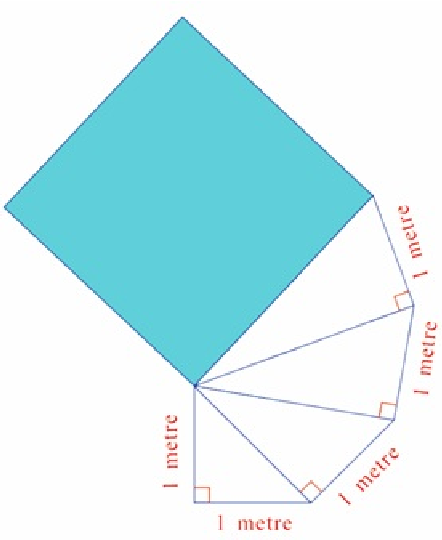
4. New Numbers

Questions Pg-64

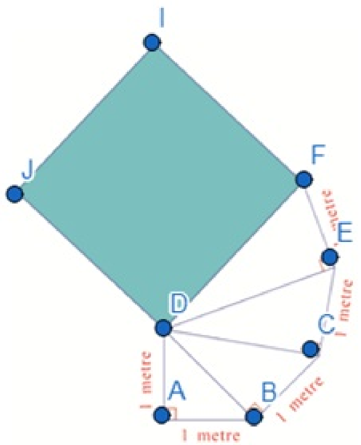
1. Question

In the picture, the square on the hypotenuse of the top most right triangle is drawn.

Calculate the area and the length of a side of the square .



Answer



Applying Pythagoras Theorem for right – angled triangle,

Base^2 + Perpendicular^2 = Hypoteneous^2

Which gives us in following right – angled triangles: –

In Δ BAD,

BA⊥ AD

⇒ $BA^2 + AD^2 = BD^2$

⇒ BD = $\sqrt{12 + 12}$

⇒ = √2 metre

In Δ DBC, DB⊥ BC

⇒ $DB^2 + BC^2 = DC^2$

⇒ DC = $\sqrt{\left[\left(\sqrt{2}\right)^2 + 12\right]}$ = √3 metre

In Δ CDE, DC⊥ CE

⇒ $DC^2 + CE^2 = DE^2$

⇒ $DE = \sqrt{\left[\left(\sqrt{3}\right)^2 + 12\right]}$ = 2 metre

In Δ DEF, DE⊥ EF

⇒ $DE^2 + EF^2 = DF^2$

⇒ DF = $\sqrt{2^2 + 1^2}$ = √5 metre....(Ans.)

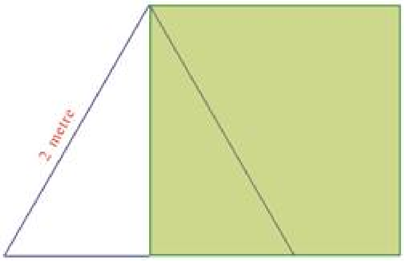
The area of the square = length of any of its side

= DF^2

= $(\sqrt{5})^2 = 5$ $metre^2$

2. Question

A square is drawn on the altitude of an equilateral triangle of side 2 metres.



i) What is the area of the square?

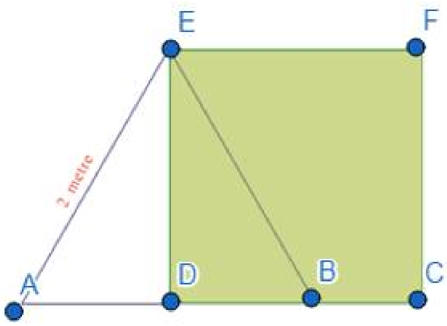
ii) What is the altitude of the triangle?

iii) What are the lengths of the other two sides of the triangle shown below?

Answer

(Avoid the very naming given in the following figures. As it was not there in the main problem,it is solver's own choice)

(i) The figure is given below:



In Δ ABC,AB = BC = CA = 2 metre.

CD is the altitude. As it is a equilateral triangle, CD is also a median.

(proof: In ΔACD and ΔBCD, –

∠ADC = ∠BDC = 90^0

AC = BC

AD is the common side

So, ΔACD≅ Δ BCD(R – H – S)

⇒ AD = BD…hence, CD is a median ,too.)

Thus, AD = BD = AB/2 = 2/2 = 1 metre

In right – angled Δ BCD,

Using Pythagoras theorem , –

$CD^2 = BC^2 - BD^2$

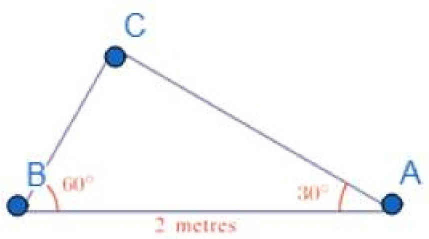
$= 2^2 - 1^2$

$= 3$ metre^2….(1)

So, area of the square $= CD^2 = 3$ metre^2

(ii) altitude of the Δ ABC = CD = √3 metre [from (1)]

(iii)



Let's name it Δ ABC, where the angles and sides are shown in the figure.

As, we know, opposite side of $60°$istwice than that of $30°$(can be proved by trigonometry,to be learnt in future)

⇒AC = 2BC

As ∠ACB = $90°$, using Pythagoras Theorem, –

$AC^2 + BC^2 = 4BC^2 + BC^2$

$= 5BC^2$

$= BA^2$

$= 2^2$

= 4

⇒ BC = √(0.8) metre

AC = 2 AC = 2 × √(0.8) metre…

3. Question

We have seen in Class 7 that any odd number can be written as the difference of two perfect squares. (The lesson, **Identities**) Usingthis, draw lines of lengths $\sqrt{7}$ and $\sqrt{11}$ centimetres.

Answer

→ Any odd number can be written in the form 2k – 1,k being a natural number.

Now,$2k - 1 = k^2 - (k - 1)^2$

● For,

2k – 1 = 7

⇒2k = 8

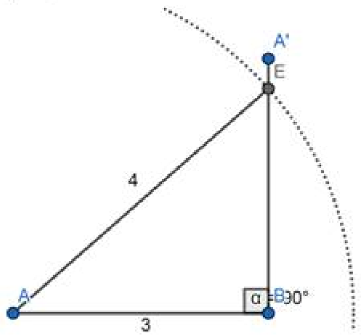
⇒ k = 4

So, $\sqrt{7} = 4^2 - 3^2$

So, if we draw a line of 4 units, say CD, then,

Then,

$BE = \sqrt{(4^2 - 3^2)} = \sqrt{7}$



Draw a random line and cut a length of 3c.m. s off it, to have AB.

→ Then draw a normal BA’ on AB.

→ Draw a circular arc with A as the centre and CD as centre, which eventually intersects BA’ at E.

→ Join B and E; and, A and E.

See, In Δ ABE, –

$AB \perp BE$

$BE^2 = AE^2 - AB^2 = 4^2 - 3^2 = 7$

⇔ E = $\sqrt{7}$ c.m.

● And,

$2k - 1 = 11$

$\Rightarrow 2k = 12$

$\Rightarrow k = 6$

So, $\sqrt{11} = \sqrt{(6^2 - 5^2)}$

Draw a random line and cut a length of 5 c.m. s off it, to have AB.

→ Then draw a normal BA’ on AB.

→ Draw a circular arc with A as the centre and CD as centre, which eventually intersects BA’ at E.

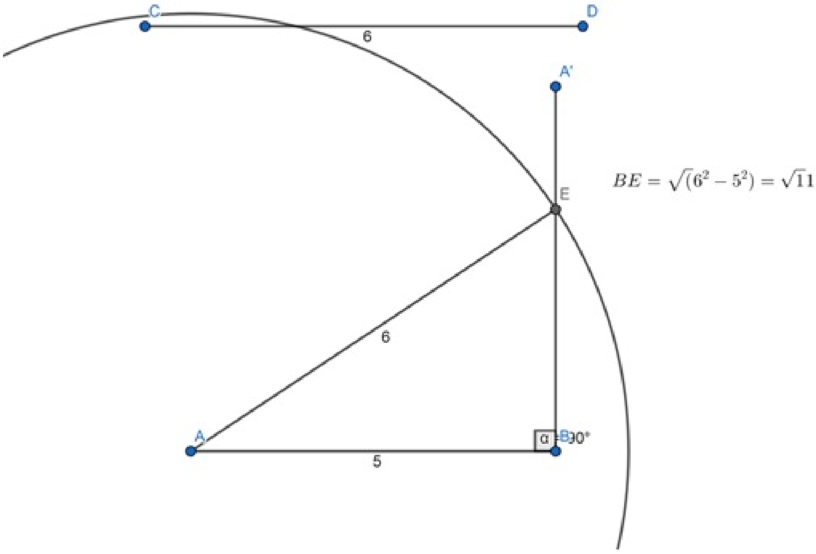
→ Join B and E; and,A and E.

See, In Δ ABE, –

$AB \perp BE$

$BE^2 = AE^2 - AB^2 = 6^2 - 5^2 = 11$

óBE = $\sqrt{11}$ c.m.



4. Question

Explain two different methods of drawing a line of length $\sqrt{13}$ centimetres.

Answer

(i) As, any odd number, $2k – 1 = k^2 – (k – 1)^2$

For,13 = 2k – 1 ⇔ 2k = 14 ⇔ k = 7

So,

$\sqrt{13} = \sqrt{(7^2 – 6^2)}$

(units are in c.m.)

Draw a random line and cut a length of 6 c.m. s off it, to have AB.

→ Then draw a normal BA’ on AB.

→ Draw a circular arc with A as the centre and CD as centre, which eventually intersects BA’ at E.

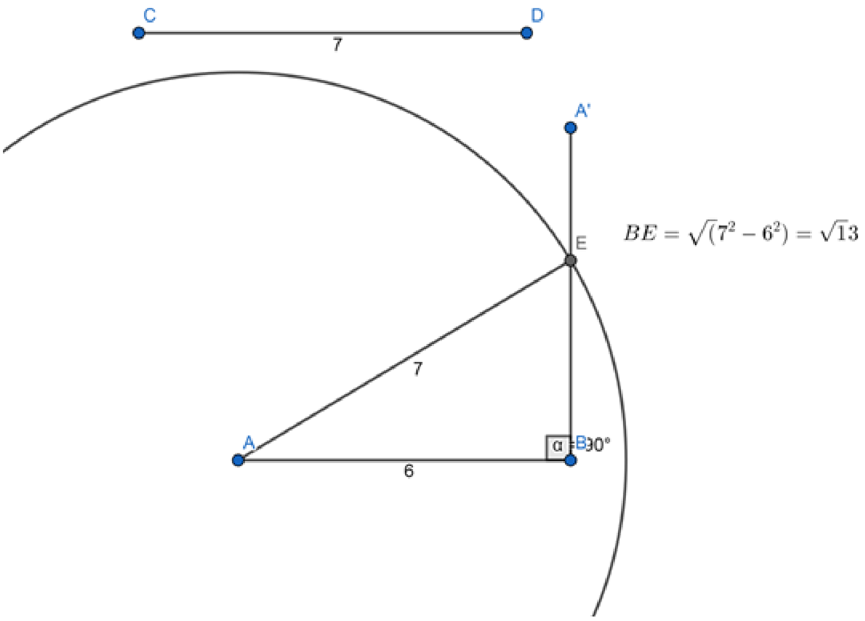
→ Join B and E; and, A and E.

See, In Δ ABE, –

AB⊥ BE

$BE^2 = AE^2 – AB^2 = 7^2 – 6^2 = 13$

óBE = √13 c.m.



(ii) Also observe, $13 = 3^2 + 2^2$

ó $\sqrt{13} = \sqrt{(3^2 + 2^2)}$

(units are in c.m.)

Draw a random line and cut a length of 3 c.m. off it, to have AB.

→ Then draw a normal BA’ on AB.

→ Draw a circular arc with B as the centre and radius of CD (2 c.m.), which eventually intersects BA’ at E.

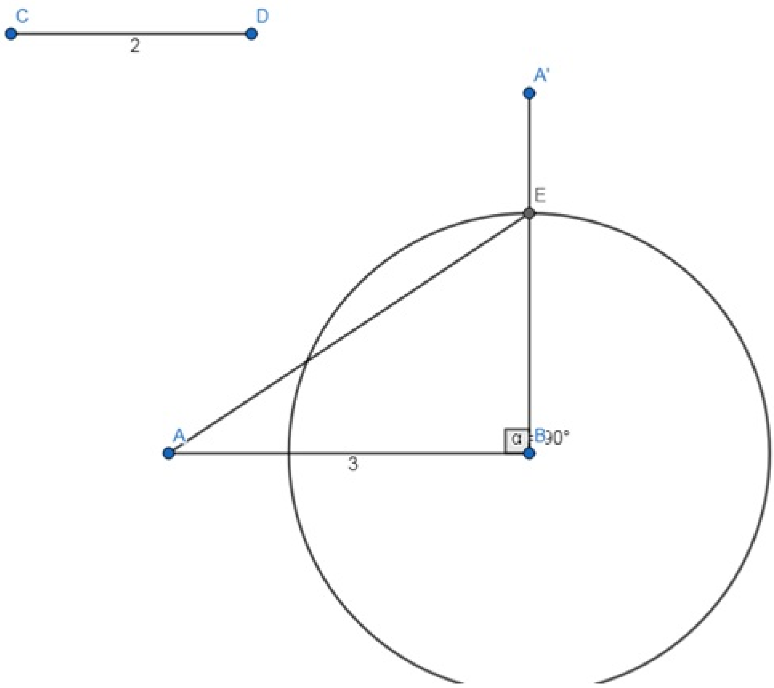
→ Join B and E ; and , A and E.

See, In Δ ABE, –

AB⊥ BE

$BE^2 + AB^2 = AE^2 = 2^2 + 3^2 = 13$

óAE = √13



5. Question

Find three fractions larger than √2 and less than √3.

Answer

→ Say, in general a is a fraction such that,

√2<a< √3[note, a>0]

$\Rightarrow 2 < a^2 < 3$

$\Rightarrow \frac{2k^2}{k^2} < a^2 < \frac{3k^2}{k^2}$,for k being a natural number.

So, for a particular k, our job is to find a perfect square(of a fraction) between $\frac{2k^2}{k^2}$ and $\frac{3k^2}{k^2}$.

Put, k = 2,3,5 (as three fractions are asked,three primes are chosen, else, like the case of k = 2 and k = 4 same fraction a may occur) – – –

● $\frac{8}{4} < a^2 < \frac{12}{4}$

$\Rightarrow a^2 = \frac{9}{4}$

$\Rightarrow a = \frac{3}{2}$

$[\text{ as}, a > 0]$

● $\frac{18}{9} < a^2 < \frac{27}{9}$

$\Rightarrow a^2 = \frac{25}{9}$

$\Rightarrow a = \frac{5}{3}$

● , $\frac{32}{16} < a^2 < \frac{48}{16}$

$\Rightarrow a^2 = \frac{36}{16}$

$\Rightarrow a = \frac{6}{4} = \frac{3}{2}$]

● $\frac{50}{25} < a^2 < \frac{75}{25}$

$\Rightarrow a^2 = \frac{64}{25}$

$\Rightarrow a = \frac{8}{5}$

So, the three fractions are:

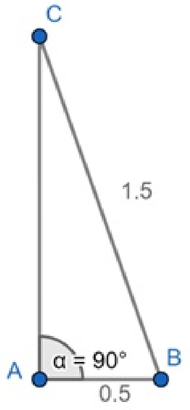
$\sqrt{2} < \frac{3}{2}, \frac{5}{3}, \frac{8}{5} < \sqrt{3}$

Questions Pg-67

1. Question

The hypotenuse of a right triangle is $1\frac{1}{2}$ metres and another side is $\frac{1}{2}$ metre. Calculate its perimeter correct to a centimetre.

Answer



In the given figure, hypotenuse, BC = 1.5 m and AB = 0.5 m.

<u>We know that in a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.</u>

⇒ In ΔABC, $BC^2 = AB^2 + AC^2$

⇒ $1.5^2 = 0.5^2 + AC^2$

⇒ $2.25 = 0.25 + AC^2$

⇒ $AC^2 = 2.25 - 0.25 = 2$

∴ AC = √2 m

⇒ √2 ≈ 1.41 m (correct to a centimetre)

<u>We know that perimeter of a polygon is the sum of all its sides.</u>

⇒ Perimeter of given triangle = 1.5 + 0.5 + √2

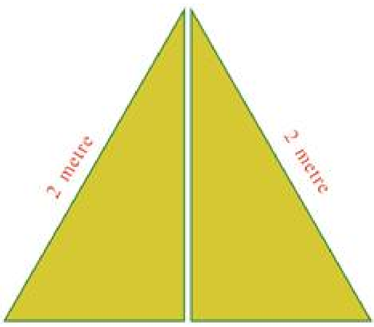
= 2 + √2

⇒ 2 + √2 ≈ 2 + 1.41 = 3.41 m

∴ Perimeter = 3.41 metres (correct to a centimetre)

2. Question

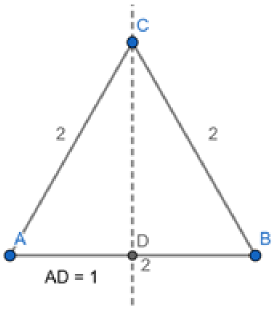
The picture shows an equilateral triangle cut into halves by a line through vertex.



i) What is the perimeter of a part?

ii) How much less than the perimeter of the whole triangle is this?

Answer



Consider ΔACD,

We know that in a right angled triangle, the square of tshe hypotenuse is equal to the sum of the squares of the other two sides.

⇒ $AC^2 = AD^2 + CD^2$

⇒ $2^2 = 1^2 + CD^2$

⇒ $4 = 1 + CD^2$

⇒ $CD^2 = 4 – 1 = 3$

∴ AC = √3 m

⇒ √3 ≈ 1.73 m

We know that perimeter of a polygon is the sum of all its sides.

(i) Here, ΔACD is a part.

⇒ Perimeter = 2 + 1 + √3

= 3 + √3

⇒ 3 + √3 = 3 + 1.73 = 4.73 m

∴ Perimeter of a part of the given equilateral triangle = 4.73 metres

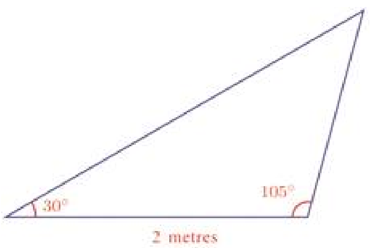
(ii) Perimeter of the whole triangle = 2 + 2 + 2 = 6 metres

The perimeter of a part is less than the whole by,

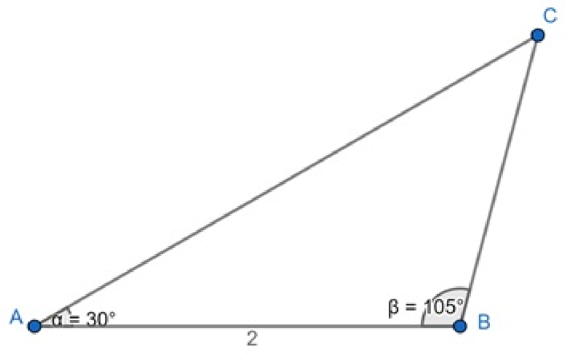
⇒ 6 – 4.73 = 1.27 metres

3. Question

Calculate the perimeter of the triangle shown below.



Answer



We know that sum of angles of a triangle is 180°.

⇒ ∠A + ∠B + ∠C = 180°

⇒ 30° + 105° + ∠C = 180°

⇒ ∠C = 180° - 135°

∴ ∠C = 45°

We know that the sine rule is $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.

In the triangle above, we have

⇒ c = 2 m;

a, b =?

Consider $\frac{a}{\sin A} = \frac{c}{\sin C}$,

$$\Rightarrow a = \frac{c \sin A}{\sin C}$$

$$= \frac{2 \sin 30°}{\sin 45°}$$

$$= \frac{2\left(\frac{1}{2}\right)}{\frac{1}{\sqrt{2}}}$$

$$= \frac{1}{\frac{1}{\sqrt{2}}}$$

∴ a = √2 ≈ 1.41 m

Now consider $\frac{b}{\sin B} = \frac{c}{\sin C}$,

⇒ $b = \frac{c \sin B}{\sin C}$

$$= \frac{2 \sin 105°}{\sin 45°}$$

$$= \frac{2\left(\sqrt{2}\left(\frac{\sqrt{3}+1}{4}\right)\right)}{\frac{1}{\sqrt{2}}}$$

$$= \frac{\sqrt{2}\left(\frac{\sqrt{3}+1}{2}\right)}{\frac{1}{\sqrt{2}}}$$

$$= \sqrt{2}\left(\frac{\sqrt{3}+1}{2}\right) \times \sqrt{2}$$

$= \sqrt{3} + 1$

$\therefore b = \sqrt{3} + 1 \approx 1.73 + 1 = 2.73$ m

<u>We know that perimeter of a polygon is the sum of all its sides.</u>

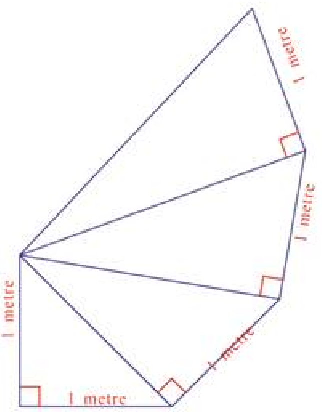
$\therefore$ Perimeter = a + b + c

= 1.41 + 2.73 + 2

= 6.14 m

4. Question

We have seen how we can draw a series of right triangles as in the picture.



i) What are the lengths of the sides of the tenth triangle?

ii) How much more is the perimeter of the tenth triangle than the perimeter of the ninth triangle?

iii) How do we write in algebra, the difference in perimeter of the n^{th} triangle and that of the triangle just before it?

Answer

<u>We know that in a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.</u>

Consider the 1^{st} triangle.

$\Rightarrow$ Hypotenuse of 1^{st} triangle, $x^2 = 1^2 + 1^2$

= 2

$\therefore$ Hypotenuse, $x = \sqrt{2}$ m

Consider 2^{nd} triangle.

$\Rightarrow$ Hypotenuse of 2^{nd} triangle, $y^2 = (\sqrt{2})^2 + 1^2$

= 2 + 1

= 3

$\therefore$ Hypotenuse, $y = \sqrt{3}$ m

Consider 3^{rd} triangle.

⇒ Hypotenuse of 3rd triangle, $z^2 = (\sqrt{3})^2 + 1^2$

= 3 + 1

= 4

∴ Hypotenuse, y = √4 m

And so on.

(i) The lengths of the 10th triangle will be √10 m, 1 m and hypotenuse.

Let hypotenuse be a.

⇒ $a^2 = (\sqrt{10})^2 + 1^2$

= 10 + 1

= 11

⇒ a = √11 m

∴ The lengths of the sides of the 10th triangle are √10 m, 1 m and √11 m.

(ii) We know that perimeter of a polygon is the sum of all its sides.

The perimeter of 10th triangle = √10 + 1 + √11

[√10 ≈ 3.16; √11 ≈ 3.31]

∴ Perimeter = 3.16 + 1 + 3.31

= 7.47 m

The perimeter of 9th triangle = √9 + 1 + √10

= 3 + 1 + 3.16

= 7.16 m

Perimeter of 10th triangle – Perimeter of 9th triangle = 7.47 – 7.16 = 0.31

∴ The perimeter of the 10th triangle is 0.31 m more than the perimeter of 9th triangle.

(iii) Perimeter of nth triangle = √n + 1 + √(n+1)

Perimeter of (n-1)th triangle = √(n – 1) + 1 + √(n – 1 + 1)

= √(n - 1) + 1 + √n

Difference between perimeters of nth triangle and (n – 1)th triangle = √n + 1 + √(n+1) – [√(n - 1) + 1 + √n]

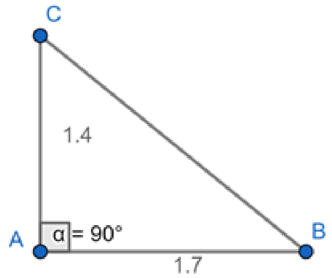
= √n + 1 + √(n+1) – √(n - 1) - 1 - √n

= √(n + 1) - √(n – 1)

5. Question

What is the hypotenuse of the right triangle with perpendicular sides √2 centimetres and √3 centimetres? How much larger than the hypotenuse is the sum of the perpendicular side?

Answer



<u>We know that in a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.</u>

⇒ In ΔABC, $BC^2 = AB^2 + AC^2$

⇒ $BC^2 = (\sqrt{3})^2 + (\sqrt{2})^2$

⇒ $BC^2 = 3 + 2$

⇒ $BC^2 = 5$

∴ BC = $\sqrt{5}$ cm (Hypotenuse)

⇒ $\sqrt{5} \approx 2.23$ cm,

$\sqrt{2} \approx 1.41$,

and $\sqrt{3} \approx 1.73$

Sum of perpendicular sides = 1.41 + 1.73 = 3.14 cm

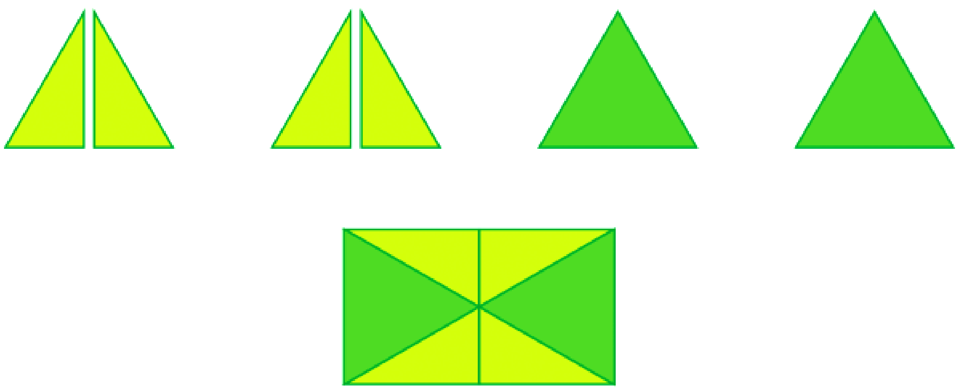
⇒ 3.14 – 2.23 = 0.91 cm

∴ The sum of perpendicular sides is 0.91 cm more than the hypotenuse.

Questions Pg-72

1. Question

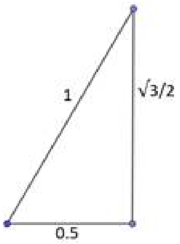
Of four equal equilateral triangles, two are cut vertically into halves and two whole are put together to make a rectangle:



If a side of a triangle is 1 metre, what is the area and perimeter of the rectangle?

Answer

First we find the sides of the equally cut triangles formed from the original equilateral triangle.



Breadth of the rectangle = Side of equilateral triangle = 1m

Length of rectangle = $(\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}) = \sqrt{3}$

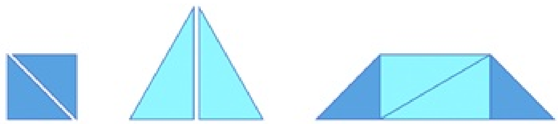
Perimeter of rectangle = $1 + \sqrt{3} + 1 + \sqrt{3} = (2 + 2\sqrt{3})$ m

Area of rectangle = Length × Breadth

⇒ Area of rectangle = $\sqrt{3} \times 1 = \sqrt{3}$ m^2

2. Question

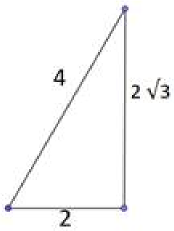
A square and an equilateral triangle of sides twice as long are cut and the pieces are rearranged to form a trapezium, as shown below:



If a side of a square is 2 centimetres, what are the area and perimeter of the trapezium?

Answer

First we find the sides of the equally cut triangles formed from the original equilateral triangle.



Slant height of trapezium = Diagonal of square = $2\sqrt{2}$ cm

Smaller side of trapezium = $2\sqrt{3}$ cm

Perimeter of trapezium = $2\sqrt{2} + 2\sqrt{3} + 2\sqrt{2} + (2 + 2\sqrt{3} + 2)$

⇒ Perimeter of trapezium = $(4\sqrt{2} + 4\sqrt{3} + 4)$ cm

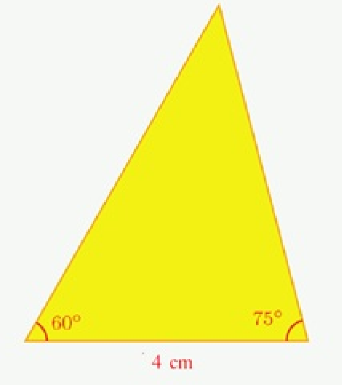
Area of trapezium = $\frac{1}{2}(\text{Sum of parallel sides})(\text{Height})$

⇒ Area of trapezium = $\frac{1}{2}(2\sqrt{3} + 2 + 2\sqrt{3} + 2)(2)$

⇒ Area of trapezium = $(4\sqrt{3} + 4)$ cm^2

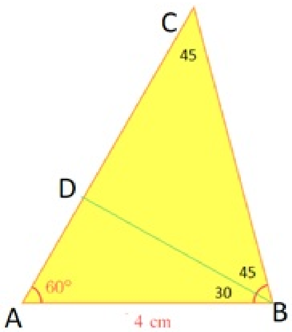
3. Question

Calculate the perimeter and area of the triangle in the picture.



Answer

We name the vertices of the triangle as A, B and C. We draw a perpendicular from B on AC as BD.



In ΔABD,

⇒ ∠A + ∠ABD + ∠ADB = 180° (Sum of all angles of a triangle)

⇒ 60 + ∠ABD + 90 = 180°

⇒ ∠ABD + 150 = 180°

⇒ ∠ABD = 30°

$$\frac{AD}{AB} = \cos 60°$$

⇒ AD = AB cos60°

⇒ AD = 4×(1/2) = 2cm …………………….(1)

$$\frac{BD}{AB} = \sin 60°$$

⇒ BD = AB sin60°

⇒ BD = 4×(√3/2) = 2√3 cm ……………………(2)

∠B = ∠ABD + ∠CBD

⇒ 75 = 30 + ∠CBD

⇒ 45 = ∠CBD

In Δ BCD,

∠BCD = ∠CBD = 45°

CD = BD (Sides opposite to equal angles are equal)

⇒ CD = 2√3 cm (From eq(2)) ……………………(3)

BC = √2 BD (ΔBCD is a right isosceles triangle)

⇒ BC = √2(2√3) = 2√6 …………………….(4)

AC = AD + DC

⇒ AC = $(2 + 2\sqrt{3})$ cm

Perimeter = AB + BC + CA

⇒ Perimeter = $4 + 2\sqrt{6} + (2 + 2\sqrt{3})$

⇒ Perimeter = $6 + 2\sqrt{6} + 2\sqrt{3}$

Area = $\frac{1}{2}(\text{Base})(\text{Height})$

⇒ Area = $\frac{1}{2}(\text{AC})(\text{BD})$

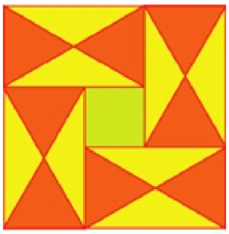
⇒ Area = $\frac{1}{2}(2 + 2\sqrt{3})(2\sqrt{3})$

⇒ Area = $(2 + 2\sqrt{3})(\sqrt{3})$

⇒ Area = $(6 + 2\sqrt{3})$ cm^2

4. Question

All red triangles in the picture are equilateral. What is the ratio of the sides of the outer and inner squares?



Answer

Let the side of red equilateral triangles be 1m.

In such rectangles, breadth = 1m

And length = $(\sqrt{3}/2) + (\sqrt{3}/2) = \sqrt{3}$ m (Sum of height of red triangle)

Side of outer square = $(\sqrt{3} + 1)$ m

Side of inner square = $(\sqrt{3} – 1)$ m

∴ Ratio of side of outer to inner squares = $\frac{\sqrt{3}+1}{\sqrt{3}-1}$

5. Question

From the pair of numbers given below, pick out those whose product is a natural number or a fraction:

(i) $\sqrt{3}$, $\sqrt{12}$

(ii) $\sqrt{3}$, $\sqrt{1.2}$

(iii) $\sqrt{5}$, $\sqrt{8}$

(iv) $\sqrt{0.5}$, $\sqrt{8}$

(v) $\sqrt{7}\frac{1}{2}$, $\sqrt{3}\frac{1}{3}$

Answer

We know that $\sqrt{x} \times \sqrt{y} = \sqrt{xy}$

(i) $\sqrt{3} \times \sqrt{12} = \sqrt{3\times12} = \sqrt{36} = 6$ Natural number

(ii) $\sqrt{3} \times \sqrt{1.2} = \sqrt{3\times1.2} = \sqrt{3.6} = 6\sqrt{0.1}$ Not a natural number or fraction

(iii) $\sqrt{5} \times \sqrt{8} = \sqrt{5\times8} = \sqrt{40} = 4\sqrt{10}$ Not a natural number or fraction

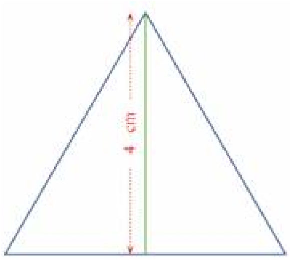
(iv) $\sqrt{0.5} \times \sqrt{8} = \sqrt{0.5\times8} = \sqrt{4} = 2$ Natural number

(v) $\sqrt{7\frac{1}{2}} \times \sqrt{3\frac{1}{3}} = \sqrt{\frac{15}{2}\times\frac{10}{3}} = \sqrt{25} = 5$ Natural number

Questions Pg-75

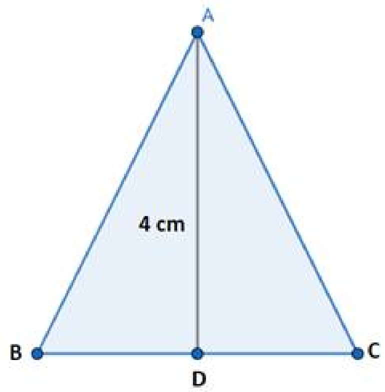
1. Question

Calculate the length of the sides of the equilateral triangle on the right, correct to a millimetre.



Answer

The figure is attached below:



An equilateral triangle has all sides of equal length,

i.e.,

AB = BC = AC

AD is perpendicular to BC,

∴ D is the midpoint of BC

⇒ BD = DC = 1/2 BC = 1/2 AB =1/2 AC

Now using Pythagoras theorem,

$AD^2 + BD^2 = AB^2$

⇒ $4^2 + (1/2\ AB)^2 = AB^2$

⇒ $AB^2 + 1/4\ AB^2 = 4^2$

⇒ $\frac{5}{4}AB^2 = 4^2$

$\Rightarrow AB^2 = \frac{16 \times 4}{5}$

$\Rightarrow AB = \sqrt{\frac{16 \times 4}{5}}$

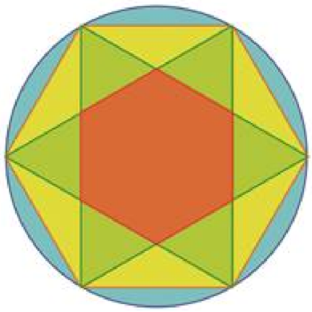
$\Rightarrow AB = \frac{8}{\sqrt{5}}$

$\Rightarrow AB = \frac{8}{2.23} = 3.6cm$

Hence, sides of triangle are 3.6 cm

2. Question

The picture shows the vertices of a regular hexagon connected by lines.



i) Prove that the inner red hexagon is also regular.

ii) How much of a side of the large hexagon is a side of the small hexagon?

iii) How much of the area of the large hexagon is the area of the small hexagon?

Answer

i) A regular hexagon is made up of 6 equilateral triangles.

Therefore, the green coloured triangles inside the yellow hexagon are equilateral triangles, that is all their sides are equal.

⇒ All the sides that comprises the hexagon are equal.

Hence, the inner red hexagon is also regular.

ii) Let the side of the inner red hexagon be x

⇒ the sides of the triangle will also be x

Let the side of outer yellow hexagon be y

Using Pythagoras theorem,

$x^2 + y^2 = (2x)^2$

$\Rightarrow y^2 = (2x)^2 - x^2$

$\Rightarrow y^2 = 4x^2 - x^2$

$\Rightarrow y^2 = 3x^2$

$\Rightarrow y = x\sqrt{3}$

Hence, the outer hexagon’s side is √3 times the inner hexagon’s side.

iii) Now

Area of larger hexagon $= \frac{3\sqrt{2}}{2}(x\sqrt{3})^2$

⇒ Area of larger hexagon $= 3 \times \frac{3\sqrt{2}}{2}x^2$

And,

Area of smaller hexagon $= \frac{3\sqrt{2}}{2}x^2$

Hence, the outer hexagon's area is 3 times the inner hexagon's area.

3. Question

Prove that $\left(\sqrt{2}+1\right)\left(\sqrt{2}-1\right)=1$. Use this to compute $\frac{1}{\sqrt{2}-1}$ correct to two decimal places.

Answer

$(\sqrt{2} + 1)(\sqrt{2} - 1) = (\sqrt{2})^2 - (1)^2$ [$\because (a+b)(a-b) = a^2-b^2$]

$\Rightarrow (\sqrt{2} + 1)(\sqrt{2} - 1) = 2 - 1$

$\Rightarrow (\sqrt{2} + 1)(\sqrt{2} - 1) = 1$

Hence, $(\sqrt{2} + 1)(\sqrt{2} - 1) = 1$

Now,

$$\frac{1}{\sqrt{2}-1} = \frac{1}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1}$$

$$\Rightarrow \frac{1}{\sqrt{2}-1} = \frac{\sqrt{2}+1}{\left(\sqrt{2}-1\right)\left(\sqrt{2}+1\right)}$$

$$\Rightarrow \frac{1}{\sqrt{2}-1} = \frac{\sqrt{2}+1}{1}$$

$$\Rightarrow \frac{1}{\sqrt{2}-1} = 1.41 + 1$$

$$\Rightarrow \frac{1}{\sqrt{2}-1} = 2.41$$

4. Question

Compute $\frac{1}{\sqrt{2}+1}$ correct to two decimal places.

Answer

Now,

$$\frac{1}{\sqrt{2}+1} = \frac{1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1}$$

$$\Rightarrow \frac{1}{\sqrt{2}+1} = \frac{\sqrt{2}-1}{\left(\sqrt{2}+1\right)\left(\sqrt{2}-1\right)}$$

$$\Rightarrow \frac{1}{\sqrt{2}+1} = \frac{\sqrt{2}-1}{1}$$

[$\because (\sqrt{2} + 1)(\sqrt{2} - 1) = 1$]

$$\Rightarrow \frac{1}{\sqrt{2}-1} = 1.41 - 1$$

$$\Rightarrow \frac{1}{\sqrt{2}-1} = 0.41$$

5. Question

Prove that $\sqrt{2\frac{2}{3}} = 2\sqrt{\frac{2}{3}}$ and $\sqrt{3\frac{3}{8}} = 3\sqrt{\frac{3}{8}}$. Can you find other numbers like this?

Answer

$$\sqrt{2\frac{2}{3}} = \sqrt{\frac{2 \times 3 + 2}{3}}$$

$$\Rightarrow \sqrt{2\frac{2}{3}} = \sqrt{\frac{6+2}{3}}$$

$$\Rightarrow \sqrt{2\frac{2}{3}} = \sqrt{\frac{8}{3}}$$

$$\Rightarrow \sqrt{2\frac{2}{3}} = \sqrt{\frac{2 \times 2 \times 2}{3}}$$

$$\Rightarrow \sqrt{2\frac{2}{3}} = 2\sqrt{\frac{2}{3}}$$

Similarly,

$$\sqrt{3\frac{3}{8}} = \sqrt{\frac{3 \times 8 + 3}{8}}$$

$$\Rightarrow \sqrt{3\frac{3}{8}} = \sqrt{\frac{24+3}{8}}$$

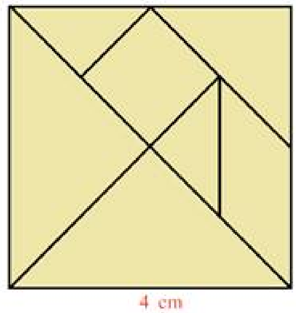
$$\Rightarrow \sqrt{3\frac{3}{8}} = \sqrt{\frac{27}{8}}$$

$$\Rightarrow \sqrt{3\frac{3}{8}} = \sqrt{\frac{3 \times 3 \times 3}{8}}$$

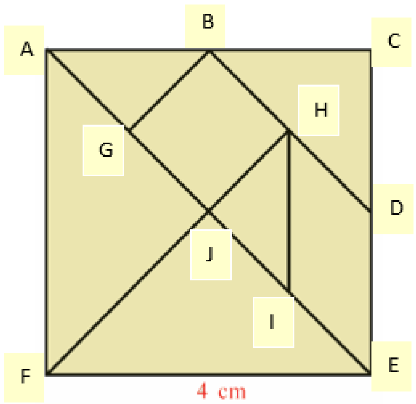
$$\Rightarrow \sqrt{3\frac{3}{8}} = 3\sqrt{\frac{3}{8}}$$

6. Question

The picture shows a tan gram of 7 pieces made by cutting a square of side 4 centimetres. Calculate the length of the sides of each piece.



Answer



AC = CE = EF = AF = 4 cm

∵ B is midpoint of AC

AB = BC = 1/2 AC

⇒ AB = BC = 1/2 × 4 cm

⇒ AB = BC = 2 cm

Similarly,

∵ D is midpoint of AC

CD = DE = 1/2 CE

⇒ CD = DE = 1/2 × 4 cm

⇒ CD = DE = 2 cm

AE is diagonal of ACEF,

∴ By Pythagoras theorem,

$AF^2 + EF^2 = AE^2$

⇒ $4^2 + 4^2 = AE^2$

⇒ $AE^2 = 16 + 16$

⇒ $AE^2 = 32$

⇒ AE= √32

⇒ AE= 4√2 cm

∵ J is midpoint of AE,

∴ AJ = JE = 1/2 AE

⇒ AJ = JE = 1/2 × 4√2 cm

⇒ AJ = JE = 2√2 cm

∵ G is midpoint of AJ,

∴ $AG = JG = 1/2\ AJ$

⇒ $AG = JG = 1/2 \times 2\sqrt{2}$ cm

⇒ $AG = JG = \sqrt{2}$ cm

∵ BGJH is a square

$BG = JH = HB = JG = \sqrt{2}$ cm

Also,

$HD = HB = \sqrt{2}$ cm

And,

$IE = IJ = GJ = \sqrt{2}$ cm

∵ HD and IE are equal and parallel

∴ HDEI is a parallelogram

⇒ $HI = DE = 2$ cm