Chapter 3
SPECIAL CURVES

3.1 INTRODUCTION

In several articles of daily use we notice that they have straight edges which may form various rectilinear figures such as triangle, square, rohmbus, parallelogram, trapezium etc. Look around or recall from your past experience such articles and name them. Then we also have articles/objects whose shape is circular, spherical, cylindrical, prisms and pyramids, conical etc.

You have studied about the various orbits of the planets of solar system. What is their shape, called as ? Oval objects are also quite common. Name a few. In Engineering we shall come across many objects/ parts which are combination of these various shapes. We are going to study about these special curves.

3.2 ELLIPSE

One the most common curve in Engineering will be the oval/an egg shape. This we shall call as Ellipse. Let us know about the ellipse in details. Now study the figure (3.1) shown and answer the simple questions by choosing the correct option from the choices given. (Major Axis, Minor Axis, Centre, Semi-Major Axis, Semi-minor axis, Focus.

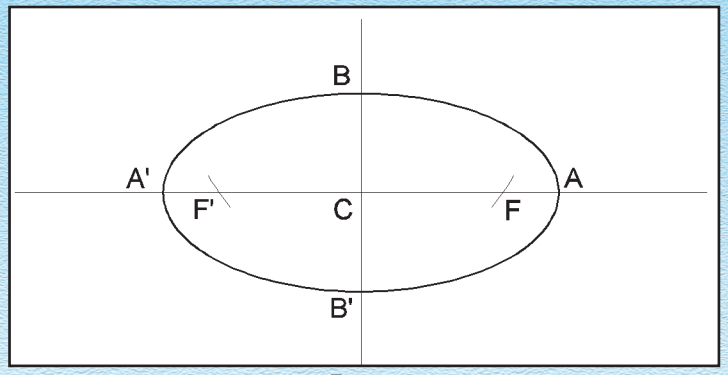

Fig. 3.1

Q1. The point C is the ..... of ellipse.

Q2. Length A–A' is the ..... of ellipse.

Q3. Length B–B' is the ..... of ellipse.

Q4. Length CA = CA' and is called ..... of the ellipse.

Q5. Length CB = CB' and is called ..... of the ellipse.

Q6. The points F and F' each is known as ..... of the ellipse. We call them focii while refering to both.

In engineering we are required to draw the curve ellipse. We shall now learn how to contruct the ellipse by various methods.

3.2.1 CONSTRUCTIONS OF THE ELLIPSE BY VARIOUS METHODS

(i) By the concentric circles method :

Example : Draw an ellipse whose major axis = 80 mm and minor-axis = 50 mm by the concentric circles method.

Solution : Refer Fig. 3.2 (i) Draw a circle of ϕ 80 mm and another circle of ϕ 50 mm. (ii) Divide both the circles in 12 equal parts either by 30° and 60° angles or by compass method of constructing 60° angles. (iii) From divisions (1, 2 .....) of the outer circle draw vertical lines as shown (iv) Similarly from the inner circle divisions draw horizonal to intersect the vertical lines already drawn. (v) The point of intersection of these vertical and horizonal lines will give us (a, b, c, d .....) points on ellipse (vi) Draw a smooth ellipse passing through these points either by free hand or by French curves (It is better to practice free hand drawing of curves.) In good drawing there should not be any kink or dip on the entire curve).

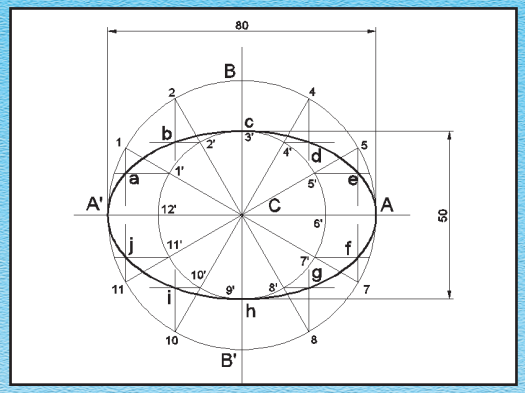

Fig. 3.2

(ii) Intersecting arcs methods :

Example : Draw an ellipse whose semi major axis is 35 mm and semi-minor axis is 20 mm by intersecting arcs method.

Solution : Refer Fig. 3.3 : (i) On a horizontal line mark centre C of the ellipse. Cut CA = CA′ = 35 mm on this line (ii) Draw a perpendicular line through C on AA′ to obtain B and B′ such that CB = CB′ = 20 mm (iii) Now B as centre and radius = semi major axis = 35 mm obtain the focii F and F′ on the major axis A-A′. (iv) Take random points 1, 2, 3, approximately, equidstant between C and F on CA. Now take A′1, A′2 and A′3 as radii and centre as F′ draw arcs above and below line CA (v) Now take A1, A2 and A3 as radii and F as F′ as centre, cut the previous arcs drawn to obtain points of intersection (1, 2, 3, 5, 6 & 7) (vi) Similarly the points are to be obtained on the left. Draw a smooth ellipse through all these points of intersecting arcs.

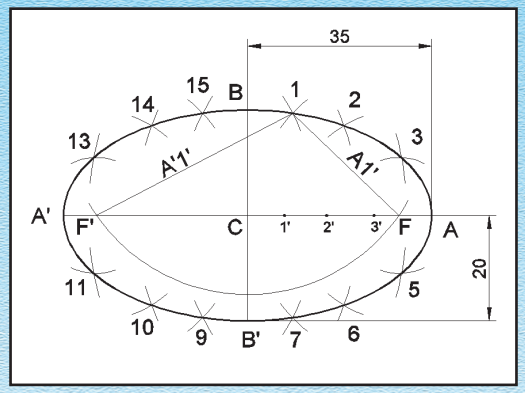

Fig. 3.3

(iii) Intersecting lines method :

Example : Draw an ellipse whose major axis = 60 mm and the minor axis = 40 mm by intersecting lines method.

Solution : Refer Fig. 3.4 : (i) Draw a rectangle (60 × 40) such that AA′ is the major axis and BB′ is the minor axis which are obtained by joining the mid-points of opposite sides of the rectangle. Mark centre as C. (ii) Now divide both the semi-major axes into four equal parts and name the points 1, 2 and 3 (iii) Similarly divide both the sides of rectangle parallel to semi minor axes in four parts, naming them 1′, 2′ and 3′ (iv) Join these points on sides with B and B′ as shown. Further Join1, 2 and 3 with B and B′ as shown (v) Extend these lines to

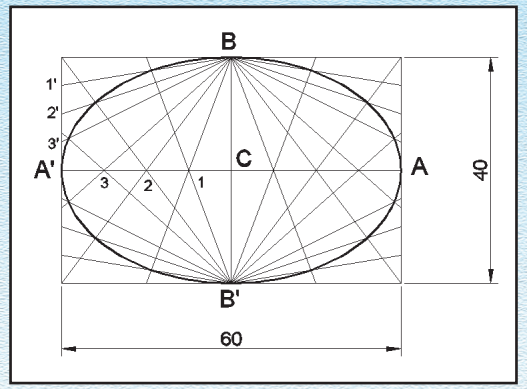

Fig. 3.4

obtain the points of intersections as shown. These points of intersection lie on ellipse. (vi) Similarly join the other points as well in the remaining three quarters (vii) Through all these points draw a smooth curve, preferably by free hand or by French curve.

3.3 PARABOLA

You must have seen the reflectors of headlights of the automobiles and the antennas of the rador and satellite transmission or reception. Can you gueses the shape of these? Yes your guess should be a "Parabola". We shall now be understanding the various parameters of the parabola. Try to fillin the blanks from the following options. (Axis, A, F, LR, ZZ', YY') Refer Fig. 3.7

1. The line about which the parabola is symeterial is XX' and is called.....
2. The vertical conjugate axis is .....
3. The point..... of intersection with the axis is called ..... of the parabola.
4. The line through focus is ..... and is called Latus Rectum.
5. The Focus is point ..... .
6. The directrix is the line left to the conjugate axis is ..... in the given figure.

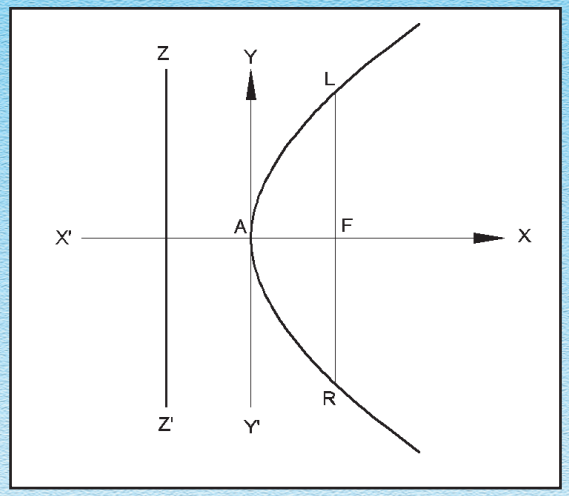

Fig. 3.7

Let us learn the various methods of drawing a parabola.

Construction of Parabola

(a) By Intersecting Lines : Draw the parabola whose width is 70 mm and the depth (height) is 30 mm.

Solution : Refer Fig. 3.5 : (i) Draw a rectangle whose length = 70 mm and the height is 30 mm (ii) Draw the central line VC perpendicular to AB at point C – the centre of AB (iii) Divide the line segments DV, VE, AD and BE each into four equal parts (iv) Name the points on the horizonal lines as 1, 2, 3 and 1', 2', 3' on vertical lines (VE and EB) (v) Join 1', 2' and 3' with V as shown on both sides (vi) From 1, 2 and 3 draw the vertical lines to intersect the lines drawn earlier (v) Match the corresponding points as shown. Mark them as fine dots (vi) Through these points draw a smooth curve. This is the parabola required.

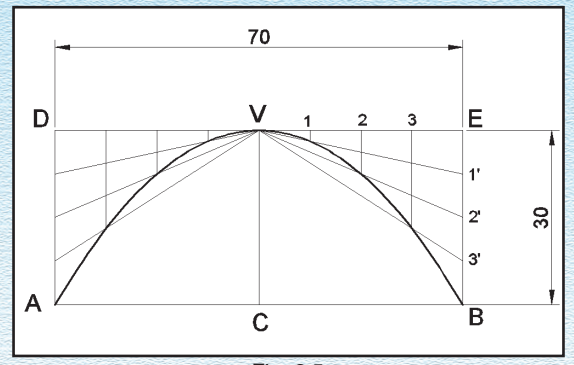

Fig. 3.5

(b) By Intersecting arcs : Draw the parabola when distance between a fixed line (called directrix) and the focus F is given = 25 mm.

Solution : Refer Fig. 3.6 : (i) Draw any vertical line (ZZ′) at any point A (preferebly in middle). This is known as directrix. (ii) On point A draw a perpendicular line AB. (iii) Cut-off AF = 25 mm and bisect it to get point V. (V is called the vertex and F is known as Focus). (iv) Take 1, 2, 3, 4 and 5 points at random on AB, preferably at increasing distances from V (v) Draw vertical lines, above and below of AB through 1, 2, F, 3, 4 and 5 points. (vi) Now take distances A1, A2, AF, A3, A4 and A5 as radii for arcs and "F as centre" and cut arcs on above and below the vertical lines drawn through 1, 2, F, 3, 4, and 5 points as shown to get pair of points (1′1′, 2′2′, 3′3′, 4′4′, 5′5′ and LR) (vii) Draw a smooth curve passing through all these points. This is Parabola.

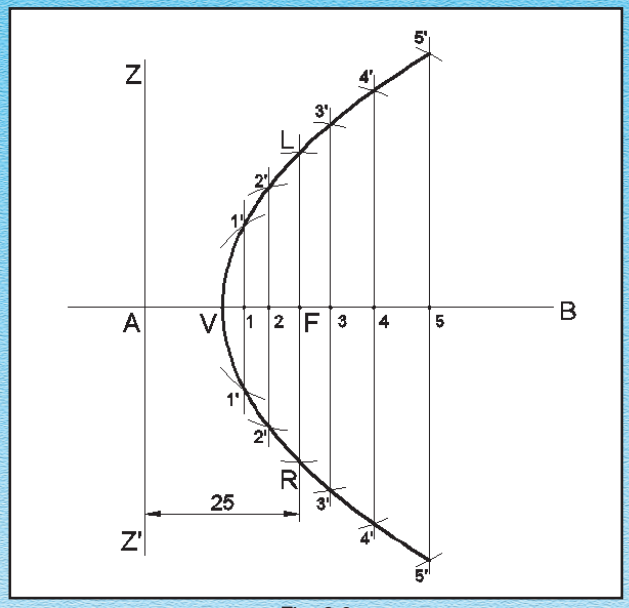

Fig. 3.6

3.4 INVOLUTE

Involute of a circle : You know that the straight line is the shortest distance between two points. So the line can be considered as the movement of the point under some conditions. In the same way if a wheel is taken and is wound tight with a non-elastic string or wire and given "One complete round". Let one end of the string or wire be "stuck" to the wheel while "the loose" end is wrapped round to meet the fixed end. Can you tell how much should be the length of the string? Now unwind the string/wire while the wheel is fixed, keeping also the loose end always tight. The loose (free) end will now trace a curve, very typical and used in Engineering in the teeth of the gears. We call this as Involute of a circle. Let us learn how to draw it ?

3.4.1 DRAWING THE INVOLUTE OF A CIRCLE

Example : Draw the Involute of a circle whose diameter is 20 mm.

Solution : Refer Fig. 3.8 (i) Draw the circle of dia. 20 mm and divide it into twelve equal parts. This is easily done either constructing 30° and 60° angles by compass or by set square|protractor after drawing horizontal and vertical diameters. (ii)Name these points as 1′, 2′, 3′, 4′ .....12′ as shown in

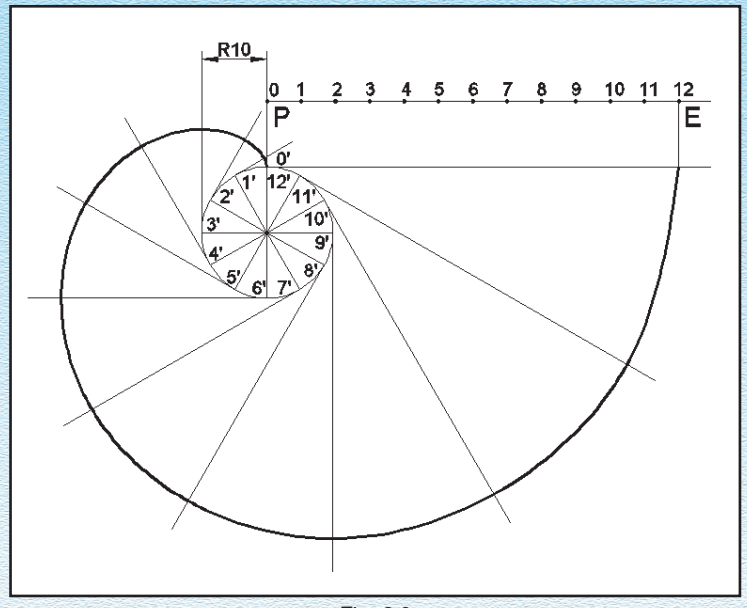

Fig. 3.8

the Fig. (iii) Now on each point 1′, 2′, 3′, ... 12′ construct tangents (iv) On PE parallel to the last tangent line on 12′ mark off arcs of 1/12 of the perimeter (1′–2′ distance or between any two successive points on the circumsference). (v) Name these points 1, 2, 3, 4 ..... 12 on PE parallel to the tangent at 12′. You will find the total length = PE, which is the circumference of the circle. (vi) On each of the tangents at 1′, 2′ ..... 12′ cut – off arcs of increasing length (01, 02, 03, 04 .....011, 012) (vii) Draw by free hand a smooth curve to pass through these points of intersection of the tangents with the arc length cut off as (01, 02, 03 ..... 011, 012). ..... This curve is the Involute of the circle.

3.5 CYCLOID : We are using wheels for means of transport on road. Let us consider a wheel rotating about its centre "without slip" on a plane straight road. If a feather of a bird get, stuck to the wheel on a certain point and remain stuck for some time. If we consider one complete rotation of such a wheel the feather will trace the typical curve called as cycloid. This curve is used in the teeth of the gears, especially straight gear called as rack in engineering.

3.5.1 CONSTRUCTION OF CYCLOID : Let us learn how to draw it.

Let us draw the cycloid of the circle whose diameter = 30 mm.

Refer Fig. 3.9 (i) Draw the circle of diameter = 30 mm and divide it into six equal parts by the compass method as explained in the construction of the involute (ii) Through the point 0′, 1′, 2′, 3′ .....6′ marked on the circle draw horizontal lines parallel to each other (iii) on the horizontal line through 0′, mark 12 points (0, 1, 2, 3..... 12) each of 1/12 of circumference (0′1′) (iv) Through these points (1, 2, 3.....11) draw vertical lines (v) Find the "corresponding points of intersection" of the horizontal and vertical lines (vi) Through these points draw a smooth free hand curve known as Cycloid.

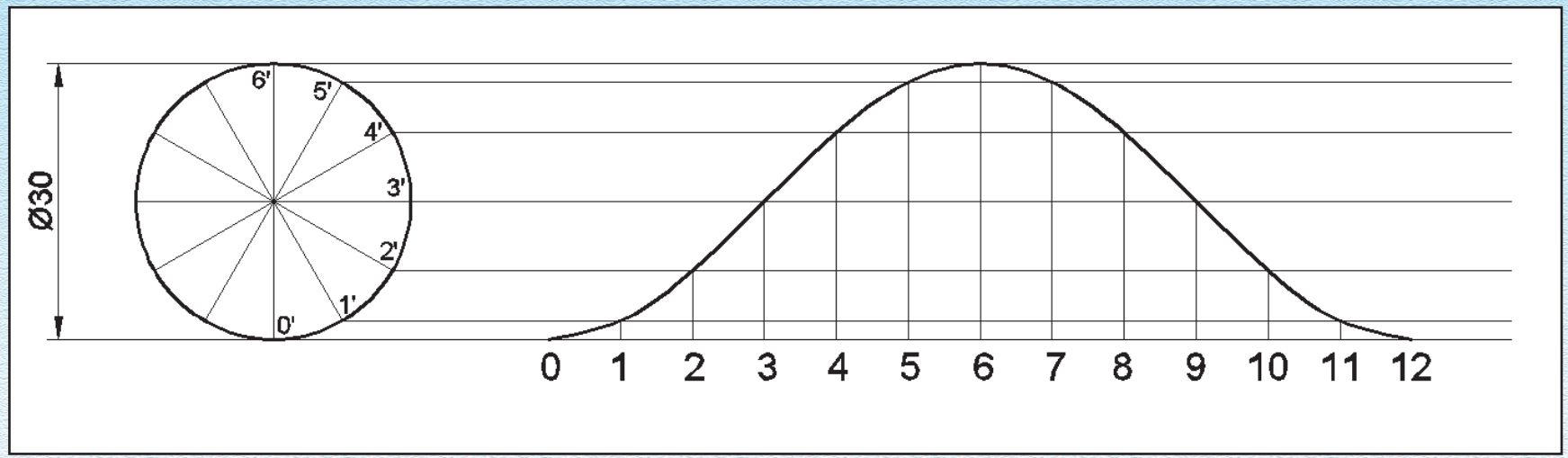

Fig. 3.9

3.6 THE HELIX : You must have heard about Screws, Nuts and Bolts. Many of you must have used them in the making of some of the models of cranes, cart, car etc. of Mechano sets. They are used to join two parts, temporarily. Did you ever study the spring like features of these parts, especially the one winding its way on the length? Some of you may be able to name it also. These are commonly known as "threads".

ACTIVITY : Take a screw or a bolt insert the tip of your pencil in its groove in the centre of the length of screw/bolt. Now hold the pencil and turn the screw/bolt in either direction (i.e. clock wise/anticlockwise. Give a few complete turns. Observe what happens to the pencil. You will notice that it will move either in the forward/backward direction. This is possible due to these typical grooves known as "threads". The shape of these threads is helical. The curve that is drawn in this way is known as helix.

In Engineering we are required to join two parts. This can be done in many ways. One of the way is a pair of Nut-bolt or a screw depending on the requirement/situation. Thus they are very common in Engineering.

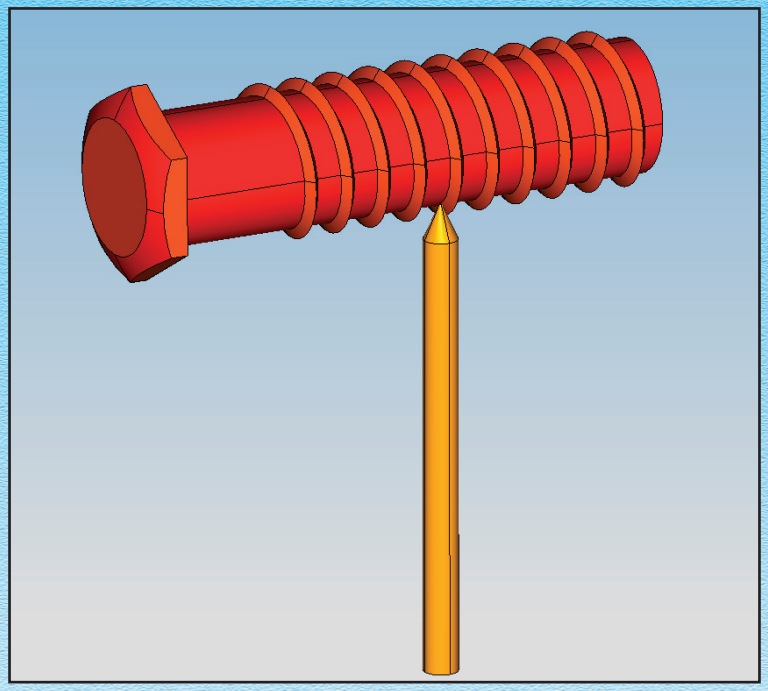

Fig. 3.10

3.6.1 CONSTRUCTION OF HELIX : Let us study the construction of helix in detail (Refer Fig. 3.11)

(i) Draw the circle of given radius say 20 mm and divide it into six equal parts (actually twelve, but the division points on the left and right are symmertical so one side will suffice) (ii) Through these points marked (0, 1, 2, 3, ...6) draw the horizontal lines as shown (iii) Now mark the distance "pitch" = 45 mm (it is also given) and divide it into twelve equal parts as shown and match the corresponding points as shown. Note that the points are symmetrical about the vertical line through 6'. (iv) Join these points by free hand. Such a curve is called helix. This is repeated all along the length of the threaded portion of the bolt/screw.

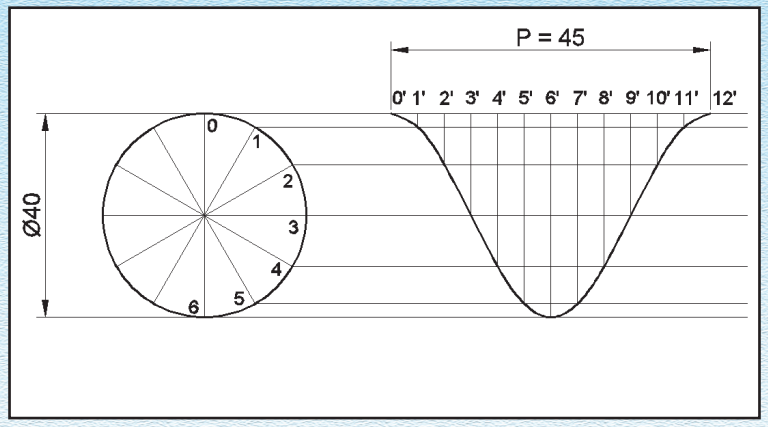

Fig. 3.11

3.7 SINE CURVE : Under the study of trigonometry you have studied the trigonometrical ratios. Let us recall the sin θ. In a right angled triangle as shown write the ratio in terms of its sides.

$$\text{sine } \theta = \frac{\text{Perpendicular}}{\text{.....}} = \frac{}{AC}$$

Let us consider θ to vary from 0° to 360° and find the values of AB, and take AC as 1 unit say 10 mm.

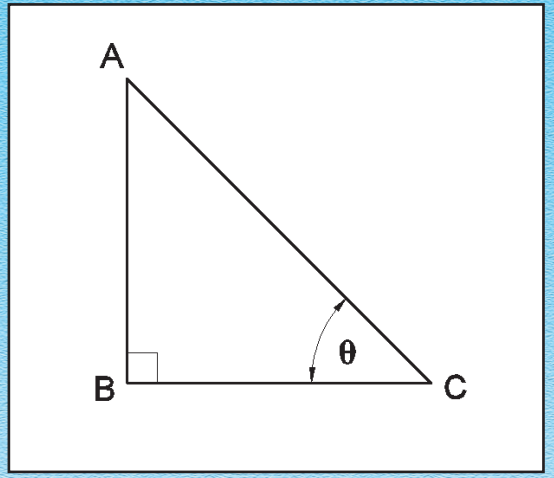

Fig. 3.12

3.7.1 CONSTRUCTION OF SINE CURVE : Refer Fig. 3.13 (i) Take a circle of given radius (10 mm) and divide it into twelve equal parts. (for simplicity only right side parts are shown) (ii) Assume a suitable scale to mark angles division (0°, 30°, 60°, 90° .....360°) on the (x-Axis) central horizontal line (iii) Match the corresponding points. Join a smooth centre through them as shown. The corresponding lengths on the y-axis shows values of sine θ. Such a curve is called the sine curve.

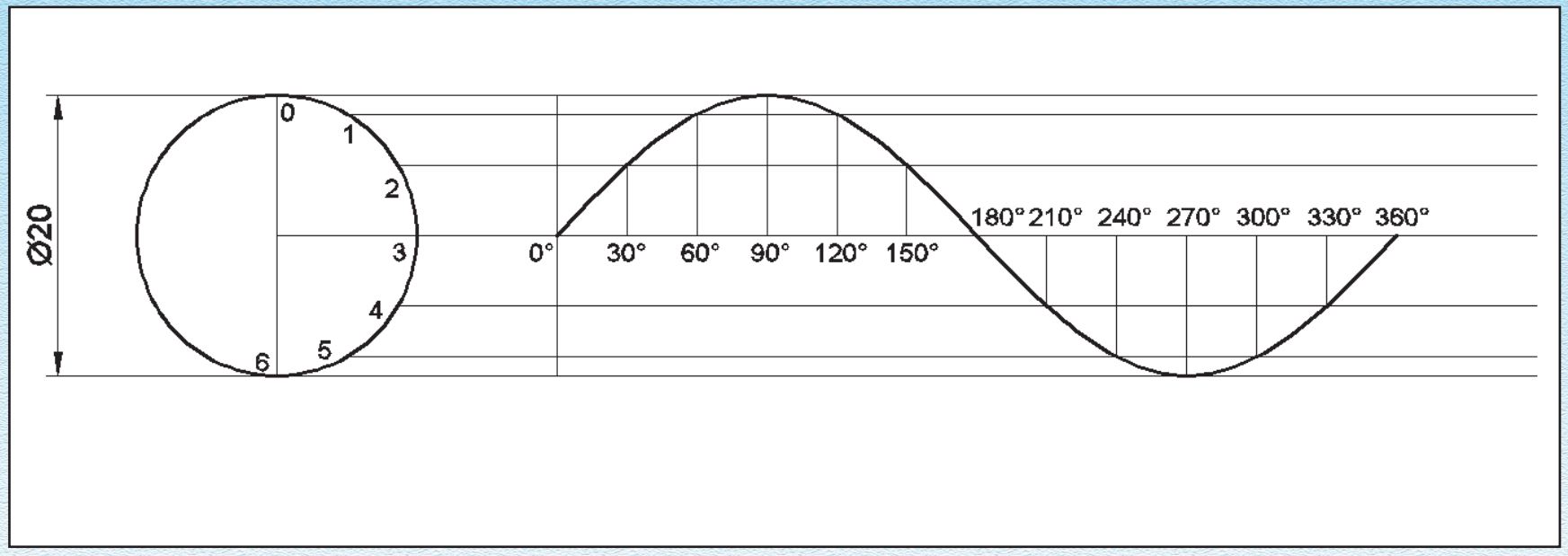

Fig. 3.13

You must have come across similar waves in the study of sound. Actually this is a continuous wave whose only one complete segment, called cycle, is shown. Such a wave in electronics / electricity is known as sinusoidal wave. Many radio waves also resemble this shape.

ASSIGNMENT

Q1. Construct the ellipse whose major axis = 70 mm and the minor axis = 40 mm by concentric circle method.

Q2. Draw an ellipse by intersecting arcs method, given the semi-major axis 40 mm and the semi-minor axis 25 mm.

Q3. Given the major-axis = 60 mm and the minor-axis = 40 mm. Draw the ellipse by intersecting lines method.

Q4. A parabola has a width = 60 mm and the depth = 25 mm. Draw this parabola by intersecting lines method.

Q5. Draw the parabola, given the distance between the directrix and the focus = 20 mm.

Q6. Draw an involute of a circle whose radius is 15 mm.

Q7. A circle is given whose radius = 20 mm. Draw the cycloid of this circle.

Q8. Draw a helix of a circle whose diameter = 40 mm and the pitch = 36 mm.

Q9. Given a circle with ϕ = 40 mm. Draw a sine - curve for it, assuming a suitable scale for degree divisions on the x-axis.

Q10. When a cone is cut in a particular way we may obtain a circle or an ellipse or a parabola as a cut surface. Fill in the blanks with a suitable alternatives given below :

(Cut parallel to base, cut at an angle to the height, cut parallel to the slant height, cut parallel to the height, cut along the height)

(a) To get a circular surface .....

(b) To obtain an elliptical surface .....

(c) To get a triangular surface .....

(d) To obtain a parabola .....

Q11. Name three machine parts used in Engineering which are (i) Circular (ii) Elliptical (iii) Parabolic.

Q12. Find out from the library /Internet where the following curves find their application in

Engineering : (i) Involute (ii) Cycloid (iii) Helix (iv) Sine - curve.