

4. Locus

The path traced by a moving point under certain geometrical condition(s) is called the locus of the moving point.

An equation is said to be the equation of locus of a moving point if :

- the coordinates of every point on the locus satisfy the equation
- the coordinates of any point satisfy the equation, then the point must lie on the locus

Working Rule For Finding Equation of the Locus of a Point :

1. If x and y coordinates of the moving points are expressed in term of a parameter, say t , then eliminate t to obtain the relation in x and y . Simplifying this relation will give the required locus.
2. If some geometrical condition(s) is(are) given, then:
 - Take the coordinates of the variable points as (h, k) .
 - Express the given geometrical condition in terms of h and k .
 - Eliminate the arbitrary variables to obtain the relation in terms of h and k , i.e. the relation must only involve h, k and known quantities.
 - When we shift the origin to a point or rotate the axes, the coordinates of the given point change with respect to that point or axes.
 - Let $f(x, y) = 0$ be the equation of a curve with respect to the origin $O(0, 0)$. When the origin is shifted to the point (a, b) , then the equation of the curve with respect to the new coordinate system will be $f(x + a, y + b) = 0$.
 - When the coordinate axes are rotated through an angle of θ about the origin, then the coordinates of point P are shifted to (X, Y) in the new system. The relation between the old and the new coordinates of P is given as $x = X \cos \theta - Y \sin \theta$ and $y = X \sin \theta + Y \cos \theta$
 - Let us consider a curve $f(x, y) = 0$ in a two dimensional system. When the system is rotated about the origin by an angle θ , then the equation of the curve, with respect to the new coordinate system, will be $f(x \cos \theta - y \sin \theta, x \sin \theta + y \cos \theta) = 0$.
 - If axes are rotated through an angle of $(-\theta)$, then the relation between the old and the new coordinates of the point P is given as $x = X \cos \theta + Y \sin \theta$ and $y = -X \sin \theta + Y \cos \theta$

Note: If the origin is shifted to the point (a, b) and axes are rotated through an angle of θ , then the equation of the curve $f(x, y) = 0$ becomes $f(x \cos \theta - y \sin \theta + a, x \sin \theta + y \cos \theta + b) = 0$.