JEE MAIN 2025

Sample Paper - 6

Time Allowed: 3 hours **Maximum Marks: 300**

General Instructions:

1. There are three subjects in the question paper consisting of Physics (Q. no. 1 to 25), Chemistry (Q, no. 26 to 50), and Mathematics (Q. no. 51 to 75).

2. Each subject is divided into two sections. Section A consists of 20 multiple-choice questions & Section B consists of 5 numerical value-type questions.

3. There will be only one correct choice in the given four choices in Section A. For each question for Section A, 4 marks will be awarded for correct choice, 1 mark will be deducted for incorrect choice questions and zero marks will be awarded for not attempted questions.

4. For Section B questions, 4 marks will be awarded for correct answers and zero for unattempted and incorrect answers.

5. Any textual, printed, or written material, mobile phones, calculator etc. is not allowed for the students appearing for the test.

6. All calculations/written work should be done in the rough sheet is provided with the Question Paper.

PHYSICS **MAX.MARKS: 100**

SECTION – I
(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which **ONLY ONE** option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and -1 if not correct.

1. A student determined young's modulus of elasticity using the formula $Y = \frac{MgL^3}{4bd^3\delta}$. The value of g is taken to be $9.8 m/s^2$, without any significant error, his observations are as following.

Physical quantity	Least count of the equipment used for measurement	Observed value
Mass (M)	1 g	2 kg
Length of bar (L)	1 mm	1 m
Breadth of bar (b)	0.1 mm	4 cm
Thickness of bar (d)	0.01 mm	0.4 cm
Depression (δ)	0.01 mm	5 mm

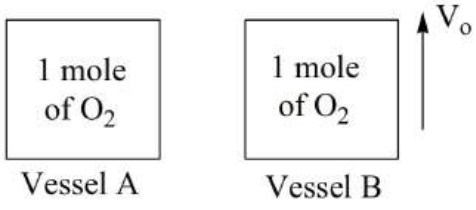
Then, the fractional error in the measurement of Y is

A) 0.0083 B) 0.0155 C) 0.155 D) 0.083

2. A box weighs 196 N on a spring balance at the north pole. Its weight recorded on the same balance if it is shifted to the equator is close to (Take $g = 10 ms^{-2}$ at the north pole and the radius of the earth = 6400 km)

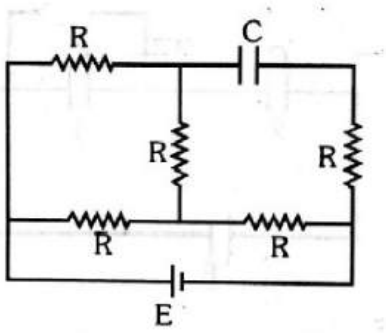
A) 195.66 N B) 194.66 N C) 194.32 N D) 195.32 N

3. Two identical vessels A and B contain one mole of O_2 each. The pressure in vessel A is 'P_0' and that in B is 'P'. The r.m.s velocity of molecules in both the vessels are equal. The vessel A is at rest and B is moving with constant speed V_0, then which of the following is correct.



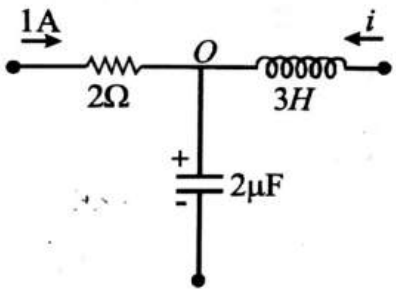
A) $P > P_0$ B) $P < P_0$ C) $P = P_0$ D) $P = P_0\left[\frac{V_{rms}^2 + V_0^2}{V_{rms}^2}\right]$

4. In the given circuit, the potential difference across the capacitor in steady state is 12V. Each resistance is of 3Ω. The cell is ideal. The emf of the cell as



A) 15V B) 9V C) 12V D) 24V

5. The potential difference (V) across the $2\mu F$ capacitor increases with time, and $\frac{dV}{dt} = 1V/s$ and $\frac{d^2V}{dt^2} = 2V/s^2$ at particular instant. The potential difference across the 3H inductor is___(Assume current through resistor is constant)



A) $4\mu V$ B) $12\mu V$ C) $6\mu V$ D) $8\mu V$

6. When voltage $v_s = 200\sqrt{2}\sin\left(\omega t + 15^0\right)$ is applied to an AC circuit the current in the circuit is found to be $i = 2\sin\left(\omega t + \frac{\pi}{4}\right)$ then average power consumed in the circuit is

A) 200 watt B) $400\sqrt{2}$ watt C) $100\sqrt{6}$ watt D) $200\sqrt{2}$ watt

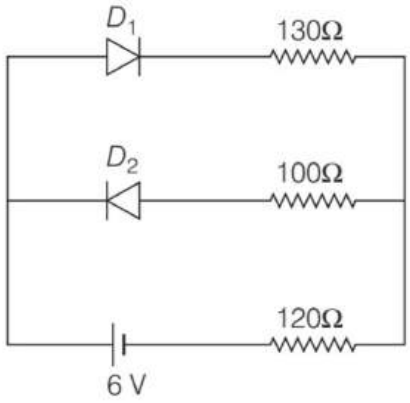
7. In the young's double-slit experiment, when a glass – plate (refractive index 1.5) of thickness t is introduced in the path of one of the interfering beams (wavelength λ), the intensity at the position where the central maximum occurred previously remains unchanged. The minimum thickness of the glass-plate is

A) 2λ B) $\frac{2\lambda}{3}$ C) $\frac{\lambda}{3}$ D) λ

8. A sonometer wire resonates with a given tuning fork forming standing waves with five antinodes between the two bridges when a mass of 9kg is suspended from the wire. When this mass is replaced by mass M, the wire resonates with the same tuning fork forming three antinodes for the same positions of the bridges. The value of M is

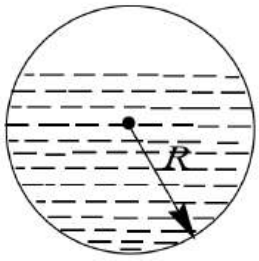
A) 25 kg B) 5 kg C) 12.5 kg D) 1/25 kg

9. The circuit contains two diodes each with a forward resistance of 50Ω and with infinite reverse resistance. If the battery voltage is 6V, the current through the 120Ω resistance is…..mA



A) 5 B) 10 C) 15 D) 20

10. A spherical glass vessel filled with liquid is kept in uniform gravity. Horizontal surface represents meniscus of liquid. Now complete system is taken to gravity free space. C is the center of sphere.



A) Vessel and liquid is wetting combination

B) Finally Liquid forms a drop

C) No effect on the liquid system

D) Finally liquid spreads over entire surface of vessel

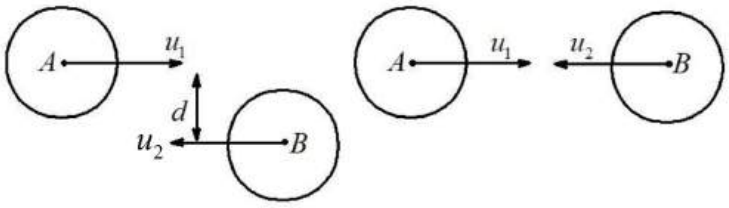
11. A plane electromagnetic wave of frequency 500 MHz is travelling in vacuum along y-direction. At a particular point in space and time, $B = 8.0\times10^{-8}\hat{z}T$. The value of electric field at this point is (speed of light $= 3\times10^{8} ms^{-1}$ $\hat{x},\hat{y},\hat{z}$ are unit vectors along x, y and z-direction)

A) $-24\hat{x}\,\frac{V}{m}$ B) $2.6\hat{x}\,\frac{V}{m}$ C) $24\hat{x}\,\frac{V}{m}$ D) $-2.6\hat{x}\,\frac{V}{m}$

12. A particle is taken from point A to point B under the influence of a force field. Now it is taken back from B to A and it is observed that the work done in taking the particle from A to B is not equal to the work done in taking it from B to A. If W_{nc} and W_{c} is the work done by non – conservative forces and conservative forces present in the system respectively, ΔU is the change in potential energy, Δk is the change in kinetic energy, then choose the correct option

A) $W_C - \Delta U = \Delta K$ B) $W_C = +\Delta U$ C) $W_{nc} + W_c = \Delta K$ D) $W_{nc} - \Delta U = -\Delta K$

13. The following set of figures show two cases of collision between two balls. Let $\vec{u}_1$ and $\vec{u}_2$ be the velocities before collision and $\vec{v}_1, \vec{v}_2$ be the velocities after collision.



Case – I Case - II

The coefficient of restitution, $e = \frac{|\vec{v}_2 - \vec{v}_1|}{|\vec{u}_1 - \vec{u}_2|}$

Student-A: Equation for 'e' holds for both cases as e is property of material of the colliding bodies.

Student-B: Equation for 'e' holds for case II only.

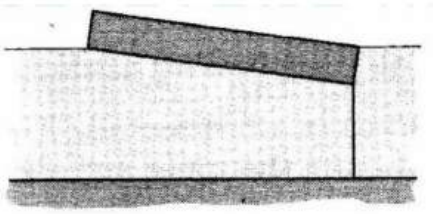
A) Student A is incorrect, student B is correct

B) Student A is correct, student B is incorrect

C) Both are incorrect

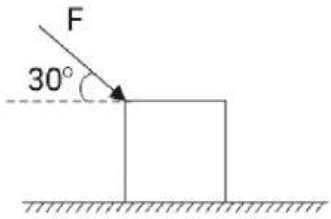
D) Both are correct

14. A uniform wooden plank floating on water is tied to the bottom of the pool such that in static equilibrium the diagonal plane of symmetry coincides with water surface, as shown in figure. The specific gravity of the wood is_____



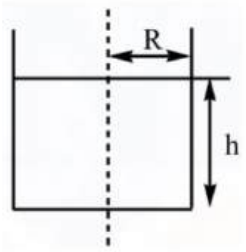
A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) $\frac{2}{3}$ D) $\frac{3}{4}$

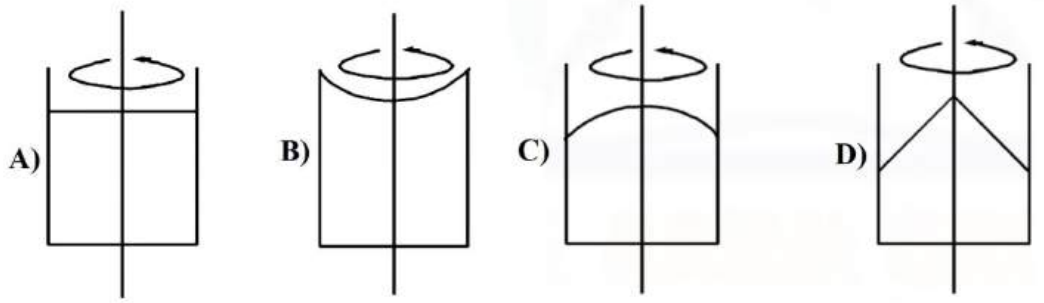
15. A body of mass 10 kg placed on a rough surface is pushed by force F making an angle of 30^0 to the horizontal. If the angle of repose (between the block and the surface) is also 30^0 , then the magnitude of minimum force F required to move the body is equal to ___ ($g = 10\ m/s^2$)



A) 100 N B) $50\sqrt{2}N$ C) $100\sqrt{2}N$ D) 50 N

16. A container is filled partially with a non-viscous fluid of density 'ρ' upto a height 'h'. Initially, the entire system is at rest. Now, the container starts rotating with a constant angular velocity ω_o about its vertical axis. Choose the appropriate plot from below which depicts the shape of free surface of the fluid after a long time.





17. One mole of an ideal monoatomic gas is expanded till the temperature of gas is doubled under the process $TV^2 = \text{const}$ (T is temperature and V is volume of gas). The initial temperature of gas is 400 K, the total work done in the process is

A) −400R B) −600R C) −300R D) −200R

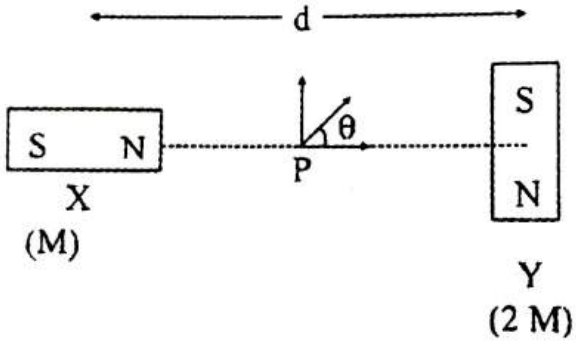
18. In a reactor, 2 kg of $_{92}U^{235}$ fuel is fully used up in 30 days. The energy released per fission is 200 MeV. Given that the Avogadro number, $N = 6.023 \times 10^{26}$ per kilo mole and $1eV = 1.6 \times 10^{-19} J$. The power output of the reactor is close to

A) 125 MW B) 35 MW C) 63 MW D) 54 MW

19. A closed organ pipe has a fundamental frequency of 1.5 kHz. The number of overtones that can be distinctly heard by a person with this organ pipe will be. (Assume that the highest frequency a person can hear is 20 kHz)

A) 7 B) 5 C) 6 D) 4

20. Two magnetic dipoles X and Y are placed at a separation d, with their axes perpendicular to each other. The dipole moment of Y is twice that of X. A particle of charge q is passing through their midpoint P, at angle $\theta = 45^0$ with the horizontal line, as shown in figure. What would be the magnitude of force on the particle at that instant? (d is much larger than the dimensions of the dipole)



A) $\left(\frac{\mu_0}{4\pi}\right)\frac{M}{\left(\frac{d}{2}\right)^3} \times qv$ B) 0 C) $\sqrt{2}\left(\frac{\mu_0}{4\pi}\right)\frac{M}{\left(\frac{d}{2}\right)} \times qv$ D) $\left(\frac{\mu_0}{4\pi}\right)\frac{2M}{\left(\frac{d}{2}\right)^3} \times qv$

SECTION-II
(NUMERICAL VALUE ANSWER TYPE)

This section contains 5 questions. The answer to each question is a Numerical value. If the Answer in the decimals , **Mark nearest Integer only.**
Marking scheme: +4 for correct answer, -1 in all other cases.

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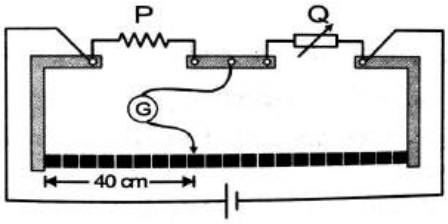
21. A stone is dropped from the top of a building. When it crosses a point 5m below the top, another stone starts to fall from a point 25 m below the top. Both stones reach the bottom of building simultaneously. The height of the building is (in meters) $\left(g = 10\,m/s^2\right)$

22. A certain mass of a solid exists at its melting temperature of 20^0C. When a heat Q is added $\frac{4}{5}$ of the material melts. When an additional Q amount of heat is added the material transforms to its liquid state at 50^0C. Find the ratio of specific latent heat of fusion (in J/g) to the specific heat capacity of the liquid (in $J\,g^{-1}\,{}^0C^{-1}$) for the material

23. A positive point charge +q is placed at the origin. There is an electric field $E_{(x)} = E_0\left(2\frac{x}{d} + 3\frac{x^2}{d^2}\right)$, that accelerates the point charge along the x-axis. The kinetic energy of the charge when it reaches the position x = 2d is $XqdE_0$ then find the value of X.

24. In a meter bridge, gaps are closed by two resistances P and Q and the balance point is obtained at 40cm. When Q is shunted by a resistance of 10Ω, the balance point shifts to 50 cm. Find the value of Q? in Ω



25. A particle of charge q and mass m starts moving from the origin under the action of an electric field $\vec{E} = E_0\hat{i}$ and magnetic field $\vec{B} = B_0\hat{i}$ with a velocity $\vec{v} = v_0\hat{j}$. The speed of the particle will become $2v_0$ after a time $t = \frac{\sqrt{P}mV_0}{qE_0}$ find the value P.

CHEMISTRY **MAX.MARKS: 100**

SECTION – I
(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which **ONLY ONE** option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and -1 if not correct.

26. Read the following Statements

1) First IP of Mg is less than that of Al

2) O^+ has lower electronegativity than that of O^-

3) Electron affinity of Cl is lesser than that of F

4) Second IP of oxygen is greater than that of N

Among the following, the correct option is

A) All are correct B) 1 only correct C) 1 and 2 only correct D) only 4 is correct

27. The correct statement regarding ClO_n^- molecular ion is:

A) On decreasing value of 'n', 'Cl – O' bond order increases

B) On increasing value of 'n', 'Cl – O' bond length increases

C) On increasing value of 'n', oxidation number of central atom increase

D) On increasing value of 'n', hybrid orbitals on central atom increase

28. Which one of the following is a correct statement?

A) Cr^{2+} is oxidising and Mn^{3+} is reducing when both have d^4 configuration

B) Many copper(I) compounds are unstable in aqueous solution and undergo disproportionation as $2Cu^+ \rightarrow Cu^{2+} + Cu$

C) Oxidation state of iron in Fe_3O_4 is fractional.

D) Actinoid contraction is lesser from element to element than lanthanoid contraction

29. Increasing value of magnetic moment of following species is

$I.\left[Fe(CN)_6\right]^{4-}$ $II.\left[Fe(CN)_6\right]^{3-}$ $III.\left[Cr(NH_3)_6\right]^{3+}$ $IV.\left[Ni(H_2O)_6\right]^{2+}$

A) $I < II < III < IV$ B) $IV < III < II < I$ C) $II < III < I < IV$ D) $I < II < IV < III$

30. The longest CO bond length will be with

A) $[Mn(CO)_6]^+$ B) $[V(CO)_6]^-$ C) $Cr(CO)_6$ D) $[Ti(CO)_6]^{2-}$

31. Assertion (A) : Boron always forms Covalent bond.

Reason (R) : The small size of B^{3+} favours formation of covalent bond

1. Both A and R are true and R is the correct explanation of A
2. Both A and R are true but R is not the correct explanation of A
3. A is true but R is false
4. A is false but R is true

32. Match the Column – I with Column-II

COLUMN - I		**COLUMN – II**	
A	NH_4Cl	P	Covalent bond
B	HNC	Q	Ionic bond
C	Liquid H_2O_2	R	Hydrogen bond
D	$CuSO_4.5H_2O$	S	Co-ordinate bond

A) $A \to p,q,s; B \to p,s; C \to p,r; D \to p,q,r,s$

B) $A \to p,r,s; B \to p,q; C \to q,r; D \to p,q,r,s$

C) $A \to p,r,s; B \to p,q; C \to q; D \to r,s$

D) $A \to p,r,s; B \to p,q,s; C \to q,s; D \to q,r,s$

33. Match **List – I** with **List – II** and select the correct answer using the code given below lists

	List – I		**List – II**
(P)	Eq.wt. $= \frac{\text{Molecular weight}}{33}$	(1)	When CrI_3 oxidizes into $Cr_2O_7^{2-}$ and IO_4^-
(Q	Eq.wt. $= \frac{\text{Molecular weight}}{27}$	(2)	When $Fe(SCN)_2$ oxidizes into Fe^{3+}, SO_4^{2-}, CO_3^{2-} and NO_3^-
(R)	Eq.wt. $= \frac{\text{Molecular weight}}{28}$	(3)	When NH_4SCN oxidizes into SO_4^{2-}, CO_3^{2-} and NO_3^-
(S)	Eq.wt. $= \frac{\text{Molecular weight}}{24}$	(4)	When As_2S_3 oxidizes into AsO_3^- and SO_4^{2-}

A) $P \to 1; Q \to 2; R \to 4; S \to 3$

B) P → 2; Q → 1; R → 3; S → 4

C) P → 2; Q → 1; R → 4; S → 3

D) P → 1; Q → 2; R → 3; S → 4

34. Based on equation $E = -2.178 \times 10^{-18} J \left(\frac{Z^2}{n^2} \right)$, certain conclusions are written. Which of them is not correct?

A) The negative sign in equation simply means that the energy of electron bounded to the nucleus is lower than it would be if the electrons were at the infinite distance from the nucleus.

B) Larger the value of n, the larger is the orbit radius.

C) Equation can be used to calculate the change in energy when the electron changes orbit.

D) For n = 1, the electron has a more energy than it does for n = 6

35. For the reaction: $2A(g) + B(g) \longrightarrow 2D(g)$

$\Delta U^{\circ}_{298} = -2.5 kcal$ and $\Delta S^{\circ}_{298} = -10.5 cal/K$. Calculate approximate ΔG°_{298} for the reaction, and predict whether the reaction may occur spontaneously.

A) 0.061 k cal, spontaneous

B) -0.033 k cal, spontaneous

C) 0.061 k cal, non spontaneous

D) 0.033 k cal, non-spontaneous

36. The equivalent conductivities of K^+, Al^{+3} and SO_4^{-2} ions x,y and Z $S\ cm^2 Eq^{-1}$ respectively. The $\Lambda^0 eq$ for $K_2SO_4\ Al_2(SO_4)_3.24H_2O$ (Potash Alum)

A) $\left(\frac{x}{4} + \frac{3y}{4} + z \right) Scm^2 Eq^{-1}$

B) $x + 3y + z\ Scm^2 Eq^{-1}$

C) $\frac{x+y+z}{8}\ Scm^2 Eq^{-1}$

D) $2x + 3y + 4z\ Scm^2 Eq^{-1}$

37. Consider the following carbocations.

I. $C_6H_5\overset{+}{C}H_2$ II. $C_6H_5CH_2\overset{+}{C}H_2$ III. $C_6H_5\overset{+}{C}HCH_3$ IV. $C_6H_5\overset{+}{C}(CH_3)_2$

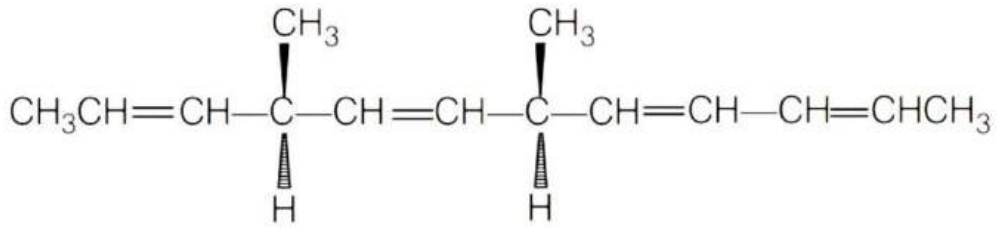
The correct sequence of the stability of these carbocation is

A) $II < I < III < IV$ B) $II < III < I < IV$ C) $III < I < II < IV$ D) $IV < III < I < II$

38. A mixture of ethyl iodide and n-propyl iodide is subjected to Wurtz reaction. The hydrocarbon which will not be formed is (exclude side reaction products)

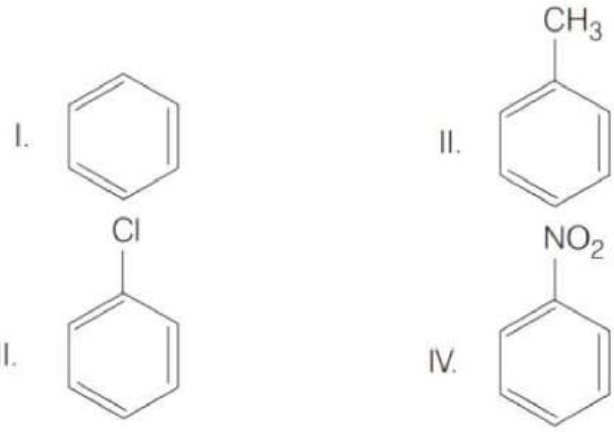
A) Butane B) Propane C) Pentane D) Hexane

39. The number of optically active products obtained from the complete ozonolysis of the given compound is



A) 0 B) 1 C) 2 D) 4

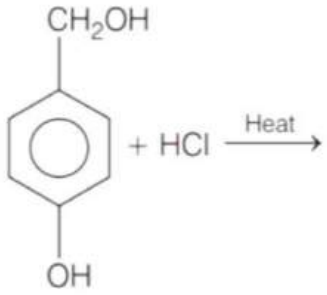
40.

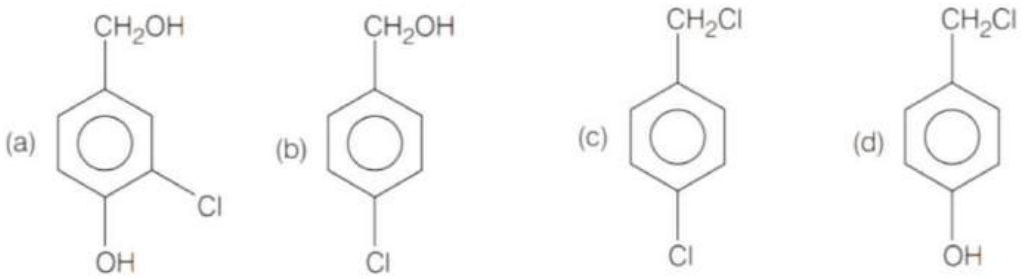


From the above compounds correct order of reactivity in electrophilic aromatic substitution reactions will be

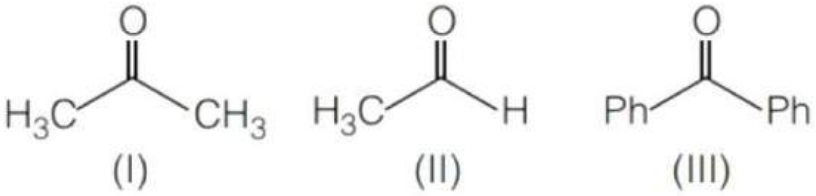
A) *II > I > III > IV* B) *IV > III > II > I* C) *I > II > III > IV* D) *II > III > I > IV*

41. The major product in the following reaction.





42. The order of reactivity of phenyl magnesium bromide with the following compounds is



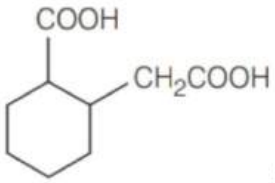
A) *II > III > I*

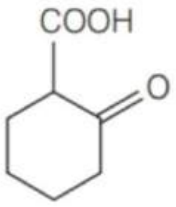
B) *I > III > II*

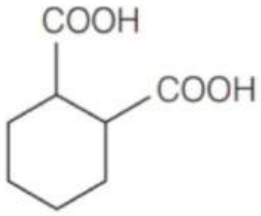
C) *II > I > III*

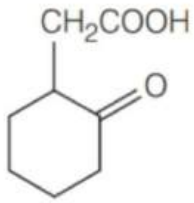
D) All react with the same rate

43. The compound that undergoes decarboxylation most readily just on heating is…

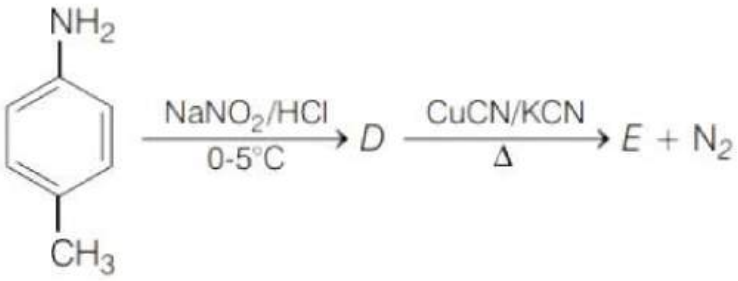
A) 

B) 

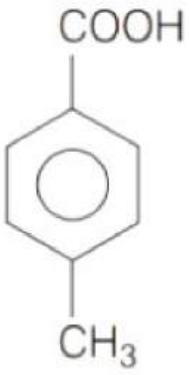
C) 

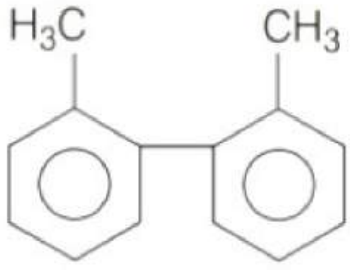
D) 

44. In the reaction,

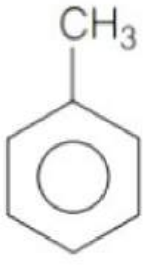


The product E is

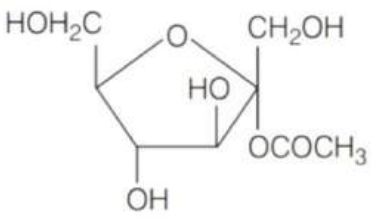
A) 

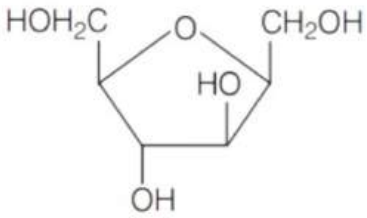
B) 

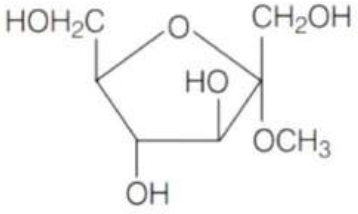
C) 

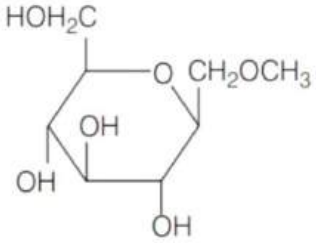
D) 

45. Which of the following compounds will behave as a reducing sugar in an aqueous KOH solution?

A) 

B) 

C) 

D) 

SECTION-II
(NUMERICAL VALUE ANSWER TYPE)

This section contains 5 questions. The answer to each question is a Numerical value. If the Answer in the decimals , **Mark nearest Integer only.**
Marking scheme: +4 for correct answer, -1 in all other cases.

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46. How many of the following reagents convert isopentyl alcohol into alkyl halide without any rearrangement?

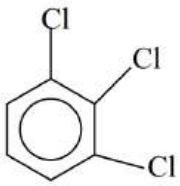
a) $HCl, ZnCl_2, \Delta$ b) $SOCl_2$ c) PCl_5 d) PCl_3 e) PBr_3 f) HI

47. How many of the following substances can act both as oxidizing and reducing agents

H_3PO_2, H_3PO_3, H_3PO_4, HNO_2, SO_2, NO, N_2O_3, NO_2, SeO_2, TeO_2

48. How many of the following molecules are non-polar?

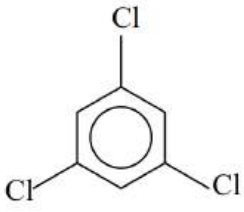
i) 

ii) 

iii) 

iv) 

v) $Cl-CH_2-Cl$

vi) 

vii) $HO-CH_2-CH_2-OH$

49. What is the molecular weight of the naturally occurring optically inactive α – amino acid?

50. The number of revolutions made by an electron in one second in H – atom 2^{nd} orbit is eight times of numbers of revolution made by electron in one second in n^{th} orbit of H – atom, then n is …….

MATHEMATICS **MAX.MARKS: 100**

SECTION – I
(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which **ONLY ONE** option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and -1 if not correct.

51. Let $I=\int_{\pi/4}^{\pi/3}\left(\frac{8\sin x-\sin 2x}{x}\right)dx$. Then

A) $\frac{2\pi}{3}<I<\frac{3\pi}{4}$ B) $\frac{\pi}{5}<I<\frac{5\pi}{12}$ C) $\frac{5\pi}{12}<I<\frac{2}{3}\pi$ D) $\frac{3\pi}{4}<I<\pi$

52. General solution of $x^2ydx=\left(x^3+y^3\right)dy,$ is (where c being arbitrary constant)

A) $\frac{x^2}{2y^2}=ln\left|x\right|+c$ B) $\frac{x^2}{2y^2}=ln\left|y\right|+c$ C) $\frac{x^3}{3y^3}=ln\left|y\right|+c$ D) $\frac{x}{y}-2ln\left|x\right|+3ln\left|y\right|=c$

53. The set of values of k $\left(k\in R\right)$ such that det $\left(adj\left(adjA\right)\right)$=16, where $A=\begin{bmatrix}k & 1 & 2\\ 0 & -1 & 1\\ 4 & 1 & 1\end{bmatrix}$, is equal to

A) $\{5,7\}$ B) $\{2,7\}$ C) $\{5,2\}$ D) $\{5,3\}$

54. Let $f\left(x\right)=3x^{10}-7x^8+5x^6-21x^3+3x^2-7,$ then the value of $\lim_{h\to 0}\frac{f\left(1-h\right)-f\left(1\right)}{h^3+3h}$ is equal to

A) $\frac{-53}{3}$ B) $\frac{22}{3}$ C) $\frac{-22}{3}$ D) $\frac{53}{3}$

55. The complex number z which satisfies the equations $\left|z\right|=1$ *and* $\left|\frac{z-\sqrt{2}\left(1+i\right)}{z}\right|=1$ is

A) 1 B) 1+i C) $\frac{1+i}{\sqrt{2}}$ D) $\frac{-1-i}{\sqrt{2}}$

56. Let A and B be two sets each containing three elements then number of subsets of $A\times B$, each having at least two and at most 7 elements, is equal to

A) 502 B) 492 C) 456 D) 1002

57. If $\vec{V_1}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{V_2}=a\hat{i}+b\hat{j}+c\hat{k}$ where $a,b,c\in\{-2,-1,0,1,2\}$, then number of possible non-zero vectors $\vec{V_2}$ such that $\vec{V_2}$ is perpendicular to $\vec{V_1}$ is

A) 10 B) 13 C) 15 D) 18

58. Let $y = y(x)$ be the solution curve of the differential equation $\sin\left(2x^2\right)\log_e\left(\tan x^2\right)dy + \left(4xy - 4\sqrt{2}x\sin\left(x^2 - \frac{\pi}{4}\right)\right)dx = 0, 0 < x < \sqrt{\frac{\pi}{2}}$, which passes through the point $\left(\sqrt{\frac{\pi}{6}}, 1\right)$. Then $\left|y\left(\sqrt{\frac{\pi}{3}}\right)\right|$ is equal to

A) 1 B) $\sqrt{3}$ C) 2 D) 3

59. If $f(x)$ is a differentiable function and satisfies $f(x+y) = f(x).\ f(y) \forall x, y \in R$ and $f(1) = 2$, then area enclosed by $3|x| + 2|y| \leq 8$ is (in sq.units)

A) $f(4)$ B) $\frac{1}{2}f(6)$ C) $\frac{1}{3}f(6)$ D) $\frac{1}{3}f(5)$

60. If roots of the equation $x^2 + ax + b = 0$ are 'c' and 'd' then one of the roots of the equation $x^2 + (2c + a)x + \left(c^2 + ac + b\right) = 0$ is always equal to $(c \neq d)$

A) c B) $d - c$ C) $2c$ D) $2d$

61. If $A_1, A_2, A_3 \ldots\ldots$ are in A.P then $\sum_{i=1}^{2n}(-1)^i\left(\frac{A_i + A_{i+1}}{A_i - A_{i+1}}\right)$ is equal to

A) 2n-1 B) n-1 C) -2n D) n+2

62. Let e be the eccentricity of hyperbola and f(e) be the eccentricity of its conjugate hyperbola then $\int\left(f(e) + f\left(f(e)\right)\right)de = g(e)$ and $g(1) = \frac{1}{2}$, then $g(e) =$

A) $\frac{1}{2}\sqrt{e^2 - 1} + \frac{e^2}{2}$ B) $\frac{1}{2}\sqrt{e^2 + 1} + \frac{e^2}{2}$ C) $\sqrt{e^2 - 1} + \frac{e^2}{2}$ D) $\sqrt{e^2 + 1} + \frac{e^2}{2}$

63. Nine balls of the same size and colour, numbered $1, 2, \ldots.9$, were put into a packet. Now A draws a ball from packet, noted that it is of number a, and puts it back. Then B also draws a ball from the pocket and noted that it is of number b. Then probability for the inequality $a - 2b + 10 > 0$ to hold is

A) $\frac{52}{81}$ B) $\frac{59}{81}$ C) $\frac{60}{81}$ D) $\frac{61}{81}$

64. A scientist is weighing each of 30 fishes. Their mean weight worked out is 30 gm and a standard deviation of 2 gm. Later it was found that the measuring scale was misaligned and always under reported every fish weight by 2 gms (2gms less than the original weight of the each fish). The ratio of correct mean and standard deviation (in gm) of fishes are respectively

A) 16 B) 18 C) 22 D) 16.5

65. Let $z_1, z_2, z_3 \in C$ such that $|z_1| = 1, |z_2 - 1 + \sqrt{3}i| = 3$ and $|z_3 - 3 - 3\sqrt{3}i| = 7$. If $|3z_1 - 2z_2 - 4z_3|$ is maximum then the value of $|z_3 - z_1| + \frac{z_3 + 3z_1}{z_2}$ is equal to

A) 8 B) 12 C) 16 D) 18

66. The number of equivalence relations in set A = {a, b, c, d} is equal to

A) 3 B) 8 C) 12 D) 15

67. Each of three identical jewellery boxes has two drawers. In each drawer of the first box there is a gold watch. In each drawer of the second box there is a silver watch. In one drawer of the third box there is a gold watch while in the other there is a silver watch. The probability of selecting any drawer after selecting a box is half. If we select a box at random, open one of the drawers and find it to contain a silver watch, then the probability that the other drawer has the gold watch in it, is

A) $\frac{1}{3}$ B) $\frac{2}{3}$ C) $\frac{1}{2}$ D) $\frac{1}{4}$

68. Let ABC be a triangle with vertices at points $A(2,3,5), B(-1,3,2)$ and $C(\lambda,5,\mu)$ in three dimensional space. If the median through A is equally inclined with the axes, then (λ,μ) is equal to

A) $(7,5)$ B) $(10,7)$ C) $(5,7)$ D) $(7,10)$

69. If the function $f(x) = axe^{bx^2}$ have the local maximum value 1 at $x = 2$, then

A) $a = \frac{\sqrt{e}}{2}$ and $b = \frac{-1}{8}$ B) $a = \frac{-\sqrt{e}}{2}$ and $b = \frac{1}{8}$

C) $a = \frac{1}{8}$ and $b = \frac{\sqrt{e}}{2}$ D) $a = \frac{-\sqrt{e}}{2}$ and $b = \frac{-1}{8}$

70. Water is pouring into a conical vessel with vertex upwards and closed base at the rate of 3 cubic meters per minute through a tiny hole at the vertex. The radius of the base is 5m and the height is 10m. When the level is 7m from the base, the rate at which water level increases is (in m/min)

A) $\frac{18}{7\pi}$ B) $\frac{9\pi}{7}$ C) $\frac{4}{3\pi}$ D) $\frac{9}{7\pi}$

SECTION-II
(NUMERICAL VALUE ANSWER TYPE)

This section contains 5 questions. The answer to each question is a Numerical value. If the Answer in the decimals , **Mark nearest Integer only.**
Marking scheme: +4 for correct answer, -1 in all other cases.

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71. The value of definite integral $\int_1^{\infty} \left(e^{x+1} + e^{3-x}\right)^{-1} dx$ is $\frac{\pi^a}{be^c} (a,b,c \in N)$ then the value of $a+b+c$ is

72. If $\underset{x\to\infty}{Lim}\dfrac{\int_0^x\left(\tan^{-1}t\right)^2dt}{\sqrt{1+x^2}}=\dfrac{\pi^2}{k}$, then k is

73. If $A=\int_1^{\sin\theta}\dfrac{t}{1+t^2}dt, B=\int_1^{\operatorname{cosec}\theta}\dfrac{1}{t\left(1+t^2\right)}dt, then\begin{vmatrix} A & A^2 & B \\ e^{A+B} & B^2 & -1 \\ 1 & A^2+B^2 & -1 \end{vmatrix}$ equals

74. The number of values of $x\in[0,n\pi], n\in I$ (the set of integers) that satisfy $\log_{|\sin x|}(1+\cos x)=2$ is_____

75. For a biased die the probabilities for the different faces to turn up are given below

Face	**1**	**2**	**3**	**4**	**5**	**6**
Probability	**0.1**	**0.32**	**0.21**	**0.15**	**0.05**	**K**

This die is tossed and you are told that either face 1 or face 2 has turned up. If the probability that it is face 1, is $\frac{a}{b}$ where a and b $(a,b\in N)$ are coprime to each other then $a+b=$

KEY SHEET

PHYSICS

1	**B**	2	**D**	3	**C**	4	**A**	5	**B**
6	**C**	7	**A**	8	**A**	9	**D**	10	**B**
11	**A**	12	**C**	13	**A**	14	**B**	15	**A**
16	**A**	17	**D**	18	**C**	19	**C**	20	**B**
21	**45**	22	**50**	23	**12**	24	**5**	25	**3**

CHEMISTRY

26	**D**	27	**C**	28	**B**	29	**D**	30	**D**
31	**A**	32	**A**	33	**C**	34	**D**	35	**D**
36	**A**	37	**A**	38	**B**	39	**A**	40	**A**
41	**D**	42	**C**	43	**B**	44	**C**	45	**A**
46	**4**	47	**9**	48	**3**	49	**75**	50	**4**

MATHEMATICS

51	**C**	52	**C**	53	**A**	54	**D**	55	**C**
56	**B**	57	**D**	58	**A**	59	**C**	60	**B**
61	**C**	62	**C**	63	**D**	64	**A**	65	**C**
66	**D**	67	**A**	68	**D**	69	**A**	70	**C**
71	**7**	72	**4**	73	**0**	74	**0**	75	**26**

SOLUTIONS
PHYSICS

1. The given formula of young's modulus of elasticity,

$$Y = \frac{mgL^3}{4bd^3\delta}$$

where, Y = Young's modulus of elasticity,

m = mass of the bar,

L = length of the bar,

b = breadth of the bar,

d = thickness of the bar

and δ = depression of the bar.

There is no error in the value of the g.

The fractional error in the measurement of Y,

$$\frac{\Delta Y}{Y} = \frac{\Delta M}{M} + 3\frac{\Delta L}{L} + \frac{\Delta b}{b} + 3\frac{\Delta d}{d} + \frac{\Delta \delta}{\delta}$$

Substituting the values in the above expression, we get

$$\frac{\Delta Y}{Y} = \frac{10^{-3}}{2} + 3\frac{(1\times 10^{-3})}{1} + \frac{0.1\times 10^{-3}}{4\times 10^{-2}}$$

$$+ 3\frac{(0.01\times 10^{-3})}{0.4\times 10^{-2}} + \frac{(0.01\times 10^{-3})}{5\times 10^{-3}}$$

5. Current through capacitor

$$\mathbf{i}_c = \frac{dq}{dt} = \frac{d}{dt}(CV) = C\left(\frac{dV}{dt}\right)$$

$$\frac{di_c}{dt} = C\left(\frac{d^2V}{d^2t}\right)$$

Current through inductor

$$\mathbf{i}_L = 1 - \mathbf{i}_c$$

$$\left|\frac{di_L}{dt}\right| = \left|-\frac{di_c}{dt}\right|$$

$$\Rightarrow V_L = L\left|\frac{di_L}{dt}\right| = L\left|\frac{di_c}{dt}\right|$$

$$V_L = LC\left|\frac{d^2V}{dt^2}\right| = 3\times 2\times 10^{-6}\,|2|$$

$$V_L = 12\mu V$$

7. According to given condition $(\mu - 1)t = n\lambda$ for minimum $t, n = 1$

So, $(\mu - 1)t_{min} = \lambda$

$$t_{min} = \frac{\lambda}{\mu - 1} = \frac{\lambda}{1.5 - 1} = 2\lambda$$

8. For a vibrating string

$$f = \frac{p}{2l}\sqrt{\frac{T}{\mu}}$$

p= no.of loops or no.of of harmonics

For the given problem.

$p\sqrt{T} = \text{constant or } p_1\sqrt{T_1} = p_2\sqrt{T_2}$

Or $\sqrt{\frac{T_2}{T_1}} = \frac{p_1}{p_2}$ or $\sqrt{\frac{Mg}{9g}} = \frac{5}{3}$ or $\frac{M}{9} = \frac{25}{9}$

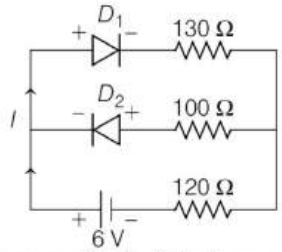
Or M = 25 kg

9. Given, forward resistance, $R_1 = 50\Omega$

Reverse resistance, R_2 = infinity

Battery voltage = 6 V

According to circuit diagram,



In this case, diode D_1 is forward biased whereas diode D_2 is reverse biased.

So, D_2 will act as open circuit.

By using Kirchhoff's voltage law,

$6 - 50I - 130I - 120I = 0$

$\Rightarrow \quad 6 = 300I$

$\Rightarrow \quad I = \frac{6}{300} = \frac{1}{50}$

$= \frac{2}{100} = 0.02\,A$

$= 20\,mA$

Hence, current through $120\,\Omega = 20$ mA

11. Given, frequency, $f = 500MHz = 5\times10^8\,Hz$

$\mathbf{B} = 8.0\times10^{-8}\,\hat{\mathbf{z}}$ T

∴ Magnitude of peak value of magnetic field is given by

$B_0 = 8\times10^{-8}$ T

We know that, $\frac{E_0}{B_0} = c$

where, E_0 is the magnitude of peak value of electric field and c is the speed of electromagnetic wave in air (or vacuum).

$\Rightarrow \quad E_0 = cB_0$

$= 3\times10^8\times8\times10^{-8} = 24\,V/m$

Since, the direction of propagation of electromagnetic wave is perpendicular to the direction of E and B both.

∴ Direction of propagation is given by $\hat{\mathbf{E}}\times\hat{\mathbf{B}}$.

As, the wave is travelling in y-direction,
and the magnetic field is in z-direction.

$\Rightarrow \quad \hat{E} \times \hat{z} = \hat{y}$

$\hat{E} = -\hat{x}$

$\therefore$ The value of electric field will be $-24\hat{x}$ V/m.

14. $mg \times \frac{h}{2} = F_b \times \frac{h}{3}$ (balance torque about the edge)

18. No of reactions $= \frac{2}{235} \times 6.023 \times 10^{26}$

Power output $= \frac{2 \times 6.023 \times 10^{26} \times 200 \times 10^6 \times 1.6 \times 10^{19}}{235 \times 30 \times 24 \times 60 \times 60}$

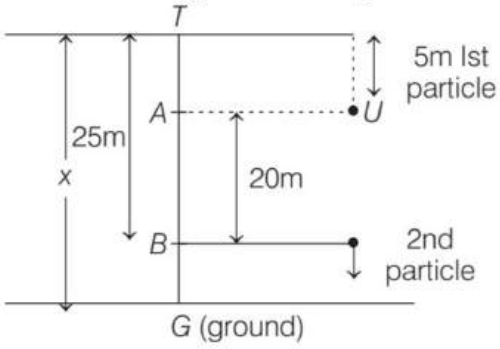
$= 63.2\, MW$

19. For closed organ pipe resonating frequency is odd multiple of fundamental frequency

$(2n+1) \times 1.5 < 20$

$\therefore$ number overtones heard = 7

21. Let the total height of building be x.



23. $V = E_0 \left[\int_0^{2d} \frac{2x}{d} dx + \int_0^{2d} \frac{3x^2}{d^2} dx \right] = 12 dE_0 \Rightarrow U = qV = 12 qdE_0$

24. $s\frac{P}{Q} = \frac{2}{3}; \frac{P(10+Q)}{10Q} = \frac{1}{1} \Rightarrow \frac{2}{3} = \frac{10}{10+Q}$

$\Rightarrow 20 + 2Q = 30. \Rightarrow Q = 5\Omega; P = \frac{10}{3}\Omega$

25. Velocity component along x direction after time t is

$v_x = \sqrt{4v_0^2 - v_0^2}$

$v_x = \frac{qE}{m} t$

CHEMISTRY

28. Cr^{2+} is reducing and Mn^{3+} is oxidizing when both have d^4 configuration. Oxidation state of iron in ferrates is +6 $[FeO_4]^{2-}$. Actinoid contraction is greater from element to element than lanthanide contraction,.

29. No. of unpaired electrons in the above species are

$[Fe(CN)_6]^{4-} = 0$ $\quad$ $[Fe(CN)_6]^{3-} = 1$

$[Cr(NH_3)_6]^{3+} = 3$ $\quad$ $[Ni(H_2O)_6]^{2+} = 2$

Strength of ligands CN^- (strong) > NH_3(strong)>H_2O(weak)

30. When electron density is pushed from metal atom into π-bond, the CO bond is weaken as electrons enter into anti bonding orbital of CO. With 2 unit negative charge on metal atom π-back bonding from metal to CO increases maximum electron density.

31. Conceptual

35. $\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ = \Delta E^\circ + \Delta(PV) - T\Delta S^\circ$

Assuming ideal gas behavior,

$$\Delta G^\circ = \Delta E^\circ + (\Delta n)RT - T\Delta S^\circ$$

Using the value of R=1.987 cal mol.K, and the fact that 2 mol of gas (D) is produced from 3 mol (2A+B),

$(\Delta n)RT = (-1\ \text{mol})(1.987\,\text{cal/mol.K})(298\,\text{K}) = -592\,\text{cal}$

$\Delta G^\circ = -2500\ \text{cal} + (-592\,\text{cal}) - (298\,\text{K})(-10.5\,\text{cal/K}) = -3.09\text{kcal} = 0.04\,\text{kcal}$

Since the value of ΔG° is positive , the indicated reaction cannot be spontaneous.

36.

$$\Lambda_m = 2x + 6y + 8z$$

$$\Lambda_{eq} = \frac{2x + 6y + 8z}{8}$$

37. The correct sequece of carbocation is $II < I < III < IV$. Electron releasing stabilizes the carbocation by dispersal of positive charge. More the number of alkyl groups, the greater the dispersal of positive charge and the more stable is the carbocation.

38. Propane will not formed as in wurtz reaction two molecules of each reacting species combine with the removla of NaX.

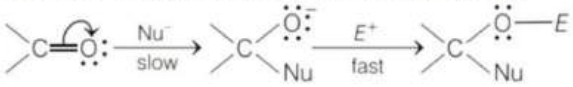
39. None of the ozonolysis products is chiral.

40. Presence of electron releasing roups, (like —*R*) increases the electron density over benzene nucleus and makes it more reactive towards electrophile. Electron withdrawing groups (like Cl^-, NO_2^-; NO_2^- is more electron withdrawing than Cl^-) make the benzene nucleus electrons deficient and hence, decrease its reactivity towards electrophile.

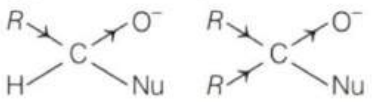
    Hence, the correct order of reactivity towards electrophilic substitution reaction will be II > I > III > IV.

41. Only aliphatic – OH is substituted by Cl^- . This is because in phenol the $C-O$ bond is stabilized by resonance.

42. In phenyl magnesium bromide, phenyl is attached with that C-atom of carbonyl group which has low electron density (higher electropositive charge). In carbonyl compounds, aldehydes are more reactive, towards nucleophile in nucleophilic addition reactions because in ketones alkyl groups is present on both sides (due to +*I* effect) which decreases the electropositive charge of carbon of carbonyl group.

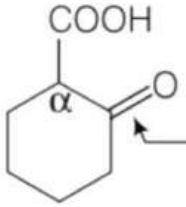


Hence, attraction of nucleophile decreases. Moreover in the tetrahedral intermediate, aldehydes have less steric repulsion than ketones and also the aldehyde increases the negative charge on oxygen less in comparison of ketones.



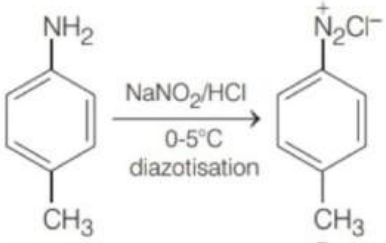
Therefore, the order of reactivity of acetone (I), acetaldehyde (II) and benzophenone (III) with PhMgBr is (II) > (I) > (III).

43. It is $a\beta$ – keto acid which undergoes decarboxylation in very mild condition, i.e. on simple heating. Ordinary carboxylic acid requires sodalime catalyst for decarboxylation (for decarboxylation β-keto acid must contain an α-H atom). Thus,



β, readily undergoes decarboxylation.

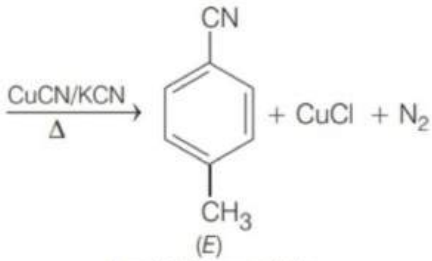
44.



4-methyl aniline

D

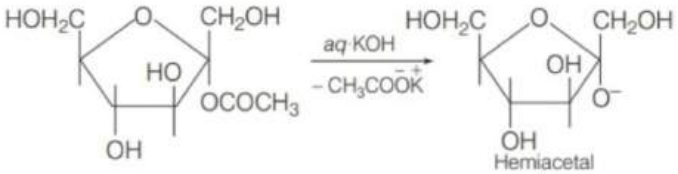
4-methyl diazonium chloride



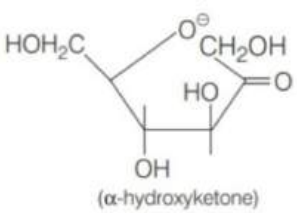
(*E*)

4-methyl benzonitrile

45. Sugars that have free aldehyde, a ketone, a hemiacetal or a hemiketal group is able to reduce an oxidising agent. These sugars are classified as reducing sugars.

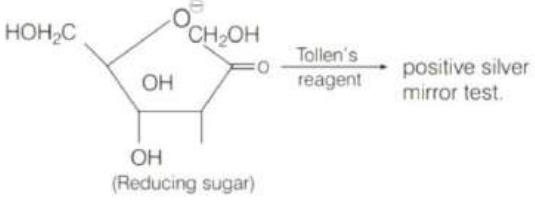


Hemiacetal



(α-hydroxyketone)

Hemiacetal can be easily reduced by oxidising agent such as Tollen's reagent



(Reducing sugar)

46. $SOCl_2, PCl_3, PCl_5, RPBr_3$

47. HNO_2, SO_2, NO, N_2O_3, NO_2, SeO_2, TeO_2 can act both as oxidizing and reducing agents

48. 3 & 7 are non-polar
Note : iii, iv,& vi are non-polar molecules

49. $80 \text{ gm } SO_2 \longrightarrow 18 \text{ gm water}(H_2O)$
$? \longrightarrow 4.5$

50. Number of revolutions per second = $\frac{6.58 \times 10^{15} z^2}{n^3}$

52. $x^2ydx=\left(x^3+y^3\right)dy$

$$\frac{y^2\left(x^2ydx-x^3dy\right)}{y^6}=y^2\frac{y^3dy}{y^6}$$

$$\frac{x^2y^3dx-x^3y^2dy}{y^6}=\frac{1}{y}dy$$

$$\int d\left(\frac{x^3}{3y^3}\right)=\int\frac{1}{y}dy+c$$

$$\frac{x^3}{3y^3}=my+c$$

$$9y^2\left(x^2ydx-x^3dy\right)=3y^5dy$$

$$3x^2ydx-3x^3dy=y^3dy$$

53.

54. $\lim\limits_{h\to0}\dfrac{f(1-h)-f(1)}{h\left(h^2+3\right)}=\dfrac{-f'(1)}{3}$

$f'(x)=30x^9-56x^7+30x^5-63x^2+6x$

$f'(1)=30-56+30-63+6=-53$

$\therefore$ Given limit $=\dfrac{53}{3}$ **Ans.**]

55.

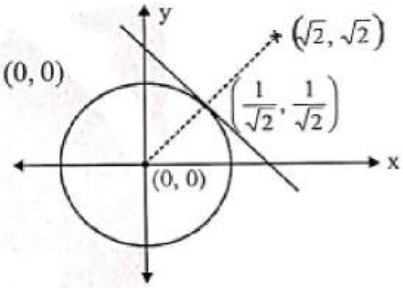
$\because\ \left|\dfrac{z-\sqrt{2}(1+i)}{z}\right|=1\Rightarrow\left|z-\sqrt{2}(1+i)\right|=|z-0|$

$\therefore$ z is on perpendicular bisector of line joining $\left(\sqrt{2},\sqrt{2}\right)$ and $(0,0)$

$\therefore$ $(0,0)$ is centre of circle $|z|=1$

$\therefore$ point z will be $\left(\dfrac{1}{\sqrt{2}},\dfrac{1}{\sqrt{2}}\right)$

$\therefore$ $z=\dfrac{1+i}{\sqrt{2}}$ **Ans.**]



56. No of elements $A\times B=9$

No of subsets of $A\times B$ with at least two & atmost seven elements is

$={}^9C_2+{}^9C_3+{}^9C_4+{}^9C_5+{}^9C_6+{}^9C_7=492$

57. $\overline{V}_1.\overline{V}_2=0$

$a+b+c=0$

$a=2,b=0,c=-2$ 6 ways

$a=1,b=0,c=1$ 6 ways

$a=2,b=-1,c=-1$ 3 ways

$a=-2,b=1,c=1$ 3 ways

18 ways

58. $\because f(x)=x^3+3\left(a^2-1\right)x+1$

$\Rightarrow f'(x)=3x^2+3\left(a^2-1\right)\geq0\forall x\in R$ for $f(x)$ to be invertible

$\therefore\ (a^2-1)\geq0\Rightarrow a\leq-1$ or $a\geq1$ **Ans.**]

59. $f(x+y)=f(x).f(y)$

$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

Get $f(x)=2^x$

Area enclosed $\frac{64}{3}=\frac{f(6)}{3}$

60. $\because$ c is a root of $x^2+ax+b=0 \Rightarrow c^2+ac+b=0$

$\therefore$ Given equation will become $x^2+(2c+a)x=0$

$\therefore$ sum of roots of given equation $= c+d=-a \Rightarrow a=-d-c$

$\therefore\ x^2+(2c+a)x=0 \Rightarrow x^2+(2c-d-c)x=0$

$\Rightarrow x\big(x+(c-d)\big)=0 \Rightarrow x=d-c$ is a root

61. $\because$ Let common difference of A.P. be d.

$\therefore \quad A_1-A_2=A_2-A_3=\ldots\ldots=A_i-A_{i+1}=-d$

$$\text{Given expression}=\frac{-1}{d}\sum_{i=1}^{2n}(-1)^i(A_i+A_{i+1})$$

$$=\frac{-1}{d}\{-(A_1+A_2)+(A_2+A_3)-(A_3+A_4)+(A_4+A_5)+\ldots\ldots\}$$

$$=-\frac{1}{d}\{2d+2d+\ldots\ldots \text{ upto n terms}\}=-2n \quad \textbf{Ans.]}$$

62. Let eccentricity of conjugate hyperbola be e'

$$\therefore\ \frac{1}{e^2}+\frac{1}{e'^2}=1 \Rightarrow \frac{1}{e'^2}=1-\frac{1}{e^2} \Rightarrow e'=\frac{e}{\sqrt{e^2-1}}$$

$$\therefore\ f(e)=\frac{e}{\sqrt{e^2-1}}$$

$$\text{and}\quad f(f(e))=\frac{f(e)}{\sqrt{f^2(e)-1}}=\frac{\frac{e}{\sqrt{e^2-1}}}{\sqrt{\frac{e^2}{e^2-1}-1}}=e$$

$$\therefore \text{Given integral}=\int_1^{\sqrt{2}}\left(\frac{e}{\sqrt{e^2-1}}+e\right)de=\left(\sqrt{e^2-1}+\frac{e^2}{2}\right)_1^{\sqrt{2}}=(1+1)-\left(0+\frac{1}{2}\right)=\frac{3}{2}$$

63. Required probablity $\lambda=\frac{2}{6}\times\frac{3}{7}+\frac{4}{6}\times\frac{4}{8}=\frac{1}{7}+\frac{1}{3}=\frac{10}{21}$

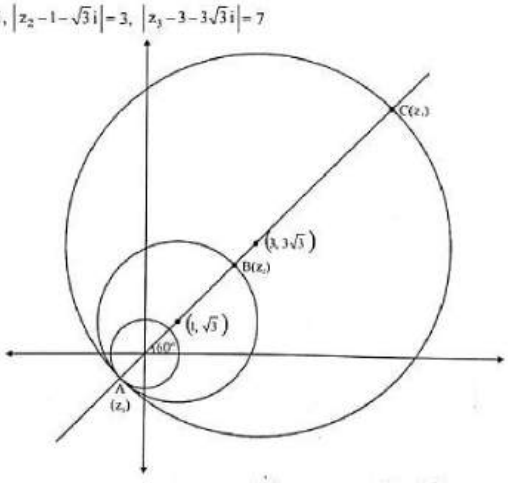
$\therefore 21\lambda=10$

64. There is no change in the standard deviation if each observation is increased by a constant number whilemean is increased by that number

New mean $30+2=32$ and standard deviation =2

65.

$|z_1| = 1,\ |z_2 - 1 - \sqrt{3}i| = 3,\ |z_3 - 3 - 3\sqrt{3}i| = 7$



$$|3z_1 - 2z_2 - 4z_3| = \left|3z_1 - 2\left(z_2 - 1 - \sqrt{3}i\right) - 4\left(z_3 - 3 - 3\sqrt{3}i\right) - 14 - 14\sqrt{3}i\right|$$

$$\leq 3|z_1| + 2\left|z_2 - 1 - \sqrt{3}i\right| + 4\left|z_3 - 3 - 3\sqrt{3}i\right| + 14 \cdot 2$$

$$\leq 3 + 6 + 28 + 28$$

$$\leq 65$$

where equality holds when z_1, z_2, z_3 and origin are collinear and z_1 lies on opposite side of z_2 and z_3.

$\therefore$ Points A, B and C must represent z_1, z_2 and z_3 respectively.

$$|z_3 - z_1| = 14$$

$$z_1 = \frac{-1}{2} - i\frac{\sqrt{3}}{2},\ z_2 = \frac{5}{2} + i\frac{5\sqrt{3}}{2},\ z_3 = \frac{13}{2} + i\frac{13\sqrt{3}}{2}$$

66. $n(A) = 3$

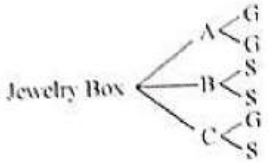
$\therefore$ Total number of relation in set $A = 2^{3\times 3} = 2^9$

and maximum number of elements in cartesian product = 9 out of which 3 ordered pair is necessary for reflexive.

So, for remaining 6 ordered pair

Number of ordered pair required $= {}^6C_0 + {}^6C_1 + {}^6C_2 + \ldots\ldots {}^6C_6 = 2^6$]

67.



A : one box and one of its drawer randomly selected and a silver watch found in it

B_1 : It is the box C; B_2 : It is the box B; B_3 : It is the box A

$$P(B_1) = P(B_2) = P(B_3) = \frac{1}{3}$$

$$P(A/B_1) = \frac{1}{2};\ P(A/B_2) = 1;\ P(A/B_3) = 0;\ P(B_1/A) = \frac{\frac{1}{2}}{\frac{1}{2}+1} = \frac{1}{2}\cdot\frac{2}{3} = \frac{1}{3}$$ Ans.]

68. Centroid is $\bar{M}\left(\frac{1+\lambda}{3},\frac{11}{3},\frac{7+\mu}{3}\right)$

$A(2,3,5)$

DR'S $\overline{AM}\left(\frac{\lambda-5}{3},\frac{2}{3},\frac{\mu-\theta}{3}\right)$

For $\overline{AM}$ to be equally inclined

$\lambda=7,\mu=10$

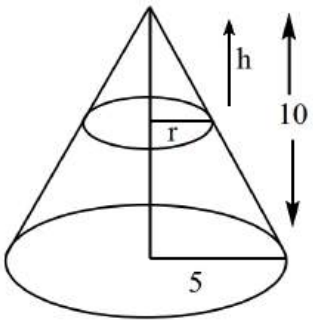
69. $f f(x)=axc^{bx^2};f(2)=1\ f'(2)=0;2ac^{4th}=1\ldots(1)$

also $\quad f'(x)=a[\,xe^{bx^2}\cdot 2bx+e^{bx^2}\,]=axe^{bx^2}[2bx^2+1]$

$f'(2)=ae^{4b}(8b+1)=0;\qquad a=0$ or $b=-1/8$ but $a\neq 0;\ a=e/2$

hence $\quad a=\frac{\sqrt{e}}{2}$ and $b=\frac{-1}{8}.\qquad]$

70.



$\frac{dv}{dt}=3m^3/\min$

$\frac{r}{h}=\frac{5}{10}=\frac{1}{2}$

$r=\frac{h}{2}$

$v=\frac{1}{3}\pi\times\frac{h^2}{4}\times h=\frac{\pi h^3}{12}$

$\frac{dv}{dt}=\frac{3\pi h^2}{12}.\frac{dh}{dt}$

$\frac{dh}{dt}=\frac{4}{3\pi}$

71. $I=\int_1^\infty\frac{1}{\left(e^{x+1}+e^{3-x}\right)}dx=\int_1^\infty\frac{1}{e\left(e^x+e^2.e^{-x}\right)}dx=\int_1^\infty\frac{e^x}{e\left(e^{2x}+e^2\right)}dx$

Let $\quad e^x=t$

$\therefore\qquad I=\frac{1}{e}\int_e^\infty\frac{1}{t^2+e^2}dt=\frac{1}{e^2}\left(\tan^{-1}\frac{t}{e}\right)_e^\infty=\frac{1}{e^2}\left(\frac{\pi}{2}-\frac{\pi}{4}\right)=\frac{\pi}{4e^2}$

72. $$\lim_{x\to\infty}\frac{\int_0^x\left(\tan^{-1}t\right)^2dt}{\sqrt{1+x^2}}$$

$$=\lim_{x\to\infty}\frac{\left(\tan^{-1}x\right)^2}{\frac{2x}{2\sqrt{1+x^2}}}=\lim_{x\to\infty}\frac{\left(\tan^{-1}x\right)^2x\sqrt{1+\frac{1}{x^2}}}{x}$$

$$=\frac{\pi^2}{4}$$

73. $$B=\int_1^{\operatorname{cosec}\theta}\frac{1}{t\left(1+t^2\right)}dt$$

Let $\frac{1}{t}=u \Rightarrow B=\int_1^{\sin\theta}\frac{-u}{1+u^2}du \Rightarrow A+B=0\Rightarrow A=-B$

$$\therefore\begin{vmatrix} A & A^2 & B \\ e^{A+B} & B^2 & -1 \\ 1 & A^2+B^2 & -1 \end{vmatrix}=0.\ \text{Ans.}]$$

74.

Solution The equation is meaningful if $|\sin x| \neq 0, 1$ and $1 + \cos x \neq 0$ so $x \neq k\pi$, $k = 0, 1, \ldots n$, $x \neq (2k + 1)\frac{\pi}{2}$. $k = 0, 1, \ldots n - 1$. Now,

$$\log_{|\sin x|}(1 + \cos x) = 2$$

$$\Leftrightarrow \quad 1 + \cos x = |\sin x|^2 = \sin^2 x = 1 - \cos^2 x$$

$$\Rightarrow \quad (1 + \cos x)(\cos x) = 0$$

$$\Leftrightarrow \quad \cos x = 0 \text{ or } \cos x = -1$$

$$\Rightarrow \quad \cos x = 0 \Rightarrow x = (2k + 1)\pi/2.$$

So there is no x which satisfy the given equation.

75. $P(1)+P(2)----P(6)=1$

$0.1+0.32+0.21+0.15+0.05+k=1$

$P(1or\,2)=0.1+0.32=0.42$

$P(req)=\frac{10}{42}=\frac{5}{21}$