INVERSE TRIGONOMETRIC FUNCTIONS

BASIC CONCEPTS

Principal value branches: *Since trigonometric functions being periodic are in general not bijective (one-one onto) and thus for existence of inverse of trigonometric function we restrict their domain and co-domain to make it bijective. This restriction of domain and range gives principal value branch of inverse trigonometric function which are as follows:*

Functions	Domain	Range (Principal value branch)
$y = \sin^{-1} x$	$[-1, 1]$	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$
$y = \cos^{-1} x$	$[-1, 1]$	$[0, \pi]$
$y = \text{cosec}^{-1} x$	$R - (-1, 1)$	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$
$y = \sec^{-1} x$	$R - (-1, 1)$	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$
$y = \tan^{-1} x$	R	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$y = \cot^{-1} x$	R	$(0, \pi)$

The value of an inverse trigonometric function which lies in its principal value branch is called the principal value of that inverse trigonometric function.

Properties of Inverse Trigonometric Functions

1. (*i*) $\sin^{-1} x + \sin^{-1} y = \sin^{-1}\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\}$, if $-1 \le x, y \le 1$ and $x^2 + y^2 \le 1$

   (*ii*) $\sin^{-1} x - \sin^{-1} y = \sin^{-1}\{x\sqrt{1-y^2} - y\sqrt{1-x^2}\}$, if $-1 \le x, y \le 1$ and $x^2 + y^2 \le 1$

2. (*i*) $\cos^{-1} x + \cos^{-1} y = \cos^{-1}\{xy - \sqrt{1-x^2}\sqrt{1-y^2}\}$, if $-1 \le x, y \le 1$ and $x + y \ge 0$

   (*ii*) $\cos^{-1} x - \cos^{-1} y = \cos^{-1}\{xy + \sqrt{1-x^2}\sqrt{1-y^2}\}$, if $-1 \le x, y \le 1$ and $x \le y$

3. (*i*) $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$, if $xy < 1$

   (*ii*) $\tan^{-1} x - \tan^{-1} y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$, if $xy > -1$

4. (*i*) $2\sin^{-1} x = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$, if $-\frac{1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}}$

   (*ii*) $2\cos^{-1} x = \cos^{-1}(2x^2 - 1)$, if $0 \le x \le 1$

   (*iii*) $2\tan^{-1} x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, if $-1 < x < 1$

5. (*i*) $3\sin^{-1}x = \sin^{-1}(3x - 4x^3)$, if $-\frac{1}{2} \le x \le \frac{1}{2}$

(*ii*) $3\cos^{-1}x = \cos^{-1}(4x^3 - 3x)$, if $\frac{1}{2} \le x \le 1$

(*iii*) $3\tan^{-1}x = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)$, if $-\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$

6. (*i*) $2\tan^{-1}x = \sin^{-1}\left(\frac{2x}{1 + x^2}\right)$, if $-1 \le x \le 1$

(*ii*) $2\tan^{-1}x = \cos^{-1}\left(\frac{1 - x^2}{1 + x^2}\right)$, if $0 \le x < \infty$

7. (*i*) $\sin^{-1}x = \cos^{-1}(\sqrt{1 - x^2}) = \tan^{-1}\left(\frac{x}{\sqrt{1 - x^2}}\right)$

$$= \cot^{-1}\left(\frac{\sqrt{1 - x^2}}{x}\right) = \sec^{-1}\left(\frac{1}{\sqrt{1 - x^2}}\right) = \text{cosec}^{-1}\left(\frac{1}{x}\right)$$

(*ii*) $\cos^{-1}x = \sin^{-1}(\sqrt{1 - x^2}) = \tan^{-1}\left(\frac{\sqrt{1 - x^2}}{x}\right)$

$$= \cot^{-1}\left(\frac{x}{\sqrt{1 - x^2}}\right) = \sec^{-1}\left(\frac{1}{x}\right) = \text{cosec}^{-1}\left(\frac{1}{\sqrt{1 - x^2}}\right)$$

(*iii*) $\tan^{-1}x = \sin^{-1}\left(\frac{x}{\sqrt{1 + x^2}}\right) = \cos^{-1}\left(\frac{1}{\sqrt{1 + x^2}}\right)$

$$= \cot^{-1}\left(\frac{1}{x}\right) = \sec^{-1}(\sqrt{1 + x^2}) = \text{cosec}^{-1}\left(\frac{\sqrt{1 + x^2}}{x}\right)$$

Important substitution

Expression	**Substitution**
$a^2 + x^2$	$x = a\tan\theta$ *or* $x = a\cot\theta$
$a^2 - x^2$	$x = a\sin\theta$ *or* $x = a\cos\theta$
$x^2 - a^2$	$x = a\sec\theta$ *or* $x = a\,\text{cosec}\,\theta$
$\sqrt{\frac{a + x}{a - x}}$ or $\sqrt{\frac{a - x}{a + x}}$	$x = a\cos 2\theta$

MULTIPLE CHOICE QUESTIONS

Choose and write the correct option in the following questions.

1. **The value of $\tan^{-1}(\sqrt{3}) + \cos^{-1}\left(-\frac{1}{2}\right)$ corresponding to principal branches is**

(*a*) $-\frac{\pi}{12}$ (*b*) 0 (*c*) π (*d*) $\frac{\pi}{3}$

2. **The value of cot ($\sin^{-1} x$) is** *[NCERT Exemplar]*

(*a*) $\frac{\sqrt{1 + x^2}}{x}$ (*b*) $\frac{x}{\sqrt{1 + x^2}}$ (*c*) $\frac{1}{x}$ (*d*) $\frac{\sqrt{1 - x^2}}{x}$

3. **The value of $\sin^{-1}\left(\cos\frac{\pi}{9}\right)$ is** **[NCERT Exemplar]**

(a) $\frac{\pi}{9}$ (b) $\frac{5\pi}{9}$ (c) $\frac{-5\pi}{9}$ (d) $\frac{7\pi}{18}$

4. **Let $\theta = \sin^{-1}(\sin(-600°))$, then value of θ is**

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$ (c) $\frac{2\pi}{3}$ (d) $\frac{-2\pi}{3}$

5. **The value of the expression $2\sec^{-1}2 + \sin^{-1}\left(\frac{1}{2}\right)$ is** **[NCERT Exemplar]**

(a) $\frac{\pi}{6}$ (b) $\frac{5\pi}{6}$ (c) $\frac{7\pi}{6}$ (d) 1

6. **The value of $\tan^2(\sec^{-1}2) + \cot^2(\operatorname{cosec}^{-1}3)$ is**

(a) 5 (b) 11 (c) 13 (d) 15

7. **The domain of the function defined by $f(x) = \sin^{-1}\sqrt{x-1}$ is** **[NCERT Exemplar]**

(a) [1, 2] (b) [–1, 1] (c) [0, 1] (d) none of these

8. **The value of $\cot\left[\cos^{-1}\left(\frac{7}{25}\right)\right]$ is** **[NCERT Exemplar]**

(a) $\frac{25}{24}$ (b) $\frac{25}{7}$ (c) $\frac{24}{25}$ (d) $\frac{7}{24}$

9. **$\sin(\tan^{-1}x)$, $|x| < 1$ is equal to**

(a) $\frac{x}{\sqrt{1-x^2}}$ (b) $\frac{1}{\sqrt{1-x^2}}$ (c) $\frac{1}{\sqrt{1+x^2}}$ (d) $\frac{x}{\sqrt{1+x^2}}$

10. **If $\cos^{-1}\alpha + \cos^{-1}\beta + \cos^{-1}\gamma = 3\pi$, then $\alpha(\beta+\gamma) + \beta(\gamma+\alpha) + \gamma(\alpha+\beta)$ equals** **[NCERT Exemplar]**

(a) 0 (b) 1 (c) 6 (d) 12

11. **$\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$ is equal to**

(a) π (b) $-\frac{\pi}{3}$ (c) $\frac{\pi}{3}$ (d) $\frac{2\pi}{3}$

12. **The value of $\sin[\cot^{-1}\{\tan(\cos^{-1}x)\}]$ is**

(a) $\sqrt{1-x^2}$ (b) 1 (c) x (d) x^2

13. **If $\theta = \sin^{-1}(\sin 600°)$ then the value of θ is**

(a) $\frac{\pi}{3}$ (b) $-\frac{\pi}{3}$ (c) 0 (d) $\frac{2\pi}{3}$

14. **$\cos^{-1}\left[\cos\frac{7\pi}{6}\right]$ is equal to**

(a) $\frac{7\pi}{6}$ (b) $\frac{5\pi}{6}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

15. **$\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$ is equal to**

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) 1

16. **$\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})$ is equal to**

(a) π (b) $-\frac{\pi}{2}$ (c) 0 (d) $2\sqrt{3}$

17. The domain of $y = \cos^{-1}(x^2 - 4)$ is

(*a*) $[-1, 1]$ (*b*) $[-\sqrt{5}, -\sqrt{3}] \cap [\sqrt{3}, \sqrt{5}]$

(*c*) $[-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$ (*d*) $[0, \pi]$

18. The domain of the function defined $f(x) = \sin^{-1} x + \cos x$ is

(*a*) ϕ (*b*) $(-\infty, \infty)$ (*c*) $[-1, 1]$ (*d*) $[0, \pi]$

19. The value of $\sin(2\sin^{-1}(0.8))$ is

(*a*) sin 1.6 (*b*) 1.6 (*c*) 0.96 (*d*) 4.8

20. $\sin^{-1}(\sin 5) > x^2 - 4x$ if

(*a*) $x = 2 - \sqrt{9 - 2\pi}$ (*b*) $x = 2 + \sqrt{9 - 2\pi}$

(*c*) $x > 2 + \sqrt{9 - 2\pi}$ (*d*) $x \in (2 - \sqrt{9 - 2\pi}, 2 + \sqrt{9 - 2\pi})$

21. If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \dfrac{3\pi}{2}$ and $f(1) = 1$, $f(p + q) = f(p).f(q)$, $\forall\ p, q \in R$, then $x^{f(1)} + y^{f(2)} + z^{f(3)} - \dfrac{x + y + z}{x^{f(1)} + y^{f(2)} + z^{f(3)}}$ equals

(*a*) 2 (*b*) 1 (*c*) 0 (*d*) 3

22. If $u = \cot^{-1}\sqrt{\tan\alpha} - \tan^{-1}\sqrt{\tan\alpha}$, then $\tan\left(\dfrac{\pi}{4} - \dfrac{u}{2}\right)$ is equal to

(*a*) $\sqrt{\tan\alpha}$ (*b*) $\sqrt{\cot\alpha}$ (*c*) $\tan\alpha$ (*d*) $\cot\alpha$

23. If $\cos^{-1} x > \sin^{-1} x$, then

(*a*) $\dfrac{1}{\sqrt{2}} < x \le 1$ (*b*) $0 \le x < \dfrac{1}{\sqrt{2}}$ (*c*) $-1 \le x < \dfrac{1}{\sqrt{2}}$ (*d*) $x > 0$

24. If $f(x) = \sin^{-1} x$, then domain of $f(x)$ is

(*a*) $x \ge 1$ or $x \le -1$ (*b*) $-1 \le x \le 1$ (*c*) $x \ge 1$ (*d*) None of these

25. The domain of the function $y = \sin^{-1}(-x^2)$ is

(*a*) $[0, 1]$ (*b*) $(0, 1)$ (*c*) $[-1, 1]$ (*d*) θ

26. If $ax + b\{\sec(\tan^{-1} x)\} = c$ and $ay + b\{\sec(\tan^{-1} y)\} = c$, then $\dfrac{x + y}{1 - xy} =$

(*a*) $\dfrac{2ac}{a - c}$ (*b*) $\dfrac{ac}{a^2 + c^2}$ (*c*) $\dfrac{2ac}{a^2 - c^2}$ (*d*) $\dfrac{a + c}{1 - ac}$

27. If $\alpha = \tan^{-1}\left\{\tan\left(\dfrac{5\pi}{4}\right)\right\}$ and $\beta = \tan^{-1}\left\{-\tan\left(\dfrac{2\pi}{3}\right)\right\}$ then

(*a*) $4\alpha = 3\beta$ (*b*) $3\alpha = 4\beta$ (*c*) $\alpha = \beta$ (*d*) none of these

28. The value of expression $\tan\left(\dfrac{\sin^{-1} x + \cos^{-1} x}{2}\right)$, where $x = \dfrac{\sqrt{3}}{2}$ is equal to

(*a*) ∞ (*b*) 1 (c) -1 (*d*) none of these

29. Domain of $\cos^{-1}[x]$ is

(*a*) $[-2, 1]$ (*b*) $(-1, 1)$ (*c*) $[-1, 2)$ (*d*) None of these

30. Let $f(x) = \sin 2x + \cos 2x$ and $g(x) = x^2 - 1$, then $g(f(x))$ will be invertible for the domain

(*a*) $x \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$ (*b*) $x \in \left[-\dfrac{\pi}{8}, \dfrac{\pi}{8}\right]$ (*c*) $x \in \left[-\dfrac{\pi}{4}, \dfrac{\pi}{4}\right]$ (*d*) $x \in \left[0, \dfrac{\pi}{4}\right]$

31. The domain of $\sin^{-1}[x]$ is given by

(*a*) $[-1, 1]$ (*b*) $[-1, 2)$ (c) $\{-1, 0, 1\}$ (*d*) None of these

32. If $\sin(\pi \cos x) = \cos(\pi \sin x)$ then x equals

(*a*) $\frac{1}{2}\sin^{-1}\frac{3}{4}$ (*b*) $\frac{1}{2}\cos^{-1}\frac{3}{4}$ (*c*) $-\frac{1}{4}\sin^{-1}\frac{3}{4}$ (*d*) $-\frac{1}{2}\cos^{-1}\frac{3}{4}$

33. If $\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$, then $4x^2 - 4xy\cos\alpha + y^2$ is equal to

(*a*) 4 (*b*) $2\sin 2\alpha$ (*c*) $-4\sin^2\alpha$ (*d*) $4\sin^2\alpha$

34. $3\sin x + 4\cos x = y^2 - 2y + 6$. If x, y are its solutions then which of the following is true?

(*a*) $y = 1$ (*b*) $x = \frac{\pi}{2} - \tan^{-1}\left(\frac{4}{3}\right)$

(*c*) $xy = \frac{\pi}{2} - \tan^{-1}\left(\frac{4}{3}\right)$ (*d*) All of these

35. If $(\cot^{-1}x)^2 - 7(\cot^{-1}x) + 10 > 0$, then x lies in the interval

(*a*) $(\cot 5, \cot 2)$ (*b*) $(-\infty, \cot 5) \cup (\cot 2, \infty)$

(*c*) $(-\infty, \cot 5)$ (*d*) $(\cot 2, \infty)$

36. The domain of $f(x) = \frac{\sin^{-1}x}{x}$ is

(*a*) $[-1, 1]$ (*b*) $\{0\}$ (*c*) $[-1, 0)$ (*d*) None of these

37. Let $f(x) = e^{\cos^{-1}\sin\left(x+\frac{\pi}{3}\right)}$, then $f\left(\frac{8\pi}{9}\right)$ equals

(*a*) $e^{\frac{7\pi}{12}}$ (*b*) $e^{\frac{13\pi}{18}}$ (*c*) $e^{\frac{5\pi}{18}}$ (*d*) $e^{\frac{\pi}{12}}$

38. If $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, the solution of the equation $\log_{\sin\theta}(\cos^2\theta - \sin^2\theta) = 2$ is given by

(*a*) $\theta = \sin^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ (*b*) $\theta = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (*c*) $\theta = n\pi$ (*d*) $\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

39. If $\sin^{-1}a + \sin^{-1}b + \sin^{-1}c = \frac{3\pi}{2}$ and $f(2) = 2$, $f(x+y) = f(x)f(y)\ \forall\ x, y \in \mathbb{R}$, then

$a^{f(2)} + b^{f(4)} + c^{f(6)} - 3\frac{(a^{f(2)}.b^{f(4)}.c^{f(6)})}{a^{f(2)} + b^{f(4)} + c^{f(6)}}$ equals

(*a*) 2 (*b*) 4 (*c*) 6 (*d*) 8

40. If $\sin^{-1}(x^2 - 7x + 12) = n\pi$, $\forall\ n \in \mathbb{Z}$, then x equals

(*a*) -4 (*b*) 4 (*c*) -3 (*d*) None of these

41. The value of x satisfies the inequality $[\tan^{-1}x]^2 - 2[\tan^{-1}x] - 3 \le 0$, where [.] represent greatest integer function, then x lies between

(*a*) $[-\tan 1, \infty)$ (*b*) $[-\tan 1, \tan 3]$ (*c*) $\left[-\frac{\pi}{4}, \tan^{-1}3\right]$ (*d*) $\left[-\frac{\pi}{2}, \tan 1\right]$

42. $\cos^{-1}\left[\cos\left(\left(-\frac{17}{15}\right)\pi\right)\right]$ is equal to

(*a*) $\frac{17\pi}{15}$ (*b*) $\frac{13\pi}{15}$ (*c*) $\frac{3\pi}{15}$ (*d*) $-\frac{17\pi}{15}$

43. Which of the following is the principal value branch of $\cos^{-1}x$?

(*a*) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ (*b*) $(0, \pi)$ (*c*) $[0, \pi]$ (*d*) $(0, \pi) - \left\{\frac{\pi}{2}\right\}$

44. Which of the following is the principal value branch of $\operatorname{cosec}^{-1}x$?

(*a*) $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ (*b*) $(0, \pi) - \left\{\frac{\pi}{2}\right\}$ (*c*) $\left\{-\frac{\pi}{2}, \frac{\pi}{2}\right\}$ (*d*) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

45. **The value of $\sin^{-1}\left[\cos\left(\frac{33\pi}{5}\right)\right]$ is**

(a) $\frac{3\pi}{5}$ (b) $\frac{-7\pi}{5}$ (c) $\frac{\pi}{10}$ (d) $\frac{-\pi}{10}$

46. **The domain of the function $\cos^{-1}(2x - 1)$ is**

(a) [0, 1] (b) [– 1, 1] (c) (– 1, 1) (d) [0, π]

47. **The value of $\cos^{-1}\left(\cos\frac{3\pi}{2}\right)$ is**

(a) $\frac{\pi}{2}$ (b) $\frac{3\pi}{2}$ (c) $\frac{5\pi}{2}$ (d) $\frac{7\pi}{2}$

48. **The value of $2\sec^{-1}2 + \sin^{-1}\left(\frac{1}{2}\right)$ is**

(a) $\frac{\pi}{6}$ (b) $\frac{5\pi}{6}$ (c) $\frac{7\pi}{6}$ (d) 1

49. **The value of $\cot\left[\cos^{-1}\left(\frac{7}{25}\right)\right]$ is**

(a) $\frac{25}{24}$ (b) $\frac{25}{7}$ (c) $\frac{24}{25}$ (d) $\frac{7}{24}$

50. **The number of real solutions of the equation $\sqrt{1 + \cos 2x} = \sqrt{2}\cos^{-1}(\cos x)$ in $\left[\frac{\pi}{2}, \pi\right]$ is**

(a) 0 (b) 1 (c) 2 (d) ∞

51. **The principal value of $\sin^{-1}\frac{1}{2}$ is**

(a) $\frac{\pi}{6}$ (b) $\frac{5\pi}{6}$ (c) $\frac{-\pi}{6}$ (d) Both (a) & (b)

52. **The principal value of $\text{cosec}^{-1}(-1)$ is**

(a) $\frac{-\pi}{2}$ (b) 0 (c) $\frac{\pi}{2}$ (d) $\frac{3\pi}{2}$

53. **The value of $\tan^{-1}(\sqrt{3}) + \cot^{-1}(-1) + \sec^{-1}\left(\frac{-2}{\sqrt{3}}\right)$ is**

(a) $\frac{-\pi}{12}$ (b) $\frac{11\pi}{12}$ (c) $\frac{5\pi}{4}$ (d) $\frac{23\pi}{12}$

54. **The value of $2\cos^{-1}\left(\frac{-1}{2}\right) + 2\sin^{-1}\left(\frac{-1}{2}\right) - \cos^{-1}(-1)$ is**

(a) 0 (b) $\frac{\pi}{2}$ (c) π (d) 2π

55. **The value of $\sec^{-1}\left(\sec\frac{4\pi}{3}\right)$ is**

(a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$ (c) $\frac{4\pi}{3}$ (d) $\frac{-\pi}{3}$

56. **The value of $\cos^{-1}(-1) + \sin^{-1}(1)$ is**

(a) $\frac{-3\pi}{2}$ (b) $\frac{\pi}{2}$ (c) π (d) $\frac{3\pi}{2}$

57. **The value of $\cos^{-1}\left(\cos\frac{5\pi}{3}\right) + \sin^{-1}\left(\sin\frac{5\pi}{3}\right)$ is equal to**

(a) 0 (b) $\frac{\pi}{2}$ (c) $\frac{10\pi}{3}$ (d) $\frac{2\pi}{3}$

58. **The value of $\cot\left[\frac{1}{2}\sin^{-1}\frac{\sqrt{3}}{2}\right]$ is**

(*a*) 1 (*b*) $\frac{1}{\sqrt{3}}$ (*c*) $\sqrt{3}$ (*d*) 0

59. **The value of $\sin^{-1}\left[-\left(\frac{1}{2}\right)\right]+\cos^{-1}\left[-\left(\frac{1}{2}\right)\right]+\cot^{-1}(-\sqrt{3})+\mathrm{cosec}^{-1}(\sqrt{2})+\tan^{-1}(-1)$ $+\sec^{-1}(\sqrt{2})$ equals**

(*a*) $\frac{9\pi}{4}$ (*b*) $\frac{19\pi}{12}$ (*c*) $\frac{3\pi}{2}$ (*d*) $\frac{\pi}{2}$

60. **The value of $\cos^{-1}(2x^2-1)$, $0 \le x \le 1$ is equal to**

(*a*) $2\cos^{-1}x$ (*b*) $2\sin^{-1}x$ (*c*) $\pi - 2\cos^{-1}x$ (*d*) $\pi + 2\cos^{-1}x$

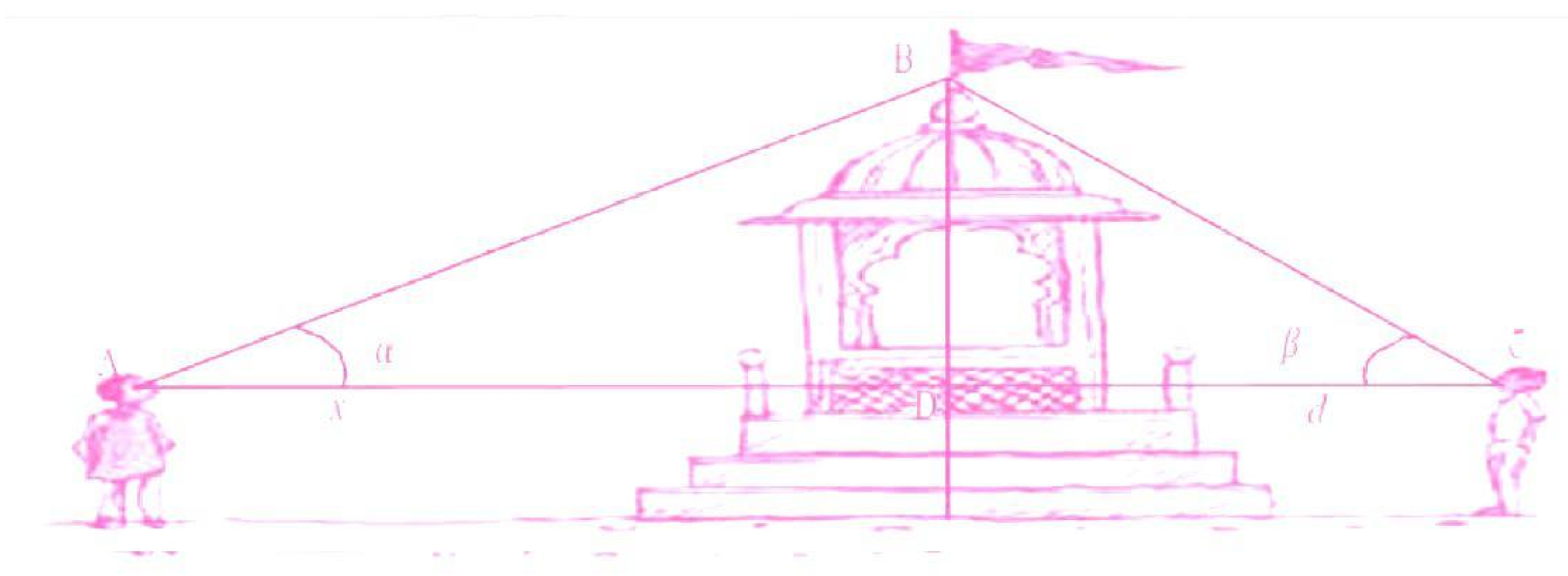
Answers

1. (*c*)	**2.** (*d*)	**3.** (*d*)	**4.** (*a*)	**5.** (*b*)	**6.** (*b*)
7. (*a*)	**8.** (*d*)	**9.** (*d*)	**10.** (*c*)	**11.** (*b*)	**12.** (*c*)
13. (*b*)	**14.** (*b*)	**15.** (*d*)	**16.** (*b*)	**17.** (*c*)	**18.** (*c*)
19. (*c*)	**20.** (*d*)	**21.** (*a*)	**22.** (*a*)	**23.** (*c*)	**24.** (*b*)
25. (*c*)	**26.** (*c*)	**27.** (*a*)	**28.** (*b*)	**29.** (*c*)	**30.** (*b*)
31. (*b*)	**32.** (*a*)	**33.** (*d*)	**34.** (*d*)	**35.** (*b*)	**36.** (*d*)
37. (*b*)	**38.** (*b*)	**39.** (*a*)	**40.** (*b*)	**41.** (*a*)	**42.** (*b*)
43. (*c*)	**44.** (*d*)	**45.** (*d*)	**46.** (*a*)	**47.** (*a*)	**48.** (*b*)
49. (*d*)	**50.** (*a*)	**51.** (*a*)	**52.** (*a*)	**53.** (*d*)	**54.** (*a*)
55. (*b*)	**56.** (*d*)	**57.** (*a*)	**58.** (*c*)	**59.** (*b*)	**60.** (*a*)

CASE-BASED QUESTIONS

Choose and write the correct option in the following questions.

1. **Read the following and answer any four questions from (*i*) to (*v*).**



Two men on either side of a temple 30 meters high observe its top at the angles of elevation α and β respectively. (as shown in the figure above). The distance between the two men is $40\sqrt{3}$ meters and the distance between the first person A and the temple is $30\sqrt{3}$ meters. Based on the above information answer the following: [*CBSE Question Bank*]

Answer the questions given below.

(*i*) $\angle CAB = \alpha =$

(*a*) $\sin^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (*b*) $\sin^{-1}\left(\frac{1}{2}\right)$ (*c*) $\sin^{-1}(2)$ (*d*) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

(ii) $\angle CAB = \alpha =$

(a) $\cos^{-1}\left(\frac{1}{5}\right)$ (b) $\cos^{-1}\left(\frac{2}{5}\right)$ (c) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (d) $\cos^{-1}\left(\frac{4}{5}\right)$

(iii) $\angle BCA = \beta =$

(a) $\tan^{-1}\left(\frac{1}{2}\right)$ (b) $\tan^{-1}(2)$ (c) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (d) $\tan^{-1}(\sqrt{3})$

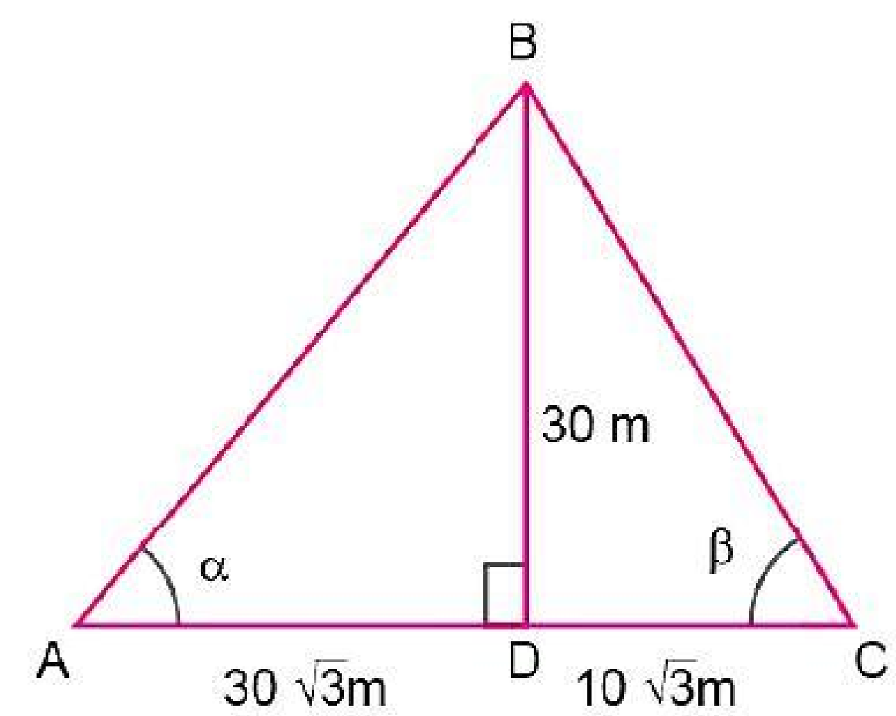
(iv) $\angle ABC =$

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{3}$

(v) Domain and Range of $\cos^{-1} x$ are respectively

(a) $(-1, 1), (0, \pi)$ (b) $[-1, 1], (0, \pi)$ (c) $[-1, 1], [0, \pi]$ (d) $(-1, 1), \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Sol. We have,



(i) Now in ΔABD(right angled)

$$\tan\alpha = \frac{BD}{AD} = \frac{30}{30\sqrt{3}} = \frac{1}{\sqrt{3}} \Rightarrow \tan\alpha = \frac{1}{\sqrt{3}} = \tan 30^\circ$$

$$\Rightarrow \alpha = 30^\circ$$

$$\Rightarrow \sin\alpha = \sin 30^\circ = \frac{1}{2}$$

$$\Rightarrow \alpha = \sin^{-1}\left(\frac{1}{2}\right)$$

$\therefore$ Option (b) is correct.

(ii) We have from (i) $\alpha = 30^\circ \Rightarrow \cos\alpha = \cos 30^\circ = \frac{\sqrt{3}}{2}$

$$\Rightarrow \alpha = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$\therefore$ Option (c) is correct.

(iii) In right ΔBCD, we have

$$\tan\beta = \frac{BD}{DC} \Rightarrow \tan\beta = \frac{30}{10\sqrt{3}} = \frac{3}{\sqrt{3}} = \sqrt{3}$$

$$\Rightarrow \beta = \tan^{-1}(\sqrt{3})$$

$\therefore$ Option (d) is correct.

(iv) In ΔABC, we have,

$$\angle ABC + \angle BAC + \angle ACB = 180^\circ$$

$$\Rightarrow \angle ABC + \alpha + \beta = 180^\circ \Rightarrow \angle ABC + 30^\circ + 60^\circ = 180^\circ \quad [\text{From (iii) } \beta = \tan^{-1}(\sqrt{3}) = 60^\circ]$$

$$\Rightarrow \angle ABC = 90^\circ$$

$$\Rightarrow \angle ABC = \frac{\pi}{2}$$

$\therefore$ Option (c) is correct.

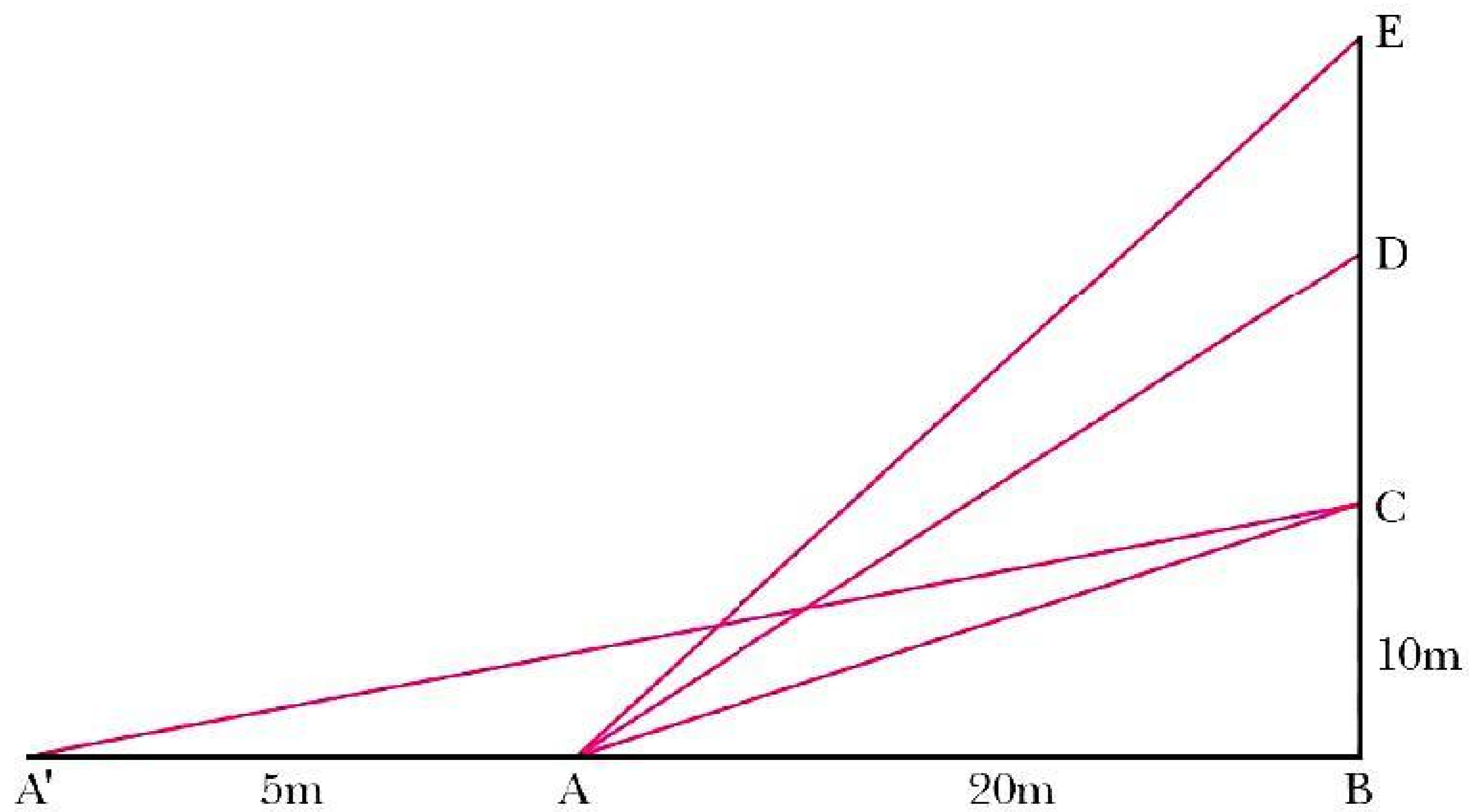
(v) Let $\cos^{-1} x = y \Rightarrow x = \cos y$

$-1 \le \cos y \le 1 \Rightarrow -1 \le x \le 1 \Rightarrow$ Domain = $[-1, 1]$

$0 \le y \le \pi \Rightarrow$ Range = $[0, \pi]$.

$\therefore$ Option (c) is correct.

2. Read the following and answer any four questions from (*i*) to (*v*).



The Government of India is planning to fix a hoarding board at the face of a building on the road of a busy market for awareness on COVID-19 protocol. Ram, Robert and Rahim are the three engineers who are working on this project. "*A*" is considered to be a person viewing the hoarding board 20 metres away from the building, standing at the edge of a pathway nearby. Ram, Robert and Rahim suggested to the firm to place the hoarding board at three different locations namely *C*, *D* and *E*. "*C*" is at the height of 10 metres from the ground level. For the viewer *A*, the angle of elevation of "*D*" is double the angle of elevation of "*C*" The angle of elevation of "*E*" is triple the angle of elevation of "*C*" for the same viewer. Look at the figure given and based on the above information answer the following: [*CBSE Question Bank*]

Answer the questions given below.

(*i*) Measure of $\angle CAB$ =

(a) $\tan^{-1}(2)$ (b) $\tan^{-1}\left(\frac{1}{2}\right)$ (c) $\tan^{-1}(1)$ (d) $\tan^{-1}(3)$

(*ii*) Measure of $\angle DAB$ =

(a) $\tan^{-1}\left(\frac{3}{4}\right)$ (b) $\tan^{-1}(3)$ (c) $\tan^{-1}\left(\frac{4}{3}\right)$ (d) $\tan^{-1}(4)$

(*iii*) Measure of $\angle EAB$ =

(a) $\tan^{-1}(11)$ (b) $\tan^{-1}3$ (c) $\tan^{-1}\left(\frac{2}{11}\right)$ (d) $\tan^{-1}\left(\frac{11}{2}\right)$

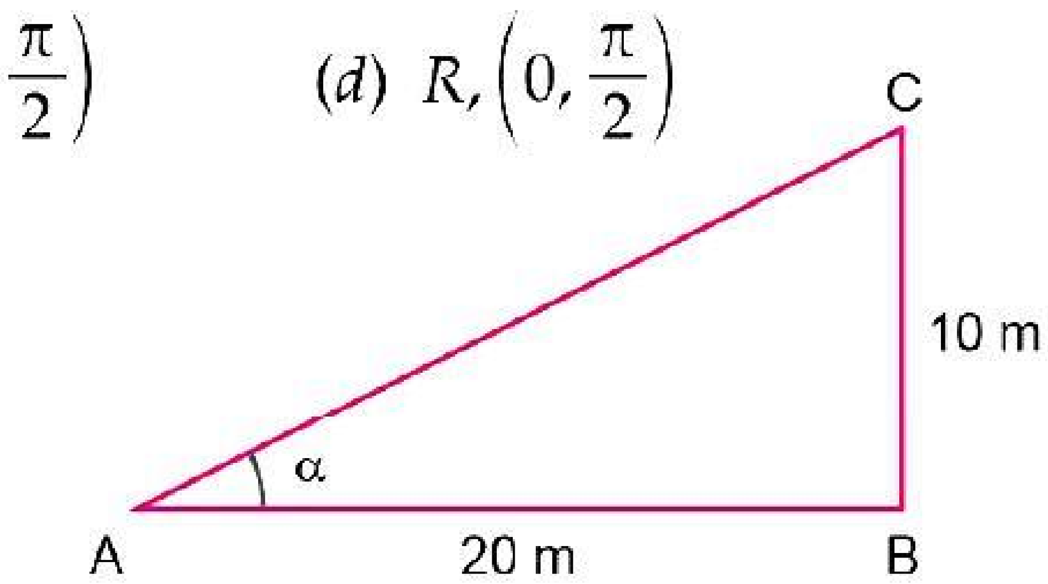
(*iv*) *A'* Is another viewer standing on the same line of observation across the road. If the width of the road is 5 meters, then the difference between $\angle CAB$ and $\angle CA'B$ is

(a) $\tan^{-1}\left(\frac{1}{12}\right)$ (b) $\tan^{-1}\left(\frac{1}{8}\right)$ (c) $\tan^{-1}\left(\frac{2}{5}\right)$ (d) $\tan^{-1}\left(\frac{11}{21}\right)$

(*v*) Domain and Range of $\tan^{-1} x$ are respectively

(a) $R^+, \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (b) $R^-, \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (c) $R, \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (d) $R, \left(0, \frac{\pi}{2}\right)$

Sol. Let $\angle CAB = \alpha$, therefor $\angle DAB = 2\alpha$ and $\angle EAB = 3\alpha$.



(i) In right ΔABC, we have

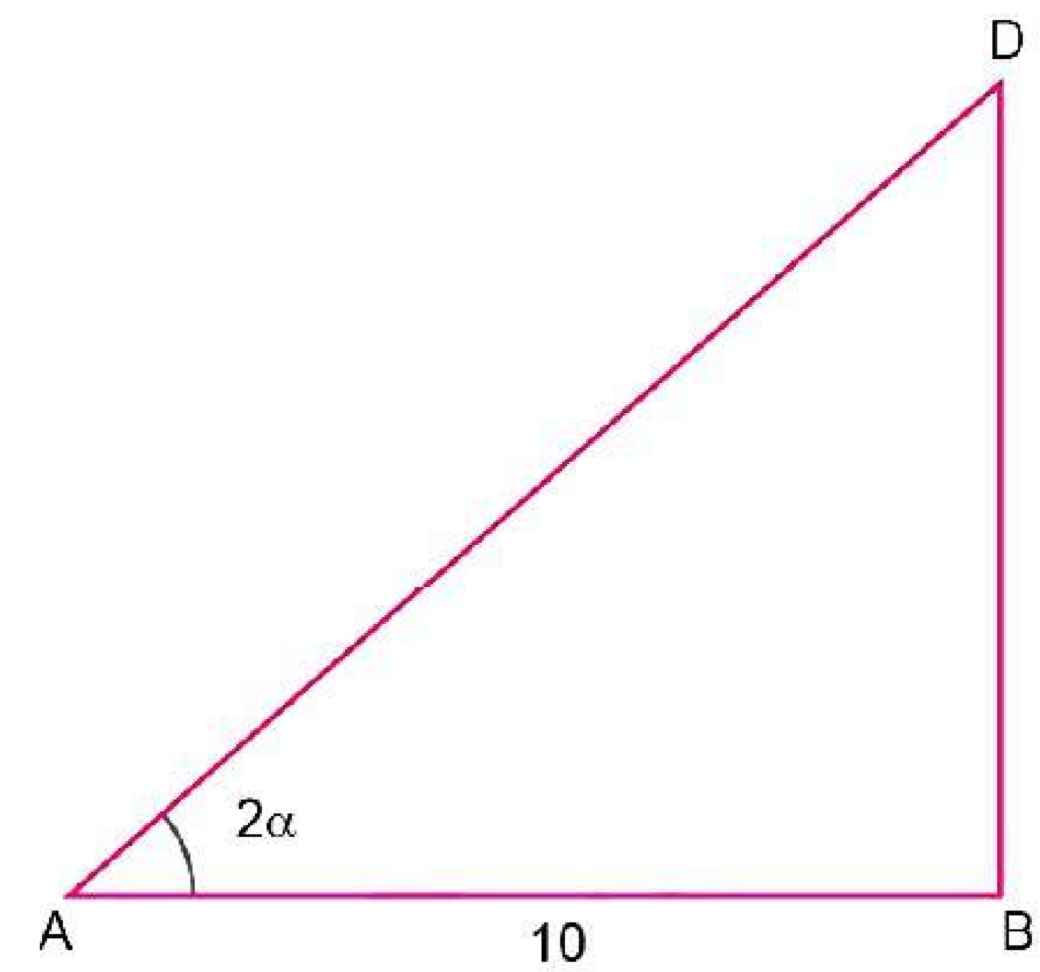
$$\tan \alpha = \frac{BC}{AB} = \frac{10}{20} = \frac{1}{2}$$

$\Rightarrow \quad \alpha = \tan^{-1}\left(\frac{1}{2}\right)$

$\Rightarrow \quad \angle CAB = \tan^{-1}\left(\frac{1}{2}\right)$

$\therefore$ Option (*b*) is correct.

(*ii*) We have, $\tan\alpha = \frac{1}{2}$ (from (*i*))



$$\therefore \quad \tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha} = \frac{2\times\frac{1}{2}}{1-\frac{1}{4}}$$

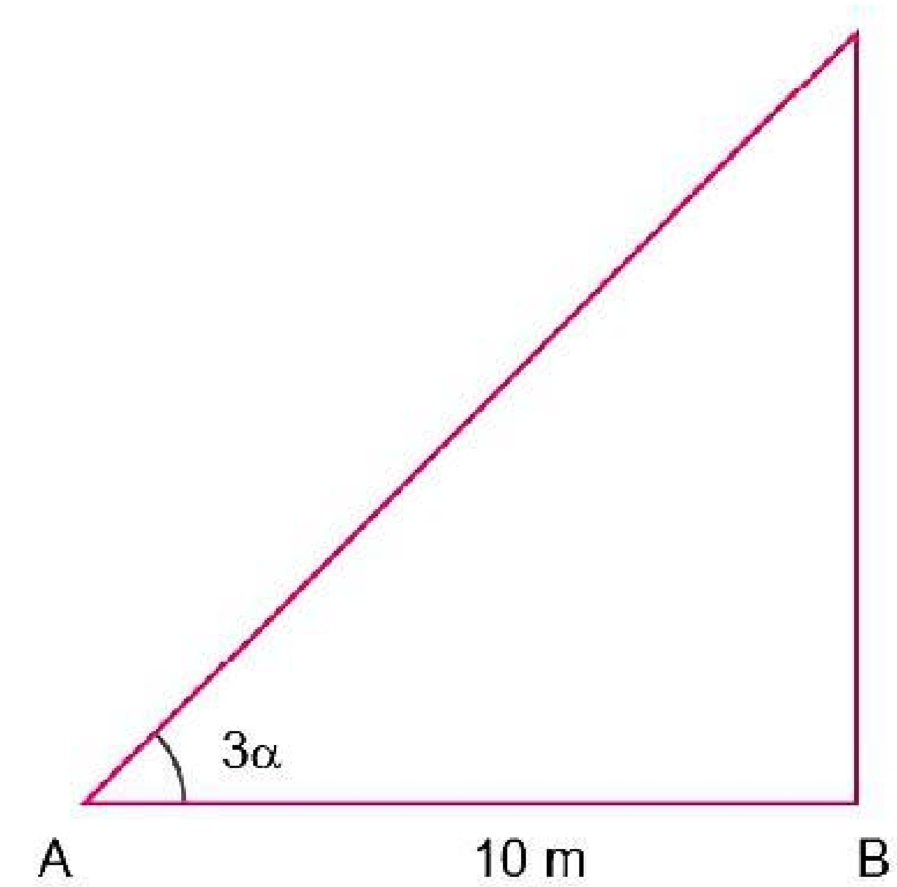
$$\tan 2\alpha = \frac{1}{\frac{3}{4}} = \frac{4}{3}$$

$\Rightarrow \quad 2\alpha = \tan^{-1}\left(\frac{4}{3}\right)$

$\Rightarrow \quad \angle DAB = \tan^{-1}\left(\frac{4}{3}\right)$

$\therefore$ Option (*c*) is correct.

(*iii*) We have, $\tan\alpha = \frac{1}{2}$



$$\therefore \quad \tan 3\alpha = \frac{3\tan\alpha - \tan^3\alpha}{1-3\tan^2\alpha}$$

$$= \frac{3\times\frac{1}{2} - \left(\frac{1}{2}\right)^3}{1-3\times\left(\frac{1}{2}\right)^2}$$

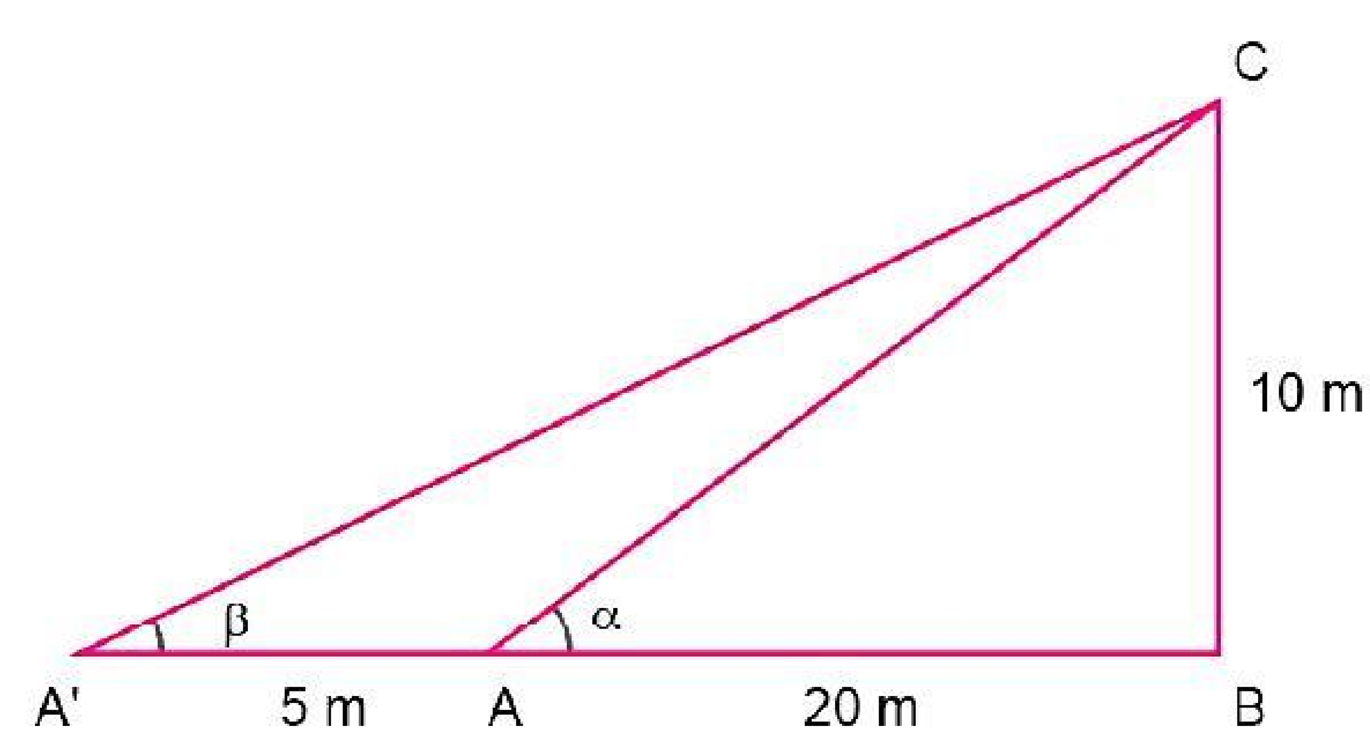
$$\tan 3\alpha = \frac{\frac{3}{2}-\frac{1}{8}}{1-\frac{3}{4}} = \frac{\frac{11}{8}}{\frac{1}{4}} = \frac{11}{8}\times 4 = \frac{11}{2}$$

$\Rightarrow \quad 3\alpha = \tan^{-1}\left(\frac{11}{2}\right) \quad \Rightarrow \quad \angle EAB = \tan^{-1}\left(\frac{11}{2}\right).$

$\therefore$ Option (*d*) is correct.

(*iv*) Let $\angle CA'B = \beta$

We have, In $A'CB$



$$\tan\beta = \frac{BC}{A'B} = \frac{10}{20+5}$$

$\Rightarrow \quad \tan\beta = \frac{10}{25} = \frac{2}{5}$

$\Rightarrow \quad \beta = \tan^{-1}\left(\frac{2}{5}\right)$

$\Rightarrow \quad \angle CA'B = \tan^{-1}\left(\frac{2}{5}\right)$ and we already know $\angle CAB = \tan^{-1}\left(\frac{1}{2}\right)$

$\therefore \quad \angle CAB - \angle CA'B = \tan^{-1}\left(\frac{1}{2}\right) - \tan^{-1}\left(\frac{2}{5}\right)$

$$= \tan^{-1}\left(\frac{\frac{1}{2}-\frac{2}{5}}{1+\frac{1}{2}\times\frac{2}{5}}\right) = \tan^{-1}\left(\frac{\frac{5-4}{10}}{\frac{12}{10}}\right)$$

$$= \tan^{-1}\left(\frac{1}{12}\right)$$

$\therefore$ Option (*a*) is correct.

(*v*) Domain and Range of $\tan^{-1} x$ are respectively $(-\infty, \infty)$ and $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ *i.e* R, $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\therefore$ Option (*c*) is correct.

ASSERTION-REASON QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(*a*) *Both A and R are true and R is the correct explanation of A.*

(*b*) *Both A and R are true but R is not the correct explanation of A.*

(*c*) *A is true but R is false.*

(*d*) *A is false and R is also false.*

1. **Assertion (A) :** Domain of $f(x) = \sin^{-1} x + \cos x$ is $[-1, 1]$.

   Reason (R) : Domain of a function is the set of all possible values for which function will be defined.

2. **Assertion (A) :** Function $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = \sin x$ is not a bijection.

   Reason (R) : A function $f : A \to B$ is said to be bijection if it is one-one and onto.

3. **Assertion (A) :** Principal value of $\tan^{-1}(-\sqrt{3})$ is $-\frac{\pi}{3}$.

   Reason (R) : $\tan^{-1} : \mathbb{R} \to \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ so for any $x \in \mathbb{R}$, $\tan^{-1}(x)$ represent an angle in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

4. **Assertion (A) :** $\sin^{-1}(-x) = -\sin^{-1} x;\ x \in [-1, 1]$

   Reason (R) : $\sin^{-1} : [-1, 1] \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ is a bijection map.

Answers

1. (*a*) **2.** (*a*) **3.** (*a*) **4.** (*b*)

HINTS/SOLUTIONS OF SELECTED MCQS

1. We have,

$$\tan^{-1}(\sqrt{3}) + \cos^{-1}\left(-\frac{1}{2}\right) = \tan^{-1}\left(\tan\frac{\pi}{3}\right) + \cos^{-1}\left(-\cos\frac{\pi}{3}\right)$$

$$= \frac{\pi}{3} + \cos^{-1}\left[\cos\left(\pi - \frac{\pi}{3}\right)\right] = \frac{\pi}{3} + \cos^{-1}\left(\cos\frac{2\pi}{3}\right)$$

$$= \frac{\pi}{3} + \frac{2\pi}{3} = \frac{3\pi}{3} = \pi$$

Option (*c*) is correct.

2. Let $\sin^{-1} x = \theta$, then $\sin\theta = x$.

$\Rightarrow \quad \operatorname{cosec}\theta = \frac{1}{x} \quad \Rightarrow \quad \operatorname{cosec}^2\theta = \frac{1}{x^2}$

$\Rightarrow \quad 1 + \cot^2\theta = \dfrac{1}{x^2} \quad \Rightarrow \quad \cot\theta = \dfrac{\sqrt{1-x^2}}{x}$

Option (*d*) is correct.

3. $\sin^{-1}\left(\cos\dfrac{\pi}{9}\right) = \sin^{-1}\left(\sin\left(\dfrac{\pi}{2} - \dfrac{\pi}{9}\right)\right) = \sin^{-1}\left(\sin\dfrac{7\pi}{18}\right) = \dfrac{7\pi}{18}$

Option (*d*) is correct.

4. $\theta = \sin^{-1}(\sin(-600)) = \sin^{-1}(-\sin 600)$

$= -\sin^{-1}(\sin 600) = -\sin^{-1}(\sin(2 \times 360 - 120))$

$= -\sin^{-1}(-\sin 120) = \sin^{-1}(\sin 120^\circ)$

$= \sin^{-1}\left(\dfrac{\sqrt{3}}{2}\right) = \dfrac{\pi}{3}$

Option (*a*) is correct.

5. We have,

$2\sec^{-1}(2) + \sin^{-1}\left(\dfrac{1}{2}\right) = 2\sec^{-1}\left(\sec\left(\dfrac{\pi}{3}\right)\right) + \sin^{-1}\left(\sin\left(\dfrac{\pi}{6}\right)\right)$

$= 2 \times \dfrac{\pi}{3} + \dfrac{\pi}{6} = \dfrac{2\pi}{3} + \dfrac{\pi}{6} = \dfrac{5\pi}{6}$

Option (*b*) is correct.

6. $\tan^2(\sec^{-1} 2) + \cot^2(\text{cosec}^{-1} 3)$

Let $\sec^{-1} 2 = \alpha \Rightarrow \sec\alpha = 2$ $\text{cosec}^{-1} 3 = \beta \Rightarrow \text{cosec}\,\beta = 3$

$\therefore \tan^2(\sec^{-1} 2) + \cot^2(\text{cosec}^{-1} 3) = \tan^2(\alpha) + \cos^2(\beta)$

$= \sec^2\alpha - 1 + \text{cosec}^2\beta - 1 = \sec^2\alpha + \text{cosec}^2\beta - 2$

$= 4 + 9 - 2 = 11$

Option (*b*) is correct.

7. $\because \quad f(x) = \sin^{-1}\sqrt{x-1}$

$\Rightarrow \quad 0 \le x - 1 \le 1 \Rightarrow 1 \le x \le 2$ $\quad [\because \sqrt{x-1} \ge 0$ and $-1 \le \sqrt{x-1} \le 1]$

$\therefore \quad x \in [1, 2]$

Option (*a*) is correct.

8. $\cot\left(\cos^{-1}\left(\dfrac{7}{25}\right)\right)$

Put $\cos^{-1}\left(\dfrac{7}{25}\right) = \alpha \Rightarrow \cos\alpha = \dfrac{7}{25}$

$\Rightarrow \sin\alpha = \sqrt{1-\cos^2\alpha} = \sqrt{1 - \dfrac{49}{625}} = \sqrt{\dfrac{625-49}{625}} = \dfrac{\sqrt{576}}{\sqrt{625}} = \dfrac{24}{25}$

$\therefore \cot\left[\cos^{-1}\left(\dfrac{7}{25}\right)\right] = \cot\alpha = \dfrac{\cos\alpha}{\sin\alpha} = \dfrac{7/25}{24/25} = \dfrac{7}{24}$

Option (*d*) is correct.

9. Let $\tan^{-1} x = \alpha \Rightarrow \tan\alpha = x$

$\Rightarrow \sec\alpha = \sqrt{1+\tan^2\alpha} = \sqrt{1+x^2}$

$$\Rightarrow \cos\alpha = \frac{1}{\sqrt{1+x^2}}$$

$$\therefore \sin\alpha = \sqrt{1-\cos^2\alpha} = \sqrt{1-\frac{1}{1+x^2}} = \sqrt{\frac{1+x^2-1}{1+x^2}}$$

$$= \frac{x}{\sqrt{1+x^2}}$$

$$\therefore \sin(\tan^{-1} x) = \sin(\alpha) = \frac{x}{\sqrt{1+x^2}}$$

Option (*d*) is correct.

10. We have, $\cos^{-1}\alpha + \cos^{-1}\beta + \cos^{-1}\gamma = 3\pi$

We know that, $0 \le \cos^{-1} x \le \pi$

$\cos^{-1}\alpha + \cos^{-1}\beta + \cos^{-1}\gamma = 3\pi$

If and only if, $\cos^{-1}\alpha = \cos^{-1}\beta = \cos^{-1}\gamma = \pi$

$\Rightarrow \cos\pi = \alpha = \beta = \gamma \quad \Rightarrow \quad -1 = \alpha = \beta = \gamma$

$\Rightarrow \alpha = \beta = \gamma = -1$

$\therefore \alpha(\beta+\gamma) + \beta(\gamma+\alpha) + \gamma(\alpha+\beta) = -1(-1-1) - 1(-1-1) - 1(-1-1)$

$= 2 + 2 + 2 = 6$

Option (*c*) is correct.

11. $\tan^{-1}\sqrt{3} = \tan^{-1}\left(\tan\frac{\pi}{3}\right) = \frac{\pi}{3}$

$\sec^{-1}(-2) = \sec^{-1}\left(-\sec\frac{\pi}{3}\right), = \sec^{-1}\left(\sec\left(\pi - \frac{\pi}{3}\right)\right)$

$= \sec^{-1}\left(\sec\frac{2\pi}{3}\right) = \frac{2\pi}{3}$

$\therefore \tan^{-1}\sqrt{3} - \sec^{-1}(-2) = \frac{\pi}{3} - \frac{2\pi}{3} = -\frac{\pi}{3}$

Option (*b*) is correct.

12. Let $\cos^{-1} x = \theta$

$\Rightarrow x = \cos\theta \Rightarrow \sec\theta = \frac{1}{x} \Rightarrow \tan\theta = \sqrt{\sec^2\theta - 1}$

$\Rightarrow \tan\theta = \sqrt{\frac{1}{x^2} - 1} \Rightarrow \tan\theta = \frac{1}{x}\sqrt{1-x^2}$

Now, $\sin[\cot^{-1}\{\tan(\cos^{-1}x)\}] = \sin[\cot^{-1}\{\tan\theta\}]$

$= \sin\left[\cot^{-1}\left\{\frac{1}{x}\sqrt{1-x^2}\right\}\right]$

Again, let $x = \sin\alpha = \sin\left[\cot^{-1}\left\{\frac{1}{\sin\alpha}\sqrt{1-\sin\alpha^2}\right\}\right]$

$= \sin\left[\cot^{-1}\left\{\frac{\cos\alpha}{\sin\alpha}\right\}\right] = \sin[\cot^{-1}(\cot\alpha)]$

$= \sin\alpha = x$

Option (*c*) is correct.

13. $\sin^{-1}(\sin 600)^\circ = \sin^{-1}[\sin(540^\circ + 60^\circ)]$

$- \sin^{-1}\left[\sin\left(3\pi + \frac{\pi}{3}\right)\right] - \sin^{-1}\left[-\sin\frac{\pi}{3}\right]$

$$= \sin^{-1}\left[\sin\left(-\frac{\pi}{3}\right)\right] = -\frac{\pi}{3}$$

Option (*b*) is correct.

14. $\cos^{-1}\left(\cos\frac{7\pi}{6}\right) = \cos^{-1}\left(\cos\left(2\pi - \frac{5\pi}{6}\right)\right) = \cos^{-1}\left(\cos\frac{5\pi}{6}\right)$

$$= \frac{5\pi}{6} \quad \left[\because \frac{5\pi}{6} \in [0, \pi]\right]$$

Option (*b*) is correct.

15. $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right] = \sin\left[\frac{\pi}{3} - \left(-\frac{\pi}{6}\right)\right] \quad \left[\because \sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}\right]$

$$= \sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right) = \sin\frac{\pi}{2} = 1$$

Option (*d*) is correct.

16. $\because \tan^{-1}(\sqrt{3}) - \cot^{-1}(-\sqrt{3})$

$$= \tan^{-1}(\sqrt{3}) - (\pi - \cot^{-1}(\sqrt{3}))$$

$$= \tan^{-1}(\sqrt{3}) - \pi + \cot^{-1}(\sqrt{3})$$

$$= \tan^{-1}(\sqrt{3}) + \cot^{-1}(\sqrt{3}) - \pi = \frac{\pi}{2} - \pi = -\frac{\pi}{2}$$

Option (*b*) is correct.

17. Given, $y = \cos^{-1}(x^2 - 4)$

$\Rightarrow \cos y = x^2 - 4$

$\Rightarrow -1 \le x^2 - 4 \le 1 \qquad [\because -1 \le \cos y \le 1]$

$\Rightarrow -1 + 4 \le x^2 - 4 + 4 \le 1 + 4$

$\Rightarrow 3 \le x^2 \le 5$

$\Rightarrow x^2 \ge 3$ and $x^2 \le 5$

$\Rightarrow x^2 - 3 \ge 0$ and $x^2 - 5 \le 0$

Now, $x^2 - 3 \ge 0 \Rightarrow x^2 - (\sqrt{3})^2 \ge 0$

$\Rightarrow (x + \sqrt{3})(x - \sqrt{3}) \ge 0$

$\Rightarrow (x + \sqrt{3}) \ge 0, (x - \sqrt{3}) \ge 0$ or $(x + \sqrt{3}) \le 0, (x - \sqrt{3}) \le 0$

$\Rightarrow x \ge -\sqrt{3}, x \ge \sqrt{3}$ or $x \le -\sqrt{3}, x \le \sqrt{3}$

$\Rightarrow x \ge \sqrt{3}$ or $x \le -\sqrt{3}$

$\rightarrow x \in (-\infty, -\sqrt{3}] \cup [\sqrt{3}, \infty) \qquad \ldots(i)$

Again, $x^2 - 5 \le 0 \Rightarrow x^2 - (\sqrt{5})^2 \le 0$

$\Rightarrow (x + \sqrt{5})(x - \sqrt{5}) \le 0$

$\Rightarrow (x + \sqrt{5}) \ge 0, (x - \sqrt{5}) \le 0$ or $(x + \sqrt{5}) \le 0, (x - \sqrt{5}) \ge 0$

$\Rightarrow x \ge -\sqrt{5}, x \le \sqrt{5}$ or $x \le -\sqrt{5}, x \ge \sqrt{5}$

$\Rightarrow -\sqrt{5} \le x \le \sqrt{5}$ or $x \le -\sqrt{5}, x \ge \sqrt{5}$ not possible

$\Rightarrow x \in [-\sqrt{5}, \sqrt{5}] \qquad \ldots(ii)$

(i) and (ii) $x \in \{(-\infty, -\sqrt{3}] \cup [\sqrt{3}, \infty)\} \cap [-\sqrt{5}, \sqrt{5}] \Rightarrow x \in [-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$

Option (c) is correct.

18. The domain of cos function is R and the domain of $\sin^{-1}$ is $[-1, 1]$

$\Rightarrow$ The domain of $\sin^{-1} x + \cos x$ is $R \cap [-1, 1] = [-1, 1]$

Option (c) is correct.

19. Let $\sin^{-1}(0.8) = \theta \Rightarrow \sin\theta = 0.8$

$\Rightarrow \cos\theta = \sqrt{1 - \sin^2\theta} \Rightarrow \cos\theta = \sqrt{1 - (0.8)^2}$

$\Rightarrow \cos\theta = \sqrt{1 - 0.64} \Rightarrow \cos\theta = \sqrt{0.36}$

$\Rightarrow \cos\theta = 0.6$

$\therefore \sin(2\sin^{-1}(0.8)) = \sin 2\theta = 2\sin\theta\cos\theta = 2 \times 0.8 \times 0.6 = 0.96$

Option (c) is correct.

20. $\because$ 1 rad $= 57.75° \Rightarrow 5$ rad $= 288.75°$

$\Rightarrow \frac{3\pi}{2} < 5 < \frac{5\pi}{2} \Rightarrow \sin^{-1}(\sin 5) = 5 - 2\pi$

Now, $\because \sin^{-1}(\sin 5) > x^2 - 4x$

$\Rightarrow 5 - 2\pi > x^2 - 4x \Rightarrow x^2 - 4x + (2\pi - 5) < 0$

$\Rightarrow 2 - \sqrt{9 - 2\pi} < x < 2 + \sqrt{9 - 2\pi})$

$\left(\begin{array}{l}\because x^2 - 4x + (2\pi - 5) = 0 \\ \Rightarrow x = 2 \pm \sqrt{9 - 2\pi}\end{array}\right)$

$\Rightarrow x \in (2 - \sqrt{9 - 2\pi}, 2 + \sqrt{9 - 2\pi})$

Option (d) is correct.

21. Since, $-1 \le \sin^{-1} x \le 1$ and given $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$

$\Rightarrow \sin^{-1} x = \sin^{-1} y = \sin^{-1} z = \frac{\pi}{2}$

$\Rightarrow x = y = z = 1$

Also, $\because f(1) = 1$

and $f(p + q) = f(p).f(q) \; \forall \; p, q \in R$, then

$\Rightarrow f(2) = f(1 + 1) = f(1) . f(1) = 1 \times 1 = 1$

$\Rightarrow f(3) = f(2 + 1) = f(2) . f(1) = 1 \times 1 = 1$

Now, $x^{f(1)} + y^{f(2)} + z^{f(3)} - \frac{x + y + z}{x^{f(1)} + y^{f(2)} + z^{f(3)}} = 1 + 1 + 1 - \frac{1 + 1 + 1}{x^1 + y^1 + z^1}$

$= 3 - \frac{3}{1 + 1 + 1} = 3 - \frac{3}{3} = 3 - 1 = 2$

Option (a) is correct.

22. Given $u = \cot^{-1}(\sqrt{\tan\alpha}) - \tan^{-1}(\sqrt{\tan\alpha})$

Put $\sqrt{\tan\alpha} = x$

$\Rightarrow u = \cot^{-1}(x) - \tan^{-1}(x) = \frac{\pi}{2} - \tan^{-1} x - \tan^{-1}(x)$

$\Rightarrow\ u = \frac{\pi}{2} - 2\tan^{-1}x \Rightarrow 2\tan^{-1}x = \frac{\pi}{2} - u$

$\Rightarrow\ \tan^{-1}x = \frac{\pi}{4} - \frac{u}{2}$

$\Rightarrow\ x = \tan\left(\frac{\pi}{4} - \frac{u}{2}\right) \Rightarrow\ \tan\left(\frac{\pi}{4} - \frac{u}{2}\right) = x = \sqrt{\tan\alpha}$

Option (*a*) is correct.

23. $\cos^{-1}x > \sin^{-1}x \Rightarrow \cos^{-1}x > \frac{\pi}{2} - \cos^{-1}x$

$\Rightarrow\ 2\cos^{-1}x > \frac{\pi}{2} \Rightarrow \cos^{-1}x > \frac{\pi}{4}$

$\Rightarrow\ x < \cos\left(\frac{\pi}{4}\right) \left[\because \cos x \text{ is decreasing function in} \left[0, \frac{\pi}{2}\right]\right]$

$\Rightarrow\ x < \frac{1}{\sqrt{2}}$

Hence $x \in \left[-1, \frac{1}{\sqrt{2}}\right]$ [As domain of $\cos^{-1}x$ [–1, 1]]

Option (*c*) is correct.

24. $\because\ \sin : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \longrightarrow [-1, 1]$ is one-one and onto function.

$\therefore$ Domain of $f(x) = \sin^{-1}x$ is [–1, 1].

Option (*b*) is correct.

25. Domain of $\sin^{-1}\theta$ is [–1, 1] *i.e.,* $\theta \in [-1, 1]$

$\therefore$ Domain of $\sin^{-1}(-x^2)$ is

$-x^2 \in [-1, 1] \Rightarrow -1 \le -x^2 \le 1$

$\Rightarrow\ 1 \ge x^2 \ge 0 \Rightarrow x \in [-1, 1]$

Domain of $\sin^{-1}(-x^2)$ is [–1, 1].

Option (*c*) is correct.

26. We have

$ax + b\sec(\tan^{-1}x) = c$

$\Rightarrow\ ax + b\sec(\sec^{-1}(\sqrt{1+x^2}) = c$

$\Rightarrow\ ax + b\sqrt{1+x^2} = c \Rightarrow b\sqrt{1+x^2} = c - ax$

$\Rightarrow\ b^2(1+x^2) = (c-ax)^2 = c^2 + a^2x^2 - 2acx$

$\Rightarrow\ (a^2 - b^2)x^2 - 2acx + (c^2 - b^2) = 0$

$\Rightarrow\ x = \frac{2ac \pm \sqrt{4a^2c^2 - 4(a^2-b^2)(c^2-b^2)}}{2(a^2-b^2)}$

Similarly,

$ay + b\sec(\tan^{-1}y) = c$

$\Rightarrow\ y = \frac{2ac \pm \sqrt{4a^2c^2 - 4(a^2-b^2)(c^2-b^2)}}{2(a^2-b^2)}$

$\therefore\ xy = \frac{c^2 - b^2}{a^2 - b^2}$ and $x + y = \frac{2ac}{a^2 - b^2}$

$$\therefore \quad \frac{x+y}{1-xy} = \frac{2ac}{a^2-c^2}$$

Option (*c*) is correct.

27. $\alpha = \tan^{-1}\left(\tan\left(\frac{5\pi}{4}\right)\right), \beta = \tan^{-1}\left(-\tan\left(\frac{2\pi}{3}\right)\right)$

$$\Rightarrow \quad \tan\alpha = \tan\frac{5\pi}{4} = \tan\left(\pi + \frac{\pi}{4}\right) = \tan\frac{\pi}{4} \quad \Rightarrow \quad \alpha = \frac{\pi}{4}$$

$$\beta = \tan^{-1}\left(-\tan\left(\frac{2\pi}{3}\right)\right) = \tan^{-1}\left(\tan\left(-\frac{2\pi}{3}\right)\right)$$

$$\Rightarrow \quad \tan\beta = \tan\left(-\frac{2\pi}{3}\right) = -\tan\left(\frac{2\pi}{3}\right) = \tan\left(\pi - \frac{2\pi}{3}\right)$$

$$\Rightarrow \quad \tan\beta = \tan\frac{\pi}{3} \Rightarrow \beta = \frac{\pi}{3}$$

$\therefore 4\alpha = 3\beta$

So option (*a*) is correct option.

28. If $x = \frac{\sqrt{3}}{2}$, then

$$\tan\left(\frac{\sin^{-1}x + \cos^{-1}x}{2}\right) = \tan\left(\frac{\sin^{-1}\frac{\sqrt{3}}{2} + \cos^{-1}\frac{\sqrt{3}}{2}}{2}\right)$$

$$= \tan\left(\frac{\frac{\pi}{3} + \frac{\pi}{6}}{2}\right) = \tan\left(\frac{\frac{\pi}{2}}{2}\right) = \tan\frac{\pi}{4} = 1$$

Option (*b*) is correct.

29. Let $f(x) = \cos^{-1}[x]$

Now, domain of $g(x) = \cos^{-1}x$ is the set

$\{x \mid -1 \le x \le 1\} = [-1, 1]$

$\therefore$ Domain of given function is $\{x : -1 \le [x] \le 1\}$

$$[x] = \begin{cases} -1 & \text{if} \quad -1 \le x \le 0 \\ 0 & \text{if} \quad 0 \le x \le 1 \\ 1 & \text{if} \quad 1 < x < 2 \end{cases}$$

$\therefore$ Domain of $\cos^{-1}[x]$ is $[-1, 2)$.

Option (*c*) is correct.

30. We have

$f(x) = \sin 2x + \cos 2x$, $g(x) = x^2 - 1$

$\therefore g(f(x)) = (f(x))^2 - 1 = (\sin 2x + \cos 2x)^2 - 1$

$= \sin^2 2x + \cos^2 2x + 2\sin 2x \times \cos 2x - 1$

$= 1 + \sin 4x - 1 = \sin 4x$

Now, $\sin 4x$ will be invertible for $-\frac{\pi}{2} \le 4x \le \frac{\pi}{2} \Rightarrow -\frac{\pi}{8} \le x \le \frac{\pi}{8}$

i.e. Domain of $g(f(x))$ for which it is invertible is $\left[-\frac{\pi}{8}, \frac{\pi}{8}\right]$

Option (*b*) is correct.

31. Domain of $\sin^{-1}x$ is $[-1, 1]$.

$\therefore$ Domain of $\sin^{-1}[x]$ is $\{x : -1 \le [x] \le 1\}$

But $[x] = \begin{cases} -1 & \forall \quad -1 \le x < 0 \\ 0 & \forall \quad 0 \le x < 1 \\ 1 & \forall \quad 1 \le x < 2 \end{cases}$

$\therefore$ Domain of $\sin^{-1}[x]$ is $[-1, 2)$

Option (*b*) is correct.

32. We have $\sin(\pi \cos x) = \cos(\pi \sin x)$

$\Rightarrow \sin(\pi \cos x) = \sin(\pi/2 \pm \pi \sin x)$

$\Rightarrow \pi \cos x = \dfrac{\pi}{2} \pm \pi \sin x \Rightarrow \pi \cos x \mp \pi \sin x = \dfrac{\pi}{2}$

$\Rightarrow \cos x \mp \sin x = \dfrac{1}{2}$

Squaring both sides, we get

$1 \pm \sin 2x = \dfrac{1}{4} \Rightarrow \pm \sin 2x = \dfrac{1}{4} - 1$

$\Rightarrow \pm \sin 2x = -\dfrac{3}{4} \Rightarrow \sin 2x = \pm \dfrac{3}{4}$

$\Rightarrow x = \dfrac{1}{2}\sin^{-1}\left(\pm \dfrac{3}{4}\right) \Rightarrow x = \pm \dfrac{1}{2}\sin^{-1}\left(\dfrac{3}{4}\right)$

Option (*a*) is correct.

33. $\cos^{-1} x - \cos^{-1} \dfrac{y}{2} = \alpha$

$\Rightarrow \cos^{-1}\left(x \times \dfrac{y}{2} + \sqrt{1 - x^2}\sqrt{1 - \dfrac{y^2}{4}}\right) = \alpha$

$\Rightarrow \dfrac{xy}{2} + \sqrt{1 - x^2}\sqrt{1 - \dfrac{y^2}{4}} = \cos \alpha$

$\Rightarrow \sqrt{1 - x^2}\sqrt{1 - \dfrac{y^2}{4}} = \cos \alpha - \dfrac{xy}{2}$

Squaring both the sides, we get

$(1 - x^2)\left(1 - \dfrac{y^2}{4}\right) = \left(\cos \alpha - \dfrac{xy}{2}\right)^2$

$\Rightarrow 1 - \dfrac{y^2}{4} - x^2 + \dfrac{x^2 y^2}{4} = \cos^2 \alpha + \dfrac{x^2 y^2}{4} - xy \cos \alpha$

$\Rightarrow 1 - \dfrac{y^2}{4} - x^2 = \cos^2 \alpha - xy \cos \alpha \Rightarrow 4 - y^2 - 4x^2 = 4\cos^2 \alpha - 4xy \cos \alpha$

$\Rightarrow 4 - 4\cos^2 \alpha = 4x^2 + y^2 - 4xy \cos \alpha$

$\Rightarrow 4(1 - \cos^2 \alpha) = 4x^2 + y^2 - 4xy \cos \alpha$

$\Rightarrow 4x^2 - 4xy \cos \alpha + y^2 = 4 \sin^2 \alpha$

Option (*d*) is correct.

34. $3 \sin x + 4 \cos x = y^2 - 2y + 6 = (y - 1)^2 + 5$

Which is possible only when $y - 1 = 0 \Rightarrow y = 1$

($\because$ Maximum value of LHS = 5 and Minimum value of RHS = 5)

$\therefore 3 \sin x + 4 \cos x = 5$

Put $3 = r \cos \theta, 4 = r \sin \theta$ such that $9 + 16 = r^2$

$\Rightarrow r^2 = 25 \Rightarrow r = 5$. Also $\tan\theta = \frac{4}{3} \Rightarrow \theta = \tan^{-1}\left(\frac{4}{3}\right)$

$\Rightarrow r\cos\theta\sin x + r\sin\theta\cos x = 5$

$\Rightarrow r\sin(x+\theta) = r \Rightarrow \sin(x+\theta) = 1$

$\Rightarrow x + \theta = \sin^{-1}(1) = \frac{\pi}{2}$

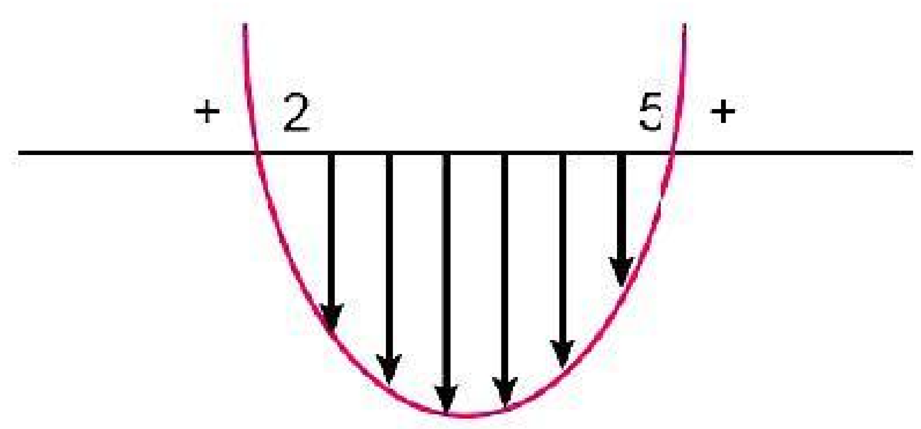
$\Rightarrow x = \frac{\pi}{2} - \theta = \frac{\pi}{2} - \tan^{-1}\left(\frac{4}{3}\right)$

$\therefore xy = \frac{\pi}{2} - \tan^{-1}\left(\frac{4}{3}\right)$

So all option are true.

Option (*d*) is correct.

35. $(\cot^{-1}x)^2 - 7(\cot^{-1}x) + 10 > 0$



$\Rightarrow (\cot^{-1}x)^2 - 2\cot^{-1}x - 5\cot^{-1}x + 10 > 0$

$\Rightarrow (\cot^{-1}x)(\cot^{-1}x - 2) - 5(\cot^{-1}x - 2) > 0$

$\Rightarrow (\cot^{-1}x - 2)(\cot^{-1}x - 5) > 0$

$\Rightarrow \cot^{-1}x < 2, \cot^{-1}x > 5 \quad [\because \cot^{-1}x \text{ is decreasing } \forall x \in \mathbb{R}]$

$\Rightarrow x \in (-\infty, \cot 5) \cup (\cot 2, \infty)$

Option (*b*) is correct.

36. Domain of $\sin^{-1}x$ is $[-1, 1]$ and domain of $\frac{1}{x}$ is $\mathbb{R} - \{0\}$

$\therefore$ Domain of $f(x) = \frac{\sin^{-1}x}{x}$ is $[-1, 1] - \{0\}$

Option (*d*) is correct.

37. $f(x) = e^{\cos^{-1}\sin\left(x+\frac{\pi}{3}\right)}$

$$f\left(\frac{8\pi}{9}\right) = e^{\cos^{-1}\sin\left(\frac{8\pi}{9}+\frac{\pi}{3}\right)}$$

$$= e^{\cos^{-1}\sin\left(\frac{11\pi}{9}\right)} = e^{\cos^{-1}\sin\left(\frac{22\pi}{18}\right)}$$

$$= e^{\cos^{-1}\sin\left(\frac{9\pi}{18}+\frac{13\pi}{18}\right)} = e^{\cos^{-1}\sin\left(\frac{\pi}{2}+\frac{13\pi}{18}\right)}$$

$$= e^{\cos^{-1}\cos\left(\frac{13\pi}{18}\right)} = e^{\frac{13\pi}{18}}$$

Option (*b*) is correct.

38. As $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ so $-1 \le \sin\theta \le 1$ but we have $0 < \sin\theta < 1$

$\log_{\sin\theta}(\cos^2\theta - \sin^2\theta) = 2$

$\Rightarrow \cos^2\theta - \sin^2\theta = (\sin\theta)^2$ and $0 < \sin\theta < 1$

$\Rightarrow \cos^2\theta = 2\sin^2\theta$ where $\sin\theta \in (0, 1)$

$\therefore \tan^2\theta = \frac{1}{2} \Rightarrow \tan\theta = \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}$

Also $\cos^2\theta - \sin^2\theta = \sin^2\theta$, $\sin\theta \in (0, 1)$

$\Rightarrow 1 - 2\sin^2\theta = \sin^2\theta$, $\sin\theta \in (0, 1)$

$\Rightarrow 1 = 3\sin^2\theta$ where $\sin\theta \in (0, 1)$

$\Rightarrow \sin^2\theta = \frac{1}{3}$, $\sin\theta \in (0, 1)$

$\Rightarrow \sin\theta = \pm\frac{1}{\sqrt{3}}$, $\sin\theta \in (0, 1)$

But $\sin\theta = -\frac{1}{\sqrt{3}}$ is not possible, as base of log is +ve and not equal to 1.

$\therefore \sin\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$.

Option (*b*) is correct.

39. Given $\sin^{-1}a + \sin^{-1}b + \sin^{-1}c = \frac{3\pi}{2}$

$\Rightarrow \sin^{-1}a = \frac{\pi}{2} = \sin^{-1}b = \sin^{-1}c \Rightarrow a = b = c = 1$

Again $f(x + y) = f(x)\, f(y), \forall\, x, y \in \mathbb{R}$ and $f(2) = 2$

$\therefore f(4) = f(2 + 2) = f(2)\, f(2) = 2 \times 2 = 4$

$f(6) = f(2 + 4) = f(2)f(4) = 2 \times 4 = 8$

$\therefore a^{f(2)} + b^{f(4)} + c^{f(6)} - \frac{3(a^{f(2)}.b^{f(4)}.c^{f(6)})}{a^{f(2)} + b^{f(4)} + c^{f(6)}}$

$= a^2 + b^4 + c^8 - \frac{3(a^2 \times b^4 \times c^8)}{a^2 + b^4 + c^8} = 1^2 + 1^4 + 1^8 - \frac{3(1^2 \times 1^4 \times 1^8)}{1^2 + 1^4 + 1^8}$

$= 3 - \frac{3 \times 1}{3} = 3 - 1 = 2$

Option (*a*) is correct.

40. We have

$\sin^{-1}(x^2 - 7x + 12) = n\pi \Rightarrow x^2 - 7x + 12 = \sin(n\pi), n \in \mathbb{Z}$

$\Rightarrow x^2 - 7x + 12 = 0 \Rightarrow x^2 - 3x - 4x + 12 = 0$

$\Rightarrow x(x - 3) - 4(x - 3) = 0 \Rightarrow (x - 3)(x - 4) = 0 \Rightarrow x = 3, 4$

Option (*b*) is correct.

41. We have $[\tan^{-1}x]^2 - 2[\tan^{-1}x] - 3 \le 0$

Put $[\tan^{-1}x] = \alpha$

$\Rightarrow \alpha^2 - 2\alpha - 3 \le 0$

$\Rightarrow \alpha^2 + \alpha - 3\alpha - 3 \le 0 \Rightarrow \alpha(\alpha + 1) - 3(\alpha + 1) \le 0$

$\Rightarrow (\alpha + 1)(\alpha - 3) \le 0$

$\Rightarrow -1 \le \alpha \le 3 \Rightarrow -1 \le [\tan^{-1} x] \le 3$

$\Rightarrow -1 \le \tan^{-1} x < 4$

$\Rightarrow \tan(-1) \le x < \tan 4 < \infty \Rightarrow -\tan 1 \le x < \infty$

$\Rightarrow x \in [-\tan 1, \infty)$

Option (*a*) is correct.

42. $\cos^{-1}\left[\cos\left(\left(-\frac{17}{15}\right)\pi\right)\right] = \cos^{-1}\left[\cos\left(\frac{17\pi}{15}\right)\right]$

$= \cos^{-1}\left[\cos\left(2\pi - \frac{13\pi}{15}\right)\right] = \cos^{-1}\left[\cos\frac{13\pi}{15}\right] = \frac{13\pi}{15}$

Option (*b*) is correct.

43. Principal value branch of $\cos^{-1} x$ is $[0, \pi]$

Option (*c*) is correct.

44. Principal value branch of cosec$^{-1} x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

Option (*d*) is correct.

45. $\sin^{-1}\left[\cos\left(\frac{33\pi}{5}\right)\right] = \sin^{-1}\left[\cos\left(6\pi + \frac{3\pi}{5}\right)\right]$

$= \sin^{-1}\left[\cos\left(\frac{3\pi}{5}\right)\right]$

$= \sin^{-1}\left[\sin\left(\frac{\pi}{2} - \frac{3\pi}{5}\right)\right] = \sin^{-1}\left[\sin\left(-\frac{\pi}{10}\right)\right]$

$= \sin^{-1}\left[-\sin\left(\frac{\pi}{10}\right)\right] = -\sin^{-1}\left[\sin\left(\frac{\pi}{10}\right)\right]$

$= -\frac{\pi}{10}, \because \frac{\pi}{10} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Option (*d*) is correct.

46. Let $\alpha = \cos^{-1}(2x - 1) \Rightarrow \cos\alpha = 2x - 1$

$\because\ -1 \le \cos\alpha \le 1 \Rightarrow -1 \le 2x - 1 \le 1$

$\Rightarrow\ 0 \le 2x \le 2 \Rightarrow 0 \le x \le 1 \Rightarrow x \in [0, 1]$

Hence domain of $\cos^{-1}(2x - 1)$ is $[0, 1]$

Option (*a*) is correct.

47. $\cos^{-1}\left(\cos\frac{3\pi}{2}\right) = \cos^{-1}\left(\cos\left(\pi + \frac{\pi}{2}\right)\right)$

$= \cos^{-1}\left(-\cos\frac{\pi}{2}\right) = \cos^{-1}(0) = \frac{\pi}{2}$

Option (*a*) is correct.

48. $2\sec^{-1} 2 + \sin^{-1}\left(\frac{1}{2}\right) = 2 \times \frac{\pi}{3} + \frac{\pi}{6}$

$= \frac{2\pi}{3} + \frac{\pi}{6} = \frac{5\pi}{6}$

Option (*b*) is correct.

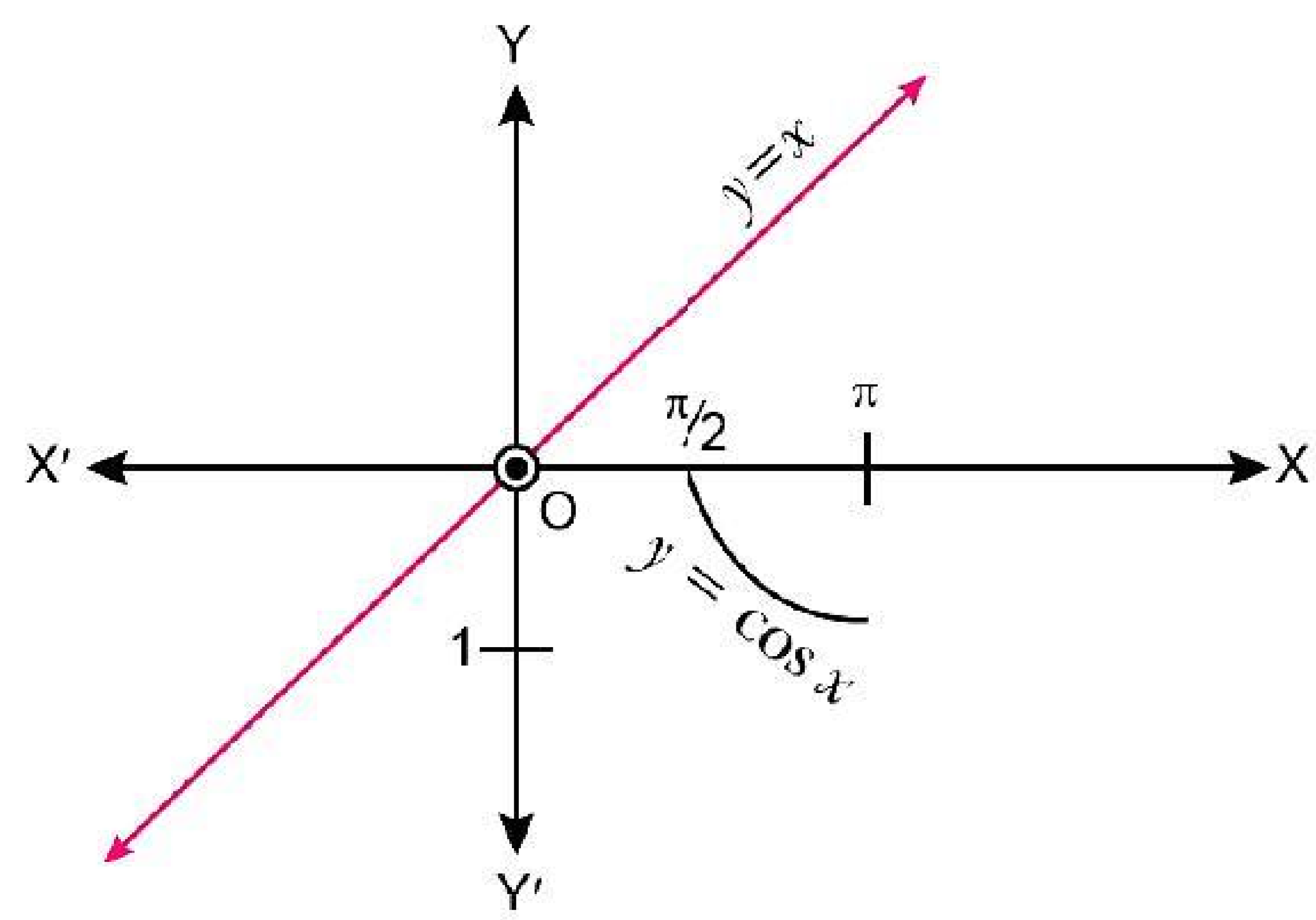
49. $\cot\left(\cos^{-1}\left(\frac{7}{25}\right)\right) = \cot\left(\cot^{-1}\left(\frac{7/25}{\sqrt{1-\left(\frac{7}{25}\right)^2}}\right)\right)$

$= \cot\left(\cot^{-1}\left(\frac{7/25}{24/25}\right)\right) = \cot\left(\cot^{-1}\left(\frac{7}{24}\right)\right)$

$= \frac{7}{24}$

Option (*d*) is correct.

50. $\sqrt{1 + \cos 2x} = \sqrt{2}\cos^{-1}(\cos x)$



$\Rightarrow \quad \sqrt{2\cos^2 x} = \sqrt{2}x,\ x \in \left[\frac{\pi}{2}, \pi\right] \subset [0, \pi]$

$\Rightarrow \quad \sqrt{2}\cos x = \sqrt{2}x$

$\Rightarrow \quad \cos x = x, \quad \because x \in \left[\frac{\pi}{2}, \pi\right]$

As in the given figure graph of $y = x$ and $y = \cos x$ does not intersect in $\left[\frac{\pi}{2}, \pi\right]$ so the number of solutions is zero.

Option (*a*) is correct.

51. $\sin^{-1}\frac{1}{2} = \alpha$, say $\Rightarrow \sin\alpha = \frac{1}{2} = \sin\frac{\pi}{6}$

$\Rightarrow \quad \alpha = \frac{\pi}{6} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\Rightarrow$ Principal value of $\sin^{-1}\frac{1}{2}$ is $\frac{\pi}{6}$

Option (*a*) is correct.

52. Let $\text{cosec}^{-1}(-1) = \alpha \Rightarrow \text{cosec}\,\alpha = -1 = \text{cosec}\left(-\frac{\pi}{2}\right)$

$\Rightarrow \quad \alpha = -\frac{\pi}{2} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

$\therefore$ Principal value of $\text{cosec}^{-1}(-1)$ is $-\frac{\pi}{2}$.

Option (*a*) is correct.

53. $\because \ \tan^{-1}(\sqrt{3}) + \cot^{-1}(-1) + \sec^{-1}\left(-\frac{2}{\sqrt{3}}\right)$

$$= \frac{\pi}{3} + \pi - \cot^{-1}(1) + \pi - \sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$$

$$= \frac{\pi}{3} + 2\pi - \frac{\pi}{4} - \frac{\pi}{6} = \frac{7\pi}{3} - \frac{5\pi}{12}$$

$$= \frac{28\pi - 5\pi}{12} = \frac{23\pi}{12}$$

Option (*d*) is correct.

54. $2\cos^{-1}\left(-\frac{1}{2}\right) + 2\sin^{-1}\left(-\frac{1}{2}\right) - \cos^{-1}(-1)$

$$= 2\left(\pi - \cos^{-1}\left(\frac{1}{2}\right)\right) - 2\sin^{-1}\left(\frac{1}{2}\right) - (\pi - \cos^{-1}1)$$

$$= 2\left(\pi - \frac{\pi}{3}\right) - 2\left(\frac{\pi}{6}\right) - (\pi - 0)$$

$$= 2\left(\frac{2\pi}{3}\right) - \frac{\pi}{3} - \pi$$

$$= \frac{4\pi}{3} - \frac{\pi}{3} - \pi = \frac{4\pi - \pi - 3\pi}{3} = 0$$

Option (*a*) is correct.

55. $\sec^{-1}\left(\sec\frac{4\pi}{3}\right) = \sec^{-1}\left(\sec\left(\pi + \frac{\pi}{3}\right)\right)$

$$= \sec^{-1}\left(-\sec\frac{\pi}{3}\right) = \sec^{-1}(-2) = \pi - \sec^{-1}2$$

$$= \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

Option (*b*) is correct.

56. $\cos^{-1}(-1) + \sin^{-1}(1) = \pi - \cos^{-1}(1) + \sin^{-1}1 = \pi - 0 + \frac{\pi}{2} = \frac{3\pi}{2}$ Option (*d*) is correct.

57. $\cos^{-1}\left(\cos\left(\frac{5\pi}{3}\right)\right) + \sin^{-1}\left(\sin\left(\frac{5\pi}{3}\right)\right)$

$$= \cos^{-1}\left(\cos\left(\pi + \frac{2\pi}{3}\right)\right) + \sin^{-1}\left(\sin\left(\pi + \frac{2\pi}{3}\right)\right)$$

$$= \cos^{-1}\left(-\cos\frac{2\pi}{3}\right) + \sin^{-1}\left(-\sin\frac{2\pi}{3}\right) = \cos^{-1}\left(-\left(-\frac{1}{2}\right)\right) + \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$$

$$= \cos^{-1}\left(\frac{1}{2}\right) - \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3} - \frac{\pi}{3} = 0$$

Option (*a*) is correct.

58. $\cot\left[\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right] = \cot\left[\frac{1}{2} \times \frac{\pi}{3}\right] = \cot^{-1}\left(\frac{\pi}{6}\right) = \sqrt{3}$

Option (*c*) is correct.

59. $\sin^{-1}\left(-\frac{1}{2}\right) + \cos^{-1}\left(-\frac{1}{2}\right) + \cot^{-1}(-\sqrt{3}) + \text{cosec}^{-1}(\sqrt{2}) + \tan^{-1}(-1) + \sec^{-1}(\sqrt{2})$

$$= -\sin^{-1}\left(\frac{1}{2}\right) + \pi - \cos^{-1}\left(\frac{1}{2}\right) + \pi - \cot^{-1}(\sqrt{3}) + \text{cosec}^{-1}(\sqrt{2}) - \tan^{-1}(1) + \sec^{-1}(\sqrt{2})$$

$$= -\frac{\pi}{6} + \pi - \frac{\pi}{3} + \pi - \frac{\pi}{6} + \frac{\pi}{4} - \frac{\pi}{4} + \frac{\pi}{4}$$

$$= -\frac{2\pi}{3} + \frac{9\pi}{4} = \frac{19\pi}{12}$$

Option (*b*) is correct.

60. $\cos^{-1}(2x^2 - 1)$

Put $x = \cos\alpha \Rightarrow \alpha = \cos^{-1} x$

$\therefore \cos^{-1}(2x^2-1) = \cos^{-1}(2\cos^2\alpha - 1) = \cos^{-1}(\cos 2\alpha)$

$$\begin{bmatrix} \because 0 \le x \le 1 \\ \Rightarrow \cos\left(-\frac{\pi}{2}\right) \le \cos\alpha \le \cos 0 \\ \Rightarrow -\frac{\pi}{2} \le \alpha \le 0 \\ \Rightarrow -\pi \le 2\alpha \le 0 \end{bmatrix}$$

$= 2\alpha = 2\cos^{-1} x$

Option (*a*) is correct.