightarrow f(x)$ is an odd function

$\therefore \quad \int_{-\pi}^{\pi} (1 - x^2) \sin x \cos^2 x \, dx = 0$

36. **Key Idea :** *Centroid divides the median in the ratio of* 2 : 1.

Given, circle has centre at origin and passing through vertices of equilateral Δ whose median is of length $3a$.



$\therefore \quad$ Radius $= 3a \cdot \frac{2}{3} = 2a$

$\Rightarrow$ Equation of circle

$(x - 0)^2 + (y - 0)^2 = (2a)^2$

$\Rightarrow \quad x^2 + y^2 = 4a^2$

37. **Key Idea :** *Equation of tangent at* (x_1, y_1) *to the ellipse*

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ is } \frac{x\,x_1}{a^2} + \frac{yy_1}{b^2} = 1 \quad \ldots(i)$$

and Equation of pair of tangent to an ellipse from (x_1, y_1) *is*

$$\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1\right)\left(\frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} - 1\right) = \left(\frac{xx_1}{a^2} + \frac{yy_1}{b^2} - 1\right)^2 \quad \ldots(ii)$$

and angle θ between them

$$= \tan^{-1}\frac{2\sqrt{h^2-ab}}{a+b} \quad \ldots(iii)$$

Given equation of ellipse $3x^2 + 2y^2 = 5$ and point (1 , 2).

It can be rewritten as, $\frac{x^2}{5/3} + \frac{y^2}{5/2} = 1$

$\therefore$ Equation of tangent at (1, 2) is

$$\frac{x}{5/3} + \frac{2y}{5/2} = 1$$

$\Rightarrow \quad 3x + 4y = 5$ [from (ii)]

$\therefore$ Joint equation of tangents

$$\left(\frac{x^2}{5/3} + \frac{y^2}{5/2} - 1\right)\left(\frac{1}{5/3} + \frac{4}{5/2} - 1\right) = (3x + 4y - 5)^2$$

(from (ii))

$\Rightarrow \quad 9x^2 - 4y^2 - 24xy + 30x + 40y - 30 = 0$

$\therefore \quad a = 9 , b = -4 , h = 12, g = 15$

(By comparing with

$ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$)

$$\therefore \theta = \tan^{-1}\left(\frac{2\sqrt{144+36}}{5}\right) = \tan^{-1}\left(\frac{2\cdot 2\cdot 3\sqrt{5}}{5}\right)$$

$\theta = \tan^{-1}(12/\sqrt{5})$

38. Given, $x^2 - xy + y^2 - 4x - 4y + 16 = 0$

$\Rightarrow \quad x^2 - (y+4)x + y^2 - 4y + 16 = 0$

For real x, $(y+4)^2 - 4(y^2 - 4y + 16) \geq 0$

$\Rightarrow \quad -3y^2 + 24y - 48 = 0$

$\Rightarrow \quad y^2 - 8y + 16 = 0$

$\Rightarrow \quad (y-4)^2 = 0 \Rightarrow y = 4$

$\therefore \quad x = 4$

$\Rightarrow \quad (x, y) = (4, 4)$

39. Given, $z = x + iy$ and

$$\left|\frac{z-5i}{z+5i}\right| = 1 \quad \because \quad \left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$$

$$\Rightarrow \quad \frac{|x+iy-5i|}{|x+iy+5i|} = 1$$

$$\Rightarrow \quad |x+iy-5i| = |x+iy+5i|$$

$$\Rightarrow \quad x^2 + (y-5)^2 = x^2 + (5+y)^2$$

$$\Rightarrow \quad -10y = 10y \Rightarrow y = 0$$

Which is the equation of axis of x.

40. $\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)$

$$= \left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)$$

$$\times \left(1-\cos\frac{3\pi}{8}\right)\times\left(1-\cos\frac{\pi}{8}\right)$$

$$\left[\because \cos\frac{5\pi}{8} = \cos\left(\pi - \frac{3\pi}{8}\right) = -\cos 3\frac{\pi}{8}\right.$$

$$\left.\text{and } \cos\frac{7\pi}{8} = \cos\left(\pi - \frac{\pi}{8}\right) = -\cos\frac{\pi}{8}\right]$$

$$= \left(1-\cos^2\frac{\pi}{8}\right)\left(1-\cos^2\frac{3\pi}{8}\right)$$

$$= \sin^2\frac{\pi}{8}\cdot\sin^2\frac{3\pi}{8} = \frac{1}{4}\left[2\sin\frac{\pi}{8}\sin\frac{3\pi}{8}\right]^2$$

$$= \frac{1}{4}\left[\cos\frac{\pi}{4} - \cos\frac{\pi}{2}\right]^2 = \frac{1}{4}\left[\frac{1}{\sqrt{2}} - 0\right]^2 = \frac{1}{4}\times\frac{1}{2} = \frac{1}{8}$$

41. Given, $x^2 - 2a|x-a| - 3a^2 = 0$

Here $\quad a \leq 0$

We know that $|x-a| = \begin{cases} x-a, & x > a \\ -(x-a), & x < a \end{cases}$

Thus $x^2 - 2a|x-a| - 3a^2 = 0$ gives two cases

Case I : When $\quad x > a$

$\therefore \quad x^2 - 2a(x-a) - 3a^2 = 0$

$\Rightarrow \quad x^2 - 2ax - a^2 = 0 \Rightarrow x = a \pm \sqrt{2}a$

Now, for $x \geq a$, $a < 0 \Rightarrow x = a(1 - \sqrt{2})$ ...(i)

$(\because x = a(1+\sqrt{2}) < a)$

Case II : When $x < a$

$\therefore \quad x^2 + 2a(x-a) - 3a^2 = 0$

$\Rightarrow \quad x^2 + 2ax - 5a^2 = 0 \quad \Rightarrow \quad x = -a \pm \sqrt{6}a$

Now, for $x < a$, $a < 0 \Rightarrow x = a(\sqrt{6} - 1)$ ...(ii)

$(\because x = -a\cdot(1+\sqrt{6}) > a)$

From (i) and (ii),

$$x = \{a(1-\sqrt{2}), a(\sqrt{6}-1)\}$$

42. Since, radius of circumcircle made by the perpendicular distance

Since, $PQ = PR \quad \therefore \quad \angle P = \frac{\pi}{3}$

43. Given $z_r = \cos\frac{\pi}{2^r} + i\sin\frac{\pi}{2^r}$; $r = 1, 2, \ldots$

$$\therefore z_1 z_2 z_3 \ldots = \left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)\left(\cos\frac{\pi}{2^2} + i\sin\frac{\pi}{2^2}\right)\ldots$$

$= e^{i\pi/2\left(1+\frac{1}{2}+\frac{1}{4}+\ldots\infty\right)} = e^{i\pi/2\frac{1}{1-1/2}} = e^{i\pi/2.2} = e^{i\pi}$

$= \cos\pi + i\sin\pi = -1$

Alternate Method :

$z_1 z_2 \ldots z_n = \cos\left(\frac{\pi}{2}+\frac{\pi}{2^2}+\frac{\pi}{2^3}+\ldots\right) + i\sin\left(\frac{\pi}{2}+\frac{\pi}{2^2}+\ldots\right)$

(By Applying De-Moivre's theorem)

$= \cos\left(\frac{\pi}{2}\left(1+\frac{1}{2}+\frac{1}{2}+\ldots\infty\right)\right) + i\sin\left(\frac{\pi}{2}\left(1+\frac{1}{2}+\ldots\infty\right)\right)$

$= \cos\left(\frac{\pi}{2}\cdot\frac{1}{1-\frac{1}{2}}\right) + i\sin\left(\frac{\pi}{2}\cdot\frac{1}{1-\frac{1}{2}}\right)$

$= \cos\pi + i\sin\pi = -1$

44. We have, $a^2x^4 + bx^3 + cx^2 + dx + f^2$

$= (ax^2 + cx + f)^2$ a perfect square

$= a^2x^4 + 2acx^3 + (2af + c^2)x^2 + 2cfx + f^2$

$\therefore b = 2ac,\ c = 2af + c^2,\ d = 2cf$ and

Again $4a^2c = 4a^2(2af + c^2) = 8a^3f + b^2$

$(\because b = 2ac)$

$\therefore \quad 4a^2c = b^2 + 8a^3f$

45. Given, $\tan(\cot x) = \cot(\tan x)$

$= \tan\left(\frac{\pi}{2} - \tan x\right)$

$\Rightarrow \quad \cot x = n\pi + \left(\frac{\pi}{2} - \tan x\right)$

$\Rightarrow \quad \cot x + \tan x = n\pi + \frac{\pi}{2}$

$\therefore \quad \frac{1}{\sin x\cos x} = \frac{\pi}{2}(2n+1)$

$\Rightarrow \quad \sin 2x = \frac{4}{(2n+1)\pi}$

46. Given a, b, c are real, $ax^2 + bx + c = 0$ has two real roots α and β where $\alpha < -2$ and $\beta > 2$

$\Rightarrow \quad f(-2) < 0$ and $f(2) > 0$

$\Rightarrow \quad 4a - 2b + c < 0 \quad \Rightarrow \quad 4 - \frac{2b}{a} + \frac{c}{a} < 0$

47. Given, $\quad f(x) = x^3 - 3x$

$\therefore \quad f'(x) = 3x^2 - 3 \quad \ldots(i)$

and $\quad f''(x) = 6x \quad \ldots(ii)$

For maxima, $f'(x) = 0$

$\Rightarrow 3x^2 - 3 = 0 \Rightarrow x = \pm 1 \quad \because x = 1 \in [0, 2]$

$\therefore x = 1$ is the only solution.

At $x = 1,\ f''(x) > 0$

$\therefore \quad x = 1$ is point of minima.

$f(0) = 0,\ f(1) = -2$ and $f(2) = 2$

$\therefore f(x)$ attains the maximum *at* $x = 2$ and maximum value $= 2$

48. **Key Idea :** *Any line perpendicular to* $ax + by = c$ *is* $bx - ay = k$.

Given line $3x + y = 3$

$\therefore$ Line perpendicular to $3x + y = 3$ is $x - 3y = \lambda$

Also, it passes through $(2, 2)$

$\Rightarrow \quad 2 - 6 = \lambda \Rightarrow \lambda = -4$

$\therefore$ Equation of line $\quad x - 3y = -4 \quad \ldots(i)$

$\therefore \quad y$ intercept $= \frac{-4}{-3} = \frac{4}{3}$

Alternate Method :

Any line through $(2, 2)$ is $\quad (y - 2) = m(x - 2) \quad \ldots(i)$

$(\because$ lines are $\perp)$

$\therefore \quad m = -\frac{1}{\text{slope of line } 3x + y = 3}$

$\Rightarrow \quad m = -\frac{1}{-3} = \frac{1}{3} \quad \ldots(ii)$

$\therefore$ From (i) and (ii)

Required line $3(y - 2) = (x - 2)$

On putting $x = 0$, we get

$y = \frac{4}{3}$ is the y intercept.

49. **Key Idea :** *If* l, m, n *be the direction cosines of a line, then* $\quad l^2 + m^2 + n^2 = 1$.

If α, β, γ are the angles which the line makes with co-ordinate axes, then

$l = \cos\alpha,\ m = \cos\beta,\ n = \cos\gamma$

$\therefore l^2 + m^2 + n^2 = \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

50. Given $\sin^{-1}x + \sin^{-1}(1 - x) = \cos^{-1}x$

$\Rightarrow \quad \sin^{-1}x + \sin^{-1}(1 - x) = \sin^{-1}\sqrt{1 - x^2}$

$\Rightarrow \quad \sin[\sin^{-1}x + \sin^{-1}(1 - x)]$

$= \sin(\sin^{-1}\sqrt{1 - x^2})$

$\Rightarrow \quad \sin[\sin^{-1}[x\sqrt{1 - (1 - x)^2} + (1 - x)\sqrt{1 - x^2}] = \sin\sin^{-1}(\sqrt{1 - x^2})$

$\Rightarrow x\sqrt{1 - (1 - x)^2} + (1 - x)\sqrt{1 - x^2} = \sqrt{1 - x^2}$

$\Rightarrow \quad x\sqrt{2x - x^2} = \sqrt{1 - x^2}x$

$\Rightarrow \quad x(\sqrt{2x - x^2} - \sqrt{1 - x^2}) = 0$

$\Rightarrow \quad x = 0$ or $2x - x^2 = 1 - x^2$

$\Rightarrow \quad x = 0$ or $x = \frac{1}{2}$