Numberr Systems

- Numberr System —

"A numberr system with radix 'r' or (base 'b') will have 'r' different numbers starting from '0' to 'r-1'."

Numberr System	radix	Digits
Binary	$r=2$	(0,1)
Ternary	$r=3$	(0,1,2)
Quaternary	$r=4$	(0,1,2,3)
octal	$r=8$	(0,1,2,3,4,5,6,7)
Decimal	$r=10$	(0,1,2,3,4,5,6,7,8,9)
Hexa Decimal	$r=16$	0,1,2,3,4,5,6,7,8,9 A B C D E F (10) (11) (12) (13) (14) (15)

- Representation of a Numberr =

**

$$(N)_r = a_n.r^n + a_{n-1}.r^{n-1} + \cdots + a_2 r^2 + a_1.r^1 + a_0.r^0 + a_{-1} r^{-1} + a_{-2} r^{-2} + \cdots$$

→ (any numberr to Decimal Numberr)

ex:

① $(123)_{10} = 1\times10^2 + 2\times10^1 + 3\times10^0$

$\Rightarrow 100 + 20 + 3$

$\Rightarrow 123$

③ $(123)_4 =$ (Quater to Deci)

$= 1\times4^2 + 2\times4^1 + 3\times4^0 \Rightarrow 16+8+3 \Rightarrow (27)_{10}$

③ $(1011.10)_2$ (Binary to Deci)

$= 1\times2^3 + 0\times2^2 + 1\times2^1 + 1\times2^0 + 1\times2^{-1} + 0\times2^{-2}$

$= 8+2+1+\frac{1}{2}+\frac{1}{4} \Rightarrow 11 + \frac{91}{4} \Rightarrow \frac{45}{4} \Rightarrow (11.5)_{10}$

④ $(1B2)_{16\,or\,h}$ (Hexa to Deci)

$B \downarrow 11$

$= 1\times16^2+11\times16^1+2\times16^0$

$= 256 + 176 + 2$

$= (434)_{10}$

⑤ $(123)_8$ (Octal to Deci)

$= 1\times8^2+2\times8^1+3\times8^0 \Rightarrow 64+16+3 \Rightarrow (83)_{10}$

• Number - Base Conversion =

(1) Decimal number system to all other Number system =

(using Division method)

ex:

① $(42)_{10} = (\ )_2 = (\ )_3 = (\ )_4 = (\ )_8 = (\ )_{16}$

→
2 | 42
2 | 21 − 0
2 | 10 − 1
2 | 5 − 0
2 | 2 − 01
1 − 0 ↑

$(101010)_2$

3 | 42
3 | 14 − 0
3 | 4 − 2
1 − 1 ↑

$(1120)_3$

4 | 42
4 | 10 − 2
2 − 2 ↑

$(222)_4$

8 | 42
5 − 3 ↑

$(52)_8$

16 | 42
2 − 10 ↑

$(2A)_{16}$.

Ex:

① $(\underbrace{157}.63)_{10} = (\ )_8$

$8 \,|\, 157$
$8 \,|\, 17 - 1$
$\quad 2 - 1$

$0.63 \times 8 = 5.04 \rightarrow 5$
$0.04 \times 8 = 0.32 \rightarrow 0$
$0.32 \times 8 = 2.56 \rightarrow 2$

$= (211.502)_8$.

- **Binary to Octal Numbersystem & HexaDecimal number =**

b = 2 (0,1)
o = 8 (0 − 7) => $2^{③}$ → 3 binary number are ref to rep one octal num.
h = 16 (0 − 15) => $2^{④}$ → 4 " " "
" " .

Ex:

① $(101)_2 = (\ )_{10} \rightarrow (5)_8$

=> $1 \times 2^2 + 1 \times 2^0$
=> $(5)_{10} = (5)_8$

② $(11001.011)_2 = (\ )_8 = (\ )_{16}$.

=> $(\underbrace{011}_{3}\underbrace{001}_{1}.\underbrace{011}_{3}) \Rightarrow (31.3)_8$

=> $(\underbrace{0001}_{1}\underbrace{1001}_{9}.\underbrace{0110}_{6}) \Rightarrow (19.6)_{16}$.

- **Questions on Number System Conversions =**

Q1 $(13)_8 = (10xy)_2$ find x & y.

=> $(1 \times 8^1 + 3 \times 8^0)_{10} \Rightarrow (8+3)_{10} \Rightarrow (11)_{10}$

2 | 11
2 | 5 − 1
2 | 2 − 1
1 − 0 ↑

$(1011)_2$

$\boxed{n=1, \ \& \ y=1}$.

OR

$$1\times8^1+3\times8^0 = 1\times2^3+0\times2^2+x\times2^1+y\times2^0$$
$$11 = 8+2x+y$$

$(11)_{10}=(11)_{10}$

when $\boxed{(n=1, y=1)}$

Q_2

$(1110)_x=(9B)_{16}$, $x=?$

$\Rightarrow 1\times x^3+1\times x^2+1\times x^1 = 9\times16^1+11\times16^0$
$\Rightarrow x^3+x^2+x = 144+11$
$\Rightarrow (x^3+x^2+x)_{10} = (155)_{10}$

$\boxed{x=5}$

$n=5$ 125 + 25 + 5 = 155

$\boxed{x(125)_5 \quad n=5 \quad \boxed{n=6}}$

Q_3 $(FADE)_{16}=(x)_{11}$

$(15\times16^3+10\times16^2+13\times16^1+14\times16^0)_{10}$
$\Rightarrow 64000$ ~~[illegible]~~ $+208+14$
$\Rightarrow (64222)_{10}$

then,
11 | 64222 ↑
11 |
:

$\rightarrow (x)_{11}$

16 8 4 2 1

A B C D E F
10 11 12 13

(Q1) $(DAD.A)_h = (\ )_{octal}$

$\underset{13}{D}\ \underset{10}{A}\ \underset{13}{D}\ .\ \underset{10}{A}$

$\Rightarrow (\underbrace{110}_{6}\underbrace{1\ 10}_{6}\underbrace{10\ 1}_{5}\underbrace{101}_{5} . \underbrace{101}_{5}\underbrace{000}_{0})_2$

$\Rightarrow (6655.50)_8$ or (0).

- **Complement :**
  - (1) Diminished radix (or) $(r-1)$'s complement.
  - (2) Radix Complement (or) r's Complement.

Applications :-

→ complements are used in digital computers to simplify the subtraction operation & for logical manipulations.

→ leads to simpler circuits, i.e expensive circuits.

- **Binary Number system :-**

$\boxed{r = 2}$.

$(r-1)$'s

→ 1's complement. $= (110)_2 = (001)$
1's comp
+1

r's

→ 2's complement. = 1's comp +1
2's comp = (010)

ex: ① $(10110.11)_2$

$$\begin{array}{r} 01001.00 \\ +\ \ 1 \\ \hline 01001.01 \end{array}$$

→ 1's comp = 01001.00

2's comp = (1's comp +1) = 01001.01 .

- **Octal Number System** =

(r = 8)

7's complement → $\overset{7\,7\,7}{-(125)_8}$ = (652)
7's comp.

8's compliment → $\overset{7\,7\,8}{-(125)_8}$ = (653)
8's com

- **Decimal number system** =

(r = 10)

9's complement → $\overset{9\,9\,9}{-(128)_{10}}$ = 871 (9's com)

10's complement → $\overset{9\,9\,10}{-(128)_{10}}$ = 872 (10's com)
or
(9's comp + 1)

- **Hexa Decimal Number System** =

15's complement → $\overset{15\,15\,15}{-(AB1)_{16}}$ = (54E) → 15's comp
+1
↓
16's complement → $\overset{15\,15\,16}{-(AB1)}$ = (54F)

- **Questions on complement's** =

Q1 Find 9's complement of following numbers –

(a) $(99088)_{10}$ (b) $\overset{9999\ \ 999}{-(1349.678)_{10}}$

→ $\overset{9\,9\,9\,9\,9}{-(99088)}$ 9's comp → $(8650.321)_{10}$

9's com → (00911)

Q2 find the 10's complement of the following –

(a) $\overset{\substack{10\ ~~0000~~ \\ ~~9\,9\,9\,10~~}}{-(1000)_{10}}$ (b) $\overset{9\,9\,9\,9\ \ 9\,9\,9\,10}{-(7658.9933)_{10}}$

10's comp = ~~89910~~ 9000 (first non zero degit sub from 10)

= (2341.00667)

Q3) Find the 9's complement of $(517)_{12}$.

→ $5 \times 12^2 + 1 \times 12^1 + 7 \times 12^0$

$= 720 + 19$

$= (739)_{10}$

$$\begin{array}{r} 999 \\ - (739) \\ \hline \end{array}$$

9's comp $= (260)$

Q4) Find 1's and 2's complement =

(a) $(10101 \cdot 110)_2$

→ 1's comp = $01010 \cdot 001$
+1
2's comp = $01010 \cdot 010$

(b) $(0110011 \cdot 001)_2$

→ 1's comp = $(1001100 \cdot 110)$
+1
2's comp = $1001100 \cdot 111$

Q5) Find 3's & 11's complement =

(a) $(112)_4$

$\overset{-333}{\to (112)_4}$

→ (221)
(3's comp)

(b) $(12A \cdot 18)_{12}$

→ $\overset{-\ 11\,11\,11 \cdot 11\,11}{(12A \cdot 18)_{12}}$

→ $1091 \cdot 103$ → $(A91 \cdot A3)$
(11's comp)

Note:

① → 1's complement (1's complement of a number) = Number.

ex:

→ $(101110 \cdot 110)_2$

1's → $(010001 \cdot 001) \xrightarrow{1's} (101110 \cdot 110)$ (number).

→ 2's compliment (2's complement of a number) = Number.

(Q3) Find the 9's complement of $(517)_{12}$:

→ $5 \times 12^2 + 1 \times 12^1 + 7 \times 12^0$

$= 720 + 19$

$= (739)_{10}$

→ $\begin{array}{r} -999 \\ (739) \end{array}$

9's comp = (260)

(Q4) Find 1's and 2's complement =

(a) $(10101.110)_2$

→ 1's comp = 01010.001
+1
2's comp = 01010.010

(b) $(0110011.001)_2$

→ 1's comp = (1001100.110)
+1
2's comp = 1001100.111

(Q5) Find ~~4~~ 3's & 11's complement =

(a) $(112)_4$

→ $\begin{array}{r} -333 \\ (112)_4 \end{array}$

→ (221)
(3's comp)

(b) $(12A.18)_{12}$

→ $\begin{array}{r} -\text{BBB}.\text{BB} \\ (12A.18)_{12} \end{array}$

→ 1091.103 → (A91.A3)
(11's comp)

Note:

→ 1's complement (1's complement of a number) = Number.

ex:

$(101110.110)_2$

1's → $(010001.001) \xrightarrow{1's} (101110.110)$
(number).

→ 2's compliment (2's compliment of a number) = Number.

- **Arithmetic operations =**

(I) Addition.
(II) Subtraction.
(III) Multiplication.

(I) Addition =

(0-1)
② **Binary addition** -

① $\begin{array}{r} 101 \\ +001 \\ \hline 110 \end{array}$ ② $\begin{array}{r} 10010 \\ +00111 \\ \hline 11001 \end{array}$

(0-7)
③ **Octal Addition** —

① $(13)_8 + (137)_8$

$\rightarrow \begin{array}{r} \overset{1}{\cancel{137}} \\ \cancel{+13} \\ \hline \cancel{150} \end{array}$ $\begin{array}{r} 0\overset{1}{1}3 \\ +137 \\ \hline (152)_8 \end{array}$ $\begin{array}{l} 8\underline{|10} \\ \;\;1-2 \\ \;(c) \end{array}$

(0-9)
① **Decimal addition** -

① $\begin{array}{r} 1\overset{1}{2}7 \\ +027 \\ \hline 154 \end{array}$ $\begin{array}{l} 10\underline{|14} \\ \;\;1-4 \\ \;(c) \end{array}$

④ **Hexa Decimal addition**

① $(A23.B)_h + (123.17)_h$

$\rightarrow \begin{array}{r} A23.B0 \\ +\overset{}{1}23.17 \\ \hline \underset{(11)}{B}46.\underset{(12)}{C}7 \end{array}$

(II) Subtraction =

① **Decimal subtraction** -

① $(192)_{10} - (092)_{10} = (100)_{10}$

$(092)_{10} - (192)_{10} = -100$

(-ve numbers will be represent by complement in Digital System)

③ Binary subtraction -

① $\begin{array}{r} (1011.1)_2 \\ -(0011.0)_2 \\ \hline (1000.1)_2 \end{array}$

$\begin{array}{r} (0_1 0_1 \cancel{1}0\cancel{1}00\;10)_2 \\ (1011.1)_2 \\ \hline 0111.01 \end{array}$ → with Borrow

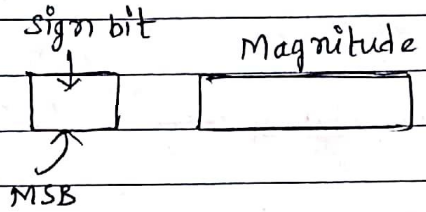
$\boxed{-(0111.01)}$ → not possible to represent like that.

• Signed binary Numbers :-

→ Un signed Numbers
0 to N
+ve ⇒ 0,1,2,3

→ signed numbers
+ve → +1, +2, ...
-ve → -1, -2, -0, +0, +1

Digital computers must represent everything with binary digits.

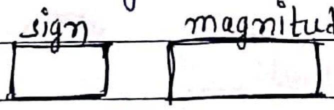


+ve → '0'
-ve → '1'

• Types of signed Representation :-

→ signed magnitude convention.
→ 1's Complement representation.
→ 2's complement representation.

(1) Signed magnitude representation :-



+15 → 0 1111 → 5 bits
(-15) → (1) 1111 → 5 bits.

(I) 1's Complement Representation =

Step 1: find +ve number in
Signed magnitude
↓
Step 2: Con 1's complement of

ex: 15 → 1111

$+15 \rightarrow 0\ 1111 \xrightarrow{1\text{'s comp}} 1\ 0000 \rightarrow -15$

(II) 2's complement representation =

Step 1: find +ve number in signed magnitude.

Step 2: find 2's complement of step (1).

ex:

+7 → 0 111

−7 → 1001

+13 → 0 1101

−13 → 1 0011

Range of binary numbers =

For n-bit -

→ For unsigned number ~~(0 — (2^n-1))~~ $\boxed{0 \text{ to } (2^n-1)}$

→ For signed number — $\boxed{-(2^{n-1}-1) \text{ to } +(2^{n-1}-1)}$

→ For 1's compliment form –

$$\boxed{-(2^{n-1}-1) \text{ to } +(2^{n-1}-1)}$$

→ For 2's compliment form –

$$\boxed{-(2^{n-1}) \text{ to } +(2^{n-1}-1)}$$

Ex: 8 4 2 1

① 1's complement form

1010 $n=4$

↓

$\rightarrow -(2^{n-1}-1)\times 1 + 0\times 2^{n-2} + 1\times 2^{n-3} + 0\times 2^{n-4}$

$\rightarrow -(2^3-1)+2^1$

$\rightarrow -7+2$

$\rightarrow -5$

$+5 \rightarrow 0101$

$\downarrow$ 1's

$-5 \rightarrow 1010$.

② 2's complement form

1010

$\rightarrow -(2^{n-1})\times 1 + 0\times 2^{n-2} + 1\times 2^{n-3} + 0\times 2^{n-4}$

$\rightarrow -8+2$

$\rightarrow -6$

$+6 \rightarrow 0110$ 2's

$-6 \rightarrow 1010$

$+17 \rightarrow 0\ 10001$

$-17 \xrightarrow{\text{sign}} 1\ 10001$

$\xrightarrow{\text{1's com}} 1\ 01110$

$+1$

$\xrightarrow{\text{2's com}} 1\ 01111$

16 8 4 2 1

- Questions On signed binary Numbers =

Q1) The 2's complement representation of -17 is -

$\rightarrow$ 17 $\rightarrow$ 10001

+17 $\rightarrow$ 010001

$-17 \xrightarrow{2's} \boxed{101111}$

Q2) 4-bit 2's complement rep of a decimal number is $\overset{3}{1}\overset{2}{0}\overset{1}{0}\overset{0}{0}$. The number is =

$\rightarrow$ $-(2^{n-1})\times 1 + 0\times 2^{n-2} + 0\times 2^{n-3} + 0\times 2^{n-4}$

$\rightarrow -2^3+0+0+0$

$\rightarrow -8$

Q3) The range of signed decimal Numbers that can be represented by 6-bit 1's complement numbers is =

$\rightarrow$ $-(2^{n-1}-1)$ to $+(2^{n-1}-1)$

$\rightarrow$ $-(2^5-1)$ to $+(2^5-1)$

$\rightarrow$ $-(32-1)$ to ~~a~~ $+(32-1)$

$\rightarrow$ $\boxed{-31 \text{ to } -31}$

• Subtraction using Complements =

(1) using r's complement =

$A - B$

$A + (-B)$

$\swarrow$ Minuend $\quad$ $\searrow$ subtrahend

Step: 1 → Add minuend (A) to r's complement of subtrahend (B).

(i) A>B, the result in (A+(-B)) will a geta carry just discard it.

(ii) A<B, the result have no carry Find result = −(2's complement of the result).

ex:

(I) $A = 1001$

$B = \overset{-2}{1}000$

$\Rightarrow A = 1001$

$-B \Rightarrow 2's = \overset{①}{1}000$

$\overline{\quad 0001}$

final result = 0001

(II) $A = 1000$

$B = \overset{-1}{1}\overset{1}{0}\overset{1}{0}\overset{2}{1}$

2's $\Rightarrow$ $A = 1000$

$-B = +0111$

$\overline{\quad 1111}$

$\overset{1112}{1111}$

$-(2\text{'s of result}) = -(0001)$.

final result = −(0001).

(2) using (r−1)'s complement =

Step: 1 → Add Minuend 'A' to (r−1)'s complement of subtrahend 'B'.

(i) I we get a carry add '1' to LSB.

(ii) No carry

Resut = −(1's complement of result).

(S.2)

The following deimal numbers are shown in sign-magnitude form: +9286 & +801.
convert them to signed-10's complement form & perform the following operations.

(Note that sum is +10,627, req Five digits and a sign)

(a) (+9286) + (+801)

+9286 → ~~ab~~ 0 09286

+801 → 0 00801

$\boxed{0\ 10087}$ (no carry)

Positive number in sign-magnitude, r's, r-1's comp same.

9 9 9 9 10
0 0801

(b) (+9286) + (−801)

→

+9286 → 0 09286

(−801) $\xrightarrow{10's}$ 9 99199

① $\boxed{0\ 08485}$

discarded

(c) (−9286) + (+801)

− 9 9 9 9 9 10
0 09286

→

−9286 $\xrightarrow{10's}$ 990714

+801 → 000801

991515

− (10's comp of result)

= − (008485)

10's & 9's
0 → +ve
9 → -ve

(d) $-(9286) + (-801)$

$\rightarrow$

$9286 \rightarrow \overset{-9}{0}\ \overset{9\,9\,9\,9\,10}{09286} \xrightarrow{10's} 990714$

$801 \rightarrow \overset{-9}{0}\ \overset{9\,9\,9\,9\,10}{00801} \xrightarrow{10's} 999199$

$\text{①}989913$

dis card

$= 989913$

$\downarrow$ 10's

$= -(010087)$

- Types of Number System =

(i) weighted number system. → (It is positionaly weighted number system) ex: Decimal, Binary, BCD codes.

(ii) Non-weighted number system.

→ (It is positionaly unweighted number system)

EX: Gray code, eccess-3 code.

- Types of codes =

(I) Non Binary codes = (The code that doesn't contain any binary numbers)

ex: The marse code.

(II) Binary codes = (The code that consists of only binary numbers)

ex: BCD codes, gray codes, execess-3 code.

(III) Alpha Numeric code =

(The code that contains numbers, alphabets and special characters.)

EX: ASCII codes, EBCDIC code.

Binary Code =

Why coding or codes ?

↓

To make Simple Operation.

Decimal	BCD or 8421 8 4 2 1
0	0 0 0 0
1	0 0 0 1
2	0 0 1 0
3	0 0 1 1
4	0 1 0 0
5	0 1 0 1
6	0 1 1 0
7	0 1 1 1
8	1 0 0 0
9	1 0 0 1
10	1 0 1 0
11	1 0 1 1
12	1 1 0 0
13	1 1 0 1
14	1 1 1 0
15	1 1 1 1

123 → 0001 0010 0011
(D) (BCD)

123 → 0001 0010 0011
123 → 0001 0010 0011
( ) ( ) ( )
↓ ↓ ↓
2 4 6

(sum=9)
Self reflative code

Decimal Digit	BCD 8421 (9)	2421 (9)	8 4 -2 -1 (9)	5211 (9)
0	0000	0000	0000	0000
1	0001	0001	0111	0001
2	0010	0010	0110	0011
3	0011	0011	0101	0101
4	0100	0100	0100	0111
5	0101	0101	1011	1000
6	0110	0110	1010	1001
7	0111	0111	1001	1011
8	1000	1110	1000	1101
9	1001	1111	1111	1111
unused combination or codes	1010	1010	0001	0010
	1011	1011	0010	0100
	1100	1100	0011	0110
	1101	1101	1100	1010
	1110	1000	1101	1100
	1111	1001	1110	1110

6 codes

0000
0001
0010
0011
0100
0101
0110
0111
1000
1001
1010
1011
1100
1101
1110
1111

- **Excess -3 code =**

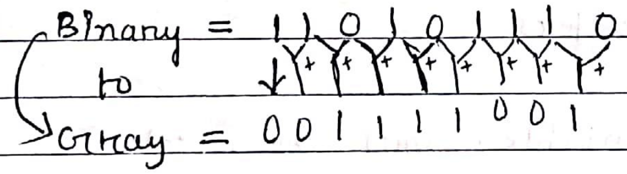
Decimal	Binary	Excess
0	0000 + 0011(3) =	0011
1	0001 + 0011 (3) =	0100
2	0010 "	0101
3	0011 "	0110
4	0100 "	0111
5	0101 "	1000
6	0110 "	1001
7	0111 "	1010
8	1000 "	1011
9	1001	1100

1101
1110
1111
0000
0001
0010
→ 6 unused code.

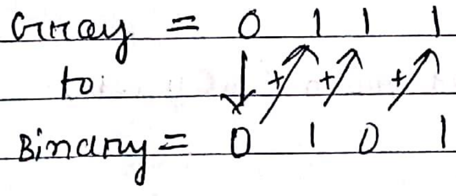
Gray Code =

The Advantage of Gray code over the straight binary number sequence is that only one bit in the code group changes in going from one number to the next.

Decimal	Binary	Gray code
0	0000	0000
1	0001	0001
2	0010	0011
3	0011	0010
4	0100	0110
5	0101	0111
6	0110	0101
7	0111	0100
8	1000	1100
9	1001	1101



Binary = 1 1 0 1 0 1 1 1 0
to
Gray = 0 0 1 1 1 1 0 0 1



Gray = 0 1 1 1
to
Binary = 0 1 0 1

- Alpha Numeric Codes =

EX: ASCII → American standard code for Information Interchange.

EBCDIC → Extended Binary Coded Decimal Interchange code.

→ ASCII Code is 7-bit code.

$\boxed{2^7 \rightarrow 128}$ (can represent)

				$b_7 b_6 b_5$							
b_4	b_3	b_2	b_1	000	001	010	011	100	101	110	111
0	0	0	0	NUL	DEL	SP	0	@	P	)	P
0	0	0	1	~~DEL~~ SOH	DC1	!	1	A	Q	a	9
	⋮	⋮									

Total 128 characters.

34 → non printable. (shift, enter, s...)
94 → remaining printable. (a, b, @ ...)

→ EBCDIC (extended virsion of ASCII code)
↳ 8 bit code.

$2^8 \rightarrow 256 \rightarrow$ Character. can represent

8 4 2 1

- **BCD Addition** = ~~BCD add~~ (Ecess-3 addition same as BCD addition)

case ①

4 → 0100
+5 → 0101
1001 → '9' ✓

case ②

4 → 0100
+8 → 1000
1100 → '12' (when more than 9 then add '6' with result)
+ 0110 (6)
0001 0010
1 2 → '12'

case ③

8 → 1000
+9 → 1001
17
1 0001 → (11) X
+ 0110 (6)
1 0111
~~4~~
1 7 ✓

Q1

173 → 0001 0111 0011
+289 → 0010 1000 1001
462
0011 1111 1100
+0110 (6) 0110 (6)
0100 0110 0010
4 6 2

Q2

184 → 0001 1000 0100
+576 → 0101 0111 0110
760
0110 1111 1010
+0110 (6) 0110 (6)
0111 0110 0000
7 6 0

Q.3

```
          1001        1000        1001   ← 989
        + 0001        0011        1000   ← 138
          1010        1100        0001
        + 0110(6)     0110(6)     0110(6)
0001      0001        0010        0111   → 1127
 1          1           2           7
```

Q.4 Represent the decimal number 5137 in –

(a) BCD
(b) excess-3 code
(c) 2421
(d) 6311

→

8421

(a) 5137 (Decimal) → 0101 0001 0011 0111 (BCD)

(b) 5137 (Decimal) →
```
  0101 0001 0011 0111 –(BCD)
+ 0011 0011 0011 0011 –(3)
  10000100 0110 1010 –(Excess-3)
```

(c) 5137 (Decimal) → 1011 0001 0011 0111 (2421).

(d) 5137 (Decimal) → 0111 0001 0100 1001 (6311).

Q.5

Find the 9's complement of decimal 5,137 and express it in 2421 code.

```
        9999
→ 9's = -5137
        4862 ──2421──→ 0100 1110 1100 1000 .
```