Class XII Session 2024-25
Subject - Applied Mathematics
Sample Question Paper - 1

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This question paper contains five sections A, B, C, D and E. Each section is compulsory.
2. Section - A carries 20 marks weightage, Section - B carries 10 marks weightage, Section - C carries 18 marks weightage, Section - D carries 20 marks weightage and Section - E carries 3 case-based with total weightage of 12 marks.
3. **Section – A:** It comprises of 20 MCQs of 1 mark each.
4. **Section – B:** It comprises of 5 VSA type questions of 2 marks each.
5. **Section – C:** It comprises of 6 SA type of questions of 3 marks each.
6. **Section – D:** It comprises of 4 LA type of questions of 5 marks each.
7. **Section – E:** It has 3 case studies. Each case study comprises of 3 case-based questions, where 2 VSA type questions are of 1 mark each and 1 SA type question is of 2 marks. Internal choice is provided in 2 marks question in each case-study.
8. Internal choice is provided in 2 questions in Section - B, 2 questions in Section – C, 2 questions in Section - D. You have to attempt only one of the alternatives in all such questions.

Section A

1. If A = $\begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $A^{-1} = kA$, then k equals **[1]**

   a) 19 b) $\frac{1}{19}$

   c) $-\frac{1}{19}$ d) -19

2. A statement made about a population parameter for testing purpose is called **[1]**

   a) statistic b) level of significance

   c) hypothesis d) parameter

3. The value of a machine purchased two years ago depreciates at the annual rate of 10%. If its present value is ₹97,200, then its value after 3 years is **[1]**

   a) ₹ 80,859 approx b) ₹ 70,859 approx

   c) ₹ 88,509 approx d) ₹ 71,859 approx

4. The solution set of the inequation 2 x + y > 5 is **[1]**

   a) whole xy-plane the points lying on the line 2x + y = 5 b) open half-plane not containing the origin

c) whole xy-plane except the points lying on the line 2x + y = 5

d) half plane that contains the origin

5. If S = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then adj A is **[1]**

a) $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

b) $\begin{bmatrix} d & c \\ b & a \end{bmatrix}$

c) $\begin{bmatrix} -d & -b \\ -c & a \end{bmatrix}$

d) $\begin{bmatrix} d & b \\ c & a \end{bmatrix}$

6. In a series of three trials, the probability of two successes is 9 times the probability of three successes. Then, the probability of success in each trial is **[1]**

a) $\frac{1}{3}$

b) $\frac{3}{4}$

c) $\frac{1}{2}$

d) $\frac{1}{4}$

7. If X has a Poisson distribution such that P(X = 1) = P(X = 2) and e^{-2} = 0.1353, then P(X = 4) is **[1]**

a) 0.0213

b) 0.0902

c) 0.9098

d) 0.9787

8. The order of the differential equation of all circles of given radius a is **[1]**

a) 1

b) 4

c) 3

d) 2

9. Two pipes A and B can fill a cistern in 10 minutes and 15 minutes respectively. Both the pipes are opened together, but after 3 minutes pipe B in turned off. How much time will the cistern take to be full? **[1]**

a) 8 minutes

b) 6 minutes

c) 11 minutes

d) 12 minutes

10. The trace of the matrix A = $\begin{bmatrix} 1 & -5 & 7 \\ 0 & 7 & 9 \\ 11 & 8 & 9 \end{bmatrix}$ is **[1]**

a) 17

b) 25

c) 3

d) 12

11. If x is the least non-negative integer satisfying 218 $\equiv$ x (mod 7), then x^2 +1 is equal to **[1]**

a) 50

b) 1

c) 2

d) 5

12. If $\frac{|x-2|}{x-2} \geq 0$, then **[1]**

a) x $\in$ $(-\infty, 2)$

b) x $\in$ $[2, \infty)$

c) x $\in$ $(2, \infty)$

d) x $\in$ $(-\infty, 2]$

13. A man can row a boat in still water at 15 km/hr and speed of water current is 5 km/hr. The distance covered by the boat downstream in 24 minutes is **[1]**

a) 4 km

b) 6 km

c) 8 km

d) 16 km

14. Comer points of the feasible region for an LPP are : (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5). Let z = 4x + 6y be the objective function. Then, Max. z - Min. z = **[1]**

a) 48
b) 42
c) 60
d) 18

15. How many of the following points satisfy the inequality 2x - 3y > - 5? **[1]**
(1, 1), (-1, 1), (1, -1), (-1, - 1), (- 2, 1), (2, -1), (-1, 2) and (-2, -1).

a) 5
b) 3
c) 4
d) 6

16. A simple random sample consists of four observations 1, 3, 5, 7. What is the point estimate of population standard deviation? **[1]**

a) 3.1
b) 2.3
c) 2.87
d) 2.58

17. $\int \frac{(\log x)^5}{x}$ is equal to **[1]**

a) $\frac{\log x^6}{6} + C$
b) $\frac{(\log x)^6}{3x^2} + C$
c) $\frac{(\log x)^6}{6} + C$
d) $\frac{\log x^6}{3x^2} + C$

18. The best-fitted trend line is one for which sum of squares of residuals or errors is: **[1]**

a) Maximum
b) Minimum
c) Positive
d) Negative

19. **Assertion (A):** If A = $\begin{bmatrix} 10 & -2 \\ -5 & 1 \end{bmatrix}$, then A^{-1} does not exist. **[1]**
Reason (R): On using elementary column operations $C_2 \to C_2 - 2C_1$ in the following matrix equation
$\begin{bmatrix} 1 & -3 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$, we have $\begin{bmatrix} 1 & -5 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & -5 \\ 2 & 0 \end{bmatrix}$.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

20. **Assertion (A):** If x is real, then the minimum value of $x^2 - 8x + 17$ is 1. **[1]**
Reason (R): If f''(x) > 0 at a critical point, then the value of the function at the critical point will be the minimum value of the function.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

Section B

21. Construct 3-yearly moving averages from the following data: **[2]**

Year:	2010	2011	2012	2013	2014	2015	2016
Imported cotton consumption in India (in '000 bales):	129	131	106	91	95	84	93

22. Mr. X took a loan of ₹2,000 for 6 months. Lender deducts ₹200 as interest while lending. Find the effective rate of interest charged by lender. **[2]**

OR

Rahul purchased an old scooter for ₹ 16000. If the cost of the scooter after 2 years depreciates to ₹14440, find the rate of depreciation.

23. Evaluate the definite integral: **[2]**

$$\int_{2}^{4} \frac{x}{x^2+1} dx$$

24. Using matrix method, solve the following system of equations: **[2]**

x - 2y + 3z = 6

x + 4y + z = 12

x - 3y + 2z = 1

OR

If the matrix $\begin{bmatrix} x+4 & x & x \\ x & x+4 & x \\ x & x & x+4 \end{bmatrix}$ is singular, find x.

25. Evaluate: -8 mod 5 **[2]**

Section C

26. Solve the initial value problem: $e^{\frac{dy}{dx}}$ = x + 1; y(0) = 3 **[3]**

OR

Show that the differential equation representing one parameter family of curves $(x^2 - y^2) = c(x^2 + y^2)^2$ is $(x^2 - 3xy^2)$ dx = $(y^2 - 3x^2y)$ dy

27. A firm anticipates an expenditure of ₹ 50,0000 for plant modernization at end of 10 years from now. How much should the company deposit at the end of year into a sinking fund earning interest 5% per annum. [Given log 1.05 = 0.0212, antilog (0.2120) = 1.629] **[3]**

28. The demand function for a commodity is p = 20 $e^{-x/10}$. Find the consumer's surplus at equilibrium price p = 2. (Given $\log_{10}$ e = 0.4343) **[3]**

29. A person buys a lottery ticket in 50 lotteries, in each of which his chance of winning a prize is What is the probability that he will win a prize $\frac{1}{100}$. **[3]**

i. at least once
ii. exactly once
iii. at least twice?

OR

Let X be a discrete random variable whose probability distribution is defined as follows:

$$P(X = x) = \begin{cases} k(x+1) & \text{for } x = 1,2,3,4 \\ 2kx & \text{for } x = 5,6,7 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant

Find:

i. k
ii. E(X)
iii. Standard deviation of X.

30. Construct 5-year Moving averages from the following data of the number of industrial failure in a country **[3]**

during 2003-2018:

Year	No. of Failures	Year	No. of Failure
2003	23	2011	9
2004	26	2012	13
2005	28	2013	11
2006	32	2014	14
2007	20	2015	12
2008	12	2016	9
2009	12	2017	3
2010	10	2018	1

31. Find the student's -t for the following variable values in a sample of eight: **[3]**

-4, -2, -2, 0, 2, 2, 3, 3 taking the mean of the universe to be zero.

Section D

32. Solve the following LPP graphically: **[5]**

Maximize Z = 5x + 3y

Subject to $3x + 5y \leq 15$

$5x + 2y \leq 10$

and, $x, y \geq 0$

OR

Solve the following LPP graphically:

Minimize Z = 3x + 5y

Subject to

$-2x + y \leq 4$

$x + y \geq 3$

$x - 2y \leq 2$

$x , y \geq 0$

33. Solve the following system of inequalities graphically: **[5]**

$3y - 2x < 4$, $x + 3y > 3$ and $x + y \leq 5$.

34. A class XII has 20 students whose marks (out of 30) are 14, 17, 25, 14, 21, 17, 17, 19, 18, 26, 18, 17, 17, 26, 19, 21, 21, 25, 14 and 19. If random variable X denotes the marks of a selected student given that the probability of each student to be selected is equally likely. **[5]**

a. Prepare the probability distribution of the random variable X.
b. Find mean, variance and standard deviation of X.

OR

Two cards are drawn successively without replacement from a well-shuffled deck of 52 cards. Compute the variance of the number of aces.

35. A start-up company invested ₹ 3,00,000 in shares for 5 years. The value of this investment was ₹ 3,50,000 at the end of second year, ₹ 3,80,000 at the end of third year and on maturity, the final value stood at ₹ 4,50,000. Calculate the Compound Annual Growth Rate (CAGR) on the investment. [Given that : $(1.5)^{\frac{1}{5}}$ = 1.084] **[5]**

Section E

36. **Read the text carefully and answer the questions:** **[4]**

A tank with a rectangular base and rectangular sides of length x metre, width y metre , open at the top is to be constructed so that the depth is 1 m and volume is $9m^3$. If the building of the tank is ₹ 70 per square metre for the base and ₹ 45 per square metre for the sides?

(a) What is the cost of the base?

(b) What is the cost of making all the sides?

(c) If ‘C’ be the total cost of the tank, then find the value of C.

OR

For what value of x, C is minimum?

37. **Read the text carefully and answer the questions:** **[4]**

The nominal rate of return shows the yield of an investment over time without accounting for negative elements such as inflation or taxes. By calculating the nominal rate of return, you can compare the performance of your assets easily, regardless of the inflation rate or differing spans of time for each investment. By obtaining a bird’s-eye view of how your assets are growing, you can make more prudent investment decisions in the future.

(a) A man invests a sum of money in ₹100 shares paying 15% dividend quoted at 20% premium. If his annual dividend is ₹540, calculate the rate of return on his investment.

(b) Mr. Satya holds 1500, ₹100 shares of a company paying 15% dividend annually quoted at 30% premium. Calculate rate of return on his investment.

(c) ₹100 shares of a company are sold at a discount of ₹ 20. If the return on the investment is 15%, find the rate of dividend declared.

OR

A company declared a dividend of 14%. Find the market value of ₹50 shares, if the return on the investment was 10%.

38. A shopkeeper has 3 varieties of pens A, B and C. Meenu purchased 1 pen of each variety for a total of ₹ 21. Jean purchased 4 pens of A variety, 3 pens of B variety and 2 pens of C variety for ₹ 60. While Shikha purchased 6 pens of A variety, 2 pens of B variety and 3 pens of C variety for ₹ 70. Using matrix method find the cost of each pen. **[4]**

OR

In an engineering workshop there are 10 machines for drilling, 8 machines for turning and 7 machines for grinding. Three types of brackets are made. Type I brackets require 0 minutes for drilling, 5 minutes for turning and 4 minutes for grinding. The corresponding times for type II and III brackets are 3, 3, 2 and 3, 2, 2, minutes respectively. How many brackets of each type should be produced per hour so that all the machines remain fully occupied during an hour? Solve by using matrix method.

Solution

Section A

1.

(b) $\frac{1}{19}$

Explanation: A = $\begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$

Using adjoint matrix

$A^{-1} = \frac{-1}{19}\begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix}$

$A^{-1} = \frac{1}{19}\begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix} = \frac{1}{19}$ A

k = $\frac{1}{19}$

2.

(c) hypothesis

Explanation: hypothesis

3.

(b) ₹ 70,859 approx

Explanation: Given, P = ₹ 97,200, i = 10% p.a.

$\Rightarrow$ i = $\frac{10}{100}$ = 0.1

So, value after 3 years

= 97,200 $\times$ $(1 - 0.1)^3$

= 97,200 $\times$ 0.729

= ₹ 70,858.80

= ₹ 70,859 (approx.)

4.

(b) open half-plane not containing the origin

Explanation: Given inequation is 2x + y > 5

Now, we convert the inequation into an intercept line equation form, we can clearly see the intercepts of the inequation on x-axis and y-axis.

2x + y > 5

[dividing the whole inequation by 5]

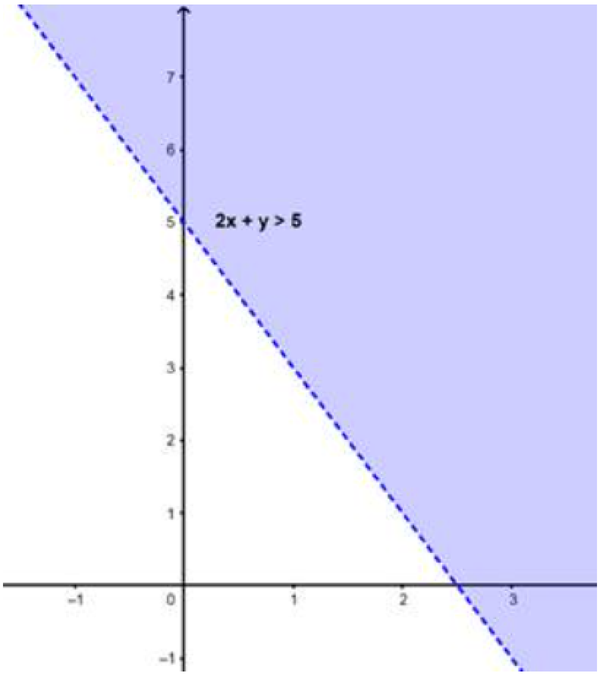
$\frac{2x}{5} + \frac{y}{5} > \frac{5}{5}$

$\frac{x}{\frac{5}{2}} + \frac{y}{5} > 1$

$\frac{x}{2.5} + \frac{y}{5} > 1$

Therefore, from the above inequation, we can say that 2.5 and 5 are the intercepts of the x-axis and y-axis respectively.

Now by plotting these on the graph, we can clearly see the graph of the inequation.



From the graph, it is clear that the solution set of the inequation.

2x + y > 5 is the open half-plane not containing the origin.

5. **(a)** $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Explanation: S = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$M_{11} = d \Rightarrow A_{11} = d$

$M_{12} = c \Rightarrow A_{12} = -c$

$M_{21} = b \Rightarrow A_{21} = -b$

$M_{22} = a \Rightarrow A_{22} = a$

$\Rightarrow$ Adj(A) = $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

6.

(d) $\frac{1}{4}$

Explanation: Given n = 3 and P(X = 2) = 9P(X = 3).

So, ${}^3C_2 p^2 \cdot q = 9 \times {}^3C_3 . p^3$

$\Rightarrow 3p^2q = 9p^3 \Rightarrow 3p^2(q - 3p) = 0$

$\Rightarrow q = 3p$

$\because p + q = 1 \Rightarrow p + 3p = 1 \Rightarrow p\ \frac{1}{4}$

7.

(b) 0.0902

Explanation: Given P(X = 1) = P(X = 2)

$\Rightarrow \lambda e^{-\lambda} = \frac{\lambda^2 e^{-\lambda}}{2!}$

$\Rightarrow \lambda^2 - 2\lambda = 0 \Rightarrow \lambda = 0, 2$

$\Rightarrow \lambda = 2$

Now, P(X = 4) $= \frac{2^4 \cdot e^{-2}}{4!} = \frac{16 \times 0.1353}{24} =$ 0.0902

8.

(d) 2

Explanation: Equation of all the circles of radius a is

$(x - h)^2 + (y - k)^2 = a^2$

where h, k are arbitrary constants.

So, the order of differential equation is 2.

9. **(a)** 8 minutes

Explanation: In one min, (A + B) fill the cistern

$= \frac{1}{10} + \frac{1}{15} = \frac{1}{6}\text{th}$

In 3 mins. (A + B) fill the cistern

$= \frac{3}{6} = \frac{1}{2}\text{th}$

Remaining part $= 1 - \frac{1}{2} = \frac{1}{2}$

$\because \frac{1}{10}$ th part in filled by A in one min.

$\therefore \frac{1}{2}$ nd part is filled by A in $10 \times \frac{1}{2}$ = 5 min

Total time = 3 + 5 = 8 min

10. **(a)** 17

Explanation: As the trace of a matrix is the sum of on – diagonal elements,

So, 1 + 7 + 9 = 17

Trace = 17

11.

(c) 2

Explanation: From the definition: a≡b (mod m)

a is said to be congruent to b modulo m, if m divides (a - b) or (a - b) is divisble by m.

$\Rightarrow$ 218≡x (mod 7)

$\Rightarrow \frac{(218-x)}{7}$

for this to be hold true, x must be 1.

$\Rightarrow x^2 + 1 = 1^2 + 1 = 2$

12.

(c) x ∈ (2, ∞)

Explanation: Since $\frac{|x-2|}{x-2} \geq 0$, for |x - 2| ≥ 0, and x - 2 ≠ 0 solution set (2,∞)

13.

(c) 8 km

Explanation: The speed of boat in still water = 15 km/hr

Speed of water current = 5 km/hr

∴ Speed in down stream = 15 + 5 = 20 km/hr

Time given = 24 min = $\frac{24}{60}$ hr = $\frac{2}{5}$ hr

∴ Distance travelled = speed × times

= 20 × $\frac{2}{5}$ = 8 km

14.

(c) 60

Explanation: Here the objective function is given by:

F = 4x + 6y

Corner points	**Z = 4x +6 y**
(0, 2)	12 (Min.)
(3, 0)	12 (Min.)
(6, 0)	24
(6, 8)	72 (Max.)
(0, 5)	30

Maximum of F - Minimum of F = 72 - 12 = 60

15. **(a)** 5

Explanation: (1, 1), (1, -1), (-1 , -1), (2, -1) and (-2, -1) satisfy the inequality 2x - 3y > - 5.

16.

(d) 2.58

Explanation: 2.58

17.

(c) $\frac{(\log x)^6}{6}+C$

Explanation: Put log x = t ⇒ $\frac{1}{x}$ dx = dt

$\therefore \int \frac{(\log x)^5}{x}dx=\int t^5dt=\frac{t^6}{6}+C=\frac{(\log x)^6}{6}+C$

18.

(b) Minimum

Explanation: The line is termed as the line of best fir from which the sum of squares of distances from the points is minimized.

19.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation: Assertion: We have, A = IA

i.e., $\begin{bmatrix}10 & -2\\ -5 & 1\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}A$

$\Rightarrow \begin{bmatrix}1 & -\frac{1}{5}\\ -5 & 1\end{bmatrix}=\begin{bmatrix}\frac{1}{10} & 0\\ 0 & 1\end{bmatrix}A$ [applying $R_1\rightarrow\frac{1}{10}R_1$]

$\Rightarrow \begin{bmatrix}1 & -\frac{1}{5}\\ 0 & 0\end{bmatrix}=\begin{bmatrix}\frac{1}{10} & 0\\ \frac{1}{2} & 1\end{bmatrix}$ [applying $R_2 \rightarrow R_2 + 5R$]

We have all zeroes in the second row of the left hand side matrix of above equation. Therefore, A^{-1} does not exist.

Reason: The given matrix equation is $\begin{bmatrix}1 & -3\\ 2 & 4\end{bmatrix}=\begin{bmatrix}1 & -1\\ 0 & 1\end{bmatrix}\begin{bmatrix}3 & 1\\ 2 & 4\end{bmatrix}$

∵ The column transformation $C_2 \rightarrow C_2 - 2C_1$ is applied.

∴ This transformation is applied on LHS and on second matrix of RHS.

Thus, we have $\begin{bmatrix}1 & -5\\ 2 & 0\end{bmatrix}=\begin{bmatrix}1 & -1\\ 0 & 1\end{bmatrix}\begin{bmatrix}3 & -5\\ 2 & 0\end{bmatrix}$.

20. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Let $f(x) = x^2 - 8x + 17$

∴ f'(x) = 2x - 8

So, f'(x) = 0, gives x = 4

Here x = 4 is the critical number

Now, f''(x) = 2 > 0 , ∀ x

So, x = 4 is the point of local minima.

∴ Minimum value of f(x) at x = 4,

f(4) = 4 × 4 - 8 × 4 + 17 = 1

Hence, we can say that both Assertion and Reason are true and Reason is the correct explanation of the Assertion.

Section B

21. Construction of 3-yearly moving average

Year	**Imported cotton consumption in India (in '000 bales)**	**3-yearly moving totals**	**3-yearly moving averages**
2010	129	-	-
2011	131	366	122.00
2012	106	328	109.33
2013	91	292	97.33
2014	95	270	90.00
2015	84	272	90.66
2016	93	-	-

22. Since the money Lender deducts ₹200 as interest while lending a loan of ₹2000 for 6 months, therefore ₹200 may be treated as interest on ₹1800 for 6 months. Consequently, interest rate per six months is

$i = \frac{200}{1800} = \frac{1}{9}$

Thus, the equivalent effective rate of interest, is given by

Now, $r_{eff} = (1 + i)^2 - 1$

$= \left(1 + \frac{1}{9}\right)^2 - 1$ = 0.23456

= 23.45%

OR

The current cost of the scooter, C_o = 16000

Cost after two years, C = 14440

Let the rate of depreciation be R, then

$C = C_0\left(1 - \frac{R}{100}\right)^T$

$\Rightarrow 14400 = 16000\left(1 - \frac{R}{100}\right)^2$

$\Rightarrow \frac{14400}{16000} = \left(1 - \frac{R}{100}\right)^2$

$\Rightarrow \left(\frac{38}{40}\right)^2 = \left(1 - \frac{R}{100}\right)^2$

$\Rightarrow \frac{38}{40} = 1 - \frac{R}{100}$

$\Rightarrow \frac{R}{100} = 1 - \frac{38}{40}$

$\Rightarrow R = \frac{2\times100}{40}$

$\Rightarrow$ R = 5%

23. Put $x^2 + 1 = t \Rightarrow 2x\,dx = dt \Rightarrow x\,dx = \frac{1}{2}dt$

When x = 2, $t = 2^2 + 1 = 5$ and when x = 4, $t = 4^2 + 1 = 17$

$\therefore \int_2^4 \frac{x}{x^2+1}dx = \frac{1}{2}\int_5^{17}\frac{dt}{t} = \frac{1}{2}[\log|t|]_5^{17}$

$= \frac{1}{2}(\log 17 - \log 5) = \frac{1}{2}\log\frac{17}{5}$

24. The given system of equations can be written as AX = B

where $A = \begin{bmatrix} 1 & -2 & 3 \\ 1 & 4 & 1 \\ 1 & -3 & 2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 6 \\ 12 \\ 1 \end{bmatrix}$.

Now, $|A| = 1(8 + 3) + 2(2 - 1) + 3(-3 - 4) = -8 \neq 0$

$\Rightarrow A^{-1}$ exists

$\Rightarrow$ the given system of equations has a unique solution $X = A^{-1}B$

$A_{11} = 11$, $A_{12} = -1$, $A_{13} = -7$,

$A_{21} = -5$, $A_{22} = -1$, $A_{23} = 1$,

$A_{31} = -14$, $A_{32} = 2$, $A_{33} = 6$.

So, $A^{-1} = \frac{1}{-8}\begin{bmatrix} 11 & -5 & -14 \\ -1 & -1 & 2 \\ -7 & 1 & 6 \end{bmatrix}$

$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{-8}\begin{bmatrix} 11 & -5 & -14 \\ -1 & -1 & 2 \\ -7 & 1 & 6 \end{bmatrix}\begin{bmatrix} 6 \\ 12 \\ 1 \end{bmatrix}$

$= \frac{1}{-8}\begin{bmatrix} 66 - 60 - 14 \\ -6 - 12 + 2 \\ -42 + 12 + 6 \end{bmatrix} = \frac{1}{-8}\begin{bmatrix} -8 \\ -16 \\ -24 \end{bmatrix}$

$\Rightarrow$ x = 1, y = 2, z = 3

OR

$\begin{vmatrix} x+4 & x & x \\ x & x+4 & x \\ x & x & x+4 \end{vmatrix} = 0$

(Operate $C_1 \to C_1 + C_2 + C_3$ and take (3x + 4) out from new C_1

$\Rightarrow (3x + 4)\begin{vmatrix} 1 & x & x \\ 1 & x+4 & x \\ 1 & x & x+4 \end{vmatrix} = 0$ (Operate $C_2 \to C_2 - xC_1$, $C_3 \to C_3 - xC_1$)

$\Rightarrow (3x + 4)\begin{vmatrix} 1 & 0 & 0 \\ 1 & 4 & 0 \\ 1 & 0 & 4 \end{vmatrix} = 0 \Rightarrow (3x + 4) \cdot 4 \cdot 4 = 0$

$\Rightarrow 16(3x + 4) = 0$

$\Rightarrow 3x + 4 = 0 \Rightarrow x = -\frac{4}{3}$

25. To find -8 mod 5, let us divide -8 by 5

$5\overline{)-8}(-2$
$\;\;-10$
$\;\;\;\;2 \longrightarrow$ Remainder

So, -8 mod 5 = 2.

(Note this step, $\because 0 \leq r < |b|$)

Section C

26. The given differential equation is,

$e^{\frac{dy}{dx}} = x + 1$

Taking log on both sides, we get,

$\frac{dy}{dx} \log e = \log (x + 1)$

$\Rightarrow \frac{dy}{dx} = \log (x + 1)$

$\Rightarrow dy = \{\log (x + 1)\}\, dx$

Integrating both sides, we get

$\int dy = \int \{\log (x + 1)\, dx$

$\Rightarrow y = \int \underset{II}{1} \times \log(\underset{I}{x} + 1) dx$

$\Rightarrow y = \log (x + 1) \int 1\, dx - \int [\frac{d}{dx}(\log x + 1) \int 1 dx]\, dx$

$\Rightarrow y = x \log (x + 1) - \int \frac{x}{x+1} dx$

$\Rightarrow y = x \log (x + 1) - \int \left(1 - \frac{1}{x+1}\right) dx$

$\Rightarrow y = x \log (x + 1) - x + \log (x + 1) + C$ ...(i)

It is given that y(0) = 3

$\therefore 3 = 0 \times \log (0 + 1) - 0 + \log (0 + 1) + C$

$\Rightarrow C = 3$

Substituting the value of C in (i), we get

$y = x \log (x + 1) + \log (x + 1) - x + 3$

$\Rightarrow y = (x + 1) \log (x + 1) - x + 3$

Hence, $y = (x + 1) \log (x + 1) - x + 3$ is the solution to the given differential equation.

OR

The given equation of one parameter family of curves is

$x^2 - y^2\, c(x^2 + y^2)^2$ ...(i)

Differentiating (i) with respect to x, we get

$2x - 2y\frac{dy}{dx} = 2c(x^2 + y^2)(2x + 2y\frac{dy}{dx})$

$\Rightarrow (x - y\frac{dy}{dx}) = 2c(x^2 + y^2)(x + y\frac{dy}{dx})$ ...(ii)

On substituting the value of c obtained from (i) in (ii), we get,

$\left(x - y\frac{dy}{dx}\right) = \frac{2\left(x^2-y^2\right)\left(x^2+y^2\right)}{\left(x^2+y^2\right)^2}\left(x + y\frac{dy}{dx}\right)$

$\Rightarrow (x^2 + y^2)(x - y\frac{dy}{dx}) = 2(x^2 - y^2)(x + y\frac{dy}{dx})$

$\Rightarrow \{x(x^2 + y^2) - 2x(x^2 - y^2)\} = \frac{dy}{dx}\{2y(x^2 - y^2) + y(x^2 + y^2)\}$

$\Rightarrow (3xy^2 - x^3) = \frac{dy}{dx}(3x^2y - y^3)$

$\Rightarrow (x^3 - 3xy^2)\, dx = (y^3 - 3x^2y)\, dy$, which is the given differential equation.

27. Given, A = ₹ 5,00,000, r - 5% and n = 10

Using formula, $A = p\left[\frac{(1+i)^n-1}{i}\right]$

where i = $\frac{r}{100}$

$\Rightarrow$ 500000 = $p\left[\frac{(1+0.05)^{10}-1}{0.05}\right]$

p = $\frac{500000\times0.05}{(1.05)^{10}-1}$

Now, let x = $(1.05)^{10}$

Taking log both sides, we get

log x = 10 log (1.05)

= 10 $\times$ 0.0212

= 0.2120

$\Rightarrow$ x = antilog (0.2120)

= 1.629

Thus, $(1.05)^{10}$ = 1.629

Now, p = $\frac{500000\times0.05}{1.629-1}$

= $\frac{25000}{0.629}$

= 39745.63

Hence, the company should deposit X 39745.63 every year into the sinking fund.

28. Given, the demand function is

p = $20e^{-x/10}$ ...(i)

and the equilibrium price p_0 = 2.

Substituting this value of p_0 = 2 in (i), we get

$2 = 20e^{-x_0/10} \Rightarrow e^{-x_0/10} = \frac{1}{10}$ ...(ii)

$\Rightarrow e^{x_0/10} = 10 \Rightarrow \log_e 10 = \frac{x_0}{10}$

$\Rightarrow$ x_0 = 10 $\log_e$ 10 $= \frac{10}{\log_{10} e} = \frac{10}{0.4343} = \frac{100000}{4343}$

$\Rightarrow$ x_0 = 23.03 ...(iii)

$\therefore$ CS $= \int_0^{x_0} 20e^{-x/10}dx - x_0 \times p_0$

$= 20\left[\frac{e^{-x/10}}{-\frac{1}{10}}\right]_0^{x_0} - 23.03 \times 2$ (using (iii))

= -200[$\left[e^{-x_0/10}\right.$ - e^0] - 46.06

$= -200\left[\frac{1}{10} - 1\right] - 46.06$ (using (ii))

= 180 - 46.06 = 133.94

Hence, consumer's surplus is 133.94

29. Let X represent the number of winning prizes in 50 lotteries. The trials are Bernoulli trials.

Clearly, X has a binomial distribution with n = 50 and p = $\frac{1}{100}$

$\therefore$ q = 1 - p = $1 - \frac{1}{100} = \frac{99}{100}$

$\therefore$ P(X = x) = ${}^nC_x q^{n-x} P^x$ = ${}^{50}C_x \left(\frac{99}{100}\right)^{50-x} \cdot \left(\frac{1}{100}\right)^x$

i. P (winning at least once) = P(X $\geq$ 1)

= 1 – P (X < 1)

= 1 - P(X = 0)

= 1 - ${}^{50}C_0\left(\frac{99}{100}\right)^{50}$

$= 1 - 1 \cdot \left(\frac{99}{100}\right)^{50}$

= 1 - $\left(\frac{99}{100}\right)^{50}$

ii. P (winning exactly once) = P (X = 1)

= ${}^{50}C_1\left(\frac{99}{100}\right)^{49} \cdot \left(\frac{1}{100}\right)^1$

= $50\left(\frac{1}{100}\right)\left(\frac{99}{100}\right)^{49}$

$= \frac{1}{2}\left(\frac{99}{100}\right)^{49}$

iii. P (at least twice) = P(x $\geq$ 2)

= 1 – P(X < 2)

= 1 - P (X $\leq$ 1)

= 1 - [P(X = 0) + P(X = 1)]

= [1 - P(X = 0) - P(X = 1)]

$= 1 - \left(\frac{99}{100}\right)^{50} - \frac{1}{2} \cdot \left(\frac{99}{100}\right)^{49}$

$= 1 - \left(\frac{99}{100}\right)^{49} \left(\frac{99}{100} + \frac{1}{2}\right]$

$= 1 - \left(\frac{99}{100}\right)^{49} \cdot \left(\frac{149}{100}\right)$

$= 1 - \left(\frac{149}{100}\right) \left(\frac{99}{100}\right)^{49}$

OR

P(X = x) = $\begin{cases} k(x+1) for\ x = 1,2,3,4 \\ 2kx\ for\ x = 5,6,7 \\ 0\ otherwise \end{cases}$

Thus, we have following table:

X	1	2	3	4	5	6	7	otherwise
P(X)	2k	3k	4k	5k	5k	12k	14k	0
XP(X)	2k	6k	12k	20k	20k	72k	98k	0
X^2P(X)	2k	12k	36k	80k	80k	432k	686k	0

add X^2P(X) in fourth row and 1st column

i. Since, $\sum P_i = 1$

$\Rightarrow$ K(2 + 3 + 4 + 5 + 10 + 12 + 14) = 1 $\Rightarrow k = \frac{1}{50}$

ii. $\because E(X) = \sum XP(X)$

$\therefore$ E(X) = 2k + 6k + 12k + 20k + 50k + 72k + 98k + 0 = 260k

$= 260 \times \frac{1}{50} = \frac{26}{5} = 5.2$ [$\because k = \frac{1}{50}$] ...(i)

iii. We know that,

Var(X) = [E(X^2)] - [E(X)]^2 = $\sum X^2 P(X) - [\sum XP(X)]^2$

=[2k + 12k + 36k + 80k + 250k + 432k + 686k + 0] - [5.2]^2 ... [using Eq. (i)]

= [1498k] - 27.04 = $\left[1498 \times \frac{1}{50}\right] - 27.04$ [$\because k = \frac{1}{50}$]

= 29.96 - 27.4 = 2.92

We know that, standard deviation of X = $\sqrt{Var(X)} = \sqrt{2.92}$ = 1.7088 = 1.7 (approx)

30. We have the following table

Year	**No. of failures**	**5-Yearly Moving Totals**	**5-Yearly Moving Averages**
2003	23	-	-
2004	26	-	-
2005	28	129	25.8
2006	32	118	23.6
2007	20	104	20.8
2008	12	86	17.2
2009	12	63	12.6
2010	10	56	11.2
2011	9	55	11.0
2012	13	57	11.4
2013	11	59	11.8

2014	14	59	11.8
2015	12	49	9.8
2016	9	39	7.8
2017	3	-	-
2018	1	-	-

31.

x	$x-\bar{x}$	$(x-\bar{x})^2$
-4	-4.25	18.0625
-2	-2.25	5.0625
-2	-2.25	5.0625
0	-0.25	0.0625
2	1.75	3.0625
2	1.75	3.0625
3	2.75	7.5625
3	2.75	7.5625
$\sum x = 2$		$\sum(x-\bar{x})^2$ = 49.5000

$\bar{x}$ = mean

$= \frac{\sum x}{n}$

= $\frac{2}{8}$

= 0.25

Now, compute the standard deviation using formula as,

$s = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}}$

$= \sqrt{\frac{49.5}{7}}$

$= \sqrt{7.071428}$

= 2.659

H_0 = The mean of universe, μ = 0, we get

$t = \frac{\bar{X}-\mu}{\frac{\sigma}{\sqrt{n}}}$

$= \frac{0.25-0}{\frac{2.659}{\frac{\sigma}{\sqrt{8}}}}$

$= \frac{0.25}{\frac{2.659}{2.828}}$

$= \frac{0.25}{0.9402}$

= 0.2659

Section D

32. Converting the given inequations into equations, we obtain the following equations:

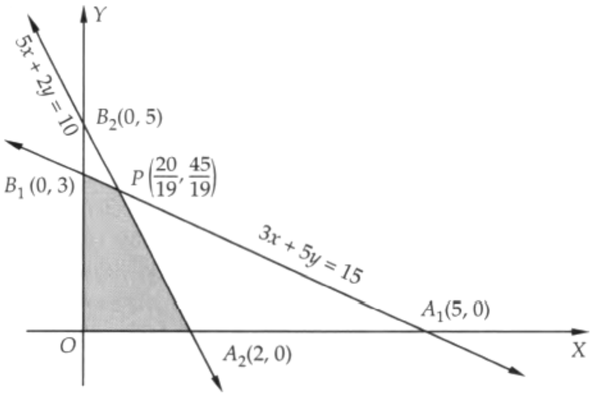
3x + 5y = 15, 5x + 2y = 10, x = 0 and y = 0

Region represented by 3x + 5y $\leq$ 15: The line 3x + 5y = 15 meets the coordinate axes at $A_1(5, 0)$ and $B_1(0, 3)$ respectively. Join these points to obtain the line 3x + 5y = 15. Clearly, (0,0) satisfies the inequation 3x + 5y $\leq$ 15. So, the region containing the origin represents the solution set of the inequation 3x + 5y < 15.

Region Represented by 5x + 2y $\leq$ 10: The line 5x + 2y = 10 meets the coordinate axes at $A_2(2, 0)$ and $B_2(0, 5)$ respectively. Join these points to obtain the graph of the line 5x + 2y = 10. Clearly, (0, 0) satisfies the inequation 5x + 2y $\leq$ 10. So, the region containing the origin represents the solution set of this inequation.

Region represented by x $\geq$ 0 and y $\geq$ 0: Since every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations x $\geq$ 0 and y $\geq$ 0. The shaded region OA_2 PB_1 in figure represents the common region of the above inequations. This region is the feasible region of the given LPP.

The coordinates of the vertices (comer-points) of the shaded feasible region are O(0, 0), A_2(2, 0), P($\frac{20}{19}$, $\frac{45}{19}$) and B_1(0, 3).



These points have been obtained by solving the equations of the corresponding intersecting lines, simultaneously.

The values of the objective function at these points are given in the following table:

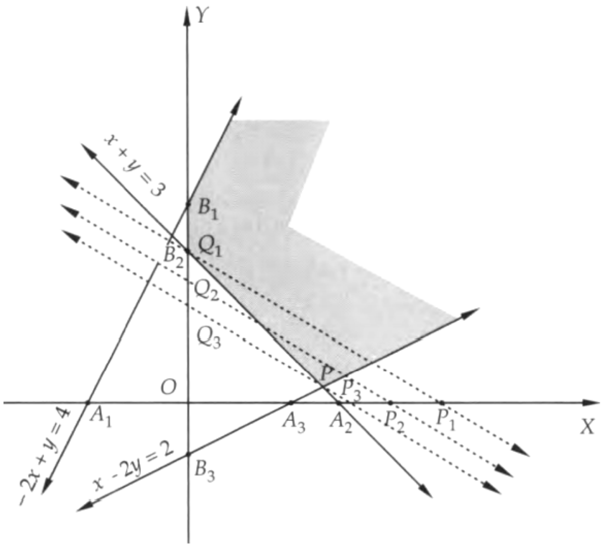
Point (x, y)	**Value of the objective function Z = 5x + 3y**
O(0, 0)	Z = $5 \times 0 + 3 \times 0 = 0$
A_2(2, 0)	Z = $5 \times 2 + 3 \times 0 = 10$
$P\left(\frac{20}{19}, \frac{45}{19}\right)$	Z = $5 \times \frac{20}{19} + 3 \times \frac{45}{19} = \frac{235}{19}$
B_1(0, 3)	Z = $5 \times 0 + 3 \times 3 = 9$

Clearly, Z is maximum at P$\left(\frac{20}{19}, \frac{45}{19}\right)$. Hence, x = $\frac{20}{19}$, y = $\frac{45}{19}$ is the optimal solution of the given LPP and the optimal value of Z is $\frac{235}{19}$.

OR

Converting the inequations into equations, we obtain the lines -2x + y = 4, x + y = 3, x - 2y = 2, x = 0 and y = 0.

These lines are drawn on a suitable scale and the feasible region of the LPP is shaded in Figure.



Now, give a value, say 15 equal to (1 c.m. of 3 and 5) to Z to obtain the line 3x + 5y = 15. This line meets the coordinate axes at P_1(5, 0) and Q_1(0, 3). Join these points by a dotted line. Move this line parallel to itself in the decreasing direction towards the origin so that it passes through only one point of the feasible region. Clearly, P_3Q_3 is such a line passing through the vertex P of the feasible region. The coordinates of P are obtained by solving the lines x - 2y = 2 and x + y = 3. Solving these equations, we get x = $\frac{8}{3}$ and y = $\frac{1}{3}$. Putting x = $\frac{8}{3}$ and y = $\frac{1}{3}$ in Z = 3x + 5y, we get

Z = $3 \times \frac{8}{3} + 5 \times \frac{1}{3} = \frac{29}{3}$

Hence, the minimum value of Z is $\frac{29}{3}$ at x = $\frac{8}{3}$, y = $\frac{1}{3}$

33. The given inequalities are

3y - 2x < 4 ...(i)

x + 3y >3 ...(ii)

and x + y $\leq$ 5 ...(iii)

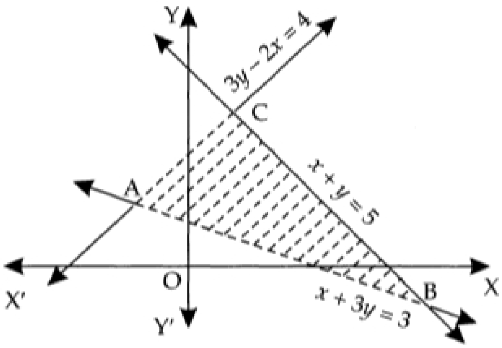
To draw the graph of 3y - 2x < 4:

We draw the straight line 3y - 2x = 4 which passes through the points (-2, 0) and $\left(0, \frac{4}{3}\right)$. The line divides the plane into two parts.

Further, as O(0, 0) satisfies the inequality 3y - 2x < 4.

($\because$ 3 $\times$ 0 - 2 $\times$ 0 = 0 < 4), therefore, the graph consists of that part of the plane divided by the line 3y - 2x = 4 which contains the origin.

Similarly, draw the graphs of other two inequalities x + 3y > 3 and x + y $\leq$ 5.



Shade the common part of the graphs of all the three given inequalities (i), (ii) and (iii).

The solution set consists of all the points in the shaded part of the coordinate plane shown in fig. The points on the line segment BC are included in the solution.

34. a. Let us prepare the following frequency table:

Marks obtained	14	17	18	19	21	25	26
No. of students	3	5	2	3	3	2	2

Total number of students = 20

Given that X = marks of a selected student.

So, P(X = 14) = $\frac{3}{20}$;

P(X = 17) = $\frac{5}{20} = \frac{1}{4}$;

P(X = 18) = $\frac{2}{20} = \frac{1}{10}$;

P(X = 19) = $\frac{3}{20}$;

P(X = 21) = $\frac{3}{20}$;

P(X = 25) = $\frac{2}{20} = \frac{1}{10}$;

P(X = 26) = $\frac{2}{20} = \frac{1}{10}$

Hence, the required probability distribution is

X	14	17	18	19	21	25	26
P(X)	$\frac{3}{20}$	$\frac{1}{4}$	$\frac{1}{10}$	$\frac{3}{20}$	$\frac{3}{20}$	$\frac{1}{10}$	$\frac{1}{10}$

b. To calculate mean, variance and standard deviation, we construct the following table:

$\mathbf{x_1}$	$\mathbf{p_i}$	$\mathbf{p_i x_i}$	$p_i x_i^2$
14	$\frac{3}{20}$	$\frac{42}{20}$	$\frac{147}{5}$
17	$\frac{1}{4}$	$\frac{17}{4}$	$\frac{289}{4}$
18	$\frac{1}{10}$	$\frac{18}{10}$	$\frac{162}{5}$
19	$\frac{3}{20}$	$\frac{57}{20}$	$\frac{1083}{20}$
21	$\frac{3}{20}$	$\frac{63}{20}$	$\frac{1323}{20}$
25	$\frac{1}{10}$	$\frac{25}{10}$	$\frac{125}{2}$
26	$\frac{1}{10}$	$\frac{26}{10}$	$\frac{338}{5}$
Total		$\frac{385}{20}$	$\frac{7689}{20}$

$\therefore$ Mean μ = $\Sigma p_i x_i$ = $\frac{385}{20} = \frac{77}{4}$ = 19.25

Variance $\sigma^2 = \Sigma p_i x_i^2 - \mu^2$ = $\frac{7689}{20} - \left(\frac{77}{4}\right)^2 = \frac{7689}{20} - \frac{5929}{16}$

$= \frac{30756-29645}{80} = \frac{1111}{80}$ = 13.89

Standard deviation $\sigma = \sqrt{\text{Variance}} = \sqrt{13.89}$ = 3.73

OR

Let A_i denote the event of getting an ace in i^{th} draw, where i = 1, 2.

Further, let X be a random variable denoting the number of aces in two draws. Then, X can take values 0, 1, 2. Then, we have,

P (X = 0) = Probability of getting no ace in two successive draws

$\Rightarrow$ P(X = 0) = $P(\overline{A_1} \cap \overline{A_2}) = P(\overline{A_1})P(\overline{A_2}/\overline{A_1}) = \frac{48}{52} \times \frac{47}{51} = \frac{564}{663}$

P (X = 1) = Probability of getting an ace in one of the two draws

$\Rightarrow$ P(X = 1) = $P\left(\left(A_1 \cap \overline{A_2}\right) \cup \left(\overline{A_1} \cap A_2\right)\right)$

$\Rightarrow$ P(X = 1) = $P\left(A_1 \cap \overline{A_2}\right) + P\left(\overline{A_1} \cap A_2\right)$

$\Rightarrow$ P(X = 1) = $P(A_1)P\left(\overline{A_2}/A_1\right) + P(\overline{A_1})P\left(A_2/\overline{A_1}\right) = \frac{4}{52} \times \frac{48}{51} + \frac{48}{52} \times \frac{4}{51} = \frac{96}{663}$

P(X = 2) = Probability of getting an ace in each draw

$\Rightarrow$ P(X = 2) = $P(A_1 \cap A_2) = P(A_1)P(A_2/A_1) = \frac{4}{52} \times \frac{3}{51} = \frac{3}{663}$

Therefore, the probability distribution of X is as follows:

X	0	1	2
P(X)	$\frac{564}{663}$	$\frac{96}{663}$	$\frac{3}{663}$

$\therefore \Sigma p_i x_i = 0 \times \frac{564}{663} + 1 \times \frac{96}{663} + 2 \times \frac{3}{663} = \frac{102}{663}$

and, $\Sigma p_i x_i^2 = \frac{564}{663} \times 0 + \frac{96}{663} \times 1 + \frac{3}{663} \times 4 = \frac{108}{663}$

Hence, Var(X) = $\Sigma p_i x_i^2 - (\Sigma p_i x_i)^2 = \frac{108}{663} - \left(\frac{102}{663}\right)^2 = \frac{108 \times 663 - (102)^2}{(663)^2} = \frac{61200}{663 \times 663} = \frac{400}{2873}$

35. Given that,

Beginning Value = BV = 300000

Ending Value = EV = 450000

Number of Years = n= 5

$\therefore CAGR = \left(\frac{EV}{BV}\right)^{\frac{1}{n}} - 1$

$= \left(\frac{450000}{3000000}\right)^{\frac{1}{5}} - 1$

$= \left(\frac{3}{2}\right)^{\frac{1}{5}} - 1$

$= (1.5)^{\frac{1}{5}} - 1$

= 1.084 - 1

= 0.084

CAGR% = 0.084 × 100 = 8.4%

Hence the compound Annual Growth Rate (CAGR) on the investment is 8.4%.

Section E

36. **Read the text carefully and answer the questions:**

A tank with a rectangular base and rectangular sides of length x metre, width y metre , open at the top is to be constructed so that the depth is 1 m and volume is $9m^3$. If the building of the tank is ₹ 70 per square metre for the base and ₹ 45 per square metre for the sides?

(i) 70xy

(ii) 90(x + y)

(iii) $90(1 - \frac{9}{x^2})$

OR

3

37. **Read the text carefully and answer the questions:**

The nominal rate of return shows the yield of an investment over time without accounting for negative elements such as inflation or taxes. By calculating the nominal rate of return, you can compare the performance of your assets easily, regardless of the inflation rate or differing spans of time for each investment. By obtaining a bird's-eye view of how your assets are growing, you can make more prudent investment decisions in the future.

(i) 12.5%

(ii) $11\frac{7}{13}$%

(iii)12%

OR

₹70

38. Let the varieties of pen A, B and C be x, y and z respectively.

According to question,

x + y + z = 21

4x + 3y + 2z = 60

6x + 2y + 3z = 70

These three equations can be written as:

$$\begin{bmatrix}1 & 1 & 1\\4 & 3 & 2\\6 & 2 & 3\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}21\\60\\70\end{bmatrix}$$

AX = B

|A| = 1(9 - 4) - 1(12 - 12) + 1(8 - 18)

= 1(5) - 1(0) + 1(-10)

= 5 - 0 - 10

= - 5

Hence, the unique solution given by x = $A^{-1}B$

$C_{11} = (-1)^{1+1}(9-4) = 5$

$C_{12} = (-1)^{1+2}(12-12) = 0$

$C_{13} = (-1)^{1+3}(8-18) = -10$

$C_{21} = (-1)^{2+1}(3-2) = -1$

$C_{22} = (-1)^{2+2}(3-6) = -3$

$C_{23} = (-1)^{2+3}(2-6) = 4$

$C_{31} = (-1)^{3+1}(2-3) = -1$

$C_{32} = (-1)^{3+2}(2-4) = 2$

$C_{33} = (-1)^{3+3}(3-4) = -1$

Adj A = $\begin{bmatrix}5 & 0 & -10\\-1 & -3 & 4\\-1 & 2 & -1\end{bmatrix}^T=\begin{bmatrix}5 & -1 & -1\\0 & -3 & 2\\-10 & 4 & -1\end{bmatrix}$

x = $A^{-1}B = \frac{1}{|A|}$ (adj A)B

X = $\frac{1}{-5}\begin{bmatrix}5 & -1 & -1\\0 & -3 & 2\\-10 & 4 & -1\end{bmatrix}\begin{bmatrix}21\\60\\70\end{bmatrix}$

X = $\frac{1}{-5}\begin{bmatrix}105-60-70\\0-180+140\\-210+240-70\end{bmatrix}$

X = $\frac{1}{-5}\begin{bmatrix}105-60-70\\0-180+140\\-210+240-70\end{bmatrix}$

$$\begin{bmatrix} X \\ y \\ Z \end{bmatrix} = \frac{1}{-5}\begin{bmatrix} -25 \\ -40 \\ -40 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix}$$

Hence, A = ₹5, B = ₹8 and C = ₹8

OR

The information provided can be summarized in the tabular form as follows:

Brackets	Drilling	Turning	Grinding
Type I	0	5	4
Type II	3	3	2
Type III	3	2	2
Time available (in minutes)	$10 \times 60 = 600$	$8 \times 60 = 480$	$7 \times 60 = 420$

Let x, y and z denote the number of brackets produced of each type. Then, the time taken by drilling machine to produce x, y and z brackets of type I, II and III respectively, is 0x + 3y + 3z minutes. But the time for which the drilling machine is available is 600 minutes.

$\therefore$ 0x + 3y + 3z = 600

Similarly, for turning and grinding machines, we obtain

5x + 3y + 2z = 480 and 4x + 2y + 2z = 420

Thus, we obtain the following system of linear equations

0x + 3y + 3z = 600

5x + 3y + 2z = 480

4x + 2y + 2z + 420

In matrix form, the above system can be written as

$$\begin{bmatrix} 0 & 3 & 3 \\ 5 & 3 & 2 \\ 4 & 2 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 600 \\ 480 \\ 420 \end{bmatrix}$$

or, AX = B where A = $\begin{bmatrix} 0 & 3 & 3 \\ 5 & 3 & 2 \\ 4 & 2 & 2 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and B = $\begin{bmatrix} 600 \\ 480 \\ 420 \end{bmatrix}$

Now, |A| = $\begin{vmatrix} 0 & 3 & 3 \\ 5 & 3 & 2 \\ 4 & 2 & 2 \end{vmatrix}$ = 0 (6 - 4) - 3(10 - 8) + 3(10 - 12) = -12 $\neq$ 0

So, A is invertible.

Let C_{ij} be cofactor of a_{ij} in A = [a_{ij}]. Then,

C_{11} = 2, C_{12} = -2, C_{13} = -2, C_{21} = 0, C_{22} = -12, C_{23} = 12, C_{31} = -3, C_{32} = 15, C_{33} = -15

$\therefore$ adj A = $\begin{bmatrix} 2 & -2 & -2 \\ 0 & -12 & 12 \\ -3 & 15 & -15 \end{bmatrix}^T = \begin{bmatrix} 2 & 0 & -3 \\ -2 & -12 & 15 \\ -2 & 12 & -15 \end{bmatrix}$

Thus, A^{-1} = $\frac{1}{|A|}$ adj A = $-\frac{1}{12}\begin{bmatrix} 2 & 0 & -3 \\ -2 & -12 & 15 \\ -2 & 12 & -15 \end{bmatrix}$

Now, AX = B

$\Rightarrow$ X = A^{-1}B

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{12}\begin{bmatrix} 2 & 0 & -3 \\ -2 & -12 & 15 \\ -2 & 12 & -15 \end{bmatrix}\begin{bmatrix} 600 \\ 480 \\ 420 \end{bmatrix} = -\frac{1}{12}\begin{bmatrix} 1200 & +0 & -1260 \\ -1200 & -5760 & +6300 \\ -1200 & +5760 & -6300 \end{bmatrix} = -\frac{1}{12}\begin{bmatrix} -60 \\ -660 \\ -1740 \end{bmatrix} = \begin{bmatrix} 5 \\ 55 \\ 145 \end{bmatrix}$

$\Rightarrow$ x = 5, y = 55 and z = 145

Hence, 5 brackets of Type-I, 55 brackets of Type-II and 145 brackets of Type-III should be produced to keep all machines fully occupied.