32. Electric Current in Conductors

Short Answer

1. Question

Suppose you have three resistors each of value 30 Ω. List all the different resistances you can obtain using them.

Answer

With three resistors if equal value, there can be the following combinations of resistors:

A. All resistors in series,

With, all resistors in series the combined resistance will be,

$$30 + 30 + 30 = 120\ \Omega$$

B. All resistors in parallel:

With all resistors in parallel, the combination of resistors will provide,

$$\frac{1}{R} = \frac{1}{30} + \frac{1}{30} + \frac{1}{30} = \frac{3}{30}$$

$$R = 10\ \Omega$$

C. Two resistors in parallel and one resistor in series

When two resistors are in parallel, their resistance combined will be,

$$\frac{1}{R_1} = \frac{1}{30} + \frac{1}{30} = \frac{2}{30}$$

$$R_1 = 15\ \Omega$$

And when this resistance when combined with the resistor in series we have,

$$15 + 30 = 45\ \Omega$$

D. Two resistors in series and one resistor in parallel

When two resistors are in series, their resistance combined will be,

$$30 + 30 = 60\ \Omega$$

And when this resistance when combined with the resistor in parallel we have

$$\frac{1}{R} = \frac{1}{60} + \frac{1}{30} = \frac{3}{60}$$

$R = 20\ \Omega$

Therefore, by combining the resistances we will have a resistances of 10 Ω , 15 Ω, 45 Ω and 20 Ω.

2. Question

A proton beam is going from east to west. Is there an electric current? If yes, in what direction?

Answer

Electric current flows in the direction of moving charges and hence the beam will constitute an electric current in the east to west direction since the charge is positive.

3. Question

In an electrolyte, positive ions move from left to right and the negative ions from right to left. If there a net current? If yes, in what direction?

Answer

Since there is a movement of charged particles, a current will be developed and the current will be in the direction of positive ions i.e. from left to right.

4. Question

In a TV tube, the electrons are accelerated from the rear to the front. What is the direction of the current?

Answer

In a TV tube, when the electrons move from rear to the front, they are moving from cathode to the anode, i.e. from negative electrode to positive electrode and therefore the current will be in the direction of anode to cathode i.e. from front to rear.

5. Question

The drift speed is defined as $v_d = \Delta\ell/\Delta t$ where $\Delta\ell$ is the distance drifted in a long time Δt. Why don't we define the drift speed as the limit of $\Delta\ell/\Delta t$ as $\Delta t \rightarrow 0$?

Answer

Since an electron, during a drift, travels in a very random and discontinuous path, we have to take an average of the distances over a long interval of time, and since limiting $\Delta t \Rightarrow 0$ corresponds to a very short time interval we don't define drift velocity as such.

6. Question

One of your friends argues that he has read in previous chapters that there can be no electric field inside a conductor. And hence there can be no current through it. What is the fallacy in this argument?

Answer

The fallacy in the argument of the friend is that in the previous chapters the focus was on stationary charges and not moving charges and hence with no moving charges there will be no electric field inside a conductor and hence no current. However, in case of moving charges or current there will be an electric field due to the applied potential difference.

7. Question

When a current is established in a wire, the free electrons drift in the direction opposite to the current. Does the number of free electrons in the wire continuously decrease?

Answer

No, since the free electrons move throughout the entire circuit the electrons, those entering the conductor from the circuit replenish which leave the conductor, simultaneously. Hence, the charge neutrality of the conductor is maintained.

8. Question

A fan with copper winding in its motor consumes less power as compared to an otherwise similar fan having aluminum winding. Explain.

Answer

Copper has a resistivity of 1.72×10^{-8} Ωm whereas aluminum has a resistivity of 2.82×10^{-8} Ωm. Since aluminum has a higher resistivity, more power is lost in heat than used to run the fan than copper and hence copper wound motor fans consume less fans than aluminum wound motor fans.

9. Question

The thermal energy developed in a current-carrying resistor is given by $U = i^2Rt$ and also by U = Vit. Should we say that U is proportional to i^2 or to i?

Answer

In the expression

$$U = Vit$$

both V and i are time dependent quantities and hence we cannot say that U is proportional to i. However, in the expression

$$U = i^2rt$$

only i is the time dependent quantity and hence we can say that U is proportional to i^2.

10. Question

Consider a circuit containing an ideal battery connected to a resistor. Do “work done by the battery” and “the thermal energy developed” represent two names of the same physical quantity?

Answer

The work done on the resistor by the battery is dissipated as heat, which is the thermal energy, developed by the resistor and hence both the phrases represent the same physical quantity.

11. Question

Is work done by a battery always equal to the thermal energy developed in electrical circuits? What happens if a capacitor is connected in the circuit?

Answer

In case of real battery, the battery also has some internal resistance and hence the work done is equal to the thermal energy developed due to both resistances in the circuit and the internal resistance of the battery. In case of a capacitor, the work done by the battery is stored as electrical energy in the capacitor and not thermal energy.

12. Question

A non-ideal battery is connected to a resistor. Is work done by the battery equal to the thermal energy developed in the resistor? Does your answer change if the battery is ideal?

Answer

A non-ideal battery has internal energy and hence the work done is equal to the thermal energy developed due to both resistances in the circuit and the internal resistance of the battery.

If the battery is ideal, then there will be no internal resistance and the work done will solely be the thermal energy developed in the resistor.

13. Question

Sometimes it is said that “heat is developed” in a resistance when there is an electric current in it. Recall that heat is defined as the energy being transferred due to the temperature difference. Is the statement under quotes technically correct?

Answer

Yes, the statement is correct. The electric current flowing inside the resistor increases the potential energy of the resistor, which increases the temperature of the resistor to be greater than the surroundings. This temperature difference is the cause of the heat.

14. Question

We often say “a current is going through the wire”. What goes through the wire, the charge or the current?

Answer

When a current is going through a wire, charges are flowing through the wire, which constitute electric current.

15. Question

Would you prefer a voltmeter or a potentiometer to measure the emf of a battery?

Answer

A Potentiometer will be preferred to measure the emf of a battery as it uses the null pointer method, hence draws very little to no current from the circuit, and hence gives an accurate measure of the emf. In a voltmeter the equivalent resistance of the circuit changes and hence the potential difference to be measured changes. To minimize this change the voltmeter resistance needs to be very high.

16. Question

Does a conductor become charged when a current is passed through it?

Answer

When a current pass through a circuit, the free electrons in the valence band of the conductor jump to the conduction band and drift throughout the conductor and as no extra electrons are provided to the conductor, the conductor does not get charged.

17. Question

Can the potential difference across a battery be greater than its emf?

Answer

No, the potential difference across a battery cannot be greater than the emf as the emf is the maximum potential difference across the terminals of the battery. The potential across the battery, however, drops when connected to a circuit due to the drop across the internal resistance of the battery.

Objective I

1. Question

A metallic resistor is connected across a battery. If the number of collisions of the free electrons with the lattice is somehow decreased in the resistor (for example, by cooling it), the current will

A. increase

B. decrease

C. remain constant

D. become zero

Answer

If the number of collisions of the free electrons with the lattice is decreased then the drift velocity of the electrons will increase.

The current is given by

$$i = neAV_d$$

Where

I is the current

n is the number of electrons

e is the charge of an electron

A is the area of cross section of a conductor

V_d is the drift velocity

From the above formula current i is directly proportional to drift velocity. So when the drift velocity is increased then the current will increase.

If the number of collisions of the free electrons is decreased then the current will increase. Option A is correct.

2. Question

Two resistors A and B have resistances R_A and R_B respectively with $R_A < R_B$. The resistivities of their materials are ρ_A and ρ_B.

A. $\rho_A > \rho_B$

B. $\rho_A = \rho_B$

C. $\rho_A < \rho_B$

D. The information is not sufficient to find the relation between ρ_A and ρ_B.

Answer

Given :

Resistance of resistor A, R_A< Resistance of resistor B, R_B

Formula used : Resistance is given by the formula

$$R = \frac{\rho l}{A}$$

Where

R is the resistance

ρ is the resistivity

l is the length

A is the area of cross section

Resistance R_A is

$$R_A = \frac{\rho_A l_A}{A_A}$$

Resistance R_B is

$$R_B = \frac{\rho_B l_B}{A_B}$$

Only the relation between resistance values of R_A and R_B is given. R is depends on ρ, *l* and With the information given we cannot conclude the relation between resistivity of the two resistors.

If information about ρ, *l* and A is given then we can say the relation between ρ_A and $\rho_{B.}$ So option D is correct.

3. Question

The product of resistivity and conductivity of a cylindrical conductor depends on

A. temperature

B. material

C. area of cross section

D. none of these

Answer

The resistivity of a conductor is given by

$$\rho = \frac{1}{\sigma}$$

Where

ρ is the resistivity of the conductor

σ is the conductivity of the conductor

The product of resistivity and conductivity is

$$\rho \times \sigma = 1$$

The product of ρ and σ is unity. So it does not depends on anything. So option D is correct.

4. Question

As the temperature of a metallic resistor is increased, the product of its resistivity and conductivity.

A. increases

B. decreases

C. remains constant

D. may increase or decrease

Answer

Resistance is depends on temperature. If temperature is increase, resistivity will increase. Increase in resistivity will leads to decrease in conductivity. The relation is given below.

$$\rho = \frac{1}{\sigma}$$

Where ρ →resistivity

σ →Conductivity

The product of resistivity and conductivity is not depend on temperature. Because resistivity and conductivity of a metallic resistor are nullifying the change in the temperature.

If the temperature of a metallic resistor is increased, the product of its resistivity and

Conductivity may increase or decrease. Option D is correct.

5. Question

In an electric circuit containing a battery, the charge (assumed positive) inside the battery

A. always goes from the positive terminal to the negative terminal

B. may go from the positive terminal to the negative terminal

C. always goes form the negative terminal to the positive terminal

D. does not move

Answer

Battery is connected to an electric circuit. We don't know what type of circuit it is. May be the battery is charging or discharging. Generally the flow of electrons is opposite to the direction of flow of positive charge. While discharging of a battery the positive charge will flows from negative terminal to positive terminal. And in charging the positive charge will flows from positive terminal to negative terminal.

Conclusion : If a battery is connected to an electric circuit the charge(positive charge) may go from positive terminal to negative terminal(in charging) or go from negative terminal to positive terminal(in discharging). So option B is correct.

6. Question

A resistor of resistance R is connected to an ideal battery. If the value of R is decreased, the power dissipated in the resistor will

A. increase B. decrease

C. remain unchanged

Answer

Resistance R is connected to a ideal battery. Internal resistance of an ideal battery is zero. So it provides constant potential difference between two terminals.

Power dissipated by the resistor is given by

Power, $P = \frac{V^2}{R}$

Where

V is the voltage or potential difference

R is the Resistance of the resistor

Power P is inversely proportional to Resistance R. If the value of R is decreased, the value of power P will be increase.

A resistor R is connected to an ideal battery. If the value R is decreased, the power dissipated in the resistor will increase. So option A is correct.

7. Question

A current passes through a resistor. Let K_1 and K_2 represent the average kinetic energy of the conduction electrons and the metal ions respectively.

A. $K_1 < K_2$

B. $K_1 = K_2$

C. $K_1 > K_2$

D. Any of these three may occur.

Answer

Kinetic energy of electrons= K_1

Kinetic energy of metal ions = K_2

Electrons are free to move and metal ions are bounded at their positions and can't move freely as electrons. Due to thermal energy metal ions are just vibrate due to collision with electrons. When a current is passes through a resistor, because of the free movement the velocity of the electrons is greater than the metal ions.

Velocity of the electrons is grater than the metal ions. So kinetic energy of electrons is greater than the kinetic energy of metal ions.

Thus, option C is the correct option.

8. Question

Two resistors R and 2R are connected in series in an electric circuit. The thermal energy developed in R and 2R are in the ratio

A. 1:2 B. 2:1

C. 1:4 D. 4:1

Answer

Two resistors are connected in series. In series, the current flows through both the resistors is same.

Formula used:

Energy is given by the formula

Energy dissipated, $E = i^2Rt$

Where

I is the current

R is the resistance

T is the time period

Energy developed in the resistor R is $E = i^2 \times R \times t = i^2Rt$

Energy developed in the resistor 2R is $E = i^2 \times 2R \times t = i^2 2Rt$

The ratio of energy developed in R and 2R is =

$$\frac{E_R}{E_{2R}} = \frac{i^2Rt}{i^2 2Rt} = \frac{1}{2}$$

When two resistors R and 2R are connected in series then the ratio of thermal energy developed is 1:2.

Conclusion : Two resistors R and 2R are connected in series, the thermal energy developed in R and 2R are in the ratio 1:2 respectively. So option A is correct.

9. Question

Two resistances R and 2R are connected in parallel in an electric circuit. The thermal energy developed in R and 2R are in the ratio

A. 1 : 2 B. 2 : 1

C. 1 : 4 D. 4 : 1

Answer

Two resistors are connected in parallel. In parallel connection voltage is same. The voltage across the two resistors is same

$$E = \frac{V^2}{R}t$$

Where E is the energy dissipated

V is the voltage

R is the resistance

T is the time

Energy developed in the resistor R is

$$E_R = \frac{V^2}{R}t$$

Energy developed in the resistor 2R is

$$E_{2R} = \frac{V^2}{2R}t$$

The ratio of energy developed in R and 2R is

$$\frac{E_R}{E_{2R}} = \frac{\frac{V^2}{R}t}{\frac{V^2}{2R}t} = \frac{2}{1}$$

When two resistors R and 2R are connected in parallel the ratio of energy developed is 2:1. Option B is correct.

10. Question

A uniform wire of resistance 50 Ω is cut into 5 equal parts. These parts are now connected in parallel. The equivalent resistance of the combination is

A. 2Ω B. 10Ω

C. 250Ω D. 6250Ω

Answer

Formula used :

Resistance of a wire is given by

Resistance, $R = \rho \frac{l}{A}$

Where

ρ→ resistivity of the material

l→ length of the wire

A → cross sectional area of the wire

l is proportional to R.

When a uniform wire of resistance 50Ω is cut into 5 equal parts then resistance of each part is

10 Ω.

Now they are connected in parallel.

Equalent resistance is calculated as

$$\frac{1}{R} = \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10}$$

$R = \frac{10}{5} = 2\Omega$

A uniform wire of resistance 50 Ω is cut into 5 equal parts. If the parts are connected in parallel then equivalent resistance is 2Ω. So option A is correct.

11. Question

Consider the following two statements:

(A) Kirchhoff's junction law follows from conservation of charge

(B) Kirchhoff's loop law follows from the conservative nature of electric field

A. Both A and B are correct

B. A is correct but B is wrong

C. B is correct but A is wrong

D. Both A and B are wrong

Answer

Kirchhoff's junction law : The sum of all the currents directed towards a node is equal to sum of all the currents leaving the same node. It follows from the conservation of charge where the charge neither be created nor be destroyed, but it just transfers from one point to another. The net quantity of charge is equal to positive charge minus negative charge.

Kirchhoff's loop law : The algebraic sum of potential differences along a closed path in a circuit is zero. It follows from the Conservative nature of electric field. Electro static force is a conservative force and the work done by it in any closed path is zero.

Kirchhoff's junction law follows from conservation of charge and Kirchhoff's loop law follows from the conservative nature of electric field. So both statements A and B are correct. Option A is correct.

12. Question

Two non-ideal batteries are connected in series. Consider the following statements:

(A) The equivalent emf is larger than either of the two emfs.

(B) The equivalent internal resistance is smaller than either of the two internal resistances.

A. Each of A and B is correct.

B. A is correct but B is wrong.

C. B is correct but A is wrong

D. Each of A and B is wrong.

Answer

Let e_1 and e_2 be the emf of battery 1 and battery 2 respectively. And r_1 and r_2 be the internal resistance of battery 1 and battery 2 respectively.

Emf is nothing but the voltage across the terminals. The two batteries are connected in series.

The equivalent emf is $e = e_1+e_2$

The equivalent internal resistance $r= r_1+r_2$

: In series connection the equivalent emf and equivalent internal resistance are becomes larger. So statement A is correct and B is wrong. Option B is correct.

13. Question

Two non-ideal batteries are connected in parallel. Consider the following statements:

(A) The equivalent emf is smaller than either of the two emfs.

(B) The equivalent internal resistance is smaller than either of the two internal resistances.

A. Both A and B are correct.

B. A is correct but B is wrong

C. B is correct but A is wrong

D. Both A and B are wrong

Answer

Let ε_1 and ε_2 be the emf of battery 1 and battery 2 respectively. And r_1 and r_2 be the internal resistance of battery 1 and battery 2 respectively.

The two batteries are connected in parallel.

Equivalent emf

$$\varepsilon_{eq} = \frac{\varepsilon_1 r_1 + \varepsilon_2 r_2}{r_1 + r_2}$$

The equivalent internal resistance

$$\frac{1}{r} = \frac{1}{r1} + \frac{1}{r2}$$

if two batteries are connected in parallel then the equivalent emf is larger than either of the two emfs and the equivalent internal resistance is smaller than either of the two internal resistances. Statement B is correct ad A is wrong. So option C is correct.

14. Question

The net resistance of an ammeter should be small to ensure that

A. it does not get overheated

B. it does not draw excessive current

C. it can measure large currents

D. it does not appreciably change the current to be measured.

Answer

We always use ammeter in series to calculate the current drawn by the element from voltage source. If the net resistance of an ammeter is high, because of the series connection it will add up the net resistance of an ammeter. Then we can't get the current values accurately. If net resistance of an ammeter is low, it only shows a very small changes in current to be measured those can be negligible.

The net resistance of the ammeter should be small. Option D is correct.

15. Question

The net resistance of a voltmeter should be large to ensure that

A. it does not get overheated

B. it does not draw excessive current

C. it can measure large potential differences

D. it does not appreciably change the potential difference to be measured.

Answer

Voltmeter is always connected in parallel to measure the voltage or potential difference across the elements. When the voltmeter is connected across some element in a circuit, it will change the overall resistance in the circuit. It will effect the current values also.

To minimize the error voltmeter should have the large net resistance. Then large resistance in parallel with small resistor will have only a very slight change.

The net resistance of a voltmeter should be large to ensure that it does not appreciably change the potential difference to be measured. Option D is correct.

16. Question

Consider a capacitor-charging circuit. Let Q_1 be the charge given to the capacitor in a time interval of 10 ms and Q_2 be the charge given in the next time interval of 10ms. Let 10μC charge be deposited in a time interval t_1 and the next 10 μC charge is deposited in the next time interval t_2.

A. $Q_1 > Q_2, t_1 > t_2$ B. $Q_1 > Q_2, t_1 < t_2$

C. $Q_1 < Q_2, t_1 > t_2$ D. $Q_1 < Q_2, t_1 < t_2$

Answer

Formula used :

Charge of a capacitor is given by

Charge of a capacitor, $Q = \varepsilon C\left(1 - e^{-\frac{t}{RC}}\right)$

Where

ε is the emf of a battery

C is the capacitance

R is the resistance of a resistor which is in series

T is the time period

charge developed on the capacitor in first interval of 10ms is

$$Q_1 = \varepsilon C\left(1 - e^{-\frac{10\times10^{-3}}{RC}}\right)$$

charge developed on the capacitor in first interval of 20ms is

$$Q_1' = \varepsilon C\left(1 - e^{-\frac{20\times10^{-3}}{RC}}\right)$$

Charge developed on the capacitor in the interval 10ms to 20ms is

$$Q_2 = Q_1 - Q_1' = \varepsilon C\left(1 - e^{-\frac{10\times10^{-3}}{RC}}\right) - \varepsilon C\left(1 - e^{-\frac{20\times10^{-3}}{RC}}\right)$$

$$Q_2 = \varepsilon C\left(e^{-\left(\frac{10\times10^{-3}}{RC}\right)} - e^{-\frac{20\times10^{-3}}{RC}}\right)$$

$$Q_2 = \varepsilon C e^{-\frac{10\times10^{-3}}{RC}}\left(1 - e^{-\frac{10\times10^{-3}}{RC}}\right)$$

Compare Q_1 with Q_2

$$\frac{Q_1}{Q_2} = \frac{\varepsilon C\left(1 - e^{-\frac{10\times10^{-3}}{RC}}\right)}{\varepsilon C e^{-\frac{10\times10^{-3}}{RC}}\left(1 - e^{-\frac{10\times10^{-3}}{RC}}\right)}$$

$$\frac{Q_1}{Q_2} = \frac{1}{e^{-\frac{10\times10^{-3}}{RC}}}$$

(Here $e^{-\frac{10\times10^{-3}}{RC}} < 1$)

$=> \; Q_1 > Q_2$

The time taking for 10μC to developed on the plates of capacitor is t_1

$$10 = \varepsilon C\left(1 - e^{-\frac{t_1}{RC}}\right) ----(1)$$

The time taking for 10μC to developed on the plates of capacitor t_2

$$20 = \varepsilon C\left(1 - e^{-\frac{t_2}{RC}}\right) ----(2)$$

Divide (1) by (2)

$$\frac{10}{20} = \frac{\varepsilon C\left(1 - e^{-\frac{t_1}{RC}}\right)}{\varepsilon C\left(1 - e^{-\frac{t_2}{RC}}\right)}$$

$$2\left(1 - e^{-\frac{t_1}{RC}}\right) = \left(1 - e^{-\frac{t_2}{RC}}\right)$$

$$2e^{-\frac{t_1}{RC}} - 1 = e^{-\frac{t_2}{RC}}$$

Take log on both sides

$$ln(2) + \frac{t_1}{RC} = \frac{t_2}{RC}$$

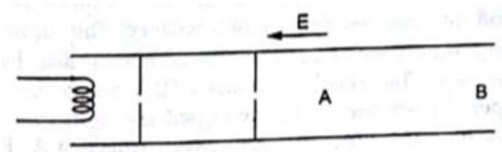
$=> \; t_2 > t_1$

Q_1 is the charge developed on the capacitor in a time interval of 10 ms and Q_2 is the charge developed on the capacitor in the next time interval of 10ms. If a10μC charge be deposited in a time interval t_1 and the next 10 μC charge is deposited in the next time interval t_2. Then $Q_1 > Q_2$, $t_1 < t_2$. So option B is correct.

Objective II

1. Question

Electrons are emitted by a hot filament and are accelerated by an electric field as shown in figure. The two stops at the left ensure that the electrons beam has a uniform cross-section.



A. The speed of he electron is more at B than at A.

B. The electric current is from left to right.

C. The magnitude of the current is larger at B than at A.

D. The current density is more at B than at B.

Answer

Electric field is in the direction form right to left.

Let velocity of an electron at stop A is V_A

velocity of an electron at stop A is V_B

Potentials are increased in the opposite direction of electric field.

So $V_B > V_A$

Potential energy of the electron at point A is

$U_A = -eV_A$

Potential energy of the electron at point B is

$U_B = -eV_B$

$U_A > U_B$

Because $V_B > V_A$

Kinetic energy of an electron at point A is K_A

Kinetic energy of an electron at point B is K_B

Applying conservation of mechanical energy , we get

$U_A + K_A = U_B + K_B$

(Mechanical energy =potential energy+kinetic energy)

=> $K_B > K_A$ (Because $U_A > U_B$)

Speed of electron is more at stop B than stop A. Option A is correct.

2. Question

A capacitor with no dielectric is connected to a battery at t = 0. Consider a point A in the connecting wires and a point B in between the plates.

A. There is no current through A.

B. There is no current through B.

C. There is a current through A as long as the charging is not complete.

D. There is a current through B as long as the charging is not complete.

Answer

capacitor is connected to a battery at t = 0. We have to consider two points. Point A is on the connecting wires and B is in between the plates. The current flows through the battery up to the capacitor is fully charged. So there is a current through point A as long as the charging is not complete.

Capacitor has no dielectric. Without a medium, how can a current flows through the plates of a capacitor. So there is no current flow through the point B.

Conclusion : A capacitor with no dielectric is connected to a battery at t = 0, no current will pass through capacitor plates and there is only current pass through connecting wire which is used to connect the capacitor with battery up to capacitor is fully charged. So option B and C are correct.

3. Question

When no current is passed through a conductor.

A. the free electrons do not move

B. the average speed of a free electron over a large period of time is zero.

C. the average velocity of a free electron over a large period of time is zero.

D. the average of the velocities of all the free electrons at an instant is zero.

Answer

When there is no current passing through a conductor then there is no charge will flow through conductor. So net charge will be zero. Because of no net charge all the electrons will be in a random motion. Due to random motion there is no net charge transfer. So the average velocity of a free electron over a large time period will be zero. If we see the average velocities of a free electrons it will also zero at an instant because of absence of net charge.

Conclusion : When no current is passed through a conductor. The average velocity of a free electron over a large period of time is zero and the average of the velocities of all the free electrons at an instant is zero. Option C and D are correct.

4. Question

Which of the following quantities do not change when a resistor connected to a battery is heated due to the current?

A. Drift speed

B. Resistivity

C. Resistance

D. Number of free electrons

Answer

Resistor is heated due to the current flowing through it. Thermal energy is increase.

Formula used :

$$i = neAV_d$$

Where i→ current

A→ area of the cross section

n→ electrons per unit area.

V_d → Drift speed

$$R = \frac{\rho l}{A}$$

R→ resistance

ρ→ resistivity

l→ length

A→ area of cross section

If thermal energy is increase the resistance will increase. Resistance is directly proportional to resistivity, so resistivity also increase. Increase in resistance leads to decrease in current. Current is directly proportional to drift velocity. So drift speed. So only the number of electrons remains same.

If the resistor connected to a battery is heated due to current flowing through it the drift speed, resistance, resistivity all are change except number of free electrons. Option D is correct.

5. Question

As the temperature of a conductor increases, its resistivity and conductivity change. The ratio of resistivity to conductivity.

A. increases

B. decreases

C. remains constant

D. may increase or decrease depending on the actual temperature.

Answer

Formula used : Resistivity is given by

$$\rho = \frac{1}{\sigma}$$

Where ρ is the resistivity

σ is the conductivity

When temperature of a conductor is increases, its resistivity will increase and conductivity will decrease.

Ration of resistivity to conductivity is

$$\frac{\rho}{\sigma} = \frac{\rho}{\frac{1}{\rho}} = \rho^2$$

Conclusion : When the temperature of conductor increases, the ratio of resistivity to conductivity will increase. Option A is correct.

6. Question

A current passes through a wire of non-uniform cross-section. Which of the following quantities are independent of the cross-section?

A. The charge crossing in a given time interval

B. Drift speed

C. Current density

D. Free-electron density

Answer

Formula used : current density is given by

current density , $j = \frac{i}{A} = neV_d$

Where

i→ current

A→ area of the cross section

n→ electrons per unit area.

V_d → Drift speed

Current density is inversely proportional to area of the Ross section. So area of the cross section is depends on current density.

Drift speed is also inversely proportional to area of the cross section. So area of the cross section is depends drift speed.

It does not depends on free electron density and the charge crossing in a given time interval.

A current passes through a wire of non uniform cross-section. Then it does not depends on The charge crossing in a given time interval and free electron density. Option A and D are correct.

7. Question

Mark out the current options.

A. An ammeter should have small resistance

B. An ammeter should have large resistance

C. A voltmeter should have small resistance

D. A voltmeter should have large resistance.

Answer

Ammeter is always connected in series with a circuit that the current to be measured. If ammeter has a large resistance, the net resistance will be high. It will effect the total measures of circuit. Then we can't get the accurate values of the current drawn from the voltage source. If the ammeter have a small resistance, it will not show an appreciable change in net resistance.

Voltmeter is always connected in parallel with the element to measure the voltage across the element. Voltmeter should have a large resistance. If voltmeter have small resistance, definitely draw the current from the source. It is not supposed to draw any current from the source. It has to measure the potential difference across the element only.

An ammeter should have small resistance and a voltmeter should have large resistance. Option A, D are correct.

8. Question

A capacitor of capacitance 500 μF is connected to a battery through a 10 kΩ resistor. The charge stored on the capacitor in the first 5s is larger than the charge stored in the next.

A. 5 s B. 50 s

C. 500 s D. 600 S

Answer

Given : C = 500 μF

R=10

Capacitor is connected to a battery thought a resistor. Initially capacitor starts charge. If it is get fully charged then it starts discharge through the element connected to it. Typically for the Charging or discharging the time constant is in the order of mille seconds. Generally for 99% of the charging of a capacitor 4 to 5 time constants are sufficient. Given that the charge stored on the capacitor is larger in the first 5s. So it larger than 5s, 50s, 500s and 600s.

A capacitor of capacitance 500 μF is connected to a battery through a 10kΩ resistor. The charge stored on the capacitor is large in the first 5s. All the options given are correct.

9. Question

A capacitor C_1 of capacitance 1 μF and a capacitor C_2 of capacitance 2 μF are separately charged by a common battery for a long time. The two capacitors are then separately discharged through equal resistors. Both the discharge circuits are connected at t = 0.

A. The current in each of the two discharging circuits is zero at t = 0.

B. The currents in the two discharging circuits at t = 0 are equal but not zero.

C. The currents in the two discharging circuits at t = 0 are unequal.

D. C_1 loses 50% of its initial charge sooner than C_2 loss 50% of its initial charge.

Answer

Given: $C_1 = 1\mu F$

$C_2 = 2\mu F$

Both the capacitors are connected to a same battery forma long time. So capacitors fully charged. The two capacitors connected to an equal resistors separately at t=0. Now the capacitors starts discharge at the same time. So at t=0 the current in both discharging circuits are equal but not zero. Capacitors are in the ratio 1:2. So C_1 loses 50% of its initial charge sooner than C_2 loss 50% of its initial charge.

If the charged capacitors $C_1 = 1\mu F, C_2 = 2\mu F$ connected to the same resistance separately then the currents in the two discharging circuits at t = 0 are equal but not zero and C_1 loses 50% of its initial charge sooner than C_2 loss 50% of its initial charge. Option B and D are correct.

Exercises

1. Question

The amount of charge passed in time t through a cross-section of a wire is Q(t) = $At^2 + Bt + C$.

(a) Write the dimensional formulae for A, B and C.

(b) If the numerical values of A, B and C are 5, 3 and 1 respectively in S.I. units, find the value of the current at t = 5 s.

Answer

a) IT^{-1}, I, IT b)53A

Given,

Charge as a function of time is $Q(t) = At^2 + Bt + C$.

The principle of homogeneity states that each term on the either side of an equation has the same dimensions.

a) Each terms on the Right Hand Side of the equation has the same unit, and hence the dimension of that of the term on the Left Hand Side.

So, each term on RHS is having same dimensions as of the quantity Charge, Q.

We know that, Charge Q is

$$Q = I \times t$$

Where I is current with dimension ‘I’ and t is time in seconds with dimension ‘T’.

Hence the dimension of Q or Q(t)is ‘IT’.

By inspection, we can see that the term C in RHS is devoid of any other quantities and hence C also has the dimension ‘IT’ (Ans.)

We know that dimension of the term At^2 is also 'IT', and t represents time (Dimension T).

So, $\dim(At^2) \approx \dim(A)T^2 \approx IT$

Or

$\dim(A) = IT^{-1}$ (Ans.)

Similarly,

$\dim(Bt) \approx \dim(B)T^1 \approx IT$

Or,

$\dim(B) = I\ (Ans.)$

So dimensions of A, B, and C are IT^{-1}, I, IT respectively.

b) The expression for the charge at time t can be rewritten by assigning values to the constants as.

$$Q(t) = 5t^2 + 3t + 1$$

We know that instantaneous current, I can be expressed as

$$I = \frac{d}{dt}Q(t)$$

By substituting the given expression in the above equation, we get,

$$I = \frac{d}{dt}(5t^2 + 3t + 1)$$

Or,

$$I = 10t + 3$$

For t=5s, I becomes $I = 10 \times 5s + 3 = 53A\ (Ans.)$

Hence the current at t=5s is 53A

2. Question

An electron gun emits 2.0×10^{16} electrons per second. What electric current does this correspond to?

Answer

3.2×10^{-3} A

Given,

The number of electrons emitted=2.0×10^{16}

Time, t, in seconds in which 2.0 × 1016 electrons are emitted=1s

Formula Used:

The current flowing from an electron gun or through a circuit, I, due to the movement of charges, q, through it can be expressed as,

$$I = \frac{q}{t}$$

Where q is the charge flowing and t is time in seconds.

Also, for n number of electrons, q is

$$q = ne$$

Where e is the charge of 1 electron= 1.6× 10^{-19}C

Hence, in the given problem, the total charge flowing from the gun is, q

$$q = ne = 2 \times 10^{16} \times 1.6 \times 10^{-19} C$$

$$= 3.2 \times 10^{-3} C$$

And the corresponding current, I is

$$I = \frac{q}{t} = \frac{3.2 \times 10^{-3} C}{1s} = 3.2 \times 10^{-3} A$$

So the corresponding current is 3.2×10^{-3} A.

3. Question

The electric current existing in a discharge tube is 2.0 μA. How much charge is transferred across a cross-section of the tube in 5 minutes?

Answer

6.0×10^{-4}C

Given,

The electric current in the tube= 2.0 μA

Time for which charge transfer is to be calculated= 5 min=300s

Formula used

The amount of charge, q, transferred in t seconds, with a current I is

$$q = It$$

Solution,

The discharge tube carries a current of 2.0 μA. So the charge transferred across the cross-section in 300s, by the above relation, is

$$q = 2.0\ \mu A \times 300s = 6 \times 10^{-4} C$$

So the charge transferred across the cross-section is 6.0×10^{-4}C

4. Question

The current through a wire depends on time as

$i = i_0 + \alpha t$,

Where i_0 = 10 A and α = 4 As^{-1}. Find the charge crossed through a section of the wire in 10 seconds.

Answer

300C

Given,

The expression of current through the wire is

$i = i_0 + \alpha t$

where i_0=10A, time for which current passes, t= 10s, and a=4 As^{-1}

Formula Used:

For a given current i, the charge q is expressed as

$$q = \int_{0S}^{tS} idt$$

Solution,

For the given expression of current, charge q is,

$$q = \int_{0S}^{tS} idt$$

Or,

$$q = \int_{0S}^{tS} (i_0 + at)dt$$

On integration,

$$q = i_0 \times t + \frac{at^2}{2}\ (\because \textit{the lower limit is zero})$$

By substituting the given values,

$$q = 10A \times 10s + \frac{4As^{-1} \times 10^2}{2}$$

$$= 300C$$

Hence, the charge crossed through a section of the wire in 10 seconds is 300C.

5. Question

A current of 1.0 A exists in a copper wire of cross-section 1.0 mm^2. Assuming one free electron per atom calculate the drift speed of the free electrons in the wire. The density of copper is 9000 kg m^{-3}.

Answer

0.074mm/s

Given,

Current in the wire, I= 1A

Cross section of the wire, A= 1mm^2=10^{-6}m^2

Density of Copper, d=9000 kg m^{-3}

Formula Used

The current due to 'n' freely bounded electrons per unit volume with a drift speed 'V_d' can be expressed as,

$$I = nAeV_d \ (eqn. 1)$$

Where 'A' is the cross-sectional area of the material through which electrons are passing; and 'e' is the charge of the electron, which is

1.6×10^{-19}C.

In the given problem, n is not directly given. But we know that 63.5 grams of Copper have Avogadro number (6.022×10^{23}) of atoms. So 'm' Kilograms have,

$$\frac{6.022 \times 10^{23} \times mKg}{63.5 \times 10^{-3}Kg}$$

Also, in terms of density, d, the mass m can be replaced in the above expression as,

$$\frac{6.022 \times 10^{23} \times (Unit\ volume \times d)Kg}{63.5 \times 10^{-3}Kg}$$

And for Unit volume, the number of atoms are,

$$\frac{\frac{6.022 \times 10^{23} \times (Unit\ volume \times d)Kg}{63.5 \times 10^{-3}Kg}}{Unit\ volume}$$

So, the number of free electrons/ atoms are,

$$\frac{6.022 \times 10^{23} \times 9000Kgm^{-3}}{63.5 \times 10^{-3}Kg} = 8.535 \times 10^{28} = n$$

From eqn.1, the expression for Drift Velocity, V_d

$$V_d = \frac{I}{nAe}$$

Substituting the known values, it becomes,

$$V_d = \frac{1A}{8.535 \times 10^{28} \times 10^{-6}m^2 \times 1.6 \times 10^{-19}C}$$

$$= 7.32 \times 10^{-5}m/s = 0.074mm/s\ (Ans.)$$

Hence the drift speed of free electrons is 0.074mm/s

6. Question

A wire of length 1 m and radius 0.1 mm has a resistance of 100 Ω. Find the resistivity of the material.

Answer

Given,

Length of wire, l = 1m

Radius of wire, $r=0.1mm=0.1\times10^{-3}m$

Resistance of the wire, R =100 Ω

Formula used,

The resistivity, ρ, of a wire with cross-sectional area A and length l is expressed as

$$\rho = \frac{RA}{l}$$

Where R is the resistance offered by the wire.

Solution,

Firs, we find the area of cross-section of the wire as,

$$A = \pi r^2$$

By substituting the value of r, Area becomes

$$A = \pi(0.1 \times 10^{-3}m)^2 = 3.14 \times 10^{-8}m^2$$

Now substitute all the given values in the expression for resistivity. So, ρ is

$$\rho = \frac{100\Omega \times 3.14 \times 10^{-8} m^2}{1m}$$

$$= 3.14 \times 10^{-6} \Omega m$$

$$= \pi \times 10^{-6} \Omega m \ (Ans.)$$

So, the resistivity of the material is $\pi \times 10^{-6}$ Ωm

7. Question

A uniform wire of resistance 100 Ω is melted and recast in a wire of length double that of the original. What would be the resistance of the wire?

Answer

400 Ω

Given,

Initial Resistance, R_1 of the wire= 100 Ω

Initial length of wire=l_1, Final length of wire=l_2= 2 l_1

Formula used

The expression for the resistance, R, of a wire is

$$R = \frac{\rho l}{A} \ (eqn. 1)$$

Where ρ is the resistivity, A is the area of cross-section and l is the length of the wire.

Hence, by knowing the final length and area we can calculate the final resistance by comparing it with that of the initial case. Also, the information that the volume of the wire would not change on recast, should be used.

Solution,

We know that the volume remains same after the recast.

If we represent the volume as a function of area and length, the above information can be expressed as,

$$A_1 \times l_1 = A_2 \times l_2$$

Where subscripts '1' and '2' denotes the initial and final cases.

It is given that, l_2= 2 l_1, so the final area A_2 can be represented as,

$$A_2 = \frac{A_1}{2}$$

We use this to compare the two resistance,

The initial resistance, R_1 is

$$R_1 = \frac{\rho_1 l_1}{A_1} \ (eqn.2)$$

Similarly, the final resistance, R_2 is

$$R_2 = \frac{\rho_2 l_2}{A_2} \ (eqn.3)$$

Since the material is the same in both cases, $\rho_1 = \rho_2$. By dividing eqn.3 by eqn.2, we get,

$$\frac{R_2}{R_1} = \frac{l_2 A_1}{l_1 A_2}$$

By substituting the relation between length and area at the initial and final cases, the above expression can be re-written as,

$$\frac{R_2}{R_1} = \frac{2 l_1 A_1}{l_1 \, {}^{A_1}/_{2}}$$

Or,

$$R_2 = 4R_1$$

Or,

$$R_2 = 4 \times 100\Omega = 400\Omega \ (\text{Ans.})$$

the resistance of the wire is 400 Ω

8. Question

Consider a wire of length 4 m and cross-sectional area 1 mm^2 carrying a current of 2 A. If each cubic meter of the material contains 10^{29} free electrons, find the average time taken by an electron to cross the length of the wire.

Answer

8.9 hours

Given,

Number of free electron per unit volume, $n=10^{29}$

Area of cross-section, $A=1\ mm^2=10^{-6}\ m^2$

Length of wire, l= 4m

Current through the wire, I=2A

Formula used

We know that the expression for drift velocity, V_d of 'n' free electrons through a wire of cross-sectional area A, for a current 'i' is

$$V_d = \frac{i}{nAe}$$

Where e is the charge of 1 electron= 1.6×10^{-19} C

V_d can be expressed in terms of the travel length, l and travel time, t as,

$$V_d = \frac{l}{t}$$

Combining the above two expressions, we can write as,

$$\frac{l}{t} = \frac{i}{nAe}$$

Or,

$$t = \frac{nAel}{i} \ (eqn.1)$$

Solution,

Substituting the given values in eqn.1, we get the time taken by the free electrons to cross the length as,

$$t = \frac{10^{29} \times 10^{-6}m^2 \times 1.6 \times 10^{-19}C \times 4m}{2A}$$

$$= 32000s = 8.89 \ hours$$

So the average time taken by an electron to cross the length of the wire is approximately 8.9 hours.

9. Question

What length of a copper wire of cross-section area 0.01 mm^2 will be needed to prepare a resistance of 1 kΩ? Resistivity of copper = 1.7×10^{-8} Ω m.

Answer

0.6km

Given,

Resistance of the wire, R=1000Ω

Resistivity of copper, = 1.7×10^{-8} Ω m

Area of cross-section, A=0.01 mm^2=0.01×10^{-6} m^2

Formula used,

The expression for the resistance, R, of a wire is

$$R = \frac{\rho l}{A}\ (eqn.1)$$

Where ρ is the resistivity, A is the area of cross-section and ‘l’ is the length of the wire.

Or the length of the wire for given area and Area of a cross section is,

$$l = \frac{AR}{\rho}\ (eqn.2)$$

Solution,

Substituting the values in eqn.2, we get the required length as,

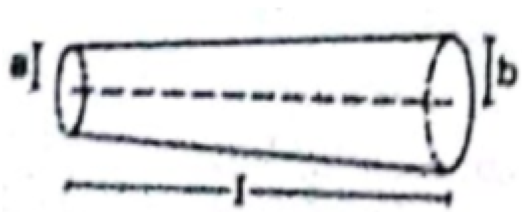
$$l = \frac{(0.01 \times 10^{-6} m^2) \times 1000\Omega}{1.7 \times 10^{-8}\Omega\text{m}}$$

$$= 588.23m \approx 0.6km\ (Ans.)$$

Hence the length of the copper wire for the given resistance is 0.6km.

10. Question

Figure shows conductor of length ℓ having a circular cross-section. The radius of cross-section varies linearly from a to b. The resistivity of the material is ρ. Assuming that b – a << ℓ, find the resistance of the conductor.



Answer

$$\frac{\rho l}{\pi ab}$$

We have to find the resistance associated with the truncated cone as shown in the figure.

For that purpose, we take an elemental area of this uniformly increasing cone and find the resistance of that element. By integrating that value from radius a to b, we would be able to find the resistance associated with the truncated cone.

The expression for the resistance, R, of a cylinder with length ‘l’ and area of cross-section A is,

$$R = \frac{\rho l}{A}$$

We assume that the cone is made up of an infinite number of cylinders with length 'dx'.

So for that element, resistance dR is

$$dR = \frac{\rho \times dx}{A} \; (eqn.1)$$

Hence the total resistance R is

$$R = \int_a^b dR \; (eqn.2)$$

Now the resistance for an element with a radius 'y' at x distance from left with length 'dx' should be found out.

The mean area of the element is,

$$A = \pi y^2$$

Hence eqn.1 becomes

$$dR = \frac{\rho \times dx}{\pi y^2} \; (eqn.3)$$

From the figure, we can write a relation connecting x and y as,

$$\frac{(b-a)}{l} = \frac{(y-a)}{x}$$

Or,

$$x(b-a) = l(y-a)$$

Differentiating the above expression, we get,

$$(b-a) = l\frac{dy}{dx}$$

On re-arranging

$$dx = \frac{l \times dy}{(b-a)}$$

Substituting this value in eqn.1 and combining with eqn.3, we get,

$$R = \int_a^b \frac{\rho \times dx}{\pi y^2}$$

$$= \int_a^b \frac{\rho \times \frac{l}{(b-a)}}{\pi y^2} dy$$

On integration,

$$R = \rho \times \frac{l}{\pi(b-a)} \int_a^b \frac{dy}{y^2}$$

Or,

$$R = \rho \times \frac{l}{\pi(b-a)} \times \left(\frac{-1}{y}\right)_a^b$$

Or,

$$R = \rho \times \frac{l}{\pi(b-a)} \times \frac{(b-a)}{ab}$$

Or,

$$R = \rho \times \frac{l}{\pi ab} \ (Ans.)$$

Hence the resistance of the truncated cone is $\frac{\rho l}{\pi ab}$

11. Question

A copper wire of radius 0.1 mm and resistance 1 kΩ is connected across a power supply of 20 V.

(a) How many electrons are transferred per second between the supply and the wire at one end?

(b) Write down the current density in the wire.

Answer

a) 1.25×10^{17} b) 6.37×10^{5} A/m^2

Given,

Radius of the wire, r= 0.1mm=0.1×10^{-3}m

Resistance of the wire, R= 1000Ω

The voltage across the wire, V= 20V

Formula used,

The wire is having a resistance, R and hence the current, I through it can be found from the equation,

$$I = \frac{V}{R}\ (eqn.1)$$

Where V denotes the potential difference across the resistance.

We know that the current, I through a resistor can be calculated from the charge, q flowing through it per seconds, or

$$I = \frac{q}{t}\ (eqn.2)$$

Where t is the time in seconds for which the current is flown.

But the charge, q can be written in terms of number of electrons, n flowing by the relation,

$$q = ne\ (eqn.3)$$

Where e is the charge of 1 electron= 1.6×10^{-19} C

Also, the current density, J through a material with constant area of cross section A, can be represented as

$$J = \frac{I}{A}\ (eqn.4)$$

Solution,

a) From the above equations, eqn.1 can be re-written with the help of eqn.2 and eqn.3 as,

$$\frac{ne}{t} = \frac{V}{R}$$

Or

$$n = \frac{Vt}{eR}$$

For time t= 1s, and by substituting the given values in the above relation, we can calculate the number of electrons passed in 1s through the copper wire with resistance 1000Ω as,

$$n = \frac{20V \times 1s}{1.6 \times 10^{-19}C \times 1000\Omega}$$

$$= 1.25 \times 10^{17}\ electrons\ (Ans.)$$

Hence the number of electrons transferred per second between the supply and the wire at one end is 1.25×10^{17}

b)

To find the current density, we have to calculate the area of cross-section, A of the wire with radius r. Hence

$$\boldsymbol{A = \pi r^2}$$

By substituting the value for r, we get the area as,

$$\boldsymbol{A = \pi(0.1 \times 10^{-3})^2 = 3.1415 \times 10^{-8} m^2}$$

Also, the current through the wire, from eqn.1, is

$$\boldsymbol{I = \frac{20V}{1000\Omega} = 0.02A}$$

Now, by substituting the known values in the expression for current density, eqn.4, we get J as

$$\boldsymbol{J = \frac{0.02A}{3.1415 \times 10^{-8} m^2} = 6.366 \times 10^5 A/m^2\ (Ans)}$$

Hence the current density in the wire is around 6.37×10^5 A/m^2

12. Question

Calculate the electric field in a copper wire of cross-section area 2.0 mm^2 carrying a current of 1 A. The resistivity of copper = 1.7×10^{-8} Ω m.

Answer

8.5mVm^{-1}

Given,

Area of cross-section of wire, A=2.0 mm^2=2×10^{-6} m^2

Current flowing through the wire, I=1A

Resistivity of copper, ρ= 1.7×10^{-8} Ω m

Formula used,

From electrostatics, we know that Electric field, E is

$$\boldsymbol{E = \frac{dV}{dL}}$$

Or it can be written as

$$\boldsymbol{E = \frac{V}{l}\ (eqn.1)}$$

Where l is the distance over which potential difference V has an effect

From Ohm's law and the concept of current density, the relation connecting resistance, R and resistivity, ρ can be written as,

$$R = \frac{\rho l}{A}\ (eqn.2)$$

Where 'l' is the length of the conductor (or wire) and 'A' is the area of cross-section of the conductor.

Solution,

We know that potential difference, V is

$$V = IR$$

Using eqn.2, the above expression can be modified as,

$$V = I \times \frac{\rho l}{A}$$

Now, in order to find the electric field by eqn.1, we can replace V in eqn.1 with the above expression as,

$$E = \frac{I \times \frac{\rho l}{A}}{l}$$

On simplification, it becomes,

$$E = \frac{I \times \rho}{A}$$

We have all the values for the above expression. So, on substitution, Electric field will be,

$$E = \frac{1A \times 1.7 \times 10^{-8}\Omega m}{2 \times 10^{-6} m^2} = 0.0085 V m^{-1}\ (Ans.)$$

Hence the electric field in a copper wire is $8.5 mVm^{-1}$

13. Question

A wire has a length of 2.0 m and a resistance of 5.0 Ω. Find the electric field existing inside the wire if it carries a current of 10 A.

Answer

$25 Vm^{-1}$

Given,

Length, l of the wire= 2m

Resistance, R of the wire=5 Ω

Current I, passing through the wire= 10A

Formula used,

From electrostatics, we know that Electric field, E is

$$\boldsymbol{E} = \frac{\boldsymbol{dV}}{\boldsymbol{dL}}$$

Or it can be written as

$$\boldsymbol{E} = \frac{\boldsymbol{V}}{\boldsymbol{l}}\ (\boldsymbol{eqn.1})$$

Where l is the distance over which potential difference V has an effect

Solution,

We know that the potential difference, V in the wire with current I passing and with a resistance R is

$$\boldsymbol{V} = \boldsymbol{IR}$$

By substituting the given values in the above expression, we get V as

$$\boldsymbol{V} = \mathbf{10}\boldsymbol{A} \times \mathbf{5\Omega} = \mathbf{50V}$$

Substituting this value of potential difference in eqn.1, we can find the electric field,

So,

$$\boldsymbol{E} = \frac{\boldsymbol{V}}{\boldsymbol{l}}$$

Or

$$\boldsymbol{E} = \frac{\mathbf{50V}}{\mathbf{2}\boldsymbol{m}} = \mathbf{25}\boldsymbol{Vm}^{-\mathbf{1}}\ (\boldsymbol{Ans.})$$

Hence the electric field existing inside the wire is $25Vm^{-1}$

14. Question

The resistances of an iron wire and a copper wire at 20°C and 3.9 Ω and 4.1 Ω respectively. At what temperature will the resistances be equal? Temperature coefficient of resistivity for iron is $5.0 \times 10^{-3}\ K^{-1}$ and for copper it is $4.0 \times 10^{-3}\ K^{-1}$. Neglect any thermal expansion.

Answer

84.5°C

Given,

Resistance of iron wire, $R_{Fe,\,i}$ at 20°C= 3.9 Ω

Resistance of Copper wire, $R_{Cu,\,i}$ at 20°C = 4.1 Ω

Initial temperature of both the wires, T_i= 20°C

Temperature coefficient of resistivity for iron, α_{Fe}=5.0 × 10^{-3} K^{-1}

Temperature coefficient of resistivity for copper, α_{Cu}=4.0 × 10^{-3} K^{-1}

Formula used,

For most of the conducting materials, the relation connecting the resistance with the change in temperature can be represented as,

$$\boldsymbol{R_f = R_i(1+\propto \Delta T)\ (eqn.1)}$$

Where R_f is the final resistance after the change in temperature, R_i is the initial temperature, α is the Temperature coefficient of resistivity and ΔT is the change in temperature from the initial temperature of the material.

Solution:

We have the expression connecting the change in temperature and the resistance of the material.

First, let us assume that the final temperature is T_f at which the resistance of both the wires will be same.

So, change in temperature ΔT is

$$\boldsymbol{\Delta T = T_f - T_i\ (eqn.2)}$$

Now the final resistance of iron wire, $R_{Fe,\,f}$ at temperature T_f can be written based on eqn.1 as,

$$\boldsymbol{R_{Fe,f} = R_{Fe,i}(1 +\propto_{Fe} \Delta T)}$$

Or by substituting the known values, we can write it as,

$$\boldsymbol{R_{Fe,f} = 3.9\ \Omega \times (1 + (5 \times 10^{-3} K^{-1}) \times \Delta T)\ (eqn.3)}$$

Similarly, the final resistance of iron wire, $R_{Cu,\,f}$ at temperature T_f can be written based on eqn.1 as,

$$\boldsymbol{R_{Cu,f} = R_{Cu,i}(1 +\propto_{Cu} \Delta T)}$$

Or by substituting the known values, we can write it as,

$$\boldsymbol{R_{Cu,f} = 4.1\ \Omega \times (1 + (4 \times 10^{-3} K^{-1}) \times \Delta T)\ (eqn.4)}$$

At the final temperature, it is given that the resistance of both the wires are same. So we can equate eqn.3 and eqn.4.

So,

$$R_{Fe,f} = R_{Cu,f}$$

Or,

$$3.9\,\Omega \times (1 + (5 \times 10^{-3} K^{-1}) \times \Delta T) = 4.1\,\Omega \times (1 + (4 \times 10^{-3} K^{-1}) \times \Delta T)$$

By solving,

$$0.2\Omega = 10^{-3}(3.9 \times 5 - 4.1 \times 4) \times \Delta T$$

Or,

$$0.2\,\Omega = 3.1 \times 10^{-3} \times \Delta T$$

Or,

$$\Delta T = 64.5°C$$

We know that ΔT is the difference between final and initial temperature, and hence T_f from eqn.2 is

$$\Delta \mathrm{T} = T_f - T_i$$

Or,

$$T_f = \Delta \mathrm{T} + T_i$$

By substituting the known values, we get T_f as

$$T_f = 64.5°C + 20°C = 84.5°C\ (Ans.)$$

Hence the temperature in which the resistances are equal is 84.5°C

15. Question

The current in a conductor and the potential difference across its ends are measured by an ammeter and a voltmeter. The meters draw negligible currents. The ammeter is accurate but the voltmeter has a zero error (that is, it does not read zero when no potential difference is applied). Calculate the zero error if the readings for two different conditions are 1.75 A, 14.4 V and 2.75 A, 22.4 V.

Answer

0.4V

Given,

Ammeter reading of current, i_1 in the 1^{st} case=1.75A

Ammeter reading of current, i_2 in the 2nd case=2.75A

Voltmeter reading for current i_1, V_1=14.4V

Voltmeter reading for current i_2, V_2=22.4V

Formula used

The ammeter shows the accurate reading while the voltmeter deviate from showing the corresponding potential difference value due to the presence of Zero error, V_e.

Hence the original potential difference can be calculated as, V_o

$$\boldsymbol{V_o = V - V_e\ (eqn.1)}$$

Where V is the shown voltage in the defected voltmeter.

We also know that the potential difference across the conductor is, V

$$\boldsymbol{V = iR\ (eqn.2)}$$

Where R is the resistance of the conductor

Solution:

For the 1st and 2nd case, the relation between absolute current and voltage can be written, from eqn.2, as

$$\boldsymbol{V_{o1} = i_1 R}$$

And

$$\boldsymbol{V_{o2} = i_2 R}$$

By dividing these expressions, we get

$$\boldsymbol{\frac{V_{o1}}{V_{o2}} = \frac{i_1}{i_2}\ (eqn.3)}$$

The terms V_{o1} and V_{o2} in above equation can be replaced by eqn.1 as

$$\boldsymbol{\frac{V_1 - V_e}{V_2 - V_e} = \frac{i_1}{i_2}}$$

By substituting the given values in the above expression, we get

$$\boldsymbol{\frac{14.4\text{V} - V_e}{22.4\text{V} - V_e} = \frac{1.75\text{A}}{2.75\text{A}}}$$

Or,

$$\frac{14.4V - V_e}{22.4V - V_e} = 0.6364$$

Or,

$$14.4V - V_e = 14.2554V - 0.6364V_e$$

Or,

$$(1 - 0.6364)V_e = (14.4V - 14.2554V)$$

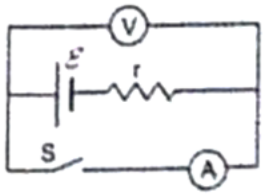
Or,

$$V_e = 0.4\ V\ (Ans.)$$

Hence the zero error associated with the measurement is 0.4V.

16. Question

Figure shows an arrangement to measure the emf ϵ and internal resistance r of a battery. The voltmeter has a very high resistance and the ammeter also has some resistance. The voltmeter reads 1.52 V when the switch S is open. When the switch is closed the voltmeter reading drops to 1.45 V and the ammeter reads 1.0 A. Find the emf and the internal resistance of the battery.



Answer

1.52V, 0.07Ω

Given,

Voltage reading, V_1 with switch is open=1.52V

Voltage reading, V_2 with switch is closed=1.45V

Current through the ammeter, i= 1A

Formula used

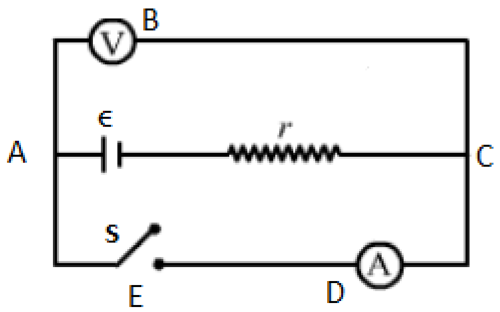
Kirchhoff's loop rule says that the algebraic sum of the voltage in a loop is always zero. Or

$$\sum V = iR$$

For a cell with an internal resistance r and emf ϵ, the voltage drop, V across it, if a current 'i' is passed through the circuit can be written using the loop rule as,

$V = \epsilon - \mathbf{ir}\ (eqn.\ 1)$

Solution:



a) When switch is open, the current would circulate through loop ABCA but not through the loop ACDEA. Since the internal resistance is very small compared to the resistance of Voltmeter, the voltage drop occurs completely across the voltmeter. This voltage drop will be measured in the meter and it will be almost equal to the emf of the cell.

Hence the e.m.f of cell= volt meter reading.

Or,

$$e.m.f = 1.52\mathbf{V}\ (\mathbf{Ans.})$$

b) When the switch is closed, a current 'i' will flow through the loop ACDEA. Now the volt meter will show the potential drop across the cell and the internal resistance combined. So, using eqn.1 , we can find the internal resistance,

$$V = \epsilon - \mathbf{ir}$$

Where V=Volt meter reading= 1.45V, i= Ammeter reading=1A, and ∈= 1.52V as we

By re-arranging, we can find the expression for internal resistance as,

$$r = \frac{\epsilon - V}{i}$$

By substituting the given values,

$$r = \frac{1.52\mathbf{V} - 1.45\mathbf{V}}{1\mathbf{A}} = 0.07\Omega\ (\mathbf{Ans.})$$

Hence the internal resistance of the cell is 0.07Ω

17. Question

The potential difference between the terminals of a battery of emf 6.0 V and internal resistance 1 Ω drops to 5.8 V when connected across an external resistor. Find the resistance of the external resistor.

Answer

29Ω

Given,

The emf of the cell, E=6V

The value of internal resistance, r= 1 Ω

The volt meter reading across the setup, V=5.8V

The value of external resistance= R Ω

Formula used:

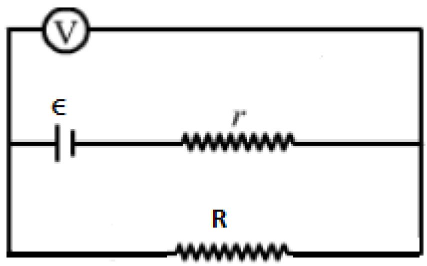


Figure shows the overview of the setup.

The voltage across the cell will be equal to that across the external resistance. So,

$$\boldsymbol{E - ir = iR = V\ (eqn.1)}$$

Solution:

The current flowing through the internal and external resistance are the same.

By substituting the given values in the eqn.1, we get i as

$$\boldsymbol{E - ir = V}$$

Or,

$$\mathbf{6V} - \boldsymbol{i} \times \mathbf{1\Omega = 5.8V}$$

Or,

$$\boldsymbol{i} = \frac{\mathbf{6V - 5.8V}}{\mathbf{1\Omega}} = \boldsymbol{0.2A}$$

This current also circulate through the external resistance R. Hence from the eqn.1, we can write its expression as,

$$\boldsymbol{iR = V}$$

By substituting the values of 'i' and V, we get

$$\boldsymbol{0.2A \times R} = \mathbf{5.8V}$$

Or,

$$R = \frac{5.8V}{0.2A} = 29\Omega\ (Ans.)$$

Hence the resistance of the external resistor is 29Ω.

18. Question

The potential difference between the terminals of a 6.0 V battery is 7.2 V when it is being charged by a current of 2.0 A. What is the internal resistance of the battery?

Answer

0.6Ω

Given,

The potential difference across the setup, V= 7.2V

The emf of the cell, E=6V

The current flowing through the circuit, i= 2A

Formula used:

When the battery is charging the potential difference, V across the cell can be written as,

$$V = E + ir\ (eqn.1)$$

Where ‘E’ is the emf of the cell and ‘r’ is internal resistance of the cell.

Solution:

As the battery is getting charged, the internal resistance can be calculated from the eqn.1, as

$$r = \frac{V - E}{i}$$

And by substituting the given values,

$$r = \frac{7.2V - 6V}{2A} = 0.6\Omega\ (Ans.)$$

Hence the internal resistance of the cell is 0.6Ω

19. Question

The internal resistance of an accumulator battery of emf 6 V is 10 Ω when it is fully discharged. As the battery gets charged up, its internal resistance decreases to 1Ω. The battery in its completely discharged state is connected to a charger which maintains a constant potential difference of 9V. Find the current through the battery

(a) just after the connections are made and

(b) after a long time when it is completely charged.

Answer

a)0.3A, b)3A

Given,

The emf of the battery, E=6V

The internal resistance of the battery, r_1 when discharged=10Ω

The internal resistance of the battery, r_2 when charged=1Ω

Potential difference provided by the charger, E_c=9V

Formula used:

When the accumulator battery is connected to a charger, the current through the internal resistance depends on the net emf available across the resistor.

Hence Net emf across the resistor in the case of charging will be the difference between the provided potential difference and the potential difference rating of the battery.

Solution:

a) When the battery is being charged, the net emf, E_{net} across the resistance, r_1 will be,

$$\boldsymbol{E_{net} = External\ potential\ difference - Rated\ Potential\ difference\ of\ battery}$$

Or,

$$\boldsymbol{E_{net} = 9V - 6V = 3V}$$

Hence from the relation, $\boldsymbol{= ir_1}$, we can find the current, i by substituting r_1 as 10 Ω,

$$\boldsymbol{i = \frac{V}{r_1}}$$

Or,

$$\boldsymbol{i = \frac{3V}{10\Omega} = 0.3A\ (Ans.)}$$

b) When the battery is completely charged, the internal resistance r_2 will be 1Ω. The net emf across the resistance will be the same, and is 3V.

Hence from the relation, $\boldsymbol{V = ir_2}$ we can find the current, i through the resistance by substituting r_2 as 1Ω,

$$i = \frac{V}{r_1}$$

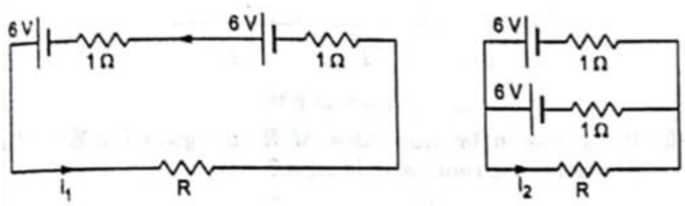
Or,

$$i = \frac{3V}{1\Omega} = 3A\ (Ans.)$$

Hence the current through the internal resistance while charging and after completely charged are 0.3A and 3A.

20. Question

Find the value of i_1/i_2 in figure if (a) R = 0.1Ω, (b) R = 1 Ω (c) R = 10 Ω. Note from our answers that in order to get more current from a combination of two batteries they should be joined in parallel if the external resistance is small and in series if the external resistance is large as compared to the internal resistances.



Answer

a) 0.57, b)1, c)1.75

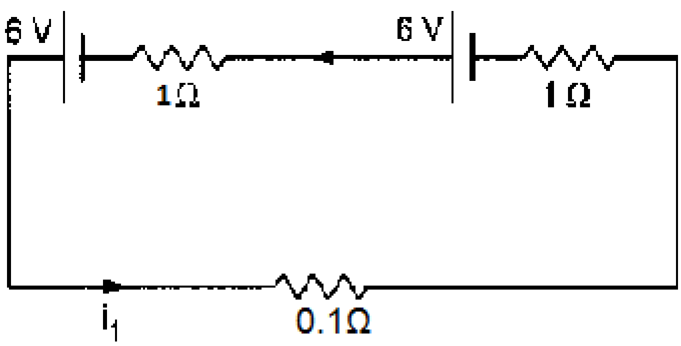
Formula used:

Kirchhoff's loop rule state that the algebraic sum of all the voltages in a loop will be zero. Or,

$$\sum V = iR$$

Solution:

a)



Given resistances are: 1 Ω,1 Ω,0.1Ω

Applying Loop rule, we can write,

$$6V + 6V = i_1 \times 0.1\Omega + i_1 \times 1\Omega + i_1 \times 1\Omega$$

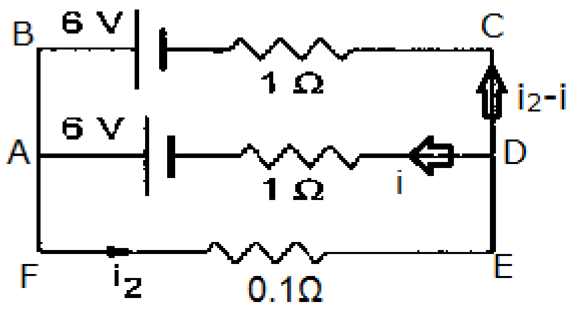
Or,

$$12V = i_1 \times 2.1\Omega$$

And i_1 will be,

$$i_1 = \frac{12V}{2.1\Omega} = 5.71A$$

In the next figure,



Let i be the current that passes through the middle branch and hence i_2-i will pass through the upper branch, as shown in figure.

By applying loop rule in AFEDA, we can write as,

$$6V = i_2 \times 0.1\Omega + i \times 1\Omega$$

Or,

$$i = 6 - 0.1i_2 \ (eqn.1)$$

Similarly, applying the loop rule in ADCBA, we can write as,

$$-6V - i \times 1\Omega + (i_2 - i) \times 1\Omega + 6V = 0$$

Or,

$$i_2 = 2i \ (eqn.2)$$

Replacing 'i' in eqn.1 using eqn.2, we get,

$$0.5i_2 = 6 - 0.1i_2$$

Or,

$$0.6i_2 = 6$$

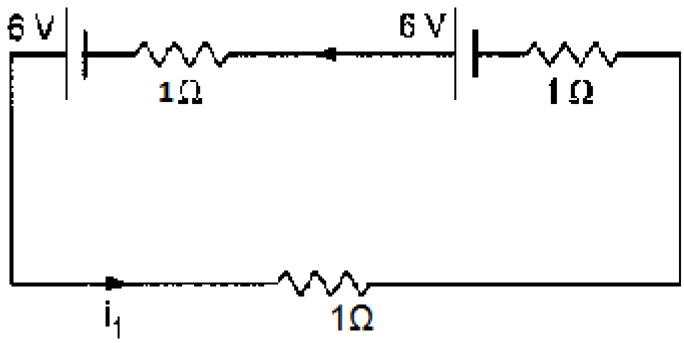
Or,

$$i_2 = 10A$$

Now i_1/i_2 is,

$$\frac{i_1}{i_2} = \frac{5.71A}{10A} = 0.57\ (Ans.)$$

b)



Given resistances are: 1 Ω,1 Ω,1Ω

Applying Loop rule, we can write,

$$\mathbf{6V + 6V} = i_1 \times \mathbf{1\Omega} + i_1 \times \mathbf{1\Omega} + i_1 \times \mathbf{1\Omega}$$

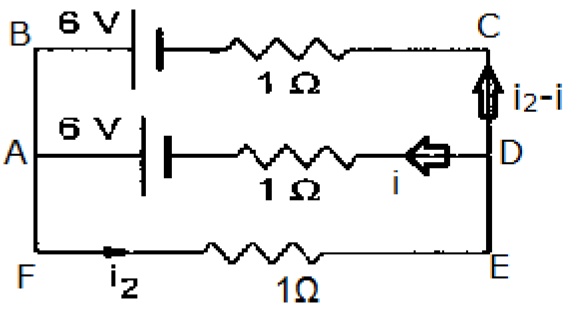
Or,

$$\mathbf{12V} = i_1 \times \mathbf{3\Omega}$$

And i_1 will be,

$$i_1 = \frac{12V}{3\Omega} = 4A$$

In the next figure,



Let i be the current that passes through the middle branch and hence i_2-i will pass through the upper branch, as shown in figure.

By applying loop rule in AFEDA, we can write as,

$$6V = i_2 \times 1\Omega + \mathrm{i} \times 1\Omega$$

Or,

$$i = 6 - i_2\ (eqn.1)$$

Similarly, applying the loop rule in ADCBA, we can write as,

$$-6V - i \times 1\Omega + (i_2 - i) \times 1\Omega + 6V = 0$$

Or,

$$i_2 = 2i\ (eqn.2)$$

Replacing 'i' in eqn.1 using eqn.2, we get,

$$0.5i_2 = 6 - i_2$$

Or,

$$1.5i_2 = 6$$

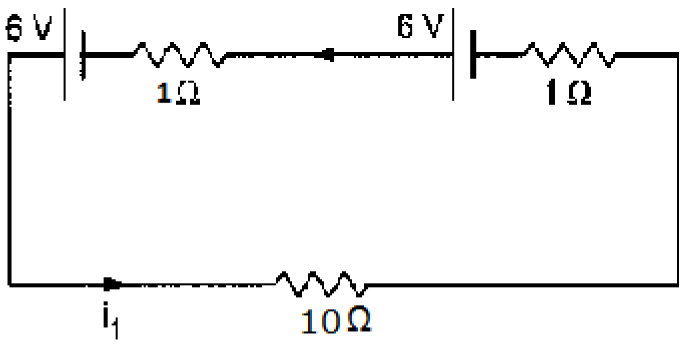
Or,

$$i_2 = 4A$$

Now i_1/i_2 is,

$$\frac{i_1}{i_2} = \frac{4A}{4A} = 1\ (Ans.)$$

c)



Given resistances are: 1 Ω,1 Ω,10Ω

Applying Loop rule, we can write,

$$6\text{V} + 6\text{V} = i_1 \times 10\Omega + i_1 \times 1\Omega + i_1 \times 1\Omega$$

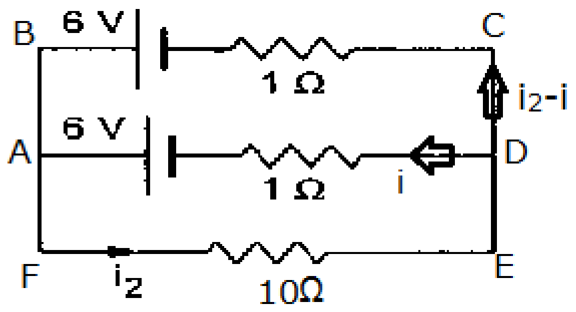
Or,

$$12\text{V} = i_1 \times 12\Omega$$

And i_1 will be,

$$i_1 = \frac{12V}{12\Omega} = 1A$$

In the next figure,



Let i be the current that passes through the middle branch and hence i_2-i will pass through the upper branch, as shown in figure.

By applying loop rule in AFEDA, we can write as,

$$6V = i_2 \times 10\Omega + \mathrm{i} \times 1\Omega$$

Or,

$$i = 6 - 10i_2\ (eqn. 1)$$

Similarly, applying the loop rule in ADCBA, we can write as,

$$-6V - i \times 1\Omega + (i_2 - i) \times 1\Omega + 6V = 0$$

Or,

$$i_2 = 2i\ (eqn. 2)$$

Replacing ‘i’ in eqn.1 using eqn.2, we get,

$$0.5i_2 = 6 - 10i_2$$

Or,

$$10.5i_2 = 6$$

Or,

$$i_2 = {}^{6}/_{10.5} = 0.57A$$

Now i_1/i_2 is,

$$\frac{i_1}{i_2} = \frac{1A}{0.57A} = 1.75\ (Ans.)$$

21. Question

Consider N = n_1n_2 identical cells, each of emf ϵ and internal resistance r. Suppose n_1 cell are joined in series to form a line and n_2 such lines are connected in parallel. The combination drives a current in an external resistance R.

(a) Find the current in the external resistance.

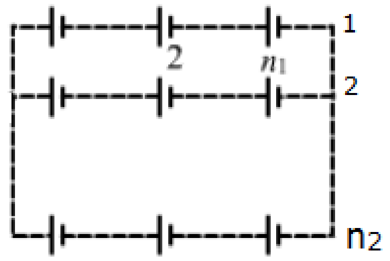
(b) Assuming that n_1 and n_2 can be continuously varied, find the relation between n_1, n_2, R and r for which the current in R is maximum.

Answer

a) $= \frac{n_1 n_2 E}{n_1 r + n_2 R}$; b) $n_1 r = n_2 R$

Solution:

a)



When n_1 cells each with emf 'E' are connected in series, the total emf, E_{net} in one branch is,

$$E_{net} = n_1 E \; (eqn. 1)$$

Since, n_2 of such the branches are connected in parallel, the Total emf in every branches are the same, E_{net}.

The resistance of n_1 cells each with resistance 'r' in series is,

$$R_o = n_1 r$$

The total resistance for such n_2 number of branches, connected in parallel, is

$$R_{eff} = \frac{n_1 r}{n_2}$$

It is given that the whole setup is connected to an external resistance R. Hence the total net resistance will be,

$$R_{net} = \frac{n_1 r}{n_2} + R \; (eqn. 2)$$

Hence the current through the external resistor is,

$$i = \frac{E_{net}}{R_{net}}$$

Or,

$$i = \frac{n_1 E}{\left(\frac{n_1 r}{n_2} + R\right)} = \frac{n_1 n_2 E}{n_1 r + n_2 R} \; (Ans.)$$

b)

We know that the relation connecting n_1, n_2, R and r is

$$i = \frac{n_1 n_2 E}{n_1 r + n_2 R}$$

To get the minimum current through the resistor R, the denominator in the above expression should be minimum.

In order to minimize the term 'n_1r+n_2R', we re-write it as,

$$n_1 r + n_2 R = \left(\sqrt{n_1 r} - \sqrt{n_2 R}\right)^2 + 2\sqrt{n_1 n_2 r}$$

SO, this term should be minimized. But, the above value is minimum only when the term in the bracket is zero.

So,

$$\sqrt{n_1 r} = \sqrt{n_2 R}$$

Or,

$$n_1 r = n_2 R$$

Hence i is maximum when $n_1r=n_2R$

22. Question

A battery of emf 100 V and a resistor of resistance 10 kΩ are joined in series. This system is used as a source to supply current to an external resistance R. If R is not greater than 100 Ω, the current through it is constant up to two significant digits. Find its value. This is the basic principle of a constant-current source.

Answer

10mA

Given,

Emf of the battery, E= 100V

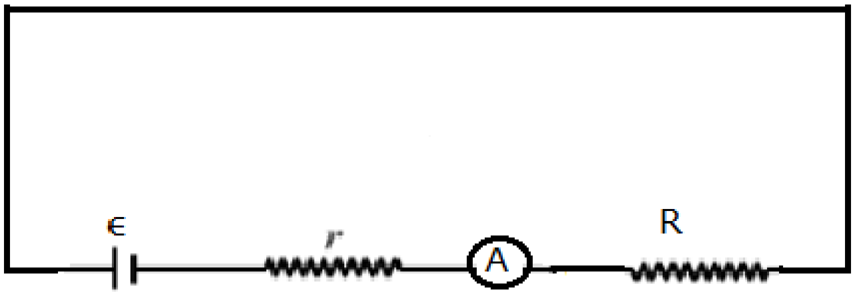
Resistance of series resistor, r= 10kΩ=10000kΩ

Resistance of external resistor, R=0-100Ω

Formula used:

A constant-current source is a power source that supply constant current to an external load, even if there is a change in load resistance.

When the battery is connected to the external resistance that vary from 0 to 100 Ω, the effective resistance will change across the potential difference provided by the battery.



Solution:

Let's find out the current when R=0Ω or when there is no external resistance is connected,

We know that, current i for a series resistor connection is

$$\boldsymbol{i = \frac{E}{r}}$$

By substituting the known values, we get i as,

$$\boldsymbol{i = \frac{100V}{10000\Omega} = 0.01A\ (Ans.)}$$

Now let's take R as 2 Ω (or a low value like 1 Ω or so)

The value of current, i is

$$\boldsymbol{i = \frac{100V}{R_{tot}}}$$

Where R_{tot} is the effective resistance across the battery. From the figure, R_{tot} can be calculated as,

$$\boldsymbol{R_{tot} = r + R}$$

Hence by putting R=2 Ω, we get i as,

$$\boldsymbol{i = \frac{100V}{r+R} = \frac{100V}{10000\Omega + 2\Omega} = 0.009998A \approx 0.01A\ (Ans.)}$$

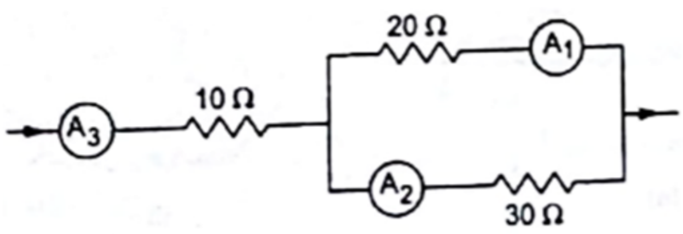
Similarly, by putting R=100Ω , the highest possible, we get the current i as,

$$\boldsymbol{i = \frac{100V}{r+R} = \frac{100V}{10000\Omega + 100\Omega} = 0.00990A \approx 0.01A\ (Ans.)}$$

So, as the principle predicted, the value of the current, 1mA, does not change much, or it stays consistent till two significant digits.

23. Question

If the reading of ammeter A_1 in figure is 2.4A, what will the ammeters A_2 and A_3 read? Neglect the resistances of the ammeters.



Answer

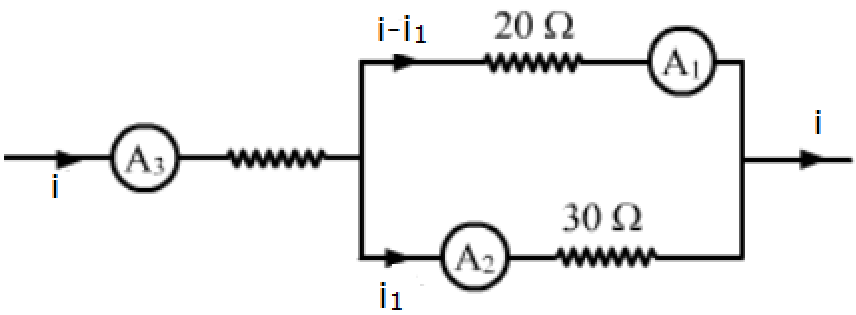
1.6A, 4A

Given,

Current through A_1 ammeter, $i-i_1$= 2.4A

Formula used:

Depending on the resistance offered by each paths of a circuit, the current split inversely. The voltage across the two branches will be same as they are connected parallel.



Solution:

The current going through Ammeter A_3 will split into two. To find the current in two branches, let's equate the voltage in each branches.

Hence,

$$(i - i_1) \times 20\Omega = i_1 \times 30\Omega$$

By substituting the value of $i-i_1$= 2.4A, we get i_1 that goes through ammeter A_2 as,

$$i_1 = \frac{2.4A \times 20\Omega}{30\Omega} = 1.6A\ (Ans.)$$

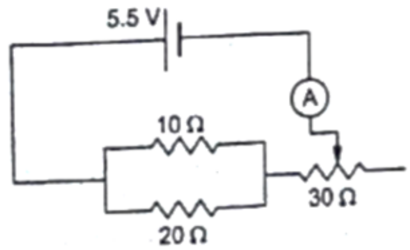
Hence the total current i, passing through A_3

$$i = (i - i_1) + i_1 = 2.4A + 1.6A = 4A\ (Ans.)$$

Hence the current through A_2 and A_3 are 1.6A and 4A.

24. Question

The resistance of the rheostat shown in figure is 30 Ω. Neglecting the meter resistance, find the minimum and maximum currents through the ammeter as the rheostat is varied.



Answer

0.15A, 0.83A

Given,

The resistance of the rheostat, R= 30Ω

The emf of the battery, E=5.5V

Formula used:

The rheostat can vary the resistance from 0Ω to maximum, and will be added as a series resistance to the given setup.

Solution:

The 10 Ω and 20 Ω connected parallel to each other. This can be reduced to a single resistance as they both connected to same potential difference. So the effective resistance, R_{eff} between 10 Ω and 20 Ω will be,

$$\boldsymbol{R_{eff}} = \frac{\mathbf{10\Omega \times 20\Omega}}{\mathbf{10\Omega + 20\Omega}} = \mathbf{6.667\Omega}$$

The rheostat resistance will be added in series to the above resistance.

The minimum current will be marked when the total resistance is maximum, which happens when rheostat resistance R=30Ω.

So the current will be,

$$\boldsymbol{i} = \frac{\boldsymbol{E}}{\boldsymbol{R_{tot}}}$$

Where R_{tot} is

$$\boldsymbol{R_{tot}} = \boldsymbol{R_{eff}} + \mathbf{30\Omega} = \mathbf{36.667\Omega}$$

Hence the minimum current, i_{min} is

$$i_{min} = \frac{5.5V}{36.667\Omega} = 0.15A\ (Ans.)$$

Similarly, maximum current, i_{max} can be obtained when Rheostat resistance R is minimum, R=0Ω.

So, R_{tot} is

$$R_{tot} = R_{eff} + 0\Omega = 6.667\Omega$$

Hence the current is,

$$i_{max} = \frac{E}{R_{tot}} = \frac{5.5V}{6.667\Omega} = 0.83A\ (Ans.)$$

Hence the current in the ammeter vary from 0.15A to 0.83A

25. Question

Three bulbs, each having a resistance of 180 Ω, are connected in parallel to an ideal battery of emf 60 V. Find the current delivered by the battery when

(a) all the bulbs are switched on,

(b) two of the bulbs are switched on and

(c) only one bulb is switched on.

Answer

a)1A, b)0.67A, c)0.33A

Given,

Emf of the battery, E=60V

Resistance of each bulb, r=180Ω

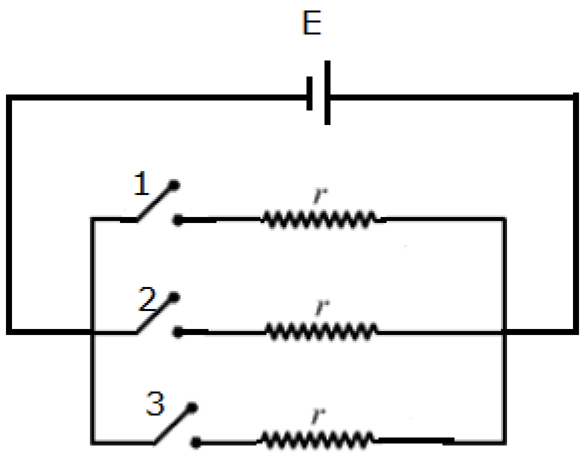
Formula used:

The potential difference across each bulb will be equal as they are connected in parallel across the same cell.

Also, we know that, for parallel connection of resistors, r with equal resistance, the effective resistance, R_{eff} will be

$$R_{eff} = r/n$$

Where n is the number of resistors.



a)

When all the switches are closed (switched on), the current will split equally across each of the resistors. And the total current will be the ratio between the potential difference and the effective resistance.

The effective resistance will be,

$$R_{eff} = r/3$$

Or,

$$R_{eff} = \frac{180\Omega}{3} = 60\Omega$$

Hence the current,i will be

$$i = \frac{E}{R_{eff}} = \frac{60V}{60\Omega} = 1A\ (Ans.)$$

b)

In this case two resistors (or bulbs) are connected in parallel. So the effective resistance will be,

$$R_{eff} = r/2$$

Or,

$$R_{eff} = \frac{180\Omega}{2} = 90\Omega$$

Hence the current, i will be

$$i = \frac{E}{R_{eff}} = \frac{60V}{90\Omega} = 0.67A\ (Ans.)$$

c)

In this case one resistor (or bulb) only connected to the battery. So the effective resistance will be,

$R_{eff} = r = 180\Omega$

Hence the current, i will be

$$i = \frac{E}{R_{eff}} = \frac{60V}{180\Omega} = 0.33A\ (Ans.)$$

The current in the setup due to the connection of 3 bulbs, 2 bulbs and 1 bulb is respectively 1.0A, 0.67A and 0.33A

26. Question

Suppose you have three resistors of 20Ω, 50Ω and 100Ω. What minimum and maximum resistances can you obtain from these resistors?

Answer

170Ω and 12.5Ω

Given, resistances $R_1 = 20\Omega, R_1 = 50\Omega, R_1 = 100\Omega$

Maximum resistance occurs when the three resistances are connected in series.

hence, $R = R_1 + R_2 + R_3$

$R = 20 + 50 + 100 = 170\Omega$

Minimum resistance is when they are parallel to each other.

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\Rightarrow \frac{1}{R} = \frac{1}{20} + \frac{1}{50} + \frac{1}{100} = \frac{5+2+1}{100} = \frac{8}{100}$$

$\Rightarrow R = 100/8 = 12.5\Omega$

27. Question

A bulb is made using two filaments. A switch selects whether the filaments are used individually or in parallel. When used with a 15 V battery, the bulb can be operated at 5W, 10W or 15W. What should be the resistances of the filaments?

Answer

45Ω and 22.5Ω

Given, Voltage of the battery, $V = 15V$

Power operated P=5W, 10W or 15W

We know, $P = \frac{V^2}{R}$

Now, $R = \frac{V^2}{P}$

$$\Rightarrow R_1 = \frac{15^2}{5} = 45\Omega$$

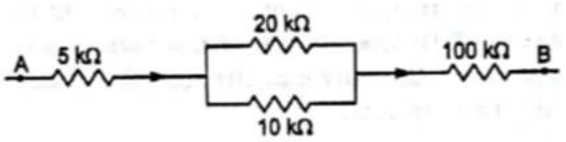
$$\Rightarrow R_2 = \frac{15^2}{10} = 22.5\Omega$$

$$\Rightarrow R_1 = \frac{15^2}{15} = 15\Omega$$

The parallel combination of resistances is always less than individual resistances. Therefore, 15Ω has to be parallel combination while 45Ω and 22.5Ω are the resistances.

28. Question

Figure shows a part of a circuit. If a current of 12 mA exists in the 5 kΩ resistor, find the currents in the other three resistors. What is the potential difference between the points A and B?



Answer

4mA, 8mA, 1340V.

Here, current in $5k\Omega$ resistor is $\text{I} = 12\text{mA}$.

Let current through $20k\Omega$ be I_1 and that from $10k\Omega$ be I_2 such that $\text{I}_2 = \text{I} - \text{I}_1$

now, the potential across $20k\Omega$ and $10k\Omega$ has to be same as they are connected in parallel.

$$\Rightarrow 20\text{I}_1 = 10(\text{I} - \text{I}_1)$$

$$\Rightarrow 30\text{I}_1 = 10\text{I}$$

$$\Rightarrow \text{I}_1 = \frac{10\text{I}}{30} = \frac{\text{I}}{3} = \frac{12}{3} = 4\text{mA}$$

$$\Rightarrow \text{I}_2 = \text{I} - \text{I}_1 = 12 - 4 = 8\text{mA}$$

Current flowing through $20k\Omega$ is 4mA and that from $10k\Omega$ is 8mA.

To calculate potential difference, we need to find the equivalent resistance.

$$R = 5 + \frac{20 \times 10}{20 + 10} + 100 = 111.67k\Omega$$

The current is $I = 12mA$

Therefore, potential between A and B is
$V = IR = 12 \times 10^{-3} \times 111.67 \times 10^3 = 1340V$

29. Question

An ideal battery sends a current of 5A in a resistor. When another resistor of value 10 Ω is connected in parallel, the current through the battery is increased to 6A. Find the resistance of the first resistor.

Answer

2Ω

Given, current in resistor, $I = 5A$

Current in a parallel setup, $I' = 6A$

Let the resistor be R

Voltage in the first case, $V_1 = IR = 5R$ ————(1)

Let R' be the equivalent resistance.

Voltage in the second case, $V_2 = I'R' = 6 \times \frac{10R}{10+R}$ ————(2)

now, (1) should be equal to (2).

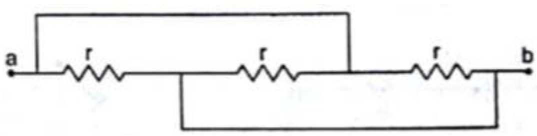
$$\Rightarrow 5R = 6 \times \frac{10R}{10 + R}$$

$$\Rightarrow 50 + 5R = 60$$

$$\Rightarrow R = 2\Omega$$

30. Question

Find the equivalent resistance of the network shown in figure between the points a and b.

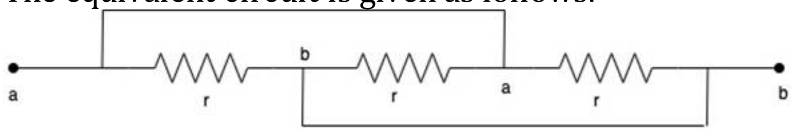


Answer

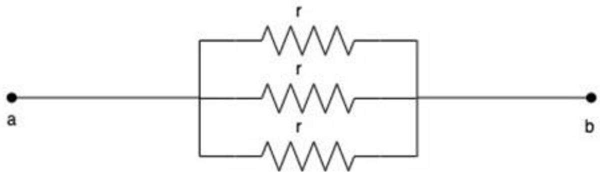
r/3

Given, resistance of each resistor is r.

The equivalent circuit is given as follows:



$$\equiv$$



Therefore, equivalent resistance is $\frac{1}{R} = \frac{1}{r} + \frac{1}{r} + \frac{1}{r} = \frac{3}{r}$

$\Rightarrow R = \frac{r}{3}$

31. Question

A wire of resistance 15.0 Ω is bent to form a regular hexagon ABCDEFA. Find the equivalent resistance of the loop between the points A and B, B A and C and C A and D.

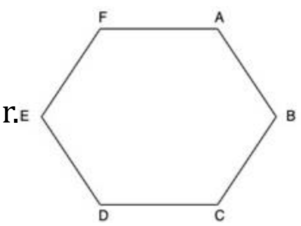
Answer

2.08Ω, 3.33Ω and 3.75Ω

Resistance of wire is 15Ω.

Resistance of each arm is $R = \frac{15}{6}\Omega$

The following figure shows the setup of resistor.



All other arms are collectively in series and parallel to arm AB.

The equivalent resistance of all other arms are: $R' = 5R = 5 \times \frac{15}{6} = \frac{75}{6}\Omega$

Hence, the resistance across AB is $R_{tot} = \frac{\frac{15}{6} \times \frac{75}{6}}{\frac{15}{6} + \frac{75}{6}} = \frac{25}{12}\Omega = 2.08\Omega$

BAll other arms are collectively in series and parallel to arm AC. AC has two series resistors.

The equivalent resistance of all other arms are: $R' = 4R = 4 \times \frac{15}{6} = \frac{60}{6}\Omega = 10\Omega$

Hence, the resistance across AB is $R_{tot} = \frac{\frac{30}{6}\times 10}{\frac{30}{6}+10} = \frac{10}{3}\Omega = 3.33\Omega$

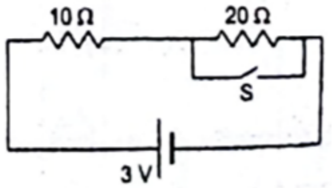
CAll other arms are collectively in series and parallel to arm AD. AD has three series resistors.

The equivalent resistance of all other arms are: $R' = 3R = 3 \times \frac{15}{6} = \frac{15}{2}\Omega$

Hence, the resistance across AB is $R_{tot} = \frac{\frac{75}{6}\times\frac{15}{2}}{\frac{75}{6}+\frac{15}{2}} = \frac{15}{4}\Omega = 3.75\Omega$

32. Question

Consider the circuit shown in figure. Find the current through the 10Ω resistor when the switch S is A. open B. closed.



Answer

0.1A and 0.3 A

Given, resistor , $R_1 = 10\Omega$ and $R_2 = 20\Omega$.

A If the switch is open, both resistors are in series.

Total resistance, $R = 10 + 20 = 30\Omega$

Current, $I = \frac{V}{R} = \frac{3}{30} = 0.1A$

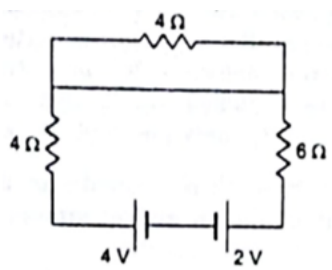
B If the switch is closed, R_2 is short circuited.

Total resistance, $R = 10\Omega$

Current, $I = \frac{V}{R} = \frac{3}{10} = 0.3A$

33. Question

Find the currents through the three resistors shown in figure.



Answer

0.2A

Here, the 4Ω resistor is short circuited. Let the current be I.

Using KVL in the loop,

$4I + 6I + 2 - 4 = 0$

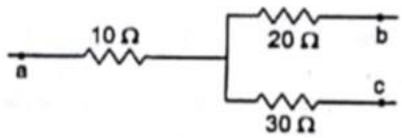
$\Rightarrow 10I = 2$

$\Rightarrow I = 0.2A$

Hence, the current through resistors is 0.2A

34. Question

Figure shows a part of an electric circuit. The potentials at the points a, b and c are 30 V, 12 V and 2 V respectively. Find the currents through the three resistors.



Answer

1A, 0.4A, 0.6A

Given, $V_a = 30V, V_b = 12V$ and $V_c = 2V$.

Let the potential at the joint be V

Current through 10Ω is $i_1 = \frac{V_a - V}{10} = \frac{30-V}{10}$

Current through 20Ω is $i_2 = \frac{V - V_c}{20} = \frac{V-12}{20}$

Current through 30Ω is $i_3 = \frac{V - V_c}{30} = \frac{V-2}{30}$

Now, Kirchhoff's junction rule, $i_1 = i_2 + i_3$

$\Rightarrow \frac{30 - V}{10} = \frac{V - 12}{20} + \frac{V - 2}{30}$

$\Rightarrow 30 - V = \frac{V - 12}{2} + \frac{V - 2}{3}$

$\Rightarrow 30 - V = \frac{3V - 36 + 2V - 4}{6}$

$\Rightarrow 11V = 220$

$\Rightarrow V = 20V$

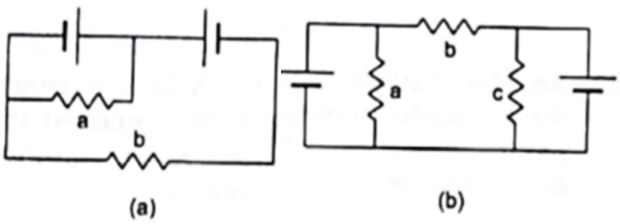
Therefore, $i_1 = \frac{V_a - V}{10} = \frac{30-20}{10} = 1A$

$$i_2 = \frac{20-12}{20} = 0.4A$$

$$i_3 = \frac{20-2}{20} = 0.6A$$

35. Question

Each of the resistors shown in figure has a resistance of 10 Ω and each of the batteries has an emf of 10 V. Find the currents through the resistors a and b in the two circuits.



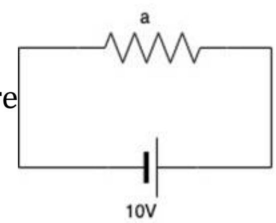
Answer

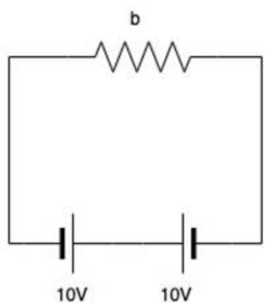
1A,0A,1A,0A

Given, resistance of resistor,$r = 10\Omega$

emf of each cell, $V = 10V$

A Equivalent circuits are

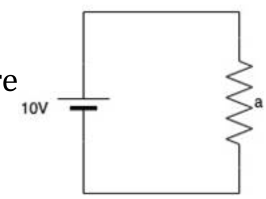


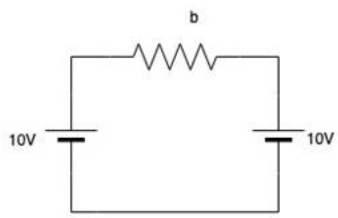


For current through a is $i = \frac{V}{R} = \frac{10}{10} = 1A$

As for b, the two emf cancel each other, thus total potential is zero. Hence, the current is zero.

B Equivalent circuits are



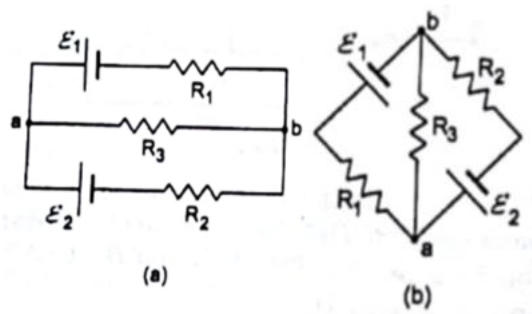


For current through a is $i = \frac{V}{R} = \frac{10}{10} = 1A$

Again for b, the two emf cancel each other, thus total potential is zero. Hence, the current is zero.

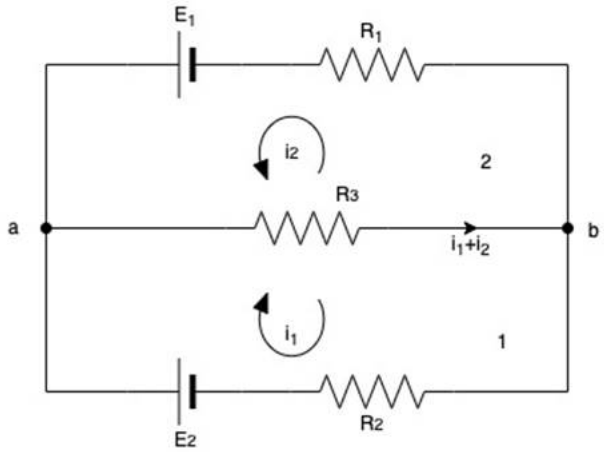
36. Question

Find the potential difference $V_a - V_b$ in the circuits shown in figure.



Answer

There are two current loops,



KVL in loop 1 gives

$$R_3(i_1 + i_2) + i_1R_2 - E_2 = 0$$

$$\Rightarrow (R_2 + R_3)i_1 + i_2R_3 = E_2 \text{ ———————(1)}$$

And in loop 2,

$$R_3(i_1 + i_2) + i_2R_2 = E_1$$

$$\Rightarrow (R_1 + R_3)i_2 + i_1R_3 = E_1 \text{ ———————(2)}$$

Solving (1) and (2)

$$i_1 = \frac{E_2(R_1 + R_3) - E_1R_3}{R_1R_2 + R_2R_3 + R_3R_1}$$

$$i_2 = \frac{E_1(R_2 + R_3) - E_2R_3}{R_1R_2 + R_2R_3 + R_3R_1}$$

Now, $V_a - V_b = (i_1 + i_2)R_3$

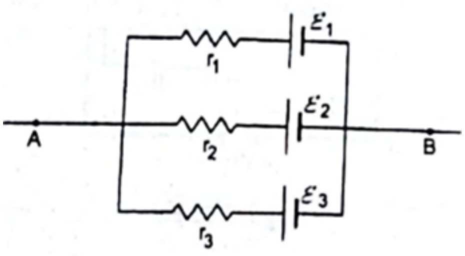
$$\Rightarrow V_a - V_b = \frac{E_1/R_1 + E_2/R_2}{1/R_1 + 1/R_2 + 1/R_3}$$

B On rotating the figure we find that this circuit is similar to A

hence, $V_a - V_b = \frac{E_1/R_1+E_2/R_2}{1/R_1+1/R_2+1/R_3}$

37. Question

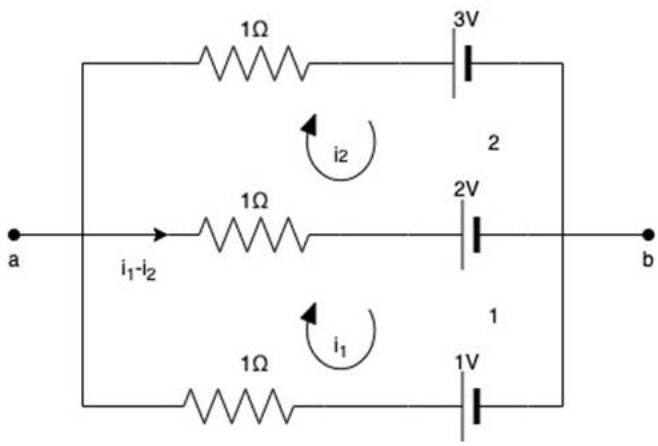
In the circuit shown in figure ϵ_1 = 3V, ϵ_2 = 2V, ϵ_3 = 1V and $r_1 = r_2 = r_3$ = 1Ω. Find the potential difference between the points A and B and the current through each branch.



Answer

2V,1A,0A,1A

Here, we will apply KVL



KVL in loop 1 gives

$i_1 + (i_1 - i_2) + 2 - 1 = 0$

$\Rightarrow 2i_1 - i_2 = -1$ ———————(1)

KVL in loop 2 gives

$i_2 - (i_1 - i_2) - 2 + 3 = 0$

$\Rightarrow 2i_2 - i_1 = -1$ ———————(2)

Solving (1) and (2) gives

$i_2 = i_1 = 1A$

Current through topmost branch is 1A.

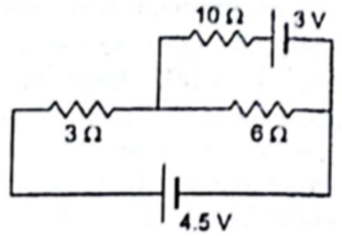
Current through middle branch is 0A.

Current through bottommost branch is 1A.

Therefore, potential difference is $V_a - V_b = E_2 - (i_1 - i_2)1 = 2V$

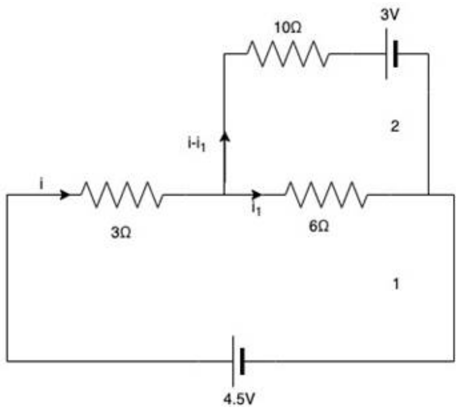
38. Question

Find the current through the 10Ω resistor shown in figure.



Answer

0A



Here, there are two loops,

KVL on loop 1 gives

$3i + 6i_1 = 4.5$ ————(1)

KVL on loop 2 gives

$10(i - i_1) + 3 - 6i_1 = 0$

$\Rightarrow 10i - 16i_1 = -3$ ————(2)

Solving (1) and (2) gives,

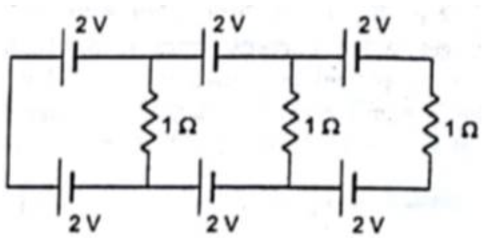
$i_1 = 0.5A$

and $i = 0.5A$

thus current through 10Ω is $i - i_1 = 0.5 - 0.5 = 0A$

39. Question

Find the current in the three resistors shown in figure.

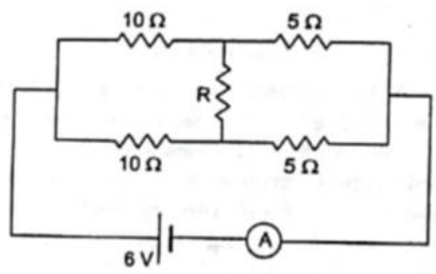


Answer

This question can be solved by critical analysis of the circuit. Three loops are present. In each loop there are 2 cells of equal emf opposing each other. Thus, the total effective emf is zero. Thus, there will be no current from either arms.

40. Question

What should be the value of R in figure for which the current in it is zero.



Answer

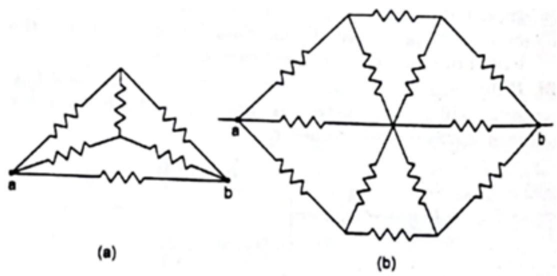
Any value of R

This is an example of balanced Wheatstone bridge.

In such a setup, the current through the middle branch is zero irrespective of the resistance. Hence, any value R would suffice for current to be zero.

41. Question

Find the equivalent resistance of the circuits shown in figure between the points a and b. Each resistor has a resistance r.



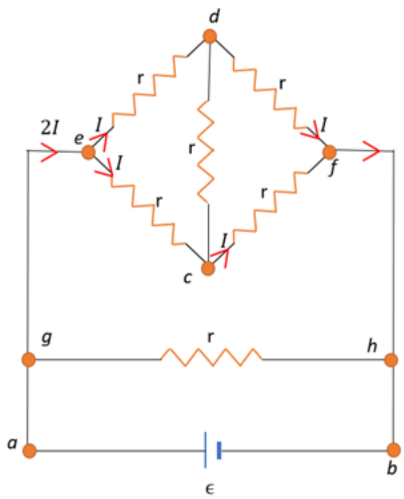
Answer

(a)

Concepts/Formula used:Resistors in Series: $R_{eq} = R_1 + R_2 + R_3 + \cdots$

Resistors in parallel: $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$

The given circuit can be rewritten as



As we wish to find the resistance between points a and b, we have proceeded to add a voltage source of emf ϵ between the points a and b.

Let the net current passing through the upper branch be 2I. We can see that the upper branch is symmetric i.e. its upper and lower portion are the identical. Thus, the current should divide equally when branching out. So, current through ec and ed is I.

Now,

$$V_d - V_e = -Ir$$

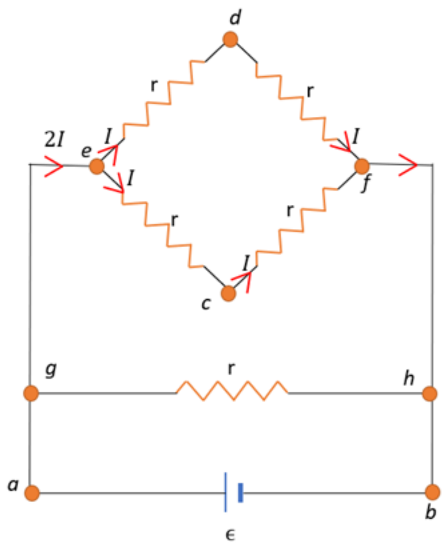
$$V_c - V_e = -\mathrm{Ir}$$

Hence,

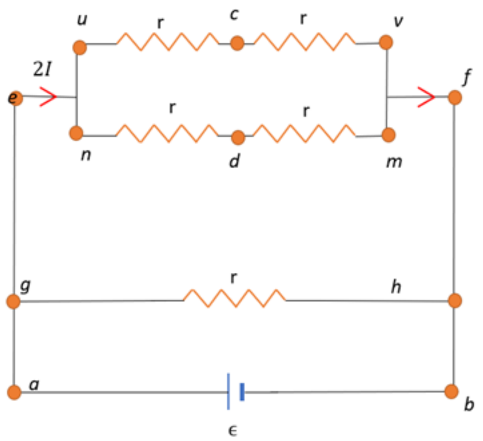
$$V_d = \mathrm{V_c}$$

As there is no potential difference across dc, there is no current passing through dc.

Hence, we can rewrite the circuit without the resistor across dc.



This circuit is equivalent to:



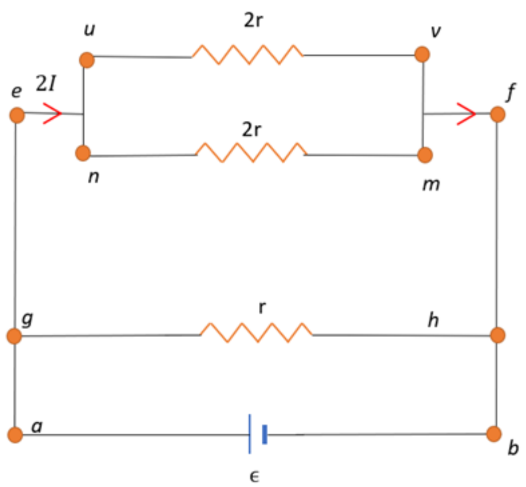
We can see that uc and cv are in series.

$$R_{uv}^{eq} = r + r = 2r$$

Also, nd and dm are is series.

$$R_{nm}^{eq} = \mathrm{r} + \mathrm{r} = 2\mathrm{r}$$

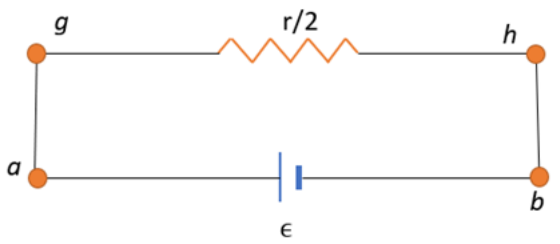
Hence, we can rewrite the circuit as follows:



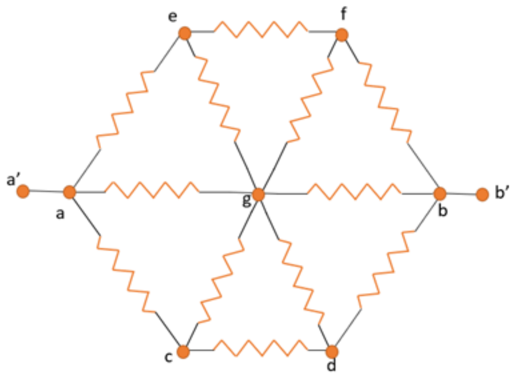
Clearly, uv and nm and gh are in parallel.

$$\frac{1}{R^{eq}} = \frac{1}{2r} + \frac{1}{2r} + \frac{1}{\mathrm{r}}$$

$$R^{eq} = \mathrm{r}/2$$



(b)

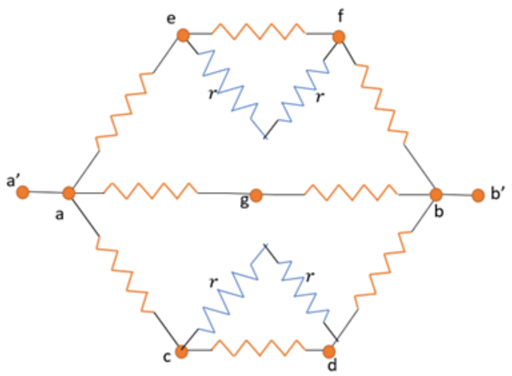


Note that because of symmetry,

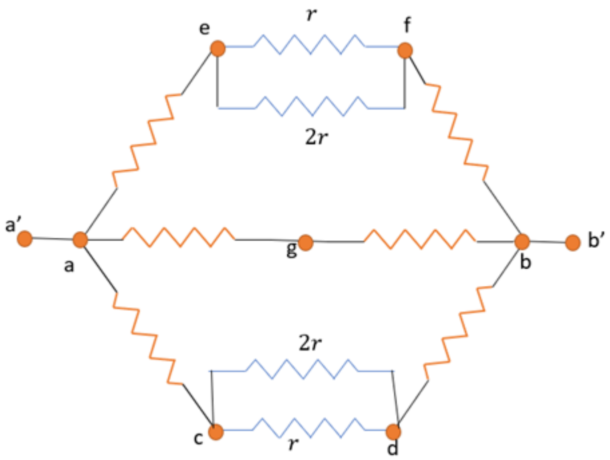
$$I_{eg} = I_{gf}$$

$$I_{cg} = I_{gd}$$

Hence, we can rewrite the circuit as cg and gd are is series and eg and gf are in series.



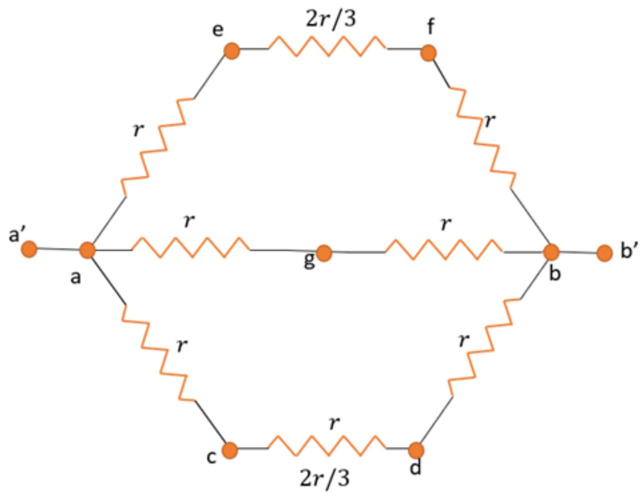
The blue resistors are in series. Net resistance = $r + r = 2r$



Now, the blue resistors are in parallel.

$$\frac{1}{R} = \frac{1}{r} + \frac{1}{2r}$$

$$R = \frac{3}{2}r$$

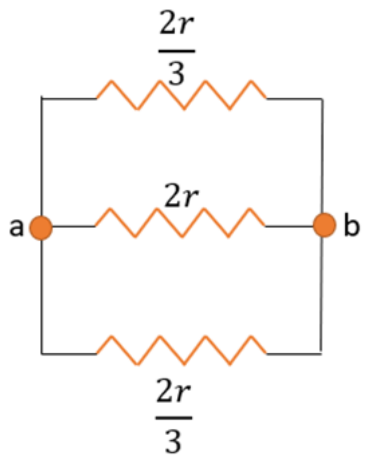


Now, the equivalent resistance of the upper branch and the lower branch is:

$$R = r + \frac{2r}{3} + r = \frac{8r}{3}$$

The middle branch: $R_{middle} = r + r = 2r$

Now, the circuit looks like this:



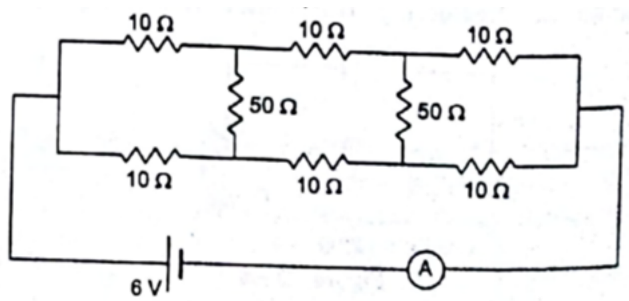
All the resistance are now in parallel:

$$\frac{1}{R_{eq}} = \frac{1}{\frac{2r}{3}} + \frac{1}{2r} + \frac{1}{\frac{2r}{3}}$$

$$R_{eq} = \frac{4r}{5}$$

42. Question

Find the current measured by the ammeter in the circuit shown in figure.



Answer

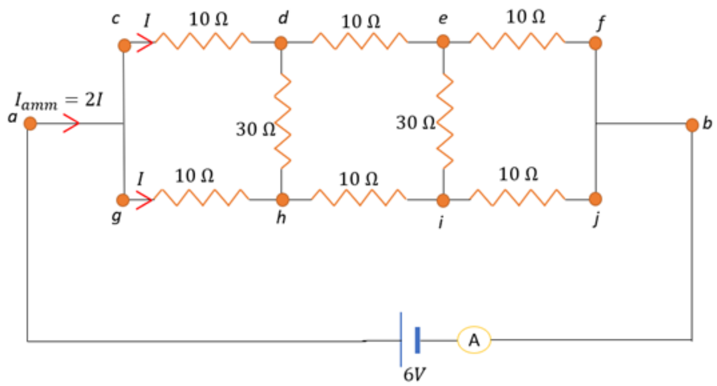
Concepts/Formula used:Resistors in Series: $R_{eq} = R_1 + R_2 + R_3 + \cdots$

Resistors in parallel: $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$

Potential Difference (V) across a resistor of resistance R when current I passes through it is given by Ohm's law:

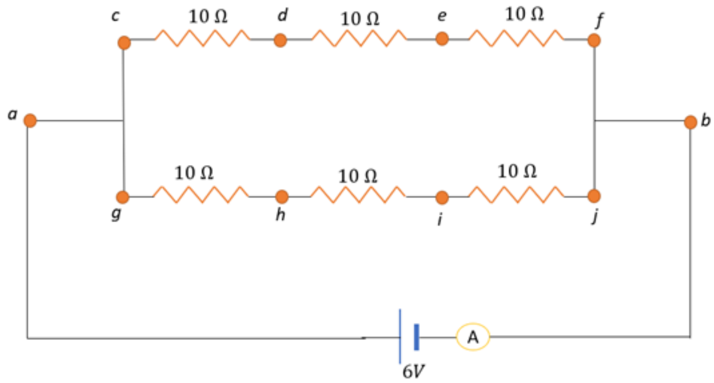
$V = IR$

Let the current through the ammeter be $I_{amm} = 2I$. We can see that the upper and lower part are completely identical. So , there is no reason they should have different currents. Hence, the current equally distributes into each branch due to symmetry.



Due to this symmetry, d and h; e and i are the same potential. Hence, no current passes through *de* and *ei*.

Hence, we can redraw the circuit by removing the resistors across *dh* and *ei*.



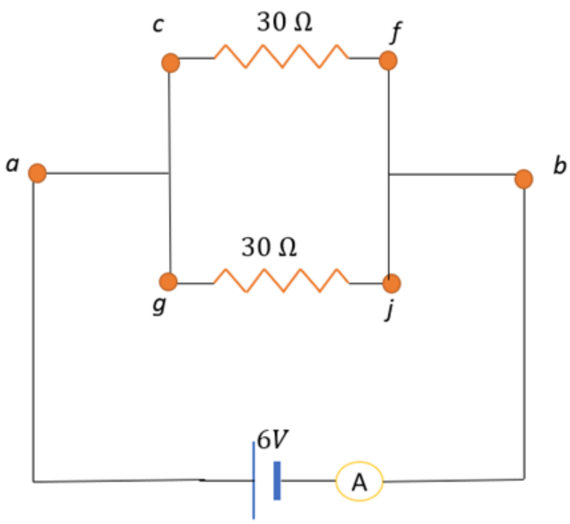
We can see that the resistors across *cd*, *de* and *ef* are in series. Hence,

$$R_{cd}^{eq} = 10\Omega + 10\Omega + 10\Omega \ = \ 30\Omega$$

Similarly,

$$R_{gj}^{eq} = 10\Omega + 10\Omega + 10\Omega \ = \ 30\Omega$$

We can rewrite the circuit as follows:

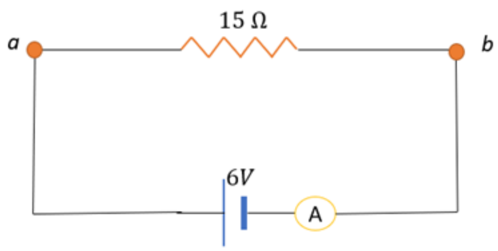


We can easily see that *cf* and *gj* are in parallel.

$$\frac{1}{R_{eq}} = \frac{1}{30\Omega} + \frac{1}{30\Omega}$$

$$R_{eq} = 15\Omega$$

Now, the circuit looks like this:

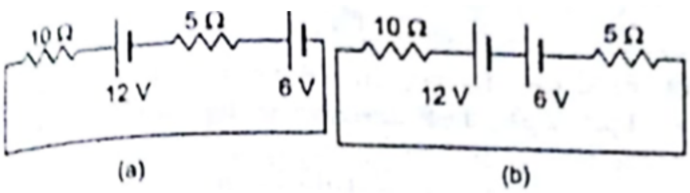


By Ohm's law,

$$I_{amm} = \frac{6V}{15\Omega} = 0.4A$$

43. Question

Consider the circuit shown in figure. Find (a) the current in the circuit, (b) the potential drop across the 5Ω resistor, (c) the potential drop across the 10Ω resistor. (d) Answer the parts (a), (b) and (c) with reference to figure.



Answer

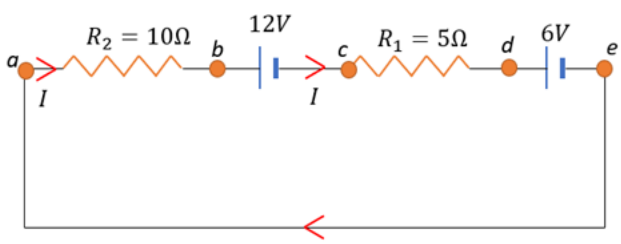
(a)

Concept/Formula used:

Kirchhoff's loop rule:

The sum of potential differences around a loop is zero.

Let the current flowing through the circuit be I.



Applying Kirchhoff's loop rule through the whole loop *abcdea*,

$-I(10\Omega) + 12V - I(5\Omega) + 6V = 0$

$-(15\Omega)I + 18V = 0$

$I = \frac{18V}{15\Omega}$

$I = 1.2A$

(b)

Concept/Formula used:

Potential Difference (V) across a resistor of resistance R when current I passes through it is given by Ohm's law:

$V = IR$

The potential difference across R_1 is:

$V_{R_1} = IR_1$

$= 1.2\text{A} \times 5\Omega$

$V_{R_1} = 6V$

(c)

The potential difference across R_2 is:

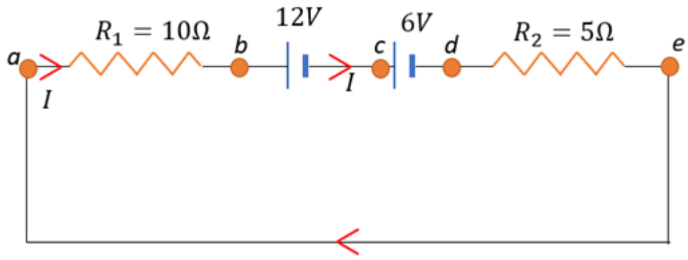
$V_{R_2} = \text{I}R_2$

$= 1.2\text{A} \times 10\Omega$

$V_{R_2} = 12V$

(d)

Let us consider the second circuit.



Applying Kirchhoff's loop rule through the whole loop *abcdea*,

$-I(10\Omega) + 12V + 6V - I(5\Omega) = 0$

$-(15\Omega)I + 18V = 0$

$I = \frac{18V}{15\Omega}$

$I = 1.2A$

The potential difference across R_1 is:

$V_{R_1} = IR_1$

$= 1.2A \times 5\Omega$

$V_{R_1} = 6V$

The potential difference across R_2 is:

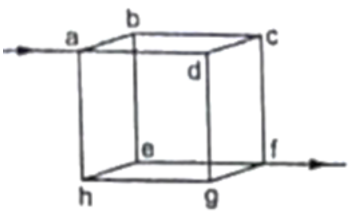
$V_{R_2} = IR_2$

$= 1.2A \times 10\Omega$

$V_{R_2} = 12V$

44. Question

Twelve wires, each having equal resistance r, are joined to form a cube as shown in figure. Find the equivalent resistance between the diagonally opposite points a and f.



Concept/Formula used:

Kirchhoff's junction rule:

The sum of currents entering a junction is equal to the sum of currents leaving it.

Kirchhoff's loop rule:

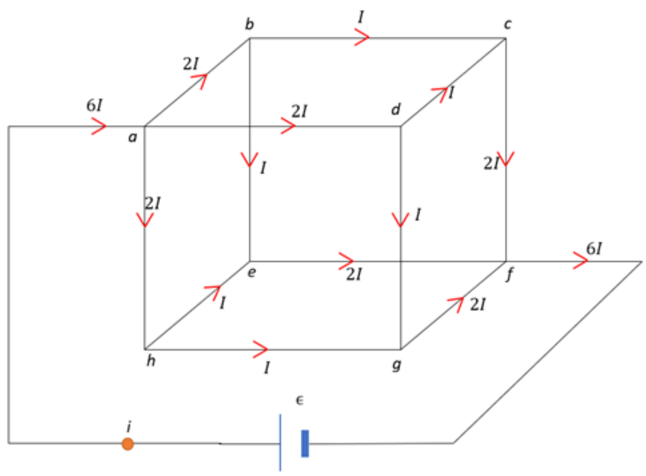
The sum of potential differences around a closed loop is zero.

Answer

As we need to find the equivalent resistance across a and f. We connect these two a source of emf ϵ.

Now, we will exploit the symmetry in the cube to find the current in each branch. Let the net current be 6I. Then, it distributes equally in all 3 branches due to symmetry. Hence, ab, ah and ad have current of 2I. Now, it further divides into two branches of current I. All this is in accordance with the junction rule.

Now, following the junction rule, we can find out the rest of the currents.



Applying Kirchhoff's loop rule on loop *adgfia*,

$$-(2I)r - Ir - (2I)r + \epsilon = 0$$

$$\epsilon = \ 5Ir \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Now, we need the find an r_{eq} such that

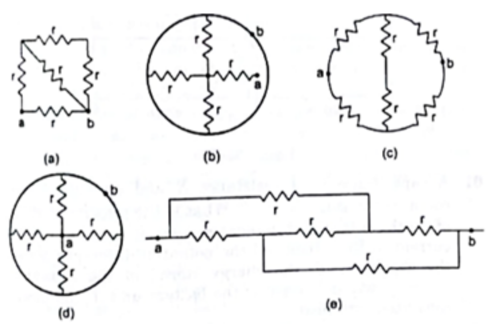
$$r_{eq} = \frac{\epsilon}{6I}$$

Using (1), we get

$$r_{eq} = \frac{5r}{6}$$

45. Question

Find the equivalent resistances of the networks shown in figure between the points a and b.

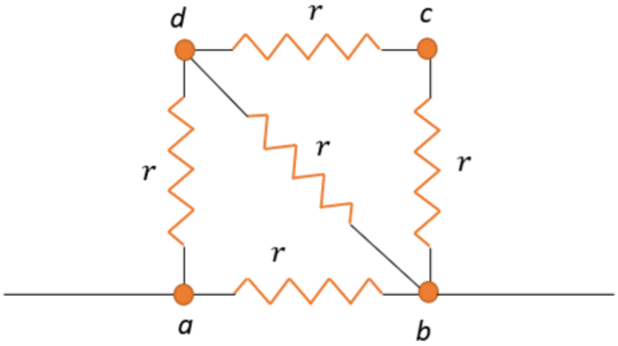


Answer

Concepts/Formula used:Resistors in Series: $R_{eq} = R_1 + R_2 + R_3 + \cdots$

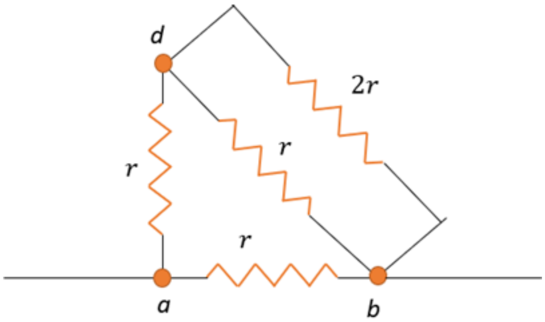
Resistors in parallel: $\dfrac{1}{R_{eq}} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3} + \ldots$

(a)



The resistors across dc and cb are in series.

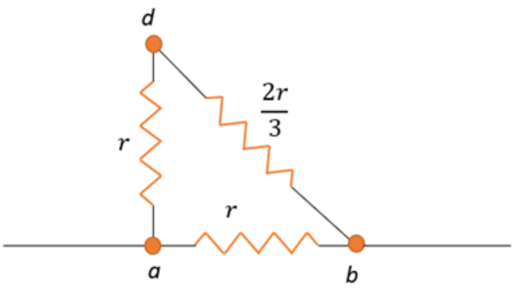
$$R_{dcb}^{eq} = r + r = 2r$$



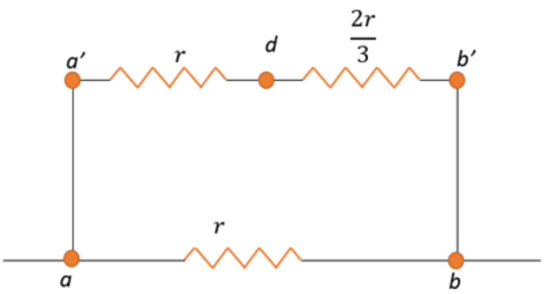
Now, the two resistors across db are in parallel.

$$\frac{1}{R_{db}^{eq}} = \frac{1}{2r} + \frac{1}{r}$$

$$R_{db}^{eq} = \frac{2r}{3}$$

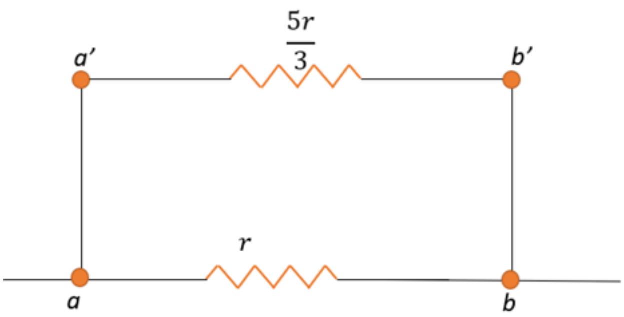


The resistor across db and ad are in series.



$$R_{a'b'}^{eq} = r + \frac{2r}{3} = \frac{5r}{3}$$

The circuit can be redrawn as follows:



The resistances *a'b'* and *ab* are in parallel.

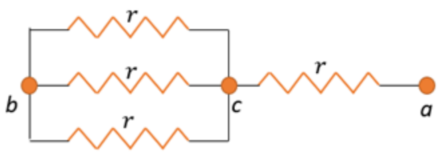
$$\frac{1}{R_{eq}} = \frac{3}{5r} + \frac{1}{r}$$

$$R_{eq} = \frac{5r}{8}$$

(b)

The point b is connecting the ends of three resistors; the other ends of the resistors are attached to a fourth one with one of its end being point a.

Hence, we can redraw the circuit as follows:

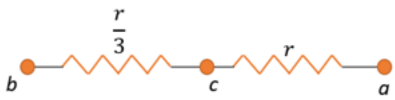


We can see that the resistors across *bc* are in parallel.

$$\frac{1}{R_{ac}^{eq}} = \frac{1}{r} + \frac{1}{r} + \frac{1}{r}$$

$$R_{ac}^{eq} = \frac{r}{3}$$

We can redraw the circuit as follows:

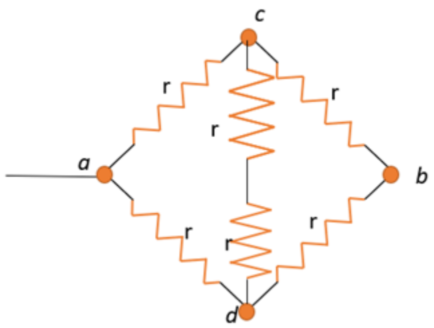


Note that the resistors across *bc* and *ca* are in series.

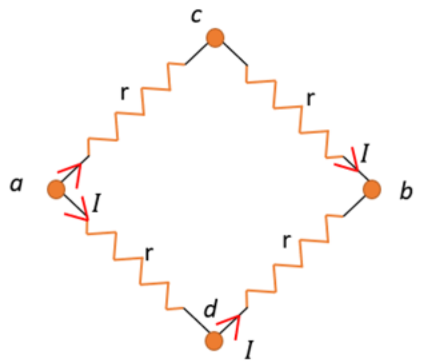
$$R_{eq} = \frac{r}{3} + r$$

$$R_{eq} = \frac{4r}{3}$$

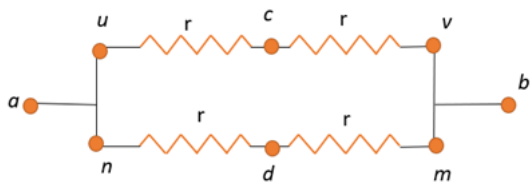
(c) We can redraw the given circuit as follows:



Note that there is symmetry in the circuit; the upper and the lower parts are identical. Hence, the potential at d should be the same as potential at c. Consequently, there is no potential difference across dc. We can neglect the resistors across *dc.*



We can redraw this circuit as follows:

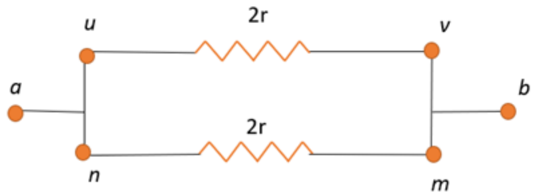


We can easily see that *uc* and *cv* are in series.

$$R_{uv}^{eq} = r + r = 2r$$

Similarly,

$$R_{nm}^{eq} = \mathrm{r} + \mathrm{r} = 2\mathrm{r}$$



Now, resistors across *uv* and *nm* are in parallel.

$$\frac{1}{R_{eq}} = \frac{1}{2r} + \frac{1}{2r}$$

$R_{eq} = r$

(d)

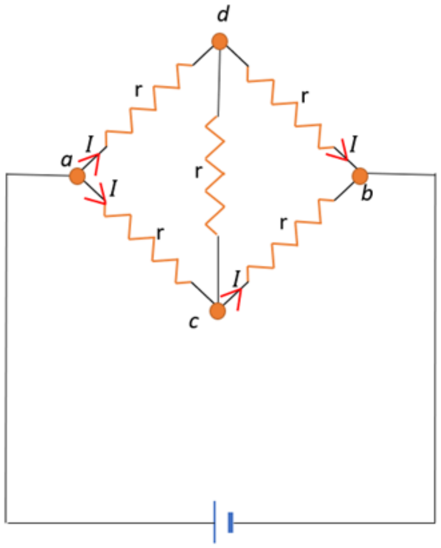
Note that one end of all four resistors is connected to one point a. The other ends are connected to a circle with no resistance. Hence, all points in the circle are same i.e. point b. Thus, all the other ends are connected to the same point b. This simply describes resistors in parallel.

$$\frac{1}{R_{eq}} = \frac{1}{r} + \frac{1}{r} + \frac{1}{r} + \frac{1}{r}$$

$$R_{eq} = \frac{r}{4}$$

(e)

The circuit can be drawn as follows:



As we wish to find the resistance between points a and b, we have proceeded to add a voltage source of emf ϵ between the points a and b.

Let the net current coming out of the battery be 2I. We can see that the circuit is symmetric i.e. its upper and lower portion are the identical. Thus, the current should divide equally when branching out. So, current through ac and ad is I.

Now,

$$V_d - V_a = -\mathrm{Ir}$$

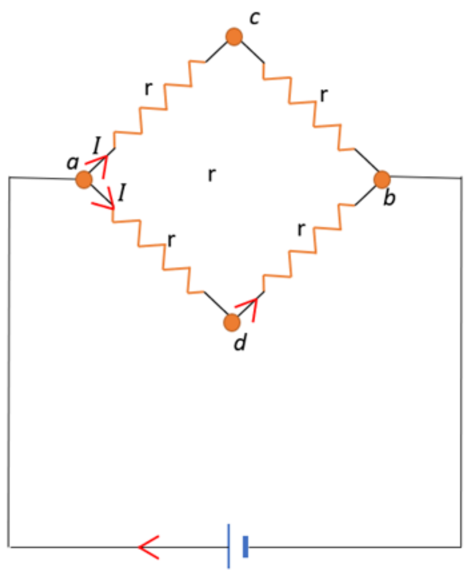
$$V_c - V_a = -\mathrm{Ir}$$

Hence,

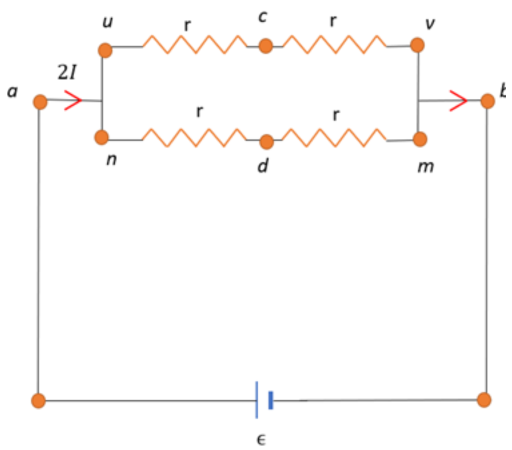
$$V_d = \mathrm{V_c}$$

As there is no potential difference across *dc*, there is no current passing through *dc*.

Hence, we can rewrite the circuit without the resistor across *dc*.



The circuit can be redrawn as:



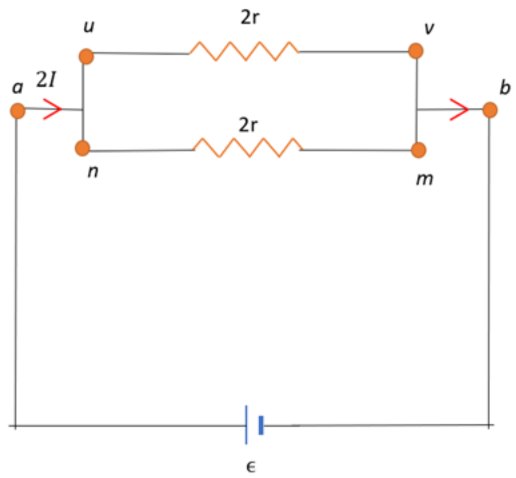
We can see that *uc* and *cv* are in series.

$$R_{uv}^{eq} = r + r = 2r$$

Also, *nd* and *dm* are is series.

$$R_{nm}^{eq} = \mathrm{r} + \mathrm{r} = 2\mathrm{r}$$

Hence, we can rewrite the circuit as follows:



Now, *uv* and *nm* are in parallel.

$$\frac{1}{R_{eq}} = \frac{1}{2r} + \frac{1}{2r}$$

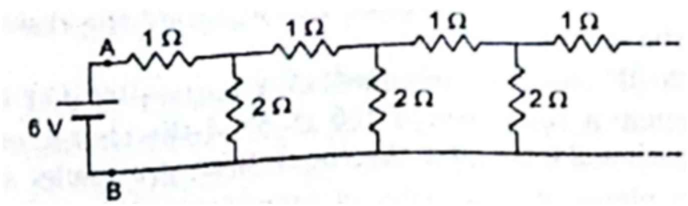
$$R_{eq} = r$$

46. Question

An infinite ladder is constructed with 1Ω and 2Ω resistors as shown in figure.

(a) Find the effective resistance between the points A and B.

(b) Find the current that passes through the 2Ω resistor nearest to the battery.



Answer

Concepts/Formula used:Resistors in Series: $R_{eq} = R_1 + R_2 + R_3 + \cdots$

Resistors in parallel: $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$

Ohm's Law:

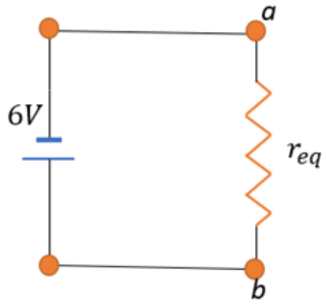
Potential Difference (V) across a resistor of resistance R when current I passes through it is given by Ohm's law:

$$V = IR$$

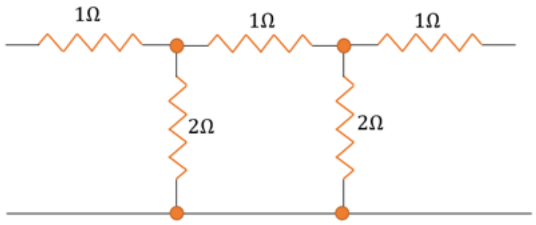
(a)

Let the equivalent resistance between A and B be r_{eq} Ω .

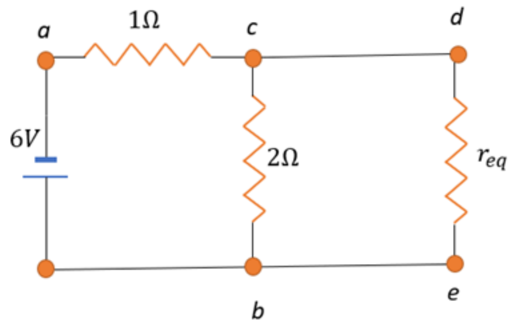
This means that we can rewrite the circuit as:



Where r_{eq} has replaced the following infinite combination:



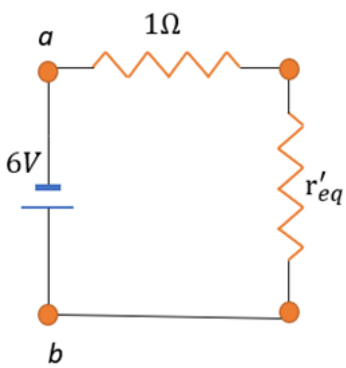
We can redraw the infinite circuit as



Note that *de* and *cb* are in parallel. The equivalent resistance is given by:

$$\frac{1}{r'_{eq}} = \frac{1}{2} + \frac{1}{r_{eq}}$$

$$\mathrm{r}'_{\mathrm{eq}} = \frac{2\mathrm{r}_{\mathrm{eq}}}{\mathrm{r}_{\mathrm{eq}} + 2}$$



Now, the 1 Ω resistor and r'_{eq} are in series.

$$R_{ab}^{eq} = r_{eq} = 1 + r'_{eq}$$

$$r_{eq} = 1 + \frac{2r_{eq}}{r_{eq} + 2}$$

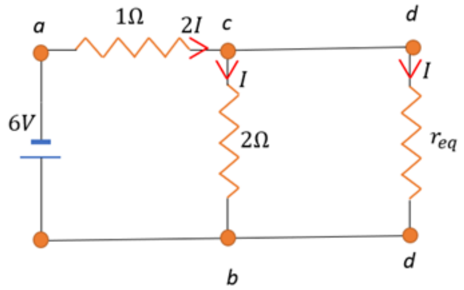
Rearranging and dropping the subscript, we get a quadratic equation:

$$r^2 - r - 2 = 0$$

The roots of this equation are 2 and -1. As resistance can't the negative the equivalent resistance between A and B is 2 Ω.

(b)

Let the net current be 2I. This current passes through the 1 Ω resistor. Than splits up equally due to symmetry as there are two 2Ω resistors.



Now,

$$I_{net} = 2I = \frac{6V}{2\Omega}$$

$$I = 1.5\ A$$

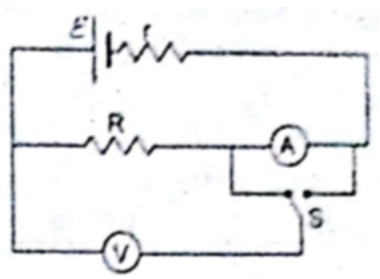
Hence, the current passing through the nearest 2 Ω resistor is 1.5A.

47. Question

The emf ϵ and the internal resistance r of the battery shown in figure are 4.3 V and 1.0 Ω respectively. The external resistance R is 50Ω. The resistances of the ammeter and voltmeter are 2.0Ω and 200Ω respectively.

(a) Find the readings of the two meters.

(b) The switch is thrown to the other side. What will be the readings of the two meters now?



Answer

Given:

Emf, $\epsilon = 4.3V$

External resistor, $R = 50\Omega$

Internal resistance, $r = 1.0\ \Omega$

Resistance of Ammeter, $R_A = 2.0\Omega$

Resistance of Voltmeter, $R_V = 200\Omega$

(a)

Concepts/Formula used:Resistors in Series: $R_{eq} = R_1 + R_2 + R_3 + \cdots$

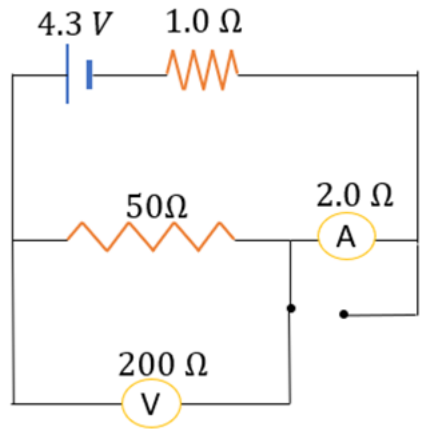
Resistors in parallel: $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ..$

Ohm's Law:

Potential Difference (V) across a resistor of resistance R when current I passes through it is given by Ohm's law:

$V = IR$

We are trying to find the equivalent resistance of the circuit first.

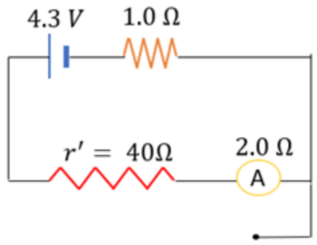


We will treat the measuring devices as ordinary resistors. Now, voltmeter and R= 50Ω are in parallel. The equivalent resistance is given by:

$$\frac{1}{r'} = \frac{1}{50\Omega} + \frac{1}{200\Omega}$$

$r' = 40\Omega$

We can redraw the circuit as follows:

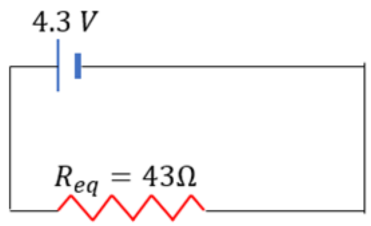


Now, we can see that all the devices are in series.

Hence,

$R_{eq} = 40\Omega + 2.0\Omega + 1.0$

$R_{eq} = 43\Omega$



Now, the current that passes through R_{eq} also passes through the three series components : internal resistor, r' and ammeter.

Hence, $I_{eq} = I_A$

Using Ohm's law,

$$I_{eq} = I_A = \frac{\epsilon}{R_{eq}}$$

$$I_A = \frac{4.3V}{43\Omega} = 0.1A$$

Now, the potential difference across R is the same as potential difference across r'.

$V_R = V_{r'} = Ir'$

$= 0.1A \times 40\Omega = 4\text{V}$

Hence, the ammeter reading is 0.1A and the voltmeter reading is 4V.

(b)

Concepts/Formula used:Resistors in Series: $R_{eq} = R_1 + R_2 + R_3 + \cdots$

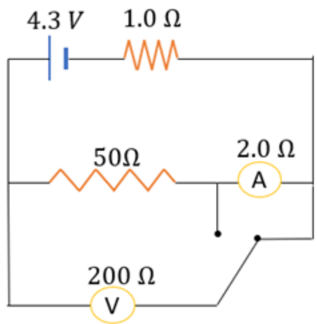
Resistors in parallel: $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$

Ohm's Law:

Potential Difference (V) across a resistor of resistance R when current I passes through it is given by Ohm's law:

$$V = IR$$

Let us find the equivalent resistance, R_{eq} of the circuit.



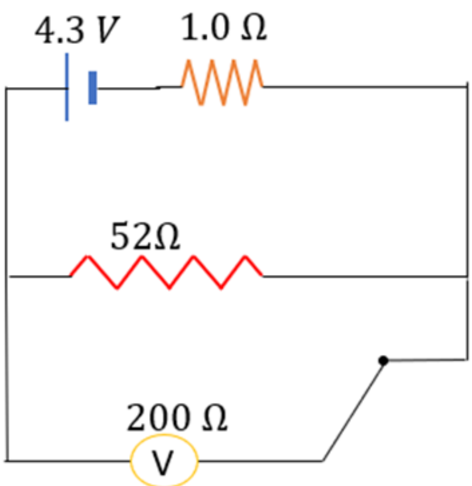
We will treat the ammeter and the voltmeter as ordinary resistors to find the equivalent resistance.

Note that the ammeter and the external resistor are in series.

Hence,

$$r' = 50\Omega + 2.0\Omega = 52$$

We can redraw the circuit as follows:

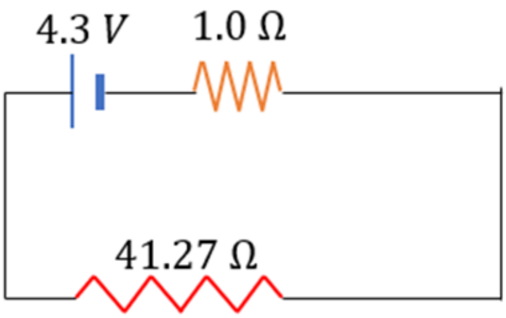


Note that the voltmeter and r' are in parallel.

$$\frac{1}{r''} = \frac{1}{52\Omega} + \frac{1}{200\Omega}$$

$$r'' \approx 41.27\ \Omega$$

The circuit can be redrawn as follows:



Now, the r'' and the internal resistance are in series.

$R_{eq} = 41.27\Omega + 1.0\Omega \approx 42.3$

By Ohm's law, the current coming out of the battery is

$$I = \frac{\epsilon}{R_{eq}} = \frac{4.3V}{42.3\Omega} \approx 0.1A$$

Now, potential difference across r'' = 41.27 Ω is the same as across r = 52 Ω .

Voltmeter Reading:

$$V_{r'} = V_{r''} = Ir'' \approx 4.3V$$

Now, the current passing through the ammeter is the one passing through r'.

$$I_A = I_{r'} = \frac{V_{r'}}{r'} = \frac{4.3V}{52\Omega} = 0.08A$$

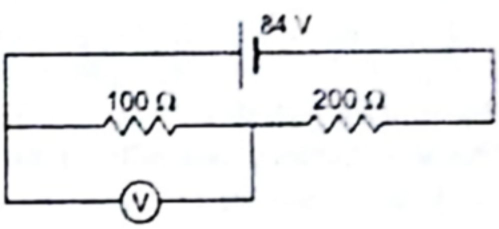
The ammeter reading is 0.08A and the voltmeter reading is about 4.3 V.

48. Question

A voltmeter of resistance 400 Ω is used to measure the potential difference across the 100Ω resistor in the circuit shown in figure.

(a) What will be the reading of the voltmeter?

(b) What was the potential difference across 100Ω before the voltmeter was connected?



Answer

Concepts/Formula used:Resistors in Series: $R_{eq} = R_1 + R_2 + R_3 + \cdots$

Resistors in parallel: $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$

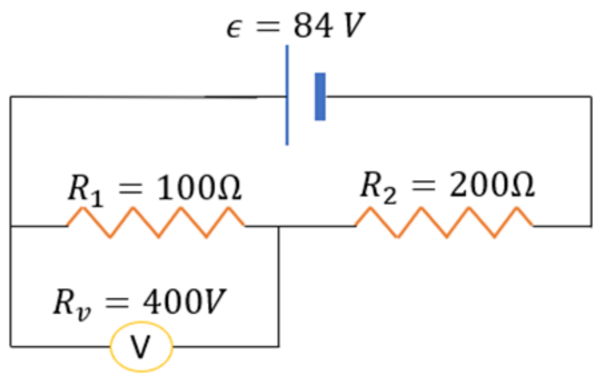
Ohm's Law:

Potential Difference (V) across a resistor of resistance R when current I passes through it is given by Ohm's law:

$V = IR$

(a)

The given circuit is:

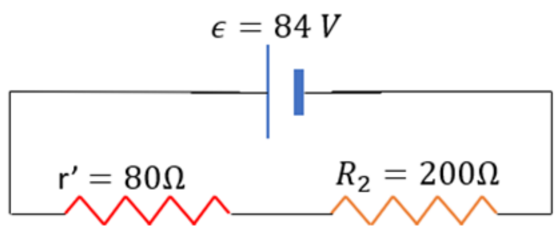


We can easily see that the voltmeter and R_1 are in parallel. The equivalent resistance, r' is given by:

$$\frac{1}{r'} = \frac{1}{100\Omega} + \frac{1}{400\Omega}$$

$$r' = 80\Omega$$

We can redraw the circuit as follows:



Note that r' and R_2 are in series. The equivalent resistance is given by:

$$R_{eq} = 80\Omega + 200\Omega = 280\Omega$$

The current coming out of the battery also passes through r'. By Ohm's law,

$$I = \frac{\epsilon}{R_{eq}}$$

$$= \frac{84V}{280\Omega} = 0.3A$$

Now, the voltmeter reads the potential different across R_1 .

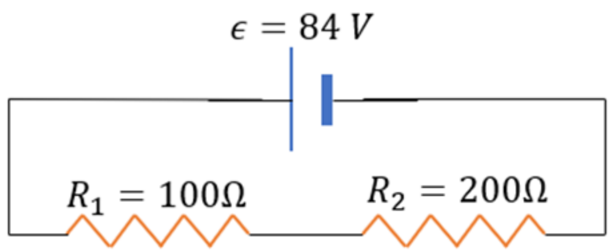
$V_{R_1} = V_{r'} = Ir'$

$V_{R_1} = 0.3A \times 80\Omega = 24V$

Hence, the voltmeter reads 24V.

(b)

The given circuit is now:



The equivalent resistance is :

$R_{eq} = 100\Omega + 200\Omega = 300\Omega$

The current is the same throughout the circuit as all components are in series and is given by:

$I = \frac{\epsilon}{R_{eq}} = \frac{84V}{300\Omega}$

$= 0.28A$

Now, the potential difference across R_1 is given by:

$V_{R_1} = IR_1$

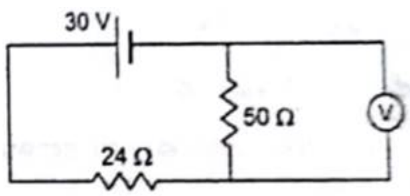
$= 0.28\ A \times 100\Omega$

$= 28V$

Hence, the potential difference across the 100Ω resistor before connecting the voltmeter was 28V.

49. Question

The voltmeter shown in figure reads 18 V across the 50 Ω resistor. Find the resistance of the voltmeter.



Answer

Concepts/Formula used:

Ohm's Law:

Potential Difference (V) across a resistor of resistance R when current I passes through it is given by Ohm's law:

$$V = IR$$

Kirchhoff's junction rule:

The sum of currents entering a junction is equal to the sum of currents leaving it.

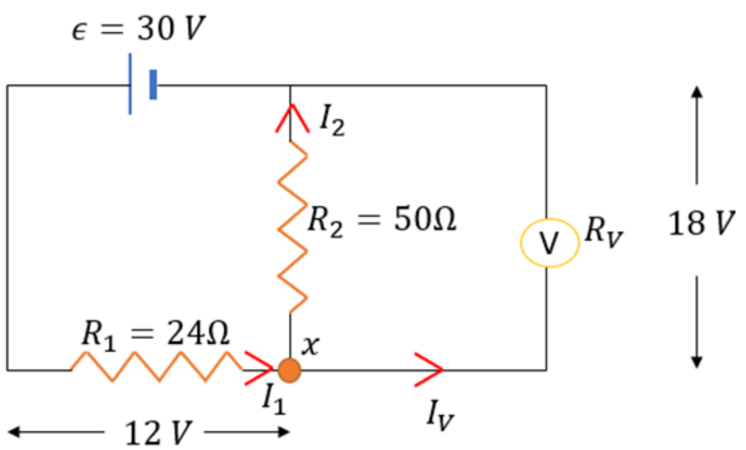
Kirchhoff's loop rule:

The sum of potential differences around a closed loop is zero.

Let us call the 24Ω resistance be R_1 and 50Ω R_2 . Let the resistance of voltmeter be R_V .

It is given that the voltage across the voltmeter, V = 18V.

The given circuit can be drawn and labelled as follows:



Applying Kirchhoff's loop rule,

$$\epsilon = V + V_1$$

$$V_1 = 20V - 18V = 12V$$

Using Ohm's law,

$$V_1 = I_1 R_1$$

$$I_1 = \frac{V_1}{R_1} = \frac{12V}{24\Omega}$$

$$= 0.5A$$

The potential difference across voltmeter and R_2 is the same as they are in parallel.

Using Ohm's law again,

$$I_2 = \frac{V}{R_2} = \frac{18V}{50V}$$

$$= 0.36A$$

Using Kirchhoff's junction rule at X,

$$I_V = I_1 - I2$$

$$= 0.5A - 0.36A = 0.14A$$

Finally, using Ohm's law for the voltmeter, we get

$$R = \frac{V}{I_V}$$

$$= \frac{18V}{0.14A} \approx 130\Omega$$

Hence, the voltmeter has resistance 130Ω.

50. Question

A voltmeter consists of a 25Ω coil connected in series with a 575Ω resistor. The coil takes 10 mA for full scale deflection. What maximum potential difference can be measured on this voltmeter?

Answer

Concepts/Formula used:

Ohm's Law:

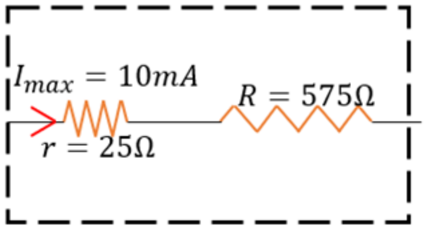
Potential Difference (V) across a resistor of resistance R when current I passes through it is given by Ohm's law:

$$V = IR$$

Note that 1A = 100mA.

Note that the current at full deflection is $I = 10mA = 0.1A$.

The voltmeter can be represented as follows:



It is given that the resistances are in series.

$R_{eq} = R + r$

$= 575\Omega + 25\Omega$

$= 600\Omega$

Potential difference is maximum when there is full scale deflection and is given by Ohm's law:

$V = IR_{eq}$

$= 0.01A \times 600\Omega$

$= 6V$

Thus, maximum potential difference that can be measured is 6V.

51. Question

An ammeter is to be constructed which can read currents up to 2.0 A. If the coil has a resistance of 25Ω and takes 1 mA for full-scale deflection, what should be the resistance of the shunt used?

Answer

Concepts/Formula used:

Ohm's Law:

Potential Difference (V) across a resistor of resistance R when current I passes through it is given by Ohm's law:

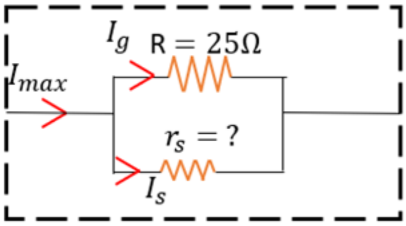
$V = IR$

The maximum current that can be measured is $I_{max} = 2A$.

The current through the coil when there is full scale deflection is $I_g = 1mA = 0.001mA$.

The resistance of the coil is $R = 25\Omega$

Let the resistance of the shunt be r_s and current when there is full-scale deflection is I_s.



By Kirchhoff's junction rule,

$$I_s + I_g = I_{max}$$

$$I_s = I_{max} - I_g$$

$$= 2A - 0.001A = 1.999A$$

Using Ohm's law, we have

$$V_g = I_g r$$

and

$$V_s = I_s r_s$$

As r_s and r are in parallel, the potential difference across them is the same.

$$V_s = V_g$$

$$I_g r = I_s r_s$$

$$r_s = \frac{I_g r}{I_s}$$

$$= \frac{0.001A \times 25\Omega}{1.999A}$$

$$= 1.25 \times 10^{-2}\ \Omega$$

Hence, the shunt resistance is 0.125Ω .

52. Question

A voltmeter coil has resistance 50.0Ω and a resistor of 1.15 kΩ is connected in series. It can read potential differences upto 12 volts. If this same coil is used to construct an ammeter which can measure currents up to 2.0 A, what should be the resistance of the shunt used?

Answer

Concepts/Formula used:

Ohm's Law:

Potential Difference (V) across a resistor of resistance R when current I passes through it is given by Ohm's law:

$V = IR$

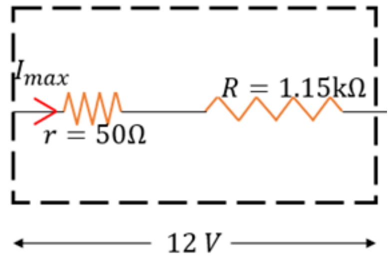
Ammeter:

It consists of a galvanometer coil in parallel with a stunt resistance.

Kirchhoff's junction rule:

The sum of currents entering a junction is equal to the sum of currents leaving it.

The given voltmeter looks like this:



The maximum potential difference that can be measured is $V_{max} = 12V$,and let the current through the voltmeter for maximum deflection be I_{max} .

Note that the coil (r) and the other resistor (R) are in series.

$R_{eq} = r + R$

$= 50\Omega + 1.15k\Omega$

$= 50\Omega + 1150\Omega$

$= 1200\Omega$

(Note that 1kΩ = 1000Ω)

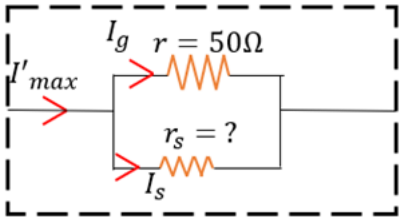
Now, using Ohm's law,

$$I_{max} = \frac{V_{max}}{R_{eq}}$$

$$= \frac{12V}{1200\Omega}$$

$= 0.01A$

The ammeter we want looks like the following diagram:



The maximum current that can be measured is $I'_{max} = 2A$.

From our previous calculations we know that the current through the coil for maximum deflection is $I_g = 0.01A$.

Note that the shunt resistance (r_s) and the coil (r) are parallel in an ammeter.

By Kirchhoff's junction rule,

$$I_s + I_g = I'_{max}$$

$$I_s = I'_{max} - I_g$$

$$= 2A - 0.01A = 1.99A$$

Using Ohm's law, we have

$$V_g = I_g r$$

and

$$V_s = I_s r_s$$

As r_s and r are in parallel, the potential difference across them is the same.

$$V_s = V_g$$

$$I_g r = I_s r_s$$

$$r_s = \frac{I_g r}{I_s}$$

$$= \frac{0.01A \times 50\Omega}{1.99A}$$

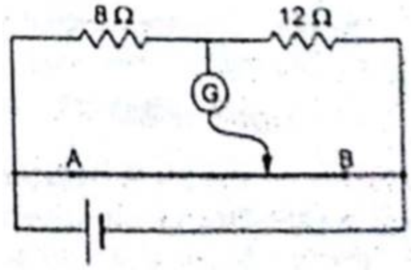
$$= 0.251\ \Omega$$

Hence, the shunt resistance is 0.251Ω.

53. Question

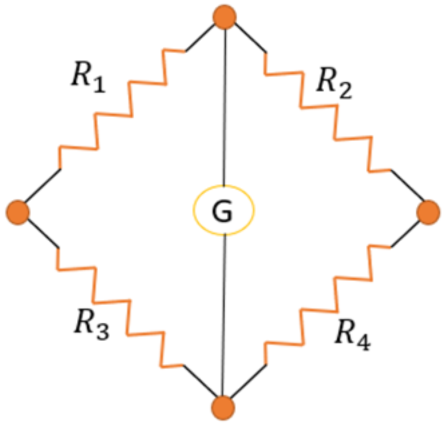
The potentiometer wire AB shown in figure is 40 cm long. Where should the free end of the galvanometer be connected on AB so that the galvanometer may show zero deflection?

Answer



Concepts/Formula used:

Wheatstone bridge:



The condition for no deflection through the galvanometer is

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

Here, and $R_1 == 8\Omega$

$R_2 = 12\Omega$

Let the potentiometer wire be at l cm from A when there is no deflection through galvanometer.

Let the resistance per cm of the potentiometer wire be be ρ .

Hence, $R_3 = \rho l$ and $R_4 = \rho(40 - l)$.

Using the equation when there is no deflection for a Wheatstone bridge/potentiometer, we have

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

$$\frac{8\Omega}{12\Omega} = \frac{\rho l}{\rho(40 - l)}$$

$$\frac{2}{3} = \frac{l}{40 - l}$$

$$80 - 2l = 3l$$

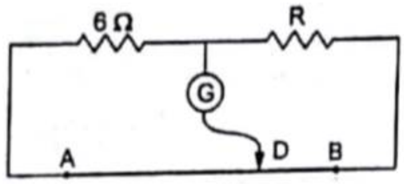
$5l = 80$

$l = 16$

Hence, the free end of the galvanometer must be 16cm from point A.

54. Question

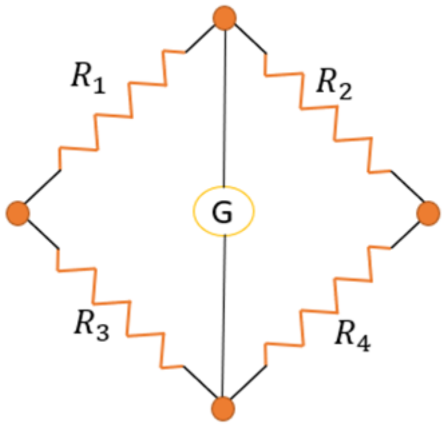
The potentiometer wire AB shown in figure is 50 cm long. When AD = 30 cm, no deflection occurs in the galvanometer. Find R.



Answer

Concepts/Formula used:

Wheatstone bridge:



The condition for no deflection through the galvanometer is

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

Here $R_1 = 6\Omega$ and $R_2 = R$.

When no deflection occurs, $AD = l\ cm = 30cm$

Let the resistance per cm of the potentiometer wire be be ρ .

Hence, $R_3 = \rho l$ and $R_4 = \rho(50 - l)$.

Using the equation when there is no deflection for a Wheatstone bridge/potentiometer, we have

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

$$\frac{6\Omega}{R} = \frac{\rho l}{\rho(50 - l)}$$

$$\frac{6\Omega}{R} = \frac{30\ cm}{20\ cm}$$

$R\ =\ 4\Omega$

55. Question

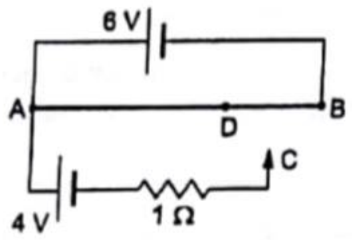
A 6-volt battery of negligible internal resistance is connected across a uniform wire AB of length 100 cm. The positive terminal of another battery of emf 4V and internal resistance 1Ω is joined to the point A as shown in figure. Take the potential at B to be zero.

(a) What are the potentials at the points A and C?

(b) At which point D of the wire AB, the potential is equal to the potential at C?

(c) If the points C and D are connected by a wire, what will be the current through it?

(d) If the 4V battery is replaced by 7.5 V battery, what would be the answers of parts (a) and (b)?



Concepts/Formula used:

Ohm's Law:

Potential Difference (V) across a resistor of resistance R when current I passes through it is given by Ohm's law:

$V\ =\ IR$

Let the area of cross section of the wire (resistor) be A and resistivity be ρ.

Then,

$$R = \rho\frac{l}{A}$$

Answer

(a)

$V_A - V_B = V_{batt} = 6V$

As $V_B = 0$,

$V_A = 6V$

Let us account for the potential differences when moving from A to C.

$V_A - 4V = V_c$

$V_c = 6V - 4V = 2V$

(b)

$V_D = V_C = 2V$

$V_A - V_D = 4V$

Let the area of cross section of wire be A and resistivity be ρ.

Then,

$$V = IR = I\rho\frac{l}{A}$$

where V is the voltage across a wire segment of length l.

As, all the quantities except for length are the same for all sections of wire AB,

$V \propto l$

Hence,

$$\frac{V_{AD}}{V_{AB}} = \frac{l_{AD}}{l_{AB}}$$

$$\frac{4V}{6V} = \frac{l_{AD}}{100cm}$$

$l_{AD} = 66.47cm$

Hence, D is 66.67cm away from A.

(c) As $V_C = V_D$, there is no potential difference across CD and hence, the current through it is zero.

(d)

(a)

$V_A - V_B = V_{batt} = 6V$

As $V_B = 0$,

$V_A = 6V$

Let us account for the potential differences when moving from A to C.

$V_A - 7.5V = V_c$

$V_c = 6V - 7.5V = -1.5V$

(b)

$V_D = V_C = -1.5V$

$V_A - V_D = 7.5V$

Let the area of cross section of wire be A and resistivity be ρ.

Then,

$$V = IR = I\rho\frac{l}{A}$$

where V is the voltage across a wire segment of length l.

As, all the quantities except for length are the same for all sections of wire AB,

$V \propto l$

Hence,

$$\frac{V_{AD}}{V_{AB}} = \frac{l_{AD}}{l_{AB}}$$

$$\frac{7.5V}{6V} = \frac{l_{AD}}{100cm}$$

$l_{AD} = 125cm$

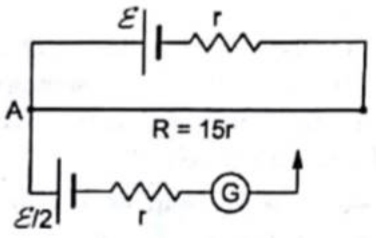
This is not possible. As the length of the wire is only 100cm. There is no such point D.

56. Question

Consider the potentiometer circuit arranged as in figure. The potentiometer wire is 600 cm long.

(a) At what distance from the point A should the jockey touch the wire to get zero deflection in the galvanometer?

(b) If the jockey touches the wire at a distance of 560 cm from A, what will be the current in the galvanometer?



Answer

Concepts/Formula used:

Ohm's Law:

Potential Difference (V) across a resistor of resistance R when current I passes through it is given by Ohm's law:

$$V = IR$$

Let the area of cross section of the wire (resistor) be A and resistivity be ρ.

Then,

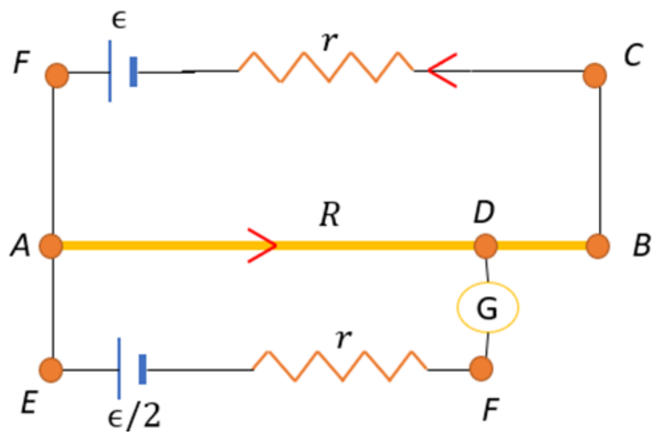
$$R = \rho\frac{l}{A}$$

Kirchhoff's loop rule:

The sum of potential differences around a closed loop is zero.

(a)

We consider the circuit when there is no deflection:



Applying Kirchhoff's rule on loop ABCF,

$$-IR - Ir + \epsilon = 0$$

$$I(R + r) = \epsilon$$

Using $R = 15r$,

$$I = \frac{\epsilon}{16r}$$

$$V_{AB} = IR$$

$$= \frac{\epsilon}{16r} 15r$$

$$= \frac{15\epsilon}{16}$$

Let the area of cross section of wire be A and resistivity be ρ.

Then,

$$V = IR = I\rho \frac{l}{A}$$

where V is the voltage across a wire segment of length l.

As, all the quantities except for length are the same for all sections of wire AB,

$$V \propto l$$

Hence,

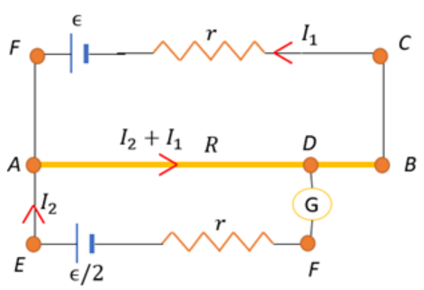
$$\frac{V_{AD}}{V_{AB}} = \frac{\epsilon}{2} \times \frac{16}{15\epsilon} = \frac{l_{AD}}{600cm}$$

$$\frac{l_{AD}}{600cm} = \frac{8}{15}$$

$$l_{AD} = 320cm$$

(b)

Let the current coming out of the main battery be I_1 and the current through the galvanometer be I_2.



Let the area of cross section of wire be A and resistivity be ρ.

Then,

$$V = IR = I\rho \frac{l}{A}$$

where V is the voltage across a wire segment of length l.

As, all the quantities except for length are the same for all sections of wire AB,

$V \propto l$

Hence,

$$\frac{R_{AD}}{R} = \frac{560cm}{600cm}$$

Using $R = 15r$

$$R_{AD} = \frac{56}{60} \times 15r = 14r$$

Now, R_{AD} and R_{DB} are in series.

$$R_{AD} + R_{DB} = R_{AB}$$

$$14r + R_{DB} = 15r$$

$$R_{DB} = r$$

Applying Kirchhoff's rule on loop ABCFA,

$$-(I_1 + I_2)(R) - I_1 r + \epsilon = 0$$

$$-(I_1 + I_2)(15r) - I_1 r + \epsilon = 016 I_1 r + 15 I_2 r = \epsilon$$ ....(1)

Applying Kirchhoff's rule on loop ADEFA,

$$-(I_1 + I_2)(R_{AD}) - I_2 r + \epsilon/2 = 0$$

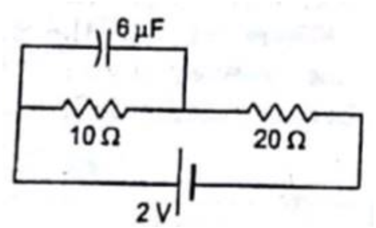
$$-(I_1 + I_2)(14r) - I_2 r + \epsilon/2 = 014 I_1 r + 15 I_2 r = \epsilon/2$$ ....(2)

From (i) and (ii)

$$I_2 = \frac{3\epsilon}{22r}$$

57. Question

Find the charge on the capacitor shown in figure.



Answer

Concepts/Formulas used:

Ohm's Law:

Potential Difference (V) across a resistor of resistance R when current I passes through it is given by Ohm's law:

$$V = IR$$

Capacitance:

If two conductors have a potential difference V between the them and have charges Q and -Q respectively on them, then their capacitance is defined as

$$C = \frac{Q}{V}$$

At steady state, no current passes through 6μF . Hence, we can ignore the capacitor to find the equivalent resistance.

Now, 10Ω and 20Ω are in series.

$$R_{eq} = 10\Omega + 20\Omega = 30\Omega$$

The current is given by Ohm's law:

$$I = \frac{V}{R} = \frac{2V}{30\Omega} = \frac{1}{15}A$$

Now, this is the same current that passes through 10Ω resistor.

The potential across it is given by:

$$V_{10\Omega} = I(10\Omega) = \frac{2}{3}V$$

Now, as the capacitor is in parallel with the 10Ω resistor the,

$$V_C = V_{10\Omega}$$

Now,

$$Q = V_c C$$

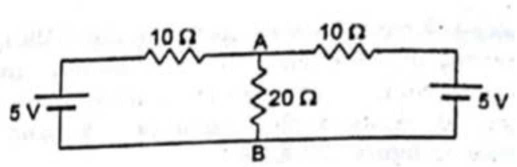
$$= \frac{2}{3}V \times 6\mu F$$

$$= 4\mu C$$

58. Question

(a) Find the current in the 20 Ω resistor shown in figure.

(b) If a capacitor of capacitance 4 μF is joined between the points A and B, what would be the electrostatic energy stored in it in steady state?



Answer

Concepts/Formula used:

Ohm's Law:

Potential Difference (V) across a resistor of resistance R when current I passes through it is given by Ohm's law:

$V = IR$

Kirchhoff's junction rule:

The sum of currents entering a junction is equal to the sum of currents leaving it.

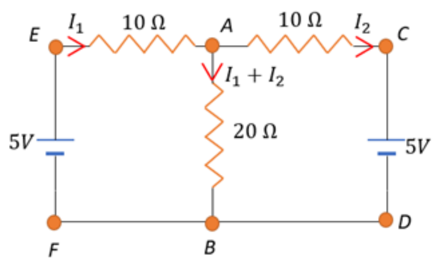
Kirchhoff's loop rule:

The sum of potential differences around a closed loop is zero.

Energy stored by a capacitor:

If the potential difference between the two conductors of the capacitor is V and its capacitance is C, its energy is given by: $U = \frac{1}{2}CV^2$

(a)



Using Kirchhoff's law on loop FEABF,

$5V + -I_1 10\Omega - (I_1 + I_2)20\Omega$

$5\text{V} = 30\text{I}_1\Omega + 20\text{I}_2\Omega 6I_1 + 4I_2 = 1A$ ....(1)

Using Kirchhoff's law on loop ACDBA,

$-I_2 10\Omega + 5V - (I_1 + I_2)20\Omega$

$5\text{V} = 30\text{I}_2\Omega + 20\text{I}_1\Omega 4I_1 + 6I_2 = 1A$ ....(2)

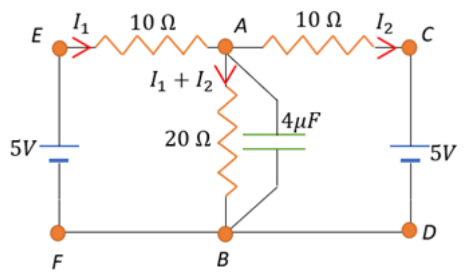
Solving (1) and (2), we get

$I_1 = I_2 = 0.1A$

Hence,

$I_{AB} = I_1 + I_2 = 0.2A$

(b)



At steady state, no current passes through the capacitor; hence, the results are the same as in (a).

$V_{20\Omega} = I_{AB} \times 20\Omega = 0.2A \times 20\Omega = 4V$

Now, the potential as the capacitor is in parallel with the 20Ω resistor,

$V_C = V_{20\Omega} = 4V$

$U = \frac{1}{2} CV^2$

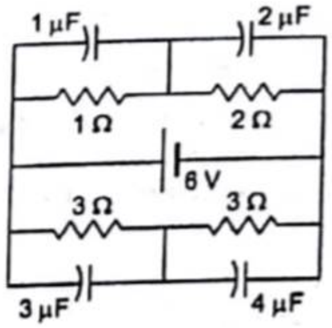
$= \frac{1}{2} \times 4\mu F \times (4V)^2$

$= 32\ \mu J$

Hence, the energy stored by the capacitor is 32μJ.

59. Question

Find the charges on the four capacitors of capacitances 1 μF, 2 μF, 3 μF and 4 μF shown in figure.



Answer

Concepts/Formula used:

Ohm's Law:

Potential Difference (V) across a resistor of resistance R when current I passes through it is given by Ohm's law:

$$V = IR$$

Kirchhoff's junction rule:

The sum of currents entering a junction is equal to the sum of currents leaving it.

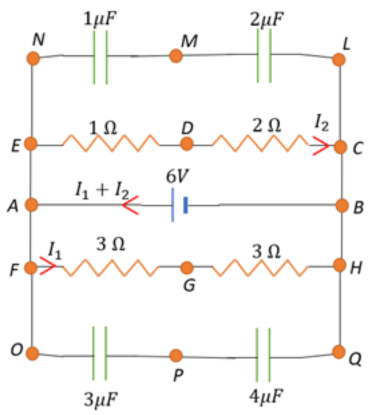
Kirchhoff's loop rule:

The sum of potential differences around a closed loop is zero.

Capacitance:

If two conductors have a potential difference V between the them and have charges Q and -Q respectively on them, then their capacitance is defined as

$$C = \frac{Q}{V}$$



Applying Kirchhoff's loop rule on ABCDEA,

$$-6V + I_2(2\Omega) + I_2(1\Omega) = 0$$

$$I_2(3\Omega) = 6V$$

$$I_2 = \frac{6V}{3\Omega} = 2A$$

Applying Kirchhoff's loop rule on ABHGFA,

$$-6V + I_1(3\Omega) + I_1(3\Omega) = 0$$

$$I_1(6\Omega) = 6V$$

$$I_1 = \frac{6V}{6\Omega} = 1A$$

Now,

$V_{FG} = I_1 \times 3\Omega = 3V$

As the 3Ω resistor and the 3μF capacitor are in parallel,

$V_{3\mu F} = V_{FG} = 3V$

Now, we know that

$Q_{3\mu F} = V_{3\mu F} \times 3\mu F = 9\mu C$

Also,

$V_{GH} = I_1 \times 3\Omega = 3V$

$V_{4\mu F} = V_{GH} = 3V$

Now, we know that

$Q_{4\mu F} = V_{4\mu F} \times 4\mu F = 12\mu C$

Now,

$V_{ED} = I_2 \times 1\Omega = 2V$

$V_{1\mu F} = V_{ED} = 2V$

Now, we know that

$Q_{1\mu F} = V_{1\mu F} \times 1\mu F = 2\mu C$

Now,

$V_{DC} = I_2 \times 2\Omega = 4V$

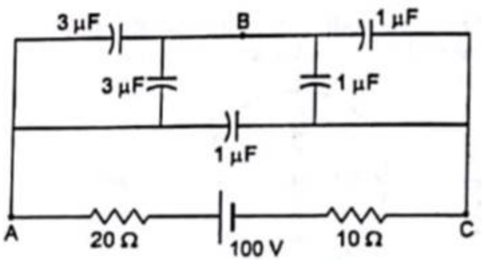
$V_{2\mu F} = V_{DC} = 4V$

Now, we know that

$Q_{2\mu F} = V_{2\mu F} \times 2\mu F = 8\mu C$

60. Question

Find the potential difference between the points A and B between the points B and C of figure in steady state.



Answer

Capacitors in parallel:

If capacitors C_1, C_2, C_3 , ... are in parallel, then the equivalent capacitance is given by:

$$C_{eq} = C_1 + C_2 + C_3 + \cdots$$

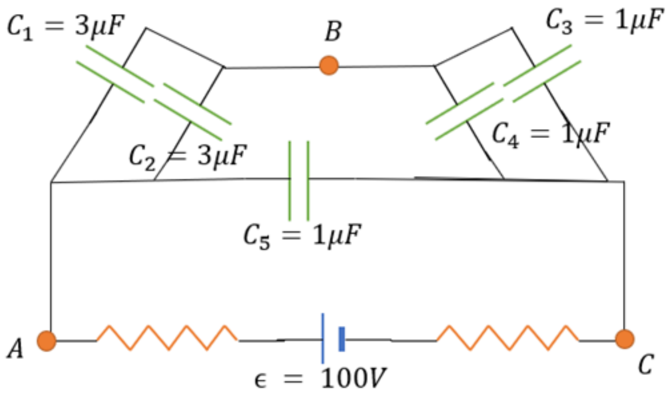
Capacitors in series:

If capacitors C_1, C_2, C_3 , ... are in sereis, then the equivalent capacitance is given by:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$$

Note that the charge is same on each capacitor in series in steady-state.

The circuit can be redrawn as follows:



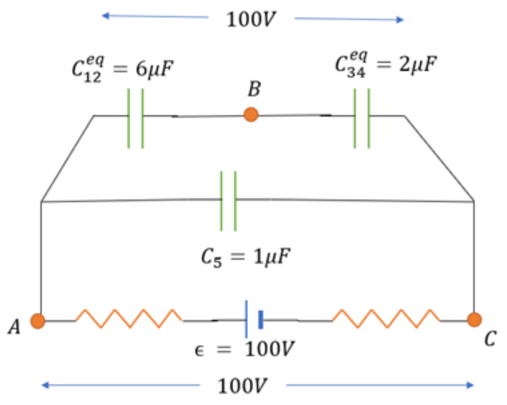
We can see that C_1 and C_2 are in parallel,

$$C_{12}^{eq} = C_1 + C_2 = 6\mu F$$

Also, we can see that C_3 and C_{24} are in parallel,

$$C_{34}^{eq} = C_3 + C_4 = 2\mu F$$

The circuit can be redrawn as follows:



Now, the two capacitors C_{12}^{eq} and C_{34}^{eq} are in series.

$$\frac{1}{C_{1234}^{eq}} = \frac{1}{6\mu F} + \frac{1}{2\mu F}$$

$$C' = C_{1234}^{eq} = \frac{3}{2}\mu F$$

Note that there is no current in equilibrium. Hence,

$$V_{C'} = \epsilon = 100V$$

As C_{12}^{eq} and C_{34}^{eq} are in series,

$$Q_{12} = Q_{34} = Q_{C'} = C'V_{c'}$$

$$= \frac{3}{2}\mu F \times 100V$$

$$= 150\ \mu C$$

Now,

$$V_{AB} = \frac{Q_{12}}{C_{12}}$$

$$= \frac{150\mu C}{6\mu F}$$

$$= 25\ V$$

Also,

$$V_{BC} = \frac{Q_{34}}{C_{34}}$$

$$= \frac{150\mu C}{2\mu F}$$

$$= 75\ V$$

61. Question

A capacitance C, a resistance R and an emf ϵ are connected in series at t = 0. What is the maximum value of

(a) the potential difference across the resistor,

(b) the current in the circuit,

(c) the potential difference across the capacitor,

(d) the energy stored in the capacitor,

(e) the power delivered by the battery and

(f) the power converted into heat.

Answer

Energy stored in a capacitor:

The energy stored in a capacitor with capacitance C , charge is given by:

$$U = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$$

where V is the potential difference across the capacitor.

Power supplied by the battery:

If a battery of emf ϵ gives a current I, then the power supplied by the battery is given by:

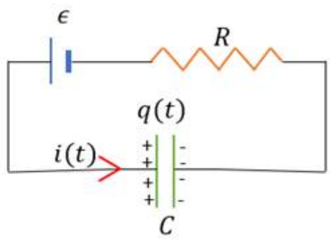
$$P = I\epsilon$$

Kirchhoff's loop rule:

The sum of potential differences around a closed loop is zero.

Potential Difference (V) across a resistor of resistance R when current I passes through it is given by Ohm's law:

$$V = IR$$



(a)

By Kirchhoff's loop rule, $\epsilon = V_c(t) + V_r(t)$

$$V_r(t) = \epsilon - V_c(t)$$

As ϵ is constant, $V_r^{max} = \epsilon - V_c^{min}$

At t = 0, there is no charge on the capacitor. Hence $V_c = 0$. So, V_r is maximum at t = 0.

$$V_r^{max} = \epsilon$$

(b) Initially, current flows through the circuit treating capacitor as short-circuit. But as charge accumulates on the capacitor the current reduces with time. It is maximum at t =0.

Using Kirchhoff's loop rule,

$\epsilon = V_c(t) + V_r(t)$

$\epsilon = V_c(t) + IR$

I is max at t = 0 and $V_c(0) = 0$

$\epsilon = I_{max}R$

$I_{max} = \frac{\epsilon}{R}$

(c)

Using Kirchhoff's loop rule,

$\epsilon = V_c(t) + V_r(t)$

$\epsilon = V_c(t) + \mathrm{I(t)R}$

As R and ϵ are constant

$V_c^{max} = \epsilon - I^{min}R$

Current is minimum at equilibrium when capacitor acts as an open switch and current is zero. Hence, $I^{min} = 0$

$V_c^{max} = \epsilon$

(d)

The energy stored by the capacitor is given by:

$U = \frac{1}{2}CV^2$

As C is constant,

U_{max} =\frac{1}{2}CV_{max}^2

We know from (c) that the maximum potential difference across the capacitor is ϵ. Hence,

$U_{max} = \frac{1}{2}\mathrm{C}\epsilon^2$

(e)

Power delivered by the battery is given by:

$P = I\epsilon$

As ϵ is a constant,

$P_{max} = I_{max}\epsilon$

Using the result from (b), we get

$$P_{max} = \frac{\epsilon}{R}\epsilon$$

$$P_{max} = \frac{\epsilon^2}{R}$$

(f)

The resistor converts energy into heat:

$$P_{heat} = I^2 R$$

As R is constant,

$$P_{heat}^{max} = I_{max}^2 R$$

Now, using the result from (b),

$$P_{heat}^{max} = \frac{\epsilon^2}{R}$$

62. Question

A parallel-plate capacitor with plate area 20 cm^2 and plate separation 1.0 mm is connected to a battery. The resistance of the circuit is 10 kΩ. Find the time constant of the circuit.

Answer

Concepts/Formulas Used:

Time constant for capacitor:

$$\tau = RC$$

Where R is the resistance through which the capacitor is being charged/discharged and C is the capacitance.

Capacitance of a parallel plate capacitor:

A capacitor consists of two conducting plates of area of cross section A each separated by a distance d. The capacitance if there is only vacuum between the plates is given by:

$$C = \frac{\epsilon_0 A}{d}$$

where ϵ_0 is the permittivity of free space.

Given,

Area of the plate, $A = 20cm^2 = 20 \times 10^{-4}m^2$

$(1cm\char`^2 = 10\char`^(-4)cm\char`^2)$

Now, distance of separation: $d = 1.0\ mm = 1.0 \times 10^{-3}m$

Resistance ,$R = 10k\Omega = 10 \times 10^3\Omega$

We know that,

$\tau = RC$

$$\tau = R\frac{\epsilon_0 A}{d}$$

$$= 10 \times 10^3\Omega \times \frac{8.854 \times 10^{-12}m^{-3}kg^{-1}s^4A^2 \times 20 \times 10^{-4}m^2}{1.0 \times 10^{-3}m}$$

$\approx 1.8 \times 10^{-7}s \approx .18\ \mu s$

63. Question

A capacitor of capacitance 10 μF is connected to a battery of emf 2V. It is found that it takes 50 ms for the charge on the capacitor to become 12.6 μC. Find the resistance of the circuit.

Answer

Concepts/Formulas used:

Charging a capacitor:

A capacitor of capacitance C is being charged using a battery of emf ϵ through a resistance R . A switch S is also connected in series with the capacitor. The switch is initially open. The capacitor is uncharged at first. At t=0, the switch is closed

The charge is at any t>0 is given by:

$$Q = C\epsilon\left(1 - e^{-\frac{t}{\tau}}\right)$$

Note that $\tau = RC$ is known as time constant.

Given,

Capacitance, $C = 10\mu F = 10 \times 10^{-6}F$

EMF of battery, $\epsilon = 2.0V$

Resistance, $R = ?$

We also know that at $= 50ms$, $Q = 12.6\mu C = 12.6 \times 10^{-6}C$

We know that,

$$Q = C\epsilon\left(1 - e^{-\frac{t}{\tau}}\right)$$

$$1 - e^{-\frac{t}{\tau}} = \frac{Q}{C\epsilon}$$

$$e^{-\frac{t}{\tau}} = 1 - \frac{Q}{C\epsilon}$$

Taking natural logarithm on both sides, we get

$$-\frac{t}{\tau} = ln\left(1 - \frac{Q}{C\epsilon}\right)$$

$$\tau = -\frac{\mathrm{t}}{ln\left(1 - \frac{Q}{C\epsilon}\right)}$$

$$= -\frac{50ms}{ln\left(1 - \frac{12.6 \times 10^{-6}C}{10 \times 10^{-6}F \times 2.0V}\right)}$$

$$= -\frac{50ms}{ln\, 0.37} = 50.3ms$$

$$= 50.3 \times 10^{-3}s$$

Now,

$$\tau = RC$$

$$R = \frac{\tau}{C}$$

$$R = \frac{50.3 \times 10^{-3}s}{10 \times 10^{-6}F}$$

$$= 5 \times 10^{3}$$

$$= 5k\Omega$$

64. Question

A 20 μF capacitor is joined to a battery of emf 6.0 V through a resistance of 100 Ω. Find the charge on the capacitor 2.0 ms after the connections are made.

Answer

Concepts/Formulas used:

Charging a capacitor:

A capacitor of capacitance C is being charged using a battery of emf ϵ through a resistance R . A switch S is also connected in series with the capacitor. The switch is initially open. The capacitor is uncharged at first. At t=0, the switch is closed

The charge is at any t>0 is given by:

$$Q = C\epsilon\left(1 - e^{-\frac{t}{\tau}}\right)$$

Note that $\tau = RC$ is known as time constant.

Given,

Capacitance, $C = 20\mu F = 20 \times 10^{-6} F$

EMF of battery, $\epsilon = 6.0V$

Resistance, $R = 100\Omega$

Now,

$$\tau = RC$$

$$= 100\Omega \times 20 \times 10^{-6} F$$

$$= 2 \times 10^{-3} s$$

$$= 2ms$$

Also,

$$C\epsilon = 20\mu F \times 6.0V$$

$$= 120\ \mu C$$

We know that,

$$Q(t) = C\epsilon\left(1 - e^{-\frac{t}{\tau}}\right)$$

At t = 2.0ms,

$$Q(2.0ms) = 120\ \mu C\ (1 - e^{-\frac{2.0ms}{20ms}})$$

$$= 120\mu C(1 - e^{-1})$$

$$= 76\ \mu C$$

Hence, the charge on the capacitor at t = 2.0ms is 76μC.

65. Question

The plates of a capacitor of capacitance 10 μF, charged to 60 μC, are joined together by a wire of resistance 10Ω at t = 0. Find the charge on the capacitor in the circuit at

(a) t = 0, (b) t = 30 μs, (c) t = 120 μs and (d) t = 1.0 ms.

Answer

Charge on Capacitor during Discharging (RC Circuit):

A capacitor of capacitance with charge C is being discharged through a resistor of resistance R. A switch S is also connected in series with the capacitor. The switch is initially open. At t=0, the switch is closed. The charge on the capacitor at any time t>0 is given by:

$$\mathbf{Q = Q_i e^{-\frac{t}{\tau}}}$$

where $\tau = RC$ and Q_i is the initial charge on the capacitor.

Note that the capacitor begins charging at t = 0.

This is simply a discharging circuit.

Given,

Capacitance, $C = 10.0\mu F = 10.0 \times 10^{-6}\ F$

Resistance, $R = 10\ \Omega$

Initial Charge, $Q_i = 60\mu C$

Now,

$$\tau = RC$$

$$= 10\Omega \times 10.0 \times 10^{-6} F$$

$$= 10^{-4} s = 100\ \mu s = 0.1\ ms$$

$(1 ms = 10^{-3} s$ and $1\mu s = 10^{-6} s)$

We know that

$$Q(t) = Q_i e^{-\frac{t}{\tau}}$$

(a) At t = 0,

$$Q(0) = Q_i = 60\mu C$$

(b) At t = 30μs

$$Q(30\mu s) = 60\mu C \times \exp\left(-\frac{30\mu s}{100\mu s}\right)$$

$$= 60\mu C \times e^{-0.3}$$

$$\approx 60\mu C \times 0.7408$$

$\approx$ **44 μC**

(c) At t = 120μs

$$\mathbf{Q(120\mu s) = 60\mu C \times\ exp\left(-\frac{120\mu s}{100\mu s}\right)}$$

$$\mathbf{= 60\mu C \times e^{-1.2}}$$

$$\mathbf{\approx 60\mu C\ \times 0.3012}$$

$$\mathbf{\approx 18\ \mu C}$$

(d) At t = 1.0ms

$$\mathbf{Q(1.0ms) = 60\mu C \times\ exp\left(-\frac{1.0\mathit{ms}}{0.1\mathit{ms}}\right)}$$

$$\mathbf{= 60\mu C \times e^{-10}}$$

$$\mathbf{\approx 60\mu C \times 4.54 \times 10^{-5}}$$

$$\mathbf{\approx 0.003\ \mu C}$$

66. Question

A capacitor of capacitance 8.0 μF is connected to a battery of emf 6.0 V through a resistance of 24Ω. Find the current in the circuit

(a) just after the connections are made and

(b) one time constant after the connections are made.

Answer

Charging a capacitor:

A capacitor of capacitance C is being charged using a battery of emf ϵ through a resistance R . A switch S is also connected in series with the capacitor. The switch is initially open. The capacitor is uncharged at first. At t=0, the switch is closed. The current through the circuit at anytime t>0 is given by:

$$\mathbf{I(\mathit{t}) = \mathit{I_0 e^{\frac{-t}{\tau}}}}$$

Where I_0 is the initial current and $\tau = RC$ is the time constant.

Given,

Capacitance, $C\ =\ 8.0\ \mu F$

EMF of battery, $\epsilon =\ 6.0V$

Resistance, $R\ =\ 24\ \Omega$

(a)

Just after the connections are made, there is no potential difference across the capacitor and it acts as a short circuit; hence, the current can simply be calculated form Ohm's law:

$$I_0 = \frac{\epsilon}{R}$$

$$= \frac{6.0V}{24\Omega}$$

$$= 0.25A$$

(b)

We know that,

$$\mathrm{I}(t) = I_0 e^{\frac{-t}{\tau}}$$

Now, at t=τ,

$$I(\tau) = I_0 e^{-\frac{\tau}{\tau}}$$

$$= \frac{I_0}{e}$$

Using the result from (a), we get,

$$I(\tau) = 0.09A$$

67. Question

A parallel-plate capacitor of plate area 40 cm^2 and separation between the plates 0.10 mm is connected to a battery of emf 2.0 V through a 16 Ω resistor. Find the electric field in the capacitor 10 ns after the connections are made.

Answer

Formulas/Concepts Used:

Capacitance:

If two conductors have a potential difference V between the them and have charges Q and -Q respectively on them, then their capacitance is defined as

$$C = \frac{Q}{V}$$

Capacitance of a Capacitor in presence of a dielectric:The capacitance of the capacitor is initially C_0 and then a dielectric medium of dielectric constant K is inserted between the plates. The new capacitance is

$C = KC_0$

Also for parallel plate capacitors,

$$C_0 = \frac{\epsilon_0 A}{l}$$

Where ϵ0 is the permittivity of free space, A is the area of plate and l is the distance between the plates.

Charging a capacitor:

A capacitor of capacitance C is being charged using a battery of emf ϵ through a resistance R . A switch S is also connected in series with the capacitor. The switch is initially open. The capacitor is uncharged at first. At t=0, the switch is closed. The current through the circuit at anytime t>0 is given by:

$$\mathrm{I}(t) = I_0 e^{\frac{-t}{RC}}$$

Where I_0 is the initial current.

The charge is given by:

$$Q = C\epsilon\left(1 - e^{-\frac{t}{\tau}}\right)$$

Note that $\tau = RC$ is known as time constant.

Given,

Area of the plate, $A = 40cm^2 = 40 \times 10^{-4}m^2$

$(1cm\char`^2 = 10\char`^(-4)cm\char`^2)$

Now, distance of separation: $d = 0.10\ mm = 0.10 \times 10^{-3}m$

Emf of battery, $\epsilon = 2.0\ V$

Resistance ,$R = 16\Omega$

Time, $t = 10ns = 10 \times 10^{-9}s$

Now,

$$C = \frac{\epsilon_0 A}{d}$$

$$\frac{8.854 \times 10^{-12}m^{-3}kg^{-1}s^4A^2 \times 40 \times 10^{-4}m^2}{0.10 \times 10^{-3}m}$$

$$= 3.542 \times 10^{-10}F$$

Time constant, $\tau = RC = 3.542 \times 10^{-10}F \times 16\Omega = 5.667 \times 10^{-9}s$

We want to find the charge on the capacitor at t = 10ns

$$Q = C\epsilon\left(1 - e^{-\frac{t}{\tau}}\right)$$

$$= 3.542 \times 10^{-10}\mathrm{F} \times 2.0\mathrm{V} \times \left(1 - exp\left(-\frac{10 \times 10^{-9}s}{5.667 \times 10^{-9}s}\right)\right)$$

$$= 3.542 \times 10^{-10}\mathrm{F} \times 2.0\mathrm{V} \times (1 - exp(-1.765))$$

$$= 5.87 \times 10^{-10}C$$

Now,

$$E = \frac{V}{d} = \frac{Q}{Cd}$$

$$= \frac{5.87 \times 10^{-10}C}{3.542 \times 10^{-10}F \times 0.10 \times 10^{-3}m}$$

$$= 1.7 \times 10^{-4}V/m$$

68. Question

A parallel-plate capacitor has plate area 20 cm^2, plate separation 1.0 mm and a dielectric slab of dielectric constant 5.0 filling up the space between the plates. This capacitor is joined to a battery of emf 6.0 V through a 100 kΩ resistor. Find the energy for the capacitor 8.9 μs after the connections are made.

Answer

Energy stored in a capacitor:

The energy stored in a capacitor with capacitance C , charge is given by:

$$U = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$$

where V is the potential difference across the capacitor.

Capacitance of a Capacitor in presence of a dielectric:The capacitance of the capacitor is initially C_0 and then a dielectric medium of dielectric constant K is inserted between the plates. The new capacitance is

$$C = KC_0$$

Also for parallel plate capacitors,

$$C_0 = \frac{\epsilon_0 A}{l}$$

Where ϵ_0 is the permittivity of free space, A is the area of plate and l is the distance between the plates.

Charging a capacitor:

A capacitor of capacitance C is being charged using a battery of emf ϵ through a resistance R . A switch S is also connected in series with the capacitor. The switch is initially open. The capacitor is uncharged at first. At t=0, the switch is closed. The current through the circuit at anytime t>0 is given by:

$$\mathbf{I}(t) = I_0 e^{\frac{-t}{RC}}$$

Where I_0 is the initial current.

The charge is given by:

$$Q = C\epsilon\left(1 - e^{-\frac{t}{\tau}}\right)$$

Note that $\tau = RC$ is known as time constant.

Given,

Area of the plate, $A = 20cm^2 = 20 \times 10^{-4}m^2$

$(1cm\text{^}2 = 10\text{^}(-4)cm\text{^}2)$

Now, distance of separation: $d = 1.0mm = 10^{-3}m$

Dielectric constant, $K = 5.0$

Emf of battery, $\epsilon = 6.0\ V$

Resistance ,$R = 100k\Omega$

Time, $t = 8.9\mu s = 8.9 \times 10^{-6}s$

Now,

$$C = KC_0 = K\frac{\epsilon_0 A}{d}$$

$$5.0 \times \frac{8.854 \times 10^{-12}m^{-3}kg^{-1}s^4A^2 \times 20 \times 10^{-4}m^2}{10^{-3}m}$$

$$= 8.854 \times 10^{-11}F$$

Time constant, $\tau = RC = 8.854 \times 10^{-11}F \times 100 \times 10^3\Omega = 8.854 \times 10^{-6}s$

We want to find the charge on the capacitor at t = 8.9μs

$$Q = C\epsilon\left(1 - e^{-\frac{t}{\tau}}\right)$$

$$= 8.854 \times 10^{-11} F \times 6.0V \times \left(1 - exp\left(-\frac{8.9 \times 10^{-6} s}{8.854 \times 10^{-6} s}\right)\right)$$

$$= 3.368 \times 10^{-10} C$$

Now, energy of the capacitor is given by:

$$U = \frac{Q^2}{2C}$$

$$= \frac{(3.368 \times 10^{-10} C)^2}{2 \times 8.854 \times 10^{-11} F}$$

$$6.4 \times 10^{-10} J$$

Hence, the capacitor stores $6.4 \times 10^{-10} J$ of energy after 8.9μs.

69. Question

A 100 μF capacitor is joined to a 24 V battery through a 1.0 MΩ resistor. Plot qualitative graphs

(a) between current and time for the first 10 minutes and

(b) between charge and time for the same period.

Answer

Charging a capacitor:

A capacitor of capacitance C is being charged using a battery of emf ϵ through a resistance R . A switch S is also connected in series with the capacitor. The switch is initially open. The capacitor is uncharged at first. At t=0, the switch is closed. The current through the circuit at anytime t>0 is given by:

$$\mathbf{I}(t) = I_0 e^{\frac{-t}{RC}}$$

Where I_0 is the initial current.

The charge is given by:

$$Q = C\epsilon\left(1 - e^{-\frac{t}{\tau}}\right)$$

Note that $\tau = RC$ is known as time constant.

Given,

Resistance of the resistor, $R = 1.0 M\Omega = 1.0 \times 10^6 \Omega$

Capacitance, $C = 100\mu F = 100 \times 10^{-6} F$

Emf of the battery, $\epsilon = 24$

Now, time constant , $\tau = RC = 100s$

(a)

The current is given by:

$$I(t) = I_0 e^{-\frac{t}{\tau}}$$

At t =0, we have y-intercept I_0.

Now,

$$I_0 = \frac{\epsilon}{R}$$

$$= \frac{24V}{1.0 \times 10^6 \Omega}$$

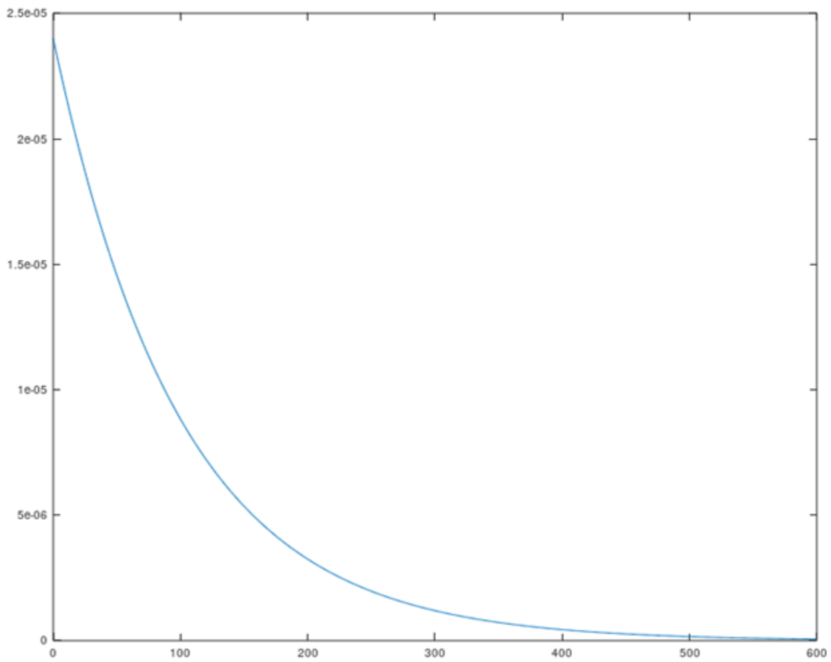
$$= 2.4 \times 10^5 \text{ A}$$

Hence, we need to plot

$$\text{I} = 2.4 \times 10^5 \text{ A } e^{-\frac{t}{100}}$$

At t = 10min = 600s,

$$I = 2.4 \times 10^5 A e^{-6} = 5.94 \times 10^{-8} A$$



(b)

Now,

$$C\epsilon = 100 \times 10^{-6}F \times 24V = 2.4 \times 10^{-3}C$$

Note that the current is in Ampere and time in seconds. The graph will represent exponential decay.

We need to plot

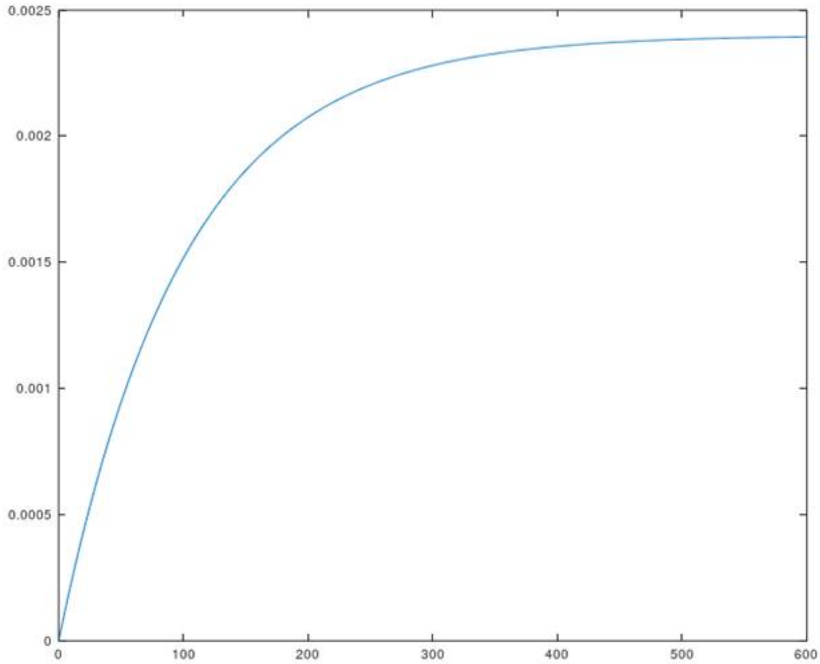
$$Q = 2.4 \times 10^{-3}\, C\left(1 - exp\left(-\frac{t}{100s}\right)\right)$$

Note that t is in seconds and Q in coulombs.

At t = 10 min = 600s,

$$Q \;=\; 0.002394C$$

The charge will keep on increasing and will almost touch the asymptope at Q = 0.0024C.



70. Question

How many time constants will elapse before the current in a charging RC circuit drops to half of its initial value? Answer the same question for a discharging RC circuit.

Answer

Current when capacitor is discharging:

A capacitor of capacitance C is being discharged through a resistance R . A switch S is also connected in series with the capacitor. The switch is initially open. At t=0, the switch is closed. The current through the circuit at anytime t>0 is given by:

$$\mathrm{I}(t) = I_0 e^{\frac{-t}{RC}}$$

Where I0 is the initial current.

Note that $\tau = RC$ is known as time constant

Current when capacitor is charging:

A capacitor of capacitance C is being charged using a battery of emf ϵ through a resistance R . A switch S is also connected in series with the capacitor. The switch is initially open. At t=0, the switch is closed. The current through the circuit at anytime t>0 is given by:

$$I(t) = I_0 e^{\frac{-t}{RC}}$$

Where I_0 is the initial current.

Note that $\tau = RC$ is known as time constant.

In both the cases wish to find time t>0 such that

$$I(t) = \frac{1}{2} I_0$$

I(t) is given by the same formula in both the cases.

$$I_0 e^{\frac{-t}{\tau}} = \frac{1}{2} I_0$$

$$e^{-\frac{t}{\tau}} = \frac{1}{2}$$

$$-\frac{t}{\tau} = ln\frac{1}{2}$$

$$t = ln\,2\ \tau$$

$$t \approx 0.69\tau$$

Hence, 0.69 time constants will elapse.

71. Question

How many time constants will elapse before the charge on a capacitor falls to 0.1% of its maximum value in a discharging RC circuit?

Answer

Concepts/Formulas used:

Charge on Capacitor during Discharging (RC Circuit):

A capacitor of capacitance with charge C is being discharged through a resistor of resistance R. A switch S is also connected in series with the capacitor. The switch is initially open. At t=0, the switch is closed. The charge on the capacitor at any time t>0 is given by:

$$Q = Q_i e^{-\frac{t}{\tau}}$$

where $\tau = RC$ and Q_i is the initial charge on the capacitor.

Note that the capacitor begins charging at t = 0.

Capacitance:

If two conductors have a potential difference V between the them and have charges Q and -Q respectively on them, then their capacitance is defined as

$$C = \frac{Q}{V}$$

Let a capacitor of capacitance C be discharged through a resistor of resistance R. Switch S in attached is series and in initially open. It is closed at t = 0.

Charge is maximum when the capacitor is fully charged. Hence,

$$Q_{max} = Q_i$$

Now, we have at any time t>0,

$$Q(t) = Q_i e^{-\frac{t}{\tau}}$$

We want to find t such that

$$Q(t) = 0.1\%\ Q_{max} = \frac{1}{1000} Q_{max}$$

$$Q_i e^{-\frac{t}{\tau}} = \frac{1}{1000} Q_i$$

$$e^{-\frac{t}{\tau}} = 0.001$$

$$t = -\tau\, ln(0.001)$$

$$t \approx 6.9\tau$$

Hence, we need 6.9 time constants.

72. Question

How many time constants will elapse before the energy stored in the capacitor reaches half of its equilibrium value in a charging RC circuit?

Answer

Concepts/Formulas used:

Charge on Capacitor during Charging (RC Circuit):

A capacitor of capacitance with charge C is being charged with a battery of emf ϵ through a resistor of resistance R. A switch S is also connected in series with the capacitor. The switch is initially open. The capacitor is uncharged at first. At t=0, the switch is closed. The charge on the capacitor at any time t>0 is given by:

$$Q = C\epsilon\left(1 - e^{-\frac{t}{\tau}}\right)$$

where $\tau = RC$

Note that the capacitor begins charging at t = 0.

Energy stored in a capacitor:

The energy stored in a capacitor with capacitance C , charge is given by:

$$U = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$$

where V is the potential difference across the capacitor.

Let a capacitor of capacitance C with no initial charge be attached to a battery of emf ϵ through a resistor of resistance R. Switch S in attached is series and in initially open. It is closed at t = 0.

Equilibrium is when no current flows and the potential across the battery is the same as the potential across the capacitor. The energy stored at equilibrium is:

$$U_{eq} = \frac{1}{2}C\epsilon^2$$

Suppose the capacitor begins charging at t = 0.

Now, at any time t > 0, the energy stored is

$$U(t) = \frac{\left(Q(t)\right)^2}{2C}$$

Substituting the value for Q(t),

$$\mathrm{U}(t) = \frac{C^2\epsilon^2\left(1 - e^{\frac{-t}{\tau}}\right)^2}{2C}$$

$$= \frac{1}{2}C\epsilon^2\left(1 - e^{\frac{-t}{\tau}}\right)^2$$

We want to find t when

$$U(t) = \frac{1}{2}U_{eq}$$

$$\frac{1}{2}C\epsilon^2\left(1-e^{\frac{-t}{\tau}}\right)^2=\frac{1}{2}\left[\frac{1}{2}C\epsilon^2\right]$$

$$\left(1-e^{\frac{-t}{\tau}}\right)^2=\frac{1}{2}$$

$$\left(1-e^{\frac{-t}{\tau}}\right)=\pm\frac{1}{\sqrt{2}}$$

$$e^{\frac{-t}{\tau}}=1\pm\frac{1}{\sqrt{2}}$$

$$\frac{-t}{\tau}=ln\left(1\pm\frac{1}{\sqrt{2}}\right)$$

$$t=-\tau\, ln\left(1\pm\frac{1}{\sqrt{2}}\right)$$

Now, $t\approx -0.539\tau$ or $t\approx 1.23\tau$

We reject the former as the capacitor begins charging at t = 0.

Hence, 1.23 time constants elapse .

73. Question

How many time constants will elapse before the power delivered by the battery drops to half of its maximum value in an RC circuit?

Answer

Concepts/Formula used:

Current when capacitor is charging:

A capacitor of capacitance C is being charged using a batteru of emf ϵ through a resistance R . A switch S is also connected in series with the capacitor. The switch is initially open. The capacitor is uncharged at first. At t=0, the switch is closed. The current through the circuit at anytime t>0 is given by:

$$\mathbf{I}(t)=I_0e^{\frac{-t}{RC}}$$

Where I_0 is the initial current.

Note that $\tau = RC$ is known as time constant

Power supplied by the battery:

If a battery of emf ϵ gives a current I, then the power supplied by the battery is given by:

$$P = I\epsilon$$

Now, at anytime t > 0,

$$P(t) = I(t)\,\epsilon$$

$$P(t) = I_0 e^{-\frac{t}{\tau}}$$

Now, power is maximum when the current is maximum i.e. when t = 0

$$P_{max} = I_0 e^0 \epsilon$$

$$= I_0 \epsilon$$

We wish to find time t such that

$$P(t) = \frac{1}{2} P_{max}$$

$$I_0 e^{-\frac{t}{\tau}} \epsilon = \frac{1}{2} I_0 \epsilon$$

$$e^{-\frac{t}{\tau}} = \frac{1}{2}$$

$$\frac{-t}{\tau} = ln\frac{1}{2}$$

$$t = -\tau\, ln\frac{1}{2}$$

$$t = \tau\, ln\, 2$$

$$\approx 0.69\,\tau$$

The time in 0.69 times the time constant.

74. Question

A capacitor of capacitance C is connected to a battery of emf ϵ at t = 0 through a resistance R. Find the maximum rate at which energy is stored in the capacitor. When does the rate has this maximum value?

Answer

Formula/Consepts Used:

Energy stored by capacitor:

For a capacitor of capacitance C , with charge Q, and potential difference V across it, the energy stored is given by:

$$U = \frac{1}{2} CV^2 = \frac{1}{2} Q^2 V$$

Charging a capacitor:

A capacitor of capacitance C is connected in series with a resistor of resistance R, a switch, and battery of emf ϵ . It is uncharged at first. The switch is closed at t = 0, then at time any time t the charge stored on the capacitor is given by

$$\mathbf{q = C\epsilon\left(1 - e^{\frac{-t}{RC}}\right)}$$

Energy stored in the capacitor is

$$U = \frac{Q^2}{2C}$$

$$= \frac{\left(C\epsilon\left(1 - e^{\frac{-t}{RC}}\right)\right)^2}{2C}$$

$$P(t) = \frac{dU}{dt} = C\frac{(\epsilon)^2}{2}2\left(1 - e^{\frac{-t}{RC}}\right)\left((-1)\frac{-1}{RC}e^{\frac{-t}{RC}}\right)$$

$$P(t) = \frac{1}{R}(\epsilon)^2\left(-e^{\frac{-2t}{RC}} + e^{\frac{-t}{RC}}\right)$$

$$\frac{dP}{dt} = \frac{1}{R}(\epsilon)^2\left(-2e^{\frac{-2t}{RC}} + e^{\frac{-t}{RC}}\right)\frac{-1}{RC}$$

For maxima,

$$\frac{dP}{dt} = 0$$

$$2e^{\frac{-2t}{RC}} = e^{\frac{-t}{RC}}$$

Taking the natural logarithm on both sides,

(Note that $\ln(e^a) = a$ and $\ln(ab) = \ln(a) + \ln(b)$

$$\ln 2 + \frac{-2t}{RC} = \frac{-t}{RC}$$

$$t = CR\ln 2$$

Now, the maximum rate is

$$P(CR\ln 2) = \frac{\epsilon^2}{R}\left(-e^{\frac{-CR\ln(2)}{RC}} + e^{\frac{-2CR\ln(2)}{RC}}\right)$$

$$= \frac{\epsilon^2}{R}\left(-e^{\ln\left(\frac{1}{4}\right)} + e^{\ln(1/2)}\right)$$

$$= \frac{\epsilon^2}{R}\left(\frac{1}{2} - \frac{1}{4}\right)$$

$$= \frac{\epsilon^2}{2R}$$

75. Question

A capacitor of capacitance 12.0 μF is connected to a battery of emf 6.00 V and internal resistance 1.00 Ω through resistanceless leads 12.0 μs after the connections are made, what will be

(a) the current in the circuit,

(b) the power delivered by the battery,

(c) the power dissipated in heat and

(d) the rate at which the energy stored in the capacitor is increasing.

Answer

Concepts/Formula used:

Current when capacitor is charging:

A capacitor of capacitance C is being charged using a battery of emf ϵ through a resistance R . A switch S is also connected in series with the capacitor. The switch is initially open. The capacitor is uncharged at first. At t=0, the switch is closed. The current through the circuit at anytime t>0 is given by:

$$\mathbf{I}(t) = I_0 e^{\frac{-t}{\tau}}$$

Where I_0 is the initial current.

Note that $\tau = RC$ is known as time constant

Power supplied by the battery:

If a battery of emf ϵ gives a current I, then the power supplied by the battery is given by:

$$P = I\epsilon$$

Energy dissipated by a resistor :

A resistor of resistance R with current I through it, dissipates energy U given by:

$$U = I^2 R \Delta t$$

in time Δt.

Its power is given by:

$$P = I^2 R$$

The capacitor is being charged

Given,

Capacitance, $C = 12.0\mu F$

Resistance, $R = 1.00\ \Omega$

Emf of the battery, $\epsilon = 6.00V$

Time, $t = 12.0\mu s$

Now, time constant, $\tau = RC = 1.00\ \Omega \times 12.0\mu F = 12.0\mu s$

(a)

The initial current is :

$$I_0 = \frac{\epsilon}{R} = \frac{6V}{1\Omega} = 6A$$

Now,

$$I(t) = I_0 e^{\frac{-t}{\tau}}$$

At t = 12.0μs

$$I(12.0\mu s) = 6A \times exp\left(-\frac{12.0\mu s}{12.0\mu s}\right)$$

$$= 6A \times e^{-1} = 2.207277A$$

$$\approx 2.21\ A$$

(b)

The power supplied by the battery is:

$$P(t) = I(t)\epsilon$$

At t = 12.0μs,

$$P(12.0\mu s) = 2.207277A \times 6.00V$$

$$\approx 13.2\ W$$

(c)

The power dissipated as heat:

$$H(t) = I^2(t)R$$

At t = 12.0μs,

$$H(12.0\mu s) = (2.207277A)^2 \times 1.00\Omega$$

$= \mathbf{4.87W}$

(d)

By conservation of energy,

Energy supplied by battery = Energy stored by capacitor + Energy dissipated as heat.

Dividing by time, gives us

Power supplied by battery = Power dissipated as heat + rate at which energy is stored in the capacitor.

Hence, using the previous results, we have,

Rate at which energy is stored in the capacitor:

$$P_c = 13.2W - 4.87W = 8.33W$$

76. Question

A capacitance C charged to a potential difference V is discharged by connecting its plates through a resistance R. Find the heat dissipated in one time constant after the connections are made. Do this by calculating $\int i_2 R\,dt$ and also by finding the decrease in the energy stored in the capacitor.

Answer

Concepts/Formulas Used:

Energy dissipated by a resistor :

A resistor of resistance R with current I through it, dissipates energy U given by:

$$U = I^2 R \Delta t$$

in time Δt.

Its power is given by:

$$P = I^2 R$$

Current when capacitor is discharging:

A capacitor of capacitance C is being charged through a resistance R , the current through the circuit is given by:

$$I(t) = I_0 e^{\frac{-t}{RC}}$$

Where I_0 is the initial current.

Energy stored by capacitor:

For a capacitor of capacitance C , with charge Q, and potential difference V across it, the energy stored is given by:

$$U = \frac{1}{2}CV^2 = \frac{1}{2}Q^2V$$

Discharging a capacitor:

A capacitor of capacitance C is connected in series with a resistor of resistance R and a switch. Before the switch is closed, it has charge Qi . If the switch is closed at t = 0, then at any time t, the charge on the capacitor is given by:

$$q = Q_i e^{\frac{-t}{\tau}}$$

where $\tau = RC$

The initial energy of the capacitor,

$$U_{i,C} = \frac{1}{2}CV^2$$

As the capacitor is discharged, it looses Charge ,and the potential difference across it also decreases.

Note that $Q_i = CV$

Now, at t= τ,

$$q = Q_i e^{\frac{-\tau}{\tau}}$$

$$= Q_i e^{-1}$$

$$= \frac{Q_i}{e}$$

$$U_f = \frac{1}{2C}q^2$$

$$= \frac{{Q_i}^2}{2Ce^2} = \frac{CV^2}{2e^2}$$

The energy lost is dedicated as heat and is equal to:

$$U_i - U_f = \frac{CV^2}{2}\left(1 - \frac{1}{e^2}\right)$$

Now let us find the energy dissipated by another method:

$$P = \frac{dU}{dt} = i^2R$$

$$dU = I^2Rdt$$

$$\int_0^U dU = \int_0^\tau I^2 R dt$$

Substituting $I(t) = I_0 e^{\frac{-t}{RC}}$,

$$U = \int_0^\tau I_0^2 e^{\frac{-2t}{RC}} dt$$

$$U = I_0^2 \int_0^\tau e^{\frac{-2t}{RC}} dt$$

$$U = I_0^2 \left[\frac{RC}{-2} e^{\frac{-2t}{RC}}\right]_0^{RC}$$

Note that $\tau = RC$ and $I_0 = \frac{V}{C}$

$$U = \frac{V^2}{C^2}\frac{(-RC)}{2}(e^{-2} - 1)$$

$$= \frac{CV^2}{2}\left(1 - \frac{1}{e^2}\right)$$

Both ways give us the same result!

77. Question

By evaluating $\int i^2 R\, dt$, show that when a capacitor is charged by connecting it to a battery through a resistor, the energy dissipated as heat equals the energy stored in the capacitor.

Answer

Concepts/Formulas Used:

Energy dissipated by a resistor :

A resistor of resistance R with current I through it, dissipates energy U given by:

$$U = I^2 R \Delta t$$

in time Δt.

Its power is given by:

$$P = I^2 R$$

Current when capacitor is charging:

A capacitor of capacitance C is being charged by a battery of emf V through a resistance R is series , the current through the circuit is given by:

$$I(t) = I_0 e^{\frac{-t}{RC}}$$

Where

$$I_0 = \frac{V}{R}$$

Suppose a capacitor of capacitance C is being charged by a battery of emf V through a resistance R.

Now, the power of the resistor is given by:

$$P = \frac{dU}{dt} = \mathrm{I}^2 R$$

$$\mathrm{dU} = I^2 \mathrm{R} dt$$

$$\int_0^U dU = \int_0^\infty I^2 R dt$$

Substituting $I(t) = I_0 e^{\frac{-t}{RC}}$,

$$\mathrm{U} = \int_0^\infty I_0^2 e^{\frac{-2t}{RC}} dt$$

$$\mathrm{U} = I_0^2 \int_0^\infty e^{\frac{-2t}{RC}} dt$$

$$\mathrm{U} = I_0^2 \left[\frac{RC}{-2} e^{\frac{-2t}{RC}}\right]_0^\infty$$

$$= I_0^2 \frac{-RC}{2} (0 - 1)$$

Substitute $I_0 = \frac{V}{R}$

$$= \frac{1}{2} CV^2$$

This is the same as the energy stored in the capacitor when it is fully charged! (Note that when the capacitor is fully charged, the potential difference across it is V.)

78. Question

A parallel-plate capacitor is filled with a dielectric material having resistivity ρ and dielectric constant K.

The capacitor is charge and disconnected from the charging source. The capacitor is slowly discharged through the dielectric. Show that the time constant of the discharge is independent of all geometrical parameters like the plate area or separation between the plates. Find this time constant.

Answer

Time constant for capacitor:

$$\tau = RC$$

Where R is the resistance through which the capacitor is being charged/discharged and C is the capacitance.

Capacitance of a Capacitor in presence of a dielectric:The capacitance of the capacitor is initially C_0 and then a dielectric medium of dielectric constant K is inserted between the plates. The new capacitance is

$$C = KC_0$$

Also for parallel plate capacitors,

$$C_0 = \frac{\epsilon_0 A}{l}$$

Where ϵ_0 is the permittivity of free space, A is the area of plate and l is the distance between the plates.

Resistance and Resistivity:

For a material of length l and uniform cross-section A and reistivity ρ, the resistance is given by:

$$R = \rho \frac{l}{A}$$

Note the area of cross section of the material is the same as the area of the capacitor plates and the length of the material is the same as the distance of separation between the plates.

Now,

$$\tau = RC$$

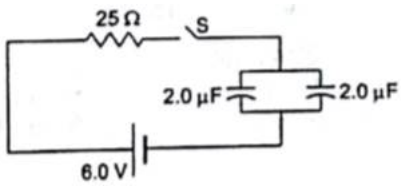
$$= RKC_0$$

$$= \rho \frac{l}{A} \times K \times \epsilon \frac{A}{l}$$

$$\tau = \rho K \epsilon$$

79. Question

Find the charge on each of the capacitors 0.20 ms after the switch S is closed in figure.



Answer

Concepts/Formulas used:

Charging a capacitor:

A capacitor of capacitance C is connected in series with a resistor of resistance R, a switch, and battery of emf ϵ . It is uncharged at first. The switch is closed at t = 0, then at time any time t the charge stored on the capacitor is given by

$$q = C\epsilon\left(1 - e^{\frac{-t}{RC}}\right)$$

Capacitors in parallel:

If capacitors C_1, C_2, C_3 , ... are in parallel, then the equivalent capacitance is given by:

$$C_{eq} = C_1 + C_2 + C_3 + \cdots$$

If the charges on the capacitors are Q1, Q2, Q3, .. are in parallel, then the charge on the capacitor with equivalent capacitance is given by:

$$Q_{eq} = Q_1 + Q_2 + Q_3 + \cdots$$

We can replace the two capacitors by another capacitor of capacitance C. As the capacitors are in parallel.

$$C = C_1 + C_2$$

$$= 2\mu F + 2\mu F = 4\mu F$$

Now,

We know that

$$q = C\epsilon\left(1 - e^{\frac{-t}{RC}}\right)$$

Here,

$$RC = 25\Omega \times 4\mu F$$

$$= 25\Omega \times 4 \times 10^{-6} F$$

$$= 10^{-4} s$$

Also, $\epsilon = 6\ V$ and $t = 0.2ms = 2 \times 10^{-4} s$

Hence,

$$q = C\epsilon\left(1 - e^{\frac{-t}{RC}}\right)$$

$$= 4\mu F \times 6V\left(1 - exp\left(\frac{-2 \times 10^{-4}s}{10^{-4}s}\right)\right)$$

$$\approx 20.752\ \mu C$$

Let the charge on both the capacitors be Q. As both have the same capacitance and potential ($Q = CV$), both must have the same charge. Note that they both are in parallel.

Hence,

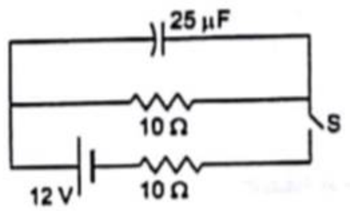
$$Q_{eq}(= q) = Q + Q = 2Q$$

$$2Q = 20.752\ \mu F$$

$$Q \approx 10.37\ \mu F$$

80. Question

The switch S shown in figure is kept closed for a long time and is then opened at t = 0. Find the current in the middle 10 Ω resistor at t = 1.0 ms.



Concepts/Formulas used:

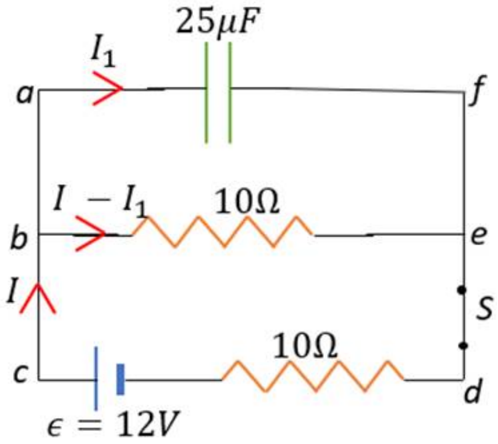
When is capacitor is discharging through as resistor of resistance R, the current through the resistor is given by:

$$I(t) = I_0 e^{\frac{-t}{RC}}$$

where I_0 is the initial current and C is the capacitance of the capacitor.

Answer

When the switch is closed and the circuit is in steady state, no current passes through the capacitor.



Applying Kirchhoff's loop rule on loop bedcb,

$$-(I - I_1)(10\Omega) - I(10\Omega) + 12V = 0$$

Substitute $I_1 = 0$ as there is no current through the capacitor at steady state.

$$-20\Omega I + 12V = 0$$

$$I = \frac{12V}{20\Omega} = 0.6A$$

Now, $V_{be} = V_{af}$ as they are in parallel

$$V_{af} = V_{be} = I(10\Omega) = 6V$$

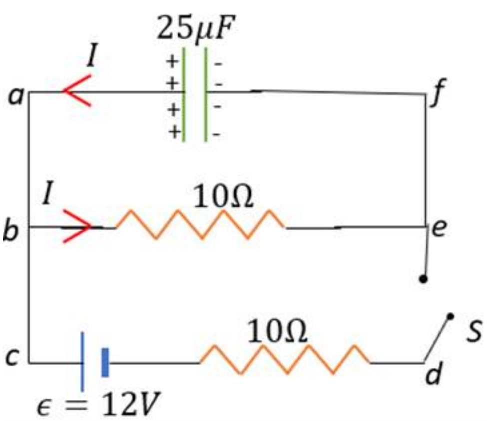
Note that V_{af} is the potential difference across the capacitor.

When the switch is just opened, the potential difference across it is the same for a moment as the charge and capacitance is the same. However, the charge and potential decrease slowly as the capacitor starts discharging.

Discharging:

We know that when discharging,

$$I(t) = I_0 e^{\frac{-t}{RC}}$$



Applying Kirchhoff's loop rule on fabef,

$$V_c - I(10\Omega) = 0$$

At $t = 0, I = I_0$ and $V_c = 6V$

$$6V - I_0(10\Omega) = 0$$

$$I_0 = 0.6A$$

Here, R = 10Ω and C = 25μF

$$\mathbf{RC = 10\Omega \times 25 \times 10^{-6}F = 2.5 \times 10^{-4}s}$$

Now,

$$I(t) = I_0 e^{\frac{-t}{RC}}$$

$$= 0.6A \times e^{\frac{-1\times10^{-3}s}{2.5\times10^{-4}s}}$$

$$= 0.011A$$

$$= 11\ mA$$

81. Question

A capacitor of capacitance 100 μF is connected across a battery of emf 6.0 V through a resistance of 20 kΩ for 4.0 s. The battery is then replaced by a thick wire. What will be the charge on the capacitor 4.0 s after the battery is disconnected?

Answer

Concepts/Formulas used:

Charging a capacitor:

A capacitor of capacitance C is connected in series with a resistor of resistance R, a switch, and battery of emf ϵ . It is uncharged at first. The switch is closed at t = 0, then at time any time t the charge stored on the capacitor is given by

$$q = C\epsilon\left(1 - e^{\frac{-t}{RC}}\right)$$

Discharging a capacitor:

A capacitor of capacitance C is connected in series with a resistor of resistance R and a switch. Before the switch is closed, it has charge Qi . If the switch is closed at t = 0, then at any time t, the charge on the capacitor is given by:

$$q = Q_i e^{\frac{-t}{RC}}$$

Given,

Emf = 6V

Capacitance, $C = 100\backslash microF = 100 \times 10^{-6}F = 10^{-4}F$

Resistance, $r = 20k\Omega = 20 \times 10^{3}\Omega = 2 \times 10^{4}\Omega$

Time for charging = time for discharging = $t = 4s$

When charging,

$$q = C\epsilon\left(1 - e^{\frac{-t}{RC}}\right)$$

$$= 10^{-4}\,\mathrm{F} \times 6\mathrm{V}\left(1 - e^{\frac{-4.0s}{2\times10^{4}\times10^{-4}F}}\right)$$

$$= 5.188 \times 10^{-4}C$$

When discharging,

$$Q_i = 5.188 \times 10^{-4}C$$

$$q = Q_i e^{\frac{-t}{RC}}$$

$$= 5.188 \times 10^{-4}C \times e^{\frac{-4.0s}{2\times10^{4}\times10^{-4}F}}$$

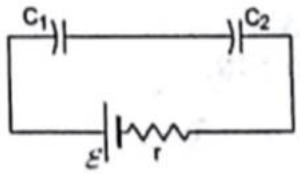
$$= 7.02 \times 10^{-5}C$$

Now, $1\mu F = 10^{-6}F$

Hence, $q \approx 70\mu F$

82. Question

Consider the situation shown in figure. The switch is closed at t = 0 when the capacitors are uncharged. Find the charge on the capacitor C_1 as a function of time t.



Answer

Concepts/Formulas used:

Kirchhoff's loop rule:

The sum of potential differences around a closed loop is zero.

Capacitance:

If two conductors have a potential difference V between the them and have charges Q and -Q respectively on them, then their capacitance is defined as

$$C = \frac{Q}{V}$$

Capacitors in series:

If capacitors C_1, C2, C3 , ... are in series, then the equivalent capacitance is given by:

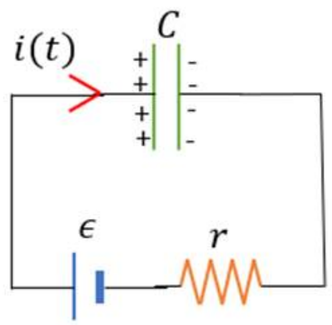
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$$

We can replace C_1 and C_2 by C_{eq}. As C_1 ans C_2 are in series,

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

Let us drop the subscript and call C_{eq} just C.

$$C = \frac{C_1 C_2}{C_1 + C_2}$$



Let the potential across the capacitor C be at time t be V_c. Let the charge at time t be q.

$$V_c = \frac{q}{C}$$

Note that as C_1 and C_2 are in series,

$$q = Q_{eq} = Q_1 = Q_2$$

Applying Kirchhoff's loop rule ,

$$\epsilon - V_c - ir = 0$$

$$\epsilon - \frac{q}{C} - ir = 0$$

$$\epsilon - \frac{q}{C} = r\frac{dq}{dt}$$

$$\frac{dq}{dt} = \frac{C\epsilon - q}{rC}$$

$$\int \frac{dq'}{q' - C\epsilon} = -\int \frac{dt'}{rC}$$

We know that $\int \frac{dx}{x+a} = ln|x + a| \; + \; Constant$

$$ln|q - C\epsilon| = -\frac{t}{rC} + B$$

Where B is a constant

$$|q - C\epsilon| = e^{-\frac{t}{rC}+B}$$

$$q - C\epsilon = \pm e^{-\frac{t}{rC}+B} = \pm e^{B} e^{-\frac{t}{rC}}$$

Let $A = \pm e^{B}$

$$q - C\epsilon = Ae^{-\frac{t}{rC}}$$

Substitute q = 0 at t = 0,

$$-C\epsilon = A$$

Substituting the value of A back,

$$q = C\epsilon\left(1 - e^{-\frac{t}{rC}}\right)$$

where $C = \frac{C_1 C_2}{C_1 + C_2}$

83. Question

A capacitor of capacitance C is given a charge Q. At t = 0, it is connected to an uncharged capacitor of equal capacitance through a resistance R. Find the charge on the second capacitor as a function of time.

Answer

Concepts/Formulas used:

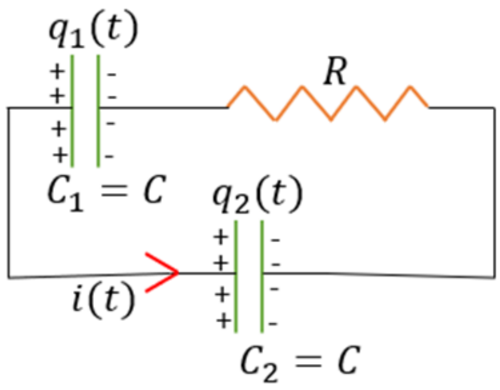
Kirchhoff's loop rule:

The sum of potential differences around a closed loop is zero.

Capacitance:

If two conductors have a potential difference V between the them and have charges Q and -Q respectively on them, then their capacitance is defined as

$$C = \frac{Q}{V}$$



Note that

$$V_{C_1} = \frac{q_1}{C}$$

and

$$V_{C_2} = \frac{q_2}{C}$$

By conservation of charge,

$$q_1 + q_2 = q_1(0) + q_2(0) = Qq_1 = Q - q_2 \qquad \text{..................(1)}$$

Now, applying Kirchhoff's loop rule, we get

$$-V_{C_1} + V_{\{C_2\}} + iR = 0$$

$$-\frac{q_1}{C} + \frac{q_2}{C} + \frac{dq_2}{dt}R = 0$$

Using (1), we get

$$\frac{2q_2 - Q}{C} = -\frac{dq_2}{dt}R$$

$$\int \frac{dq_2}{2q_2 - Q} = -\int \frac{dt}{RC}$$

$$\frac{1}{2} ln|2q_2 - Q| = -\frac{t}{RC} + A$$

where A is a constant.

$$|2q_2 - \mathrm{Q}| = e^{\frac{-2t}{RC} + 2A} = e^{2A}\mathrm{e}^{\frac{-2t}{RC}}$$

$$2q_2 - \mathrm{Q} = \pm e^{2A}\mathrm{e}^{\frac{-2t}{RC}}$$

Let $e^{2A} = \pm B$

$$2q_2 = \mathbf{B}e^{\frac{-2t}{RC}} + \mathbf{Q}$$

Substituting $q_2(0) = 0$, we get B = -Q

Hence,

$$2q_2 = -Qe^{\frac{-2t}{RC}} + \mathbf{Q}$$

$$q_2 = \frac{Q}{2}\left(1 - e^{\frac{-2t}{RC}}\right)$$

84. Question

A capacitor of capacitance C is given a charge Q. At t = 0, it is connected to an ideal battery of emf ϵ through a resistance R. Find the charge on the capacitor at time t.

Answer

Note that $q_2(0) = 0$ and $q_1(t) = Q$.

Concepts/Formulas used:

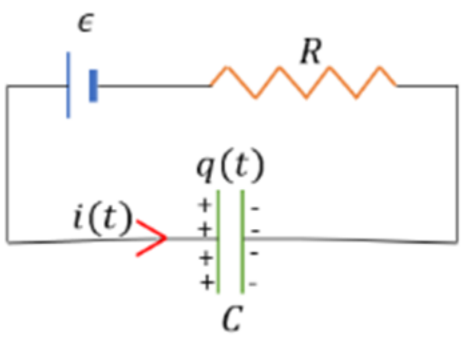
Kirchhoff's loop rule:

The sum of potential differences around a closed loop is zero.

Capacitance:

If two conductors have a potential difference V between the them and have charges Q and -Q respectively on them, then their capacitance is defined as

$$C = \frac{Q}{V}$$



Let the potential across the capacitor be at time t be V_c. Let the charge at time t be q. The initial charge is Q.

$$V_c = \frac{q}{C}$$

Applying Kirchhoff's loop rule ,

$$\epsilon - V_c - iR = 0$$

$$\epsilon - \frac{q}{C} - iR = 0$$

$$\epsilon - \frac{q}{C} = R\frac{dq}{dt}$$

$$\frac{dq}{dt} = \frac{C\epsilon - q}{RC}$$

$$\int_Q^q \frac{dq'}{q' - C\epsilon} = -\int_0^t \frac{\mathrm{dt}'}{RC}$$

We know that $\int \frac{dx}{x+a} = \ln|x+a| \ + \ C$

$$[ln|q' - C\epsilon|]_Q^q = -\left[\frac{t'}{RC}\right]_0^t$$

$$ln|q - C\epsilon| - ln|Q - C\epsilon| = -\frac{t}{RC}$$

Using the property : $ln(a) - ln(b) = ln(a/b)$, we get

$$ln\left|\frac{q - C\epsilon}{Q - C\epsilon}\right| = -\frac{t}{RC}$$

Note that at any time,

$$\epsilon \geq V_c$$

$$\epsilon \geq \frac{q}{C}$$

$$q \ - \ C\epsilon \geq \ 0$$

Thus, we can remove the modulus,

$$\ln\left(\frac{q - C\epsilon}{Q - C\epsilon}\right) = -\frac{t}{RC}$$

$$\frac{q - C\epsilon}{Q - C\epsilon} = e^{\frac{-t}{RC}}$$

$$q - C\epsilon = (Q - C\epsilon)e^{\frac{-t}{RC}}$$

$$q = C\epsilon\left(1 - e^{\frac{-t}{RC}}\right) + Qe^{\frac{-t}{RC}}$$