

MECHANICAL PROPERTIES

OF SOLID.

ELASTICITY

(Molecule of Solid)

- # The tendency of an object to regain its original position, its original configuration when Deforming force is removed. up to its Elastic limit.
- # The Tendency to oppose any change in its Shape, Size by a body.

Q Which is More elastic?? Steel or Rubber?
 Ans. Steel, because it come very speedily to its original position.

Q Why Elasticity?

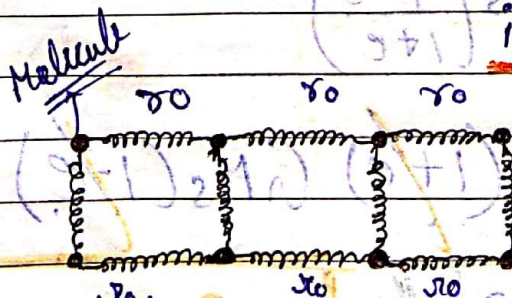
Molecule $\rightarrow$ Deforming Force (external)

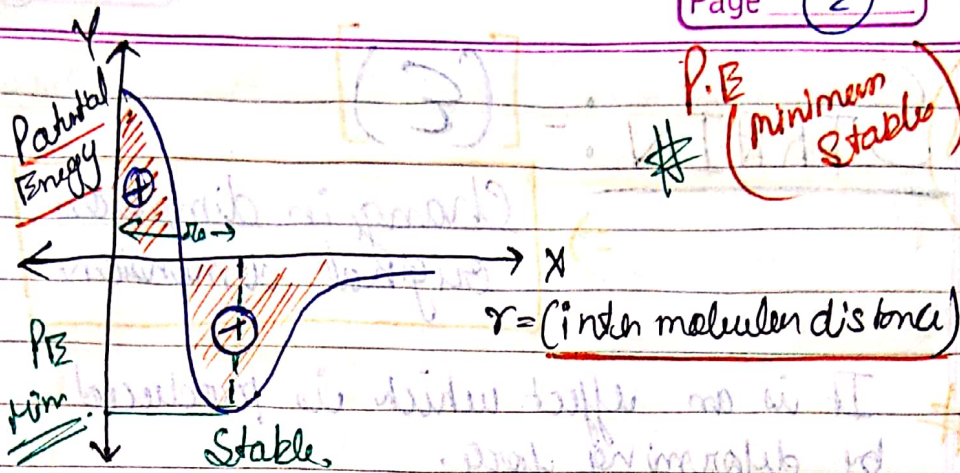


Restoring Force
intermolecular Force



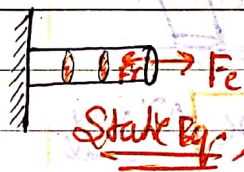
Elasticity





STRESS :-

The internal restoring force per unit cross sectional Area.



Restoring Force = Deforming Force

$F_R = F_{ext}$ 99% Calc

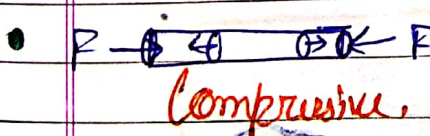
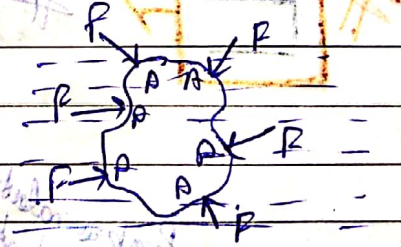
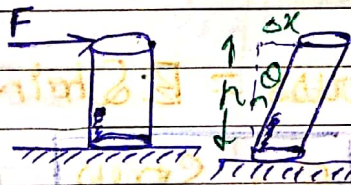
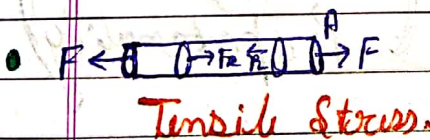
$$\sigma = \frac{F_R}{A_{cross\ section}} \quad \frac{N}{m^2} \quad \text{Scalar q. (Tensor)}$$

TYPES :-

LONGITUDINAL

SHEARING / TANGENTIAL

VOLUME STRESS



$$\frac{F_t}{A} = \sigma$$

$$\sigma = \frac{F}{A} = P$$

$$\sigma = \Delta P$$

$$\sigma = \frac{F}{A_{cross\ section}}$$

$(P - P_0)$ 2 faces
Ex. Plastic Ball etc

$\sin \theta \rightarrow \text{small} \rightarrow 0$
 $\tan \theta \rightarrow \text{small} \rightarrow 0$

STRAIN :- (ϵ)

always +ve

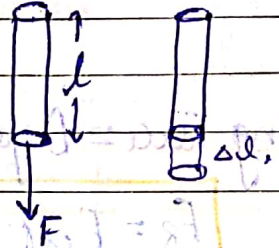
Change in dimension
 Original dimension

No limit

It is an effect which is produced by deforming force.

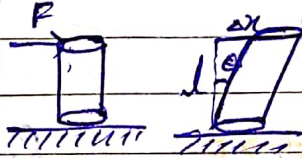
TYPES :-

LONGITUDINAL



$$\epsilon = \frac{\Delta l}{l}$$

SHEAR STRAIN



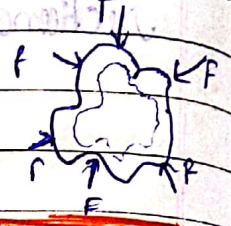
$$\epsilon = \frac{\Delta x}{l}$$

Δx very small

$$\frac{\Delta x}{l} \rightarrow 0 \text{ (radian)}$$

$$\epsilon = \frac{\Delta x}{l} \approx 0$$

VOLUME STRAIN



$$\frac{-\Delta V}{V} = \epsilon$$

Because it cannot be negative

$E \Rightarrow$ # Stress $\propto$ Strain (limit of Proportionality)

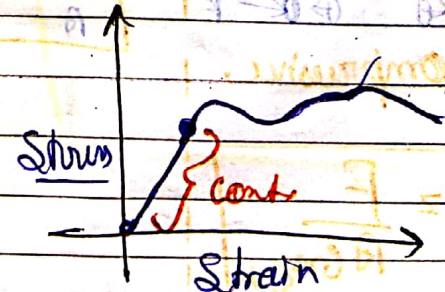
$$\text{Stress} = E \times \text{Strain}$$

Proportionality constant

$$E = \frac{\text{Stress}}{\text{Strain}}$$

$$\frac{N}{m^2}$$

$E \Rightarrow$ Modulus of Elasticity
 Property of Material



(i) YOUNG'S MODULUS OF ELASTICITY

$$\gamma = \frac{\text{Longitudinal Stress}}{\text{Longitudinal Strain}}$$

$$\gamma = \frac{F l}{A \Delta l}$$

(ii) SHEAR MODULUS OF ELASTICITY

$$G = \frac{\text{Shear Stress}}{\text{Shear Strain}}$$

MODULUS OF
RIGIDITY

$$G = \frac{F x}{A \Delta x}$$

$$G = \frac{F x}{A \theta}$$

(iii) BULK MODULUS OF ELASTICITY

$$\beta = \frac{\text{Volume Stress}}{\text{Volume Strain}}$$

COMPRESSIBILITY.

$$\beta = \frac{F}{A} \left(\frac{V}{-\Delta V} \right)$$

$$\beta = - \frac{\Delta P V}{\Delta V}$$

$$C = \frac{1}{\beta}$$

Q. Elongation of a steel bar 1m long and 1.5 cm^2 cross sectional area when, subjected to a pull of $1.5 \times 10^4 \text{ N}$. ($\gamma_s = 2 \times 10^{-11} \text{ N/m}^2$)

- a) 0.1 mm b) 0.3 mm c) 0.5 mm d) 0.2 mm

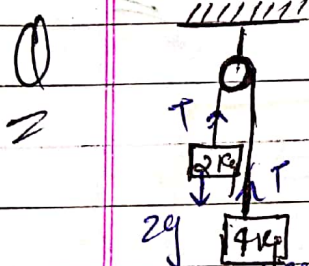
$$\gamma = \frac{\text{Stress}}{\text{Strain}} = \frac{F l}{A \Delta l}$$

$$\Delta l = \frac{F l}{A \gamma}$$

$$= \frac{1.5 \times 10^4 \times 1}{1.5 \times 10^{-4} \times 2 \times 10^{-11}} = \frac{1}{2} \times 10^{-11}$$

$$= 0.5 \times 10^{-3} \text{ m}$$

$$= 0.5 \text{ mm}$$



Wire $\rightarrow$ Breaking stress $= 2 \times 10^5 \text{ N/m}^2$

Find the min cross sectional area of wire so that it doesn't break.

- a) 2.66 cm^2 b) 1.66 cm^2 c) 3.33 cm^2 d) 1.33 cm^2

$$\text{Stress} = \frac{F}{A}$$

$$4g - T = 4a$$

$$T - 2g = 2a$$

$$2g = 6a$$

$$a = \frac{g}{3}$$

$$T = 2a + 2g$$

$$T = \frac{4g}{3}$$

$$T = \frac{2 \text{ m/m}^2}{1 \text{ m/m}^2}$$

Trick to find Tension

$$\text{Stress} = \frac{F}{A} = \frac{T}{A}$$

$$2 \times 10^5 = \frac{4g}{3A}$$

$$A = \frac{4g}{3 \times 2 \times 10^5}$$

$$A = \frac{4 \times 10^{-5}}{3 \times 22 \times 10^4 \times 10^2}$$

$$A = \frac{4 \times 10^{-4}}{3}$$

$$A = 1.33 \times 10^{-4} \text{ m}^2$$

$$A = 1.33 \text{ cm}^2$$

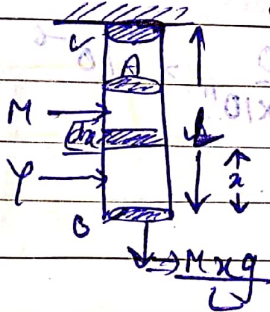
Q Find the extension in a wire of length 'L' cross section Area (A), ~~Mass~~ ρ & Young's Modulus of Elasticity γ under its own weight??

a) $\frac{\rho g L}{2 A \gamma}$

b) $\frac{M g L}{3 A \gamma}$

c) $\frac{2 M g L}{3 A \gamma}$

d) $\frac{M g L}{A \gamma}$



$$\gamma = \frac{F l}{A \Delta l}$$

$$\Delta l = \frac{F l}{A \gamma}$$

$$L \rightarrow M$$

$$x \rightarrow Mx$$

Work done in small element 'dx'

$$d\Delta l = \frac{F dx}{A \gamma}$$

$$\int d\Delta l = \int \frac{Mxg dx}{L A \gamma}$$

$$= \frac{Mg}{A \gamma} \int_0^L x dx$$

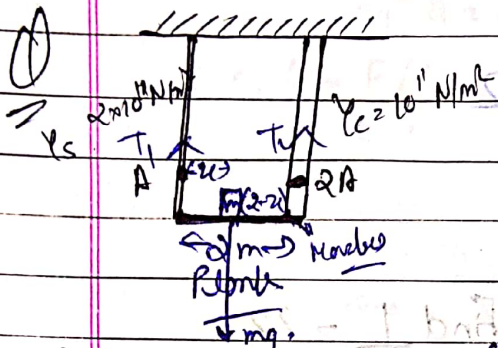
$$\Delta l = \frac{Mg}{A \gamma} \frac{L^2}{2}$$

$$\Delta l = \frac{Mg L}{2 A \gamma}$$

$$\left(\frac{T_M}{T}\right)^2 = 1 + \frac{Mg}{A\ell}$$

$$\left(\frac{T_M}{T}\right)^2 - 1 = \frac{Mg}{A\ell}$$

$$\frac{1}{\ell} = \left[\left(\frac{T_M}{T}\right)^2 - 1\right] \frac{A}{Mg}$$



Where should a mass be kept so that in steel & copper wire, we obtain

- (i) equal stress
- (ii) equal strain

(ii) a) 1.00 m

b) 0.5 m

c) 0.33 m

d) 1.33 m

e) 1.66 m

f) 0.8 m

$$(1) (\text{Stress})_s = (\text{Stress})_c$$

$$\frac{T_1}{A} = \frac{T_2}{2A}$$

$$T_2 = 2T_1$$

equilibrium $\sum \tau_{\text{net}} = 0$

$$T_1 \times x - T_2 (2-x) = 0$$

$$T_1 x - 2T_1 (2-x) = 0$$

$$x - 4 + 2x = 0$$

$$3x = 4$$

$$x = \frac{4}{3}$$

$$x = 1.33$$

$$(\text{Strain}) = (\text{Stress}) / c$$

$$\epsilon = \frac{\text{Stress}}{\text{Strain}}$$

$$\left(\frac{\text{Stress}}{\epsilon} \right)_s = \left(\frac{\text{Stress}}{\epsilon} \right)_c$$

$$\left(\frac{T}{A \epsilon_s} \right) = \left(\frac{T_2}{2A \epsilon_c} \right)$$

$$\frac{T}{2 \times 10^{-11}} = \frac{T_2}{2 \times 10^{-11}}$$

$$T_1 = T_2$$

uq, $T_{\text{act}} = 20$

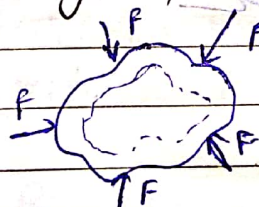
$$T_1 x - T_2 (x-1) = 0$$

$$x - 2 + x = 0$$

$$2x = 2$$

$$x = 1$$

Q Avg. dept of Indian Ocean is about 300 m.
Calculate the fractional compression $\Delta V/V$ of water at the bottom of ocean ($g = 10 \text{ m/s}^2$)
 $\rho_w = 10^3 \text{ kg/m}^3$, Bulk Mod of water $= 2.2 \times 10^9 \text{ N/m}^2$



$$\text{Volume Stress} = \frac{F}{A} = P$$

$$\text{Volume Strain} = - \frac{\Delta V}{V}$$

$$\beta = \frac{\text{Vol. Stress}}{\text{Vol. Strain}} = - \frac{PV}{\Delta V}$$

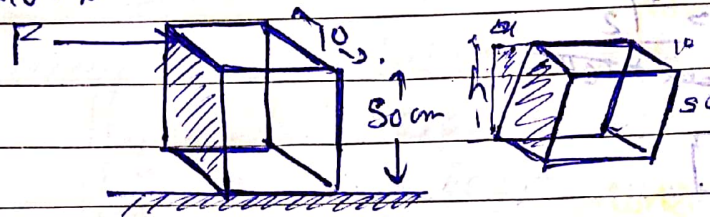
300m
depth

$$P = h \rho g$$

$$= 300 \times 10^3 \times 10 = 3 \times 10^6 \text{ N/m}^2$$

$$\frac{\Delta x}{b} = \frac{P}{B} = \frac{3 \times 10^6}{2.2 \times 10^9}$$

Q. A Square lead slab of side 50 cm & thickness 10 cm is subjected to a shearing force 9×10^4 N. How much upper edge will be displaced. If shear modulus of elasticity $G = 3.6 \times 10^9$ N/m² for lead.



$$\text{Shear Stress} = \frac{F}{A} = \frac{9 \times 10^4}{50 \times 10 \times 10^{-4}} \text{ N/m}^2$$

$$\text{Shear Strain} = \frac{\Delta x}{50}$$

$$G = \frac{9 \times 10^4}{50 \times 10 \times 10^{-4}}$$

$$G = \frac{\Delta x}{50}$$

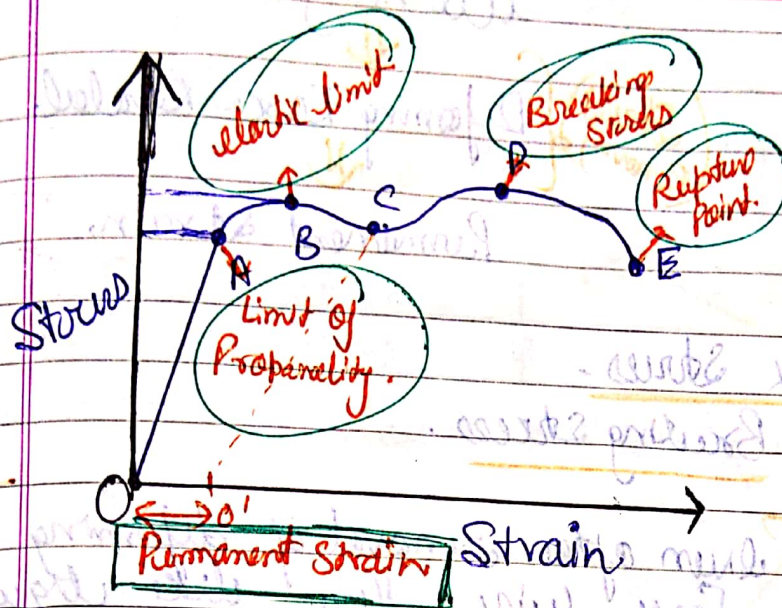
$$\Delta x = \frac{9 \times 10^4 \times 50}{3.6 \times 10^9}$$

$$\Delta x = \frac{F}{G} = \frac{9 \times 10^4}{3.6 \times 10^9} \times 50$$

$$\Delta x = \frac{9 \times 10^4 \times 50}{3.6 \times 10^9}$$

$$\begin{aligned} & \frac{9 \times 10^4}{50 \times 10 \times 10^{-4}} = \frac{\Delta x}{50} \\ & \frac{9 \times 10^4}{50 \times 10^{-3}} = \frac{\Delta x}{50} \\ & \frac{9 \times 10^7}{50} = \frac{\Delta x}{50} \\ & \Delta x = \frac{9 \times 10^7}{50} \times 50 \\ & \Delta x = 9 \times 10^7 \end{aligned}$$

STRESS-STRAIN CURVE.



$$\text{Stress} \propto \text{Strain}$$

$$\frac{F}{A} \propto \frac{\Delta l}{l}$$

OA $\rightarrow$ Stress $\propto$ Strain.

A $\rightarrow$ limit of proportionality.
(Stress)

$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{\text{Stress}}{\text{Strain}} = \text{Modulus of Elasticity.}$$

B $\rightarrow$ elastic limit
(Stress)

(Force is removed up till B, wire returns to its original state.)

A $\rightarrow$ B stress $\propto$ strain

B → D Plastic Region

Date _____
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KRISHNA

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After B → Small stress $\rightarrow$ large strain $\downarrow$

Elasticity $\times$
 $\downarrow$

(Stress Remove) $\left\{ \begin{array}{l} \text{Deforming Force Reversed} \\ \downarrow \end{array} \right.$

Permanent strain

D → Max Stress.

Breaking stress.

$\star$ Even after removal of deforming force wire does not return to original shape (thinning of wire $\uparrow \uparrow$)

E → Rupture Point

OB → Elastic Region

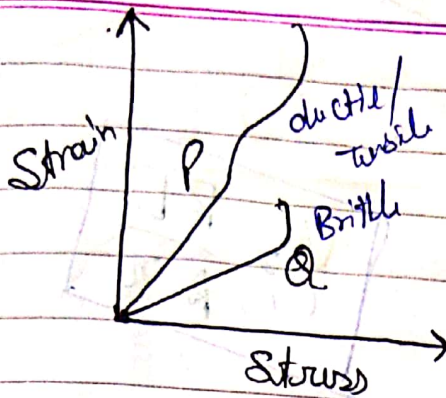
BD → Plastic Region.

1) DUCTILE / TENSILE.
Plastic Region Large.

2) BRITTLE $\rightarrow$
Plastic Region small.

3) ELASTOMER
No Plastic Region,
only have.
Elastic Region (Rubber)

Ad (13)



a) P has more tensile strength

b) P is More ductile

c) P is More brittle

~~d) γ of P is More than that of Q.~~

$$\text{Slope} = \frac{\text{Strain}}{\text{Stress}} = \frac{1}{\gamma}$$

$$(\text{Slope})_P > (\text{Slope})_Q$$

$$\left(\frac{1}{\gamma}\right)_P > \left(\frac{1}{\gamma}\right)_Q$$

$$\gamma_Q > \gamma_P$$

ENERGY STORED IN A STRETCHER WIRE

$$U = \frac{1}{2} \times \text{Stress} \times \text{Strain} \times \text{Volume}$$

$$\frac{PE}{\text{Vol.}} = \frac{1}{2} \times \text{Stress} \times \text{Strain}$$

$$E = \frac{\text{Stress}}{\text{Strain}}$$

$$\frac{PE}{\text{Vol.}} = \frac{1}{2} \times E \times (\text{Strain})^2$$

$$\frac{1}{2} \times (\text{Stress})^2 \times \text{Volume} \times \frac{1}{E} = W$$

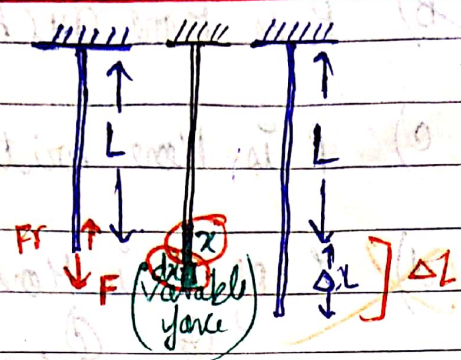
$$\text{Volume} \times (\text{Stress})^2 \times (\text{Stress}) \times \frac{1}{E} = W$$

$$\text{Volume} \times \text{Stress} \times \text{Stress} \times \frac{1}{E} = W \Rightarrow U$$

$$F = \frac{YA \Delta L}{L}$$

DERIVATION :-

$V = A \times L$
 $\Rightarrow A \times L$



$$Y = \frac{FL}{A \Delta L}$$

Small Work done in stretching wire by 'dx' amount when it is stretched upto 'x' amount

$$dw = F dx$$

$$\int dw = \int \left(\frac{YA x}{L} \right) dx$$

$$W = \frac{YA}{L} \left[\frac{x^2}{2} \right]_0^{\Delta L}$$

$$W = \frac{YA}{L} \frac{(\Delta L)^2}{2}$$

$$E = \frac{Y}{\epsilon}$$

$$W = \frac{YA}{L} \times \frac{(\Delta L)^2}{2} \times \frac{L}{L}$$

$$W = \frac{YA}{L} \times \left(\frac{\Delta L}{L} \right)^2 \times \frac{1}{2} \times L$$

$$\left(\frac{1}{2} \sigma \epsilon \right)$$

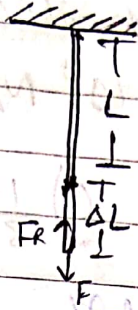
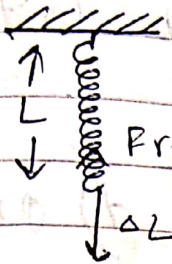
$$W = Y \times \text{volume} \times (\text{Strain})^2 \times \frac{1}{2}$$

$$W = \frac{1}{2} \times (\text{Strain}) (\text{Strain } Y) \times \text{volume}$$

P.E.
U

$$W = \frac{1}{2} \times \text{Stress} \times \text{Strain} \times \text{Volume}$$

TRICK :-



$$p = \frac{F L}{A \Delta L}$$

$$F = K \Delta L$$

$$F = \left(\frac{Y A}{L} \right) \Delta L$$

$$K \Delta L = \left(\frac{Y A}{L} \right) \Delta L$$

$$K = \frac{Y A}{L} \quad \text{Most}$$

Energy stored in a stretched spring $= \frac{1}{2} K (\Delta x)^2$

$$= \frac{1}{2} \times \frac{Y A}{L} (\Delta L)^2 \times L$$

$$= \frac{1}{2} \times Y (A L) \left(\frac{\Delta L}{L} \right)^2$$

$$= \frac{1}{2} \times Y \times \text{vol} \times (\text{Strain})^2$$

$$U = \frac{1}{2} \times \text{Stress} \times \text{Strain} \times \text{volume}$$

Q A rubber cord of length 10cm is stretched upto 12cm. If cross sectional area of cord is 7 mm^2 , find the velocity of a missile (mass 5g) which is hit upon with this rubber cord ($\gamma_{\text{Rubber}} = 5 \times 10^8 \text{ N/m}^2$).

- a) 10 m/s b) 20 m/s c) $\frac{15}{3} \text{ m/s}$ d) 0 m/s

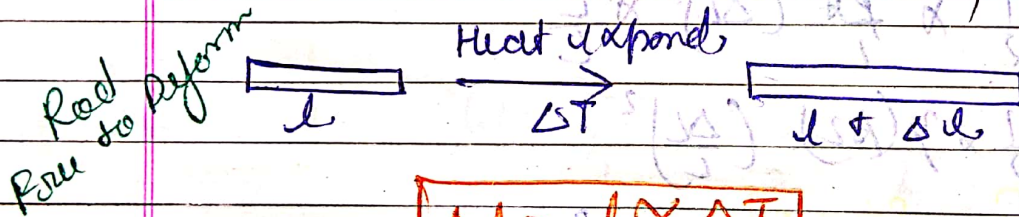
Tension / Force $\rightarrow \frac{\gamma AL}{L} = 100 \text{ N}$

Velocity $V = \sqrt{\frac{FL}{m}}$

$= \sqrt{\frac{100 \times 0.1 \times 10^{-2}}{5 \times 10^{-3}}}$

$= 20 \text{ m/s}$

THERMAL STRESSES & STRAIN



$$\Delta l = l \alpha \Delta T$$

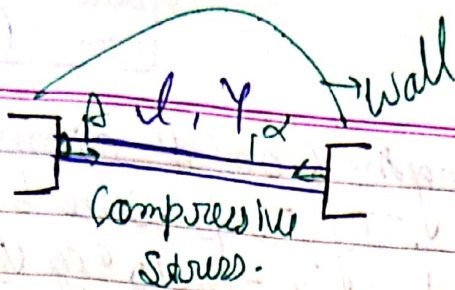
(alpha) Thermal Coefficient of linear expansion.

$\alpha \Rightarrow$ Unit temp increase,

Unit length Me.

Kitra Change Hoga

Not free to deform



Temp rise $\rightarrow \Delta T$
New length

$$= l + \Delta l$$

$$= l + l \alpha \Delta T$$

$$\text{Thermal Strain} = \frac{\Delta l}{l}$$

[original length $\rightarrow l$]

$$\text{Thermal Stress} = \gamma \text{ strain} = \gamma \frac{\Delta l}{l}$$

$$\frac{\text{Stress}}{\text{Strain}} = \gamma$$

$$\text{Stress} = \gamma \frac{l \alpha \Delta T}{l}$$

$$\text{Thermal Stress} = \gamma \alpha \Delta T$$

How much Force:-

$$\text{Stress} = \frac{F}{A}$$

$$F = \text{Stress} \times A$$

$$F = \gamma \alpha A \Delta T$$

Q A steel rod of length 6.0m & diameter 20mm is fixed as shown. If Temp. rise by 80°C find the stress in rod.
 $\gamma = 2 \times 10^6 \text{ kg/cm}^2$
 $\alpha = 12 \times 10^{-6} \text{ Per}^\circ\text{C}$

$$\Delta T = 80^\circ\text{C}$$

$$\Delta l = l \alpha \Delta T$$

$$\text{Strain} = \frac{\Delta l}{l} = \frac{l \alpha \Delta T}{l} = \alpha \Delta T$$

$$\text{Stress} = \gamma \text{ Strain}$$

$$= \gamma \alpha \Delta T$$

$$= 2 \times 10^6 \text{ kg/cm}^2 \times 12 \times 10^{-6} \times 80$$

$$= 1920 \text{ kg/cm}^2$$

Q In the previous Q. After that :-
 if it is allowed to yield (expand by 1mm)

a) 1920

b) None

c) Less

Kitna expand hoga

Karna Chahat hai.

$$\Delta l = l \alpha \Delta T$$

$$\text{Strain} = \frac{\Delta l}{l} = \frac{1 \text{ mm}}{600 \text{ cm}}$$

$$\text{Stress} = \gamma \text{ Strain}$$

$$= 2 \times 10^6 \left(\frac{1 \text{ mm}}{600 \text{ cm}} \right)$$

$$= 2 \times 10^6 \left(\alpha \Delta T - \frac{0.1 \text{ cm}}{600 \text{ cm}} \right)$$

$$= 2 \times 10^6 \left(12 \times 10^{-6} \times 80 - \frac{0.1}{600} \right)$$

$$S_{\text{true}} = \gamma \Delta T$$

$$= 1920 - 2 \times 10^6 \times \frac{1}{6000}$$

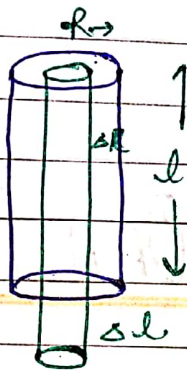
$$= 1920 - \frac{2 \times 10^3}{6} = 1920 - \frac{1000}{3}$$

$$= 1920 - 333.3$$

$$= 1586.67 \text{ Kg/cm}^2$$

POISSON'S RATIO.

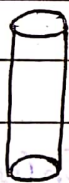
$$\sigma = \frac{\text{Lateral Strain}}{\text{Longitudinal Strain}}$$



$$\sigma = \frac{\Delta R}{R} / \frac{\Delta l}{l}$$

(+ve)

Q



→ It's length increases by 1%.

if $\sigma = \frac{1}{4}$, find the % change in its vol.

a) 1% increase

b) 1% decrease

c) 0.5% increase

d) 0.5% Decrease

$$V = \pi R^2 L$$

$$\frac{\Delta V}{V} \times 100 = 2 \left(\frac{\Delta R}{R} \times 100 \right) + \left(\frac{\Delta L}{L} \times 100 \right)$$

$$= -2 \times \frac{1}{4} \left(\frac{\Delta \phi}{\phi} \times 100 \right) + \left(\frac{\Delta L}{L} \times 100 \right)$$

$$\sigma = -\frac{\Delta R}{R} \frac{\Delta L}{L}$$

$$\frac{\Delta R}{R} = -\left(\frac{\Delta \phi}{\phi} \frac{1}{4} \right)$$

$$= -2 \times \frac{1}{4} (1\%) + 1\%$$

$$= 1 - \frac{1}{2} = 0.5\%$$

CHANGE IN DENSITY OF A LIQUID

↓
Due to Pressure

$$\boxed{\rho \uparrow \Rightarrow P \uparrow}$$

Practical (non ideal liquid)

ideal fluid

Incompressible

[Density is same]

$$\rho = \frac{m}{V}$$

$$\rho \propto \frac{1}{V}$$

$$\rho \rightarrow \rho'$$

$$V \rightarrow V + dV$$

$$\frac{\rho'}{\rho} = \frac{V}{V + dV}$$

$$= \frac{1}{1 + \frac{du}{u}}$$

$$B = - \frac{dP}{\left(\frac{du}{u}\right)}$$

$$P' = P \left(\frac{1}{1 - \frac{dP}{B}} \right)$$

$$P \uparrow \Rightarrow dP = +ve.$$

$$\left(1 - \frac{dP}{B}\right) \downarrow \Rightarrow P \uparrow \quad \#$$

FINISH

Indifferent curve

Indifference curve

$$Q = \frac{1}{|A|}$$