Sample Paper 17

Class- X Exam - 2022-23

Mathematics - Standard

Time Allowed: 3 Hours **Maximum Marks : 80**

General Instructions :

1. This Question Paper has 5 Sections A-E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

SECTION - A **20 marks**

(Section A consists of 20 questions of 1 mark each.)

1. The median class of the following distribution is:

Class	40-45	45-50	50-55	55-60	60-65	65-70	70-75
Frequency	2	3	8	6	6	3	2

(a) 45 – 50 (b) 65 – 70
(c) 40 – 45 (d) 55 – 60 1

2. An integer is chosen at random between 1 and 100. The probability that the chosen number is divisible by 10 is:

(a) $\frac{9}{98}$ (b) $\frac{8}{95}$
(c) $\frac{9}{97}$ (d) $\frac{5}{97}$ 1

3. Two different dice are rolled together. The probability of getting a sum of 10 of the numbers on the two dice is:

(a) $\frac{2}{13}$ (b) $\frac{5}{14}$
(c) $\frac{1}{12}$ (d) $\frac{1}{13}$ 1

4. The area of the largest triangle that can be inscribed in a semi–circle of radius *r* units is:

(a) $2r^2$ (b) r^2
(c) $\frac{r^2}{2}$ (d) 1 1

5. The total surface area of a quadrant of a wooden sphere of radius 3.5 cm is:

(a) 56 cm^2 (b) 35 cm^2
(c) 77 cm^2 (d) 22 cm^2 1

6. If $\sin\theta - \cos\theta = 0$, then the value of $\sin^4\theta + \cos^4\theta$ is:

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$
(c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{\sqrt{2}}$ 1

7. The ratio of the height of a tower and the length of its shadow on the ground is $\sqrt{3}:1$. What is the angle of elevation?

(a) 30° (b) 45°
(c) 60° (d) 90° 1

8. The zeroes of the polynomial $p(x) = x^3 - 4x$ is:

(a) 1, 4, 2 (b) 2, 0, 3
(c) 0, –2, 2 (d) 0, 2, 2 1

9. The first negative term of the AP: $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, \ldots\ldots$ is:

(a) 27 (b) 24

(c) 25 (d) 28 1

10. The value of k for which the equation $x^2 + 4x + k = 0$ has real roots is:

(a) $k = 4$ (b) $k \leq 4$

(c) $k > 4$ (d) $k \geq 4$ 1

11. Write the solution of the following pair of equations:

$x - 3y = 2$; $3x - y = 14$ is:

(a) $x = 5, y = 1$ (b) $x = 3, y = 5$

(c) $x = 2, y = 1$ (d) $x = 3, y = 2$ 1

12. Write a quadratic polynomial for which sum and product of the zeros are 3 and – 10 respectively.

(a) $2x^2 + 3x + 5$ (b) $x^2 - 3x - 10$

(c) $x^2 + 3x + 10$ (d) $x^2 - 5x + 4$ 1

13. The pairs of equations $x + 2y + 5 = 0$ and $5x + 10y + 25 = 0$ have:

(a) unique solution

(b) exactly two solutions

(c) infinitely many solutions

(d) No solution 1

14. The perimeter of a circle having radius 7cm is equal to:

(a) 30 cm (b) 3.14 cm

(c) 31.4 cm (d) 44 cm 1

15. The chord of a circle of radius 10 cm subtends a right angle at its centre. The length of the chord is:

(a) 10 cm (b) 20 cm

(c) $10\sqrt{2}$ cm (d) $10\sqrt{3}$ cm 1

16. If 6 times the 6th terms of on A.P. is equal to 9 times the 9h term, then find its 15th terms:

(a) 10 (b) 31

(c) 22 (d) 0 1

17. The mid-point of the line segment joining the points (–2, 4) and (6, 10) is:

(a) (2, 7) (b) (5, 2)

(c) (3, 5) (d) (4, 5) 1

18. The value of 'a', if HCF $(a, 18) = 2$ and LCM $(a, 18) = 36$, is:

(a) 2 (b) 5

(c) 7 (d) 4 1

DIRECTION: In the question number 19 and 20, a statement of assertion (A) is followed by a statement of reason (R).
Choose the correct option as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

19. **Statement A (Assertion):** The graph of the linear equations $5x + 3y = 12$ and $7x - 5y = 4$ gives a pair of intersecting lines.

Statement R (Reason): The graph of linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ gives a pair of intersecting lines if $\frac{a_1}{a_2} \uparrow \frac{b_1}{b_2}$. 1

20. **Statement A (Assertion):** If the circumference of a circle is 176 cm, then its radius is 22 cm.

Statement R (Reason): Circumference = $2\pi \times$ radius. 1

SECTION - B

10 marks

(Section B consists of 5 questions of 2 marks each.)

21. If n is a positive odd integer, then show that $n^2 - 1$ is divisible by 8.

OR

Check whether 15^n can end with digit zero for any natural number n. 2

22. If P(5, 7), Q(x, – 2) and R(– 3, y) are collinear points such that PR = 2PQ, calculate the values of x and y.

OR

Show that the roots of the quadratic equation:

$(b - c)x^2 + (c - a)x + (a - b) = 0$

are equal if $c + a = 2b$. 2

23. Determine the value of "k" so that the ratio of the zeros in the quadratic polynomial $3x^2 - kx + 14$ is 7:6. 2

24. The sum of circumferences of two circles is 132 cm. If the radius of one circle is 14 cm, find the radius of the other circle. 2

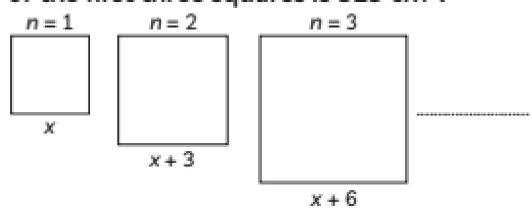
25. In a car park, there are 125 cars, $3p$ motorbikes, $2q$ lorries and 20 buses. One of the vehicles leaves the car park at random. Given that the probability that the vehicle is a motorbike is $\frac{3}{40}$ and probability that the vehicle is a bus is $\frac{1}{10}$, form a pair of linear equations in p and q. 2

SECTION - C 18 marks

(Section C consists of 6 questions of 3 marks each.)

26. Find the HCF and the LCM of 72 and 120, using prime factorisation method. 3

27. The diagram given below shows a sequence of square wire frames. The lengths of a side of these frames are 'x' cm, $(x + 3)$ cm, $(x + 6)$ cm,...... respectively. The sum of the areas of the first three squares is 525 cm^2.



(A) Express the length of a side of the n^{th} frame in terms of x and n.

(B) Find the value of x.

(C) A piece of wire is 99 cm long. It is cut and bent into a frame in the sequence. Find the length of a side of the largest frame than can be formed.

OR

If the roots of the equation $x^2 + 2cx + ab = 0$ are real and unequal, prove that the equation $x^2 - 2(a + b)x + a^2 + b^2 + 2c^2 = 0$ has no real roots. 3

28. Let A(4, 2), B(6, 5) and C(1, 4) be the vertices of $\triangle$ABC. The median AD from A meets BC in D. Find the coordinates of the point P on AD such that, AP : PD = 2 : 1. 3

29. If $\tan\theta = \frac{12}{13}$, evaluate $\frac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}$ 3

30. There are 4 green marbles, 8 white marbles, and 5 red marbles in a box. Randomly, one marble is taken out of the box. What is the probability that the marble will be one of the following:

(A) Red (B) White

(C) Not green

OR

A sphere of diameter 6 cm is dropped in a right circular cylindrical vessel partly filled with water. The diameter of the cylindrical vessel is 12 cm. If the sphere is completely submerged in water, by how much will the level of water rise in the cylindrical vessel? 3

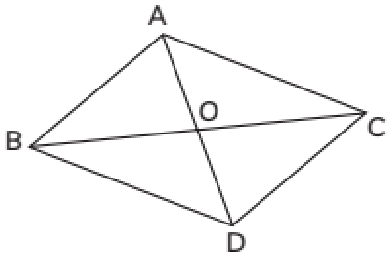
31. If $\sec\theta + \tan\theta = m$, prove that $\frac{m^2 - 1}{m^2 + 1} = \sin\theta$. 3

SECTION - D 20 marks

(Section D consists of 4 questions of 5 marks each.)

32. From the first floor of Qutab Minar, which is at a height of 25 m from the level ground, a man observes the top of a building at an angle of elevation of 30° and the angle of depression of the base of the building to be 60°. Calculate the height of the building. 5

33. In the figure, ABC and DBC are two triangles on the same base BC. If AD intersects BC at O, show that $\frac{ar(\triangle ABC)}{ar(\triangle DBC)} = \frac{AD}{DO}$

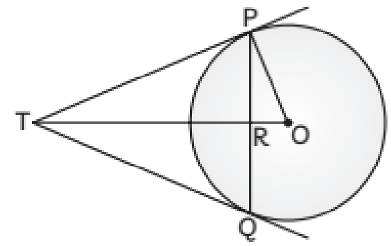


OR

Prove that the line segments joining the mid – points of the sides of a triangle form four triangles, each of which is similar to the original triangle. 5

34. A piece of cloth costs ₹ 35. If the piece were 4 m longer and each metre costs ₹ 1 less, the cost would remain unchanged. How long is the piece? 5

35. PQ is a 16 cm long chord of a circle with a 10 cm radius in given figure. At a point T, the tangents at P and Q come together. Determine TP's length.



OR

Find the mean and mode of the following frequency distribution:

Marks	0 – 9	10 – 19	20 - 29	30 - 39	40 - 49	50 - 59
Number of students	4	6	12	6	7	5

5

SECTION - E

12 marks

(Case Study Based Questions)

(Section E consists of 3 questions. All are compulsory.)

36. *Formula one Portugese Grand Prix technical team at the Algarve International Circuit are analysing last year data of drivers' performance to provide valuable inferences to commentators on how the drivers can improve this year.*

	Support staff	Lap errors		Support staff	Lap errors
Ferrari	36	41 (13%)	Force India	36	36 (11%)
Mercedes	36	61 (19%)	Toro Rosso	36	23 (7%)
Red Bull Racing	36	52 (16%)	Renault	36	16 (5%)
McLaren	36	31 (9%)	Sauber	36	13 (4%)
Williams	36	33 (10%)	Haas	-	19 (6%)

The length of time taken by 80 drivers to complete a journey is given in the table below:

Times (in minutes)	70-80	80-90	90-100	100-110	110-120	120-130
Number of drivers	4	10	14	20	24	8

On the basis of the above information, answer the following questions:

(A) In which interval does the median of the distribution lie?

OR

Find the estimate of the mean time (in minutes) taken to complete the journey. 2

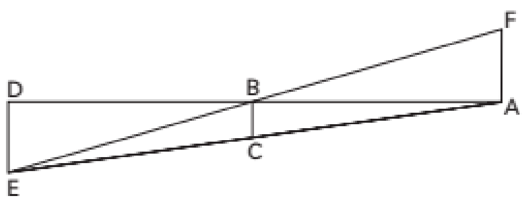
(B) One driver is chosen at random. Find the probability that he took 90 minutes or less for the journey. 1

(C) Two drivers are chosen at random. Find the probability that one took 80 minutes or less and other took more than 120 minutes for the journey. 1

37. *Google maps cartography team is working on improving the scalability quality of maps when you use the app on your phones to zoom in using 4 fingers. They are using a proprietary tool called 'MapMaker' to figure out scalability factors. A mathematical model is created for a type of object (below cross-section) to test its scalability on maps app.*

In the diagram, AC = 8 cm, CE = 4 cm and the area of the triangle BEC is 4.2 sq cm.

Another enlargement with centre E, maps ΔEBC onto ΔEFA. BC = 3.6 cm.



On the basis of the above information, answer the following questions:

(A) An enlargement, with centre A, maps ΔABC onto ΔADE, then find the scale factor of the enlargement. 1

(B) Find the length of AF. 1

(C) Calculate the area of ΔABC.

OR

Find the area of ΔEFA. 2

38. *Amrita makes biscuits. The amount of mixture required to make one biscuit is 18 cu cm. Before it is cooked, the mixture is rolled into a sphere. After the biscuit is cooked, the biscuit becomes a cylinder of radius 3 cm and height 0.7 cm (The increase in volume is due to air being trapped in the biscuit)*

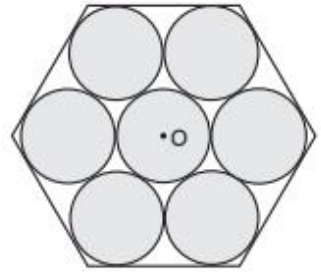


Diagram I

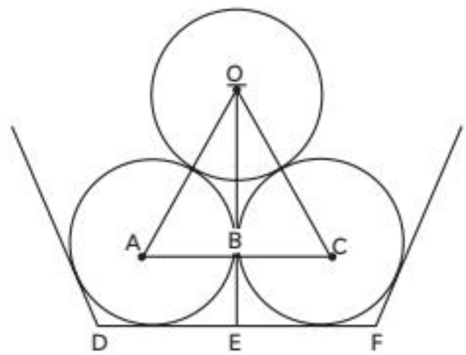


Diagram II

The cross – section of the box is a regular hexagon, containing 7 biscuits , arranged in Diagram I .

Three of the biscuits are shown in Diagram II.

O is the centre of the hexagon and of the middle biscuits. B is the point where two biscuits touch. A and C are the centres of these biscuits. E is the mid – point of the side DF of the box.

On the basis of the above information, answer the following questions:

(A) Find the length of OB. 1

(B) Find the length of OE. 1

(C) Find the volume of the biscuits after it is cooked and also find the air trapped, while cooling the biscuit.

OR

Using the concept of similarity of triangles, find the length of a side of the box. 2

SOLUTION

SECTION - A

1. *(d) 55–60*

Explanation:

Class	Frequency	Cumulative Frequency
40–45	2	2
45–50	3	8
50–55	8	13
55–60	6	19
60–65	6	25
65–70	3	28
70–75	2	30

So, $N = 30$ and $\frac{N}{2} = 15$

The cumulative frequency, just greater than 15, is 19 which belongs to class interval 55-60.

Hence, the median class is 55-60.

Caution

➥ *It is important to remember that the median class is the class interval whose cumulative frequency is greater than $\frac{N}{2}$.*

2. *(a)* $\frac{9}{98}$

Explanation: Numbers divisible by 10 between 1 and 100 are:

10, 20, 30, 40, ..., 90

So, required probability $= \frac{9}{98}$.

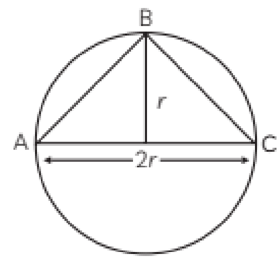
3. *(c)* $\frac{1}{12}$

Explanation: The pairs having sum of 10 are (4, 6), (5, 5) and (6, 4), out of (6 × 6), i.e. 36 pairs.

So, the required probability is $\frac{3}{36}$ or $\frac{1}{12}$.

4. *(b)* r^2

Explanation: Area of triangle $= \frac{1}{2} \times 2r \times r = r^2$

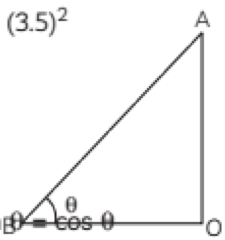


5. *(c)* $77\ cm^2$

Explanation: Radius of sphere, $r = 3.5$ cm

$$\therefore \quad \text{T.S.A.} = \frac{4\pi r^2}{4} + \frac{\pi r^2}{2} + \frac{\pi r^2}{2}$$
$$= \pi r^2 + \pi r^2$$
$$= 2\pi r^2 = 2\pi\,(3.5)^2$$
$$= 77\ cm^2$$

6. *(a)* $\frac{1}{2}$



Explanation: Given, $\sin\theta = \cos\theta$

$$\Rightarrow \quad \frac{\sin\theta}{\cos\theta} = 1$$

or $\tan\theta = 1$

$\therefore \quad \theta = 45°$

Now, $\sin^4\theta + \cos^4\theta$

$$= \sin^4 45° + \cos^4 45°$$
$$= \left(\frac{1}{\sqrt{2}}\right)^4 + \left(\frac{1}{\sqrt{2}}\right)^4$$
$$= \frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$$

7. *(c) 60°*

Explanation: Let OA be the tower, OB be its shadow and θ be the angle of elevation of the sun at that instant.

Then, in triangle OAB, we have

$$\tan\theta = \frac{AO}{OB}$$
$$\Rightarrow \quad \tan\theta = \frac{\sqrt{3}}{1}$$
$$\Rightarrow \quad \theta = 60°$$

Hence, angle of elevation of the sun is 60°.

> **Caution**
> Learn the table of trigonometric ratios for specific angles properly for solving such types of questions.

8. *(c) 0, –2, 2*

Explanation: $p(x) = x^3 - 4x$

$$= x(x^2 - 4)$$
$$= x(x + 2)(x - 2)$$

Thus, the zeros of $p(x)$ are 0, – 2 and 2.

9. *(d) 28*

Explanation: Here, $a = 20$ and $d = -\frac{3}{4}$

If the n^{th} term be the first negative term, then

$a + (n - 1) d < 0$

i.e. $20 + (n-1)\left(-\frac{3}{4}\right) < 0 \Rightarrow n > \frac{83}{3}$ or $27\frac{2}{3}$

which is true for $n = 28$.

Hence, 28^{th} is the first negative term of the given A.P.

10. *(b) $k \leq 4$*

Explanation: Equation will have real roots when $(4)^2 - 4k \geq 0$, i.e. $k \leq 4$

11. *(a) $x = 5, y = 1$*

Explanation: Given, equations are

$$x - 3y = 2 \quad \text{...(i)}$$

and $$3x - y = 14 \quad \text{...(ii)}$$

Then, $$x = 2 + 3y$$

[From equation (i)]

Put the value of 'x' in equation (ii)

$$3(2 + 3y) - y = 14$$
$$\Rightarrow \quad 6 + 9y - y = 14$$
$$\Rightarrow \quad 6 + 8y = 14$$
$$\Rightarrow \quad 8y = 8 \quad \Rightarrow \quad y = 1$$

Then, $x = 2 + 3 \times 1 = 5$

Then values of x and y are 5 and 1 respectively.

12. *(b) $x^2 - 3x - 10$*

Explanation: Sum of zeroes = 3

i.e., $\alpha + \beta = 3$

and product of zeroes $= -10$

i.e., $\alpha\beta = -10$

Then, quadratic polynomial is

$$x^2 - (\alpha + \beta)x + \alpha\beta$$

i.e., $x^2 - 3x - 10$

13. *(c) Infinitely many solutions*

Explanation: $\frac{a_1}{a_2} = \frac{1}{5}$

$$\frac{b_1}{b_2} = \frac{2}{10} = \frac{1}{5}$$

$$\frac{c_1}{c_2} = \frac{2}{25} = -\frac{1}{5}$$

This shows:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Therefore, the pair of equations has infinitely many solutions.

> **Caution**
> While finding the solution of linear equation, always compare the given equation with the standard equation i.e. $ax + by + c = 0$.

14. *(d) 44 cm*

Explanation: The perimeter of the circle is equal to the circumference of the circle.

Circumference = $2\pi r$

$$= 2 \times \frac{22}{7} \times 7$$
$$= 44 \text{ cm}$$

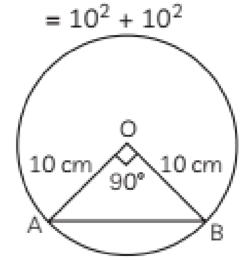
15. *(c) $10\sqrt{2}$ cm*

Explanation: Here, AB is a chord, subtending a right angle at the centre O.

Given, radius of the circle, $r = 10$ cm.

In ΔAOB, by Pythagoras theorem

$$AB^2 = OA^2 + OB^2$$
$$= 10^2 + 10^2$$



$$= 100 + 100 = 200$$

$$\therefore \quad AB = \sqrt{200} = 10\sqrt{2} \text{ cm}$$

16. *(d) 0*

Explanation: Let, the first term of an A.P. be 'a' and its common difference be 'd'.

Then, $6(a_6) = 9(a_9)$ (given)

$$6(a + 5d) = 9(a + 8d)$$

$$6a + 30d = 9a + 72d$$
$$-3a = 42d$$
$$a = -14d \quad ...(i)$$
$$15^{th} \text{ term, } a_{15} = a + 14d$$
$$= -14d + 14d \quad \text{[using (i)]}$$
$$= 0$$

Hence, the 15^{th} term of A.P. is 0.

17. *(a) 2, 7*

Explanation: The mid-point of the line segment is $\left(\frac{-2+6}{2}, \frac{4+10}{2}\right)$ *i.e.* (2,7)

18. *(d) 4*

Explanation: Given, HCF (a, 18) = 2

and LCM (a, 18) = 36

$$\text{HCF}(a, b) \times \text{LCM}(a, b) = a \times b$$
$$2 \times 36 = a \times 18$$
$$a = 4$$

Hence, value of 'a' is 4.

19. *(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)*

Explanation: The graph of linear equations $5x + 3y = 12$, and $7x - 5y = 4$ gives a pair of intersecting lines.

$$5x + 3y = 12$$
$$7x - 5y = 4$$
$$\frac{5}{7} \neq \frac{3}{-5}$$

Hence lines intersects each others.

The graph of linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ gives a pair of Intersecting lines if $\frac{a_1}{a_2} \uparrow \frac{b_1}{b_2}$.

20. *(d) Assertion (A) is false but reason(R) is true.*

Explanation: Circumference (c) = 176 cm

$\therefore \quad 2\pi r = 176$

$\therefore \quad 2 \times \frac{22}{7} \times r = 176$

$\therefore \quad \frac{44}{7} \times r = 176$

$\therefore \quad r = 176 \times \frac{7}{44} = 28$ cm

$\therefore$ The radius of the circle is 28 cm.

SECTION - B

21. We know that every positive odd integer is of the form $2q + 1$, where q is a whole number.

Put $n = 2q + 1$ and get

$$n^2 - 1 = (2q + 1)^2 - 1$$
$$= (2q + 1 - 1)(2q + 1 + 1)$$
$$= 2q(2q + 2)$$
$$= 4q(q + 1)$$

Clearly, for $q = 0, 1, 2, ..., 4q(q + 1)$ is divisible by 8.

OR

No; because $15^n = (3 \times 5)^n = 3^n \times 5^n$.

So, the only primes in the factorisation of 15^n are 3 and 5, and not 2 and 5.

Hence, 15^n cannot end with the digit 0.

⚠ Caution

➥ *It is important to know that only numbers having 2 and 5 as factors, can and with digit 0.*

22. Since P, Q, R are collinear and PQ = $\frac{1}{2}$ PR,

So, Q is the mid-point of PR.

$\Rightarrow \quad (x, -2) = \left(\frac{5-3}{2}, \frac{7+y}{2}\right)$

$\Rightarrow \quad (x, -2) = \left(1, \frac{7+y}{2}\right)$

$\Rightarrow \quad x = 1$ and $\frac{7+y}{2} = -2$

$\Rightarrow \quad x = 1$ and $y = -11$

OR

The roots of the given equation will be equal, if
$$(c - a)^2 = 4(b - c)(a - b)$$

For $c + a = 2b$, we have:

$$c - a = 2b - a - a \Rightarrow c - a = 2(b - a)$$

$\Rightarrow \quad (c - a)^2 = 4(a - b)^2 \quad ...(i)$

Also, $\quad c + a = 2b$

gives $\quad b - c = a - b$

So, $4(b - c)(a - b) = 4(a - b)^2 \quad ...(ii)$

From (i) and (ii), we have

$(c - a)^2 = 4(b - c)(a - b) = 4(a - b)^2$

23. Let the zeros are $7p$ and $6p$.

$3x^2 - k + 14$

$\therefore \quad 7p + 6p = \frac{-(-k)}{3} = \frac{k}{3}$

and $\quad 7p \times 6p = \frac{14}{3}$

$\Rightarrow \quad 42p^2 = \frac{14}{3}$

$p = 3$

$\Rightarrow \quad 39p = k$

$\therefore \quad k = 39 \times 3$

$\therefore \quad k = 117$

Caution

While finding the value of any variable, always consider the signs of constants.

24. Let $r = 14$ cm and R cm be the radii of two circles. Then,

$2\pi(14) + 2\pi R = 132$

$\Rightarrow \quad 2\pi R = 132 - 88$

$R = \frac{44 \times 7}{2 \times 22} = 7$

Thus, the radius of the other circle is 7 cm.

25. P (a motorbike)

$= \frac{3p}{125 + 3p + 2q + 20}$

i.e. $\frac{3p}{145 + 3p + 2q}$

Given probability is $\frac{3}{40}$. So

$\frac{3p}{145 + 3p + 2q} = \frac{3}{40}$

$\Rightarrow \quad 145 + 3p + 2q = 40p$

$\Rightarrow \quad 37p - 2q = 145$

Also, $\frac{20}{145 + 3p + 2q} = \frac{1}{10}$

$\Rightarrow \quad 145 + 3p + 2q = 200$

$\Rightarrow \quad 3p + 2q - 55 = 0$

Thus, the pair of linear equations are

$37p - 2q = 145$ and $3p + 2q - 55 = 0$.

SECTION - C

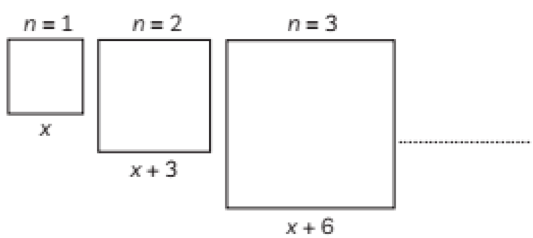
26. Here,

$72 = 2 \times 2 \times 2 \times 3 \times 3$, or $2^3 \times 3^2$

$120 = 2 \times 2 \times 2 \times 3 \times 5$, or $2^3 \times 3^1 \times 5^1$

So, HCF(72, 120) = $2^3 \times 3^1$, i.e. 24.

27.



The length of the sides of the sequence of squares are: $x, x + 3, x + 6, \ldots$

It is an A.P. with $a = x, d = 3$

(A) So, side of the nth square $= a + (n - 1)d$

$= x + (n - 1)(3)$

$= x + 3n - 3$

(B) It is given that

$x^2 + (x + 3)^2 + (x + 6)^2 = 525$

$\Rightarrow \quad 3x^2 + 18x + 45 = 525$

$\Rightarrow \quad x^2 + 6x - 160 = 0$

$\Rightarrow \quad x^2 + 16x - 10x - 160 = 0$

$\Rightarrow \quad x(x + 16) - 10(x + 16) = 0$

$\Rightarrow \quad (x + 16)(x - 10) = 0$

$\Rightarrow \quad x = 10$

$(\because x \neq -16)$

(C) Side of the square formed with a wire of length 99 cm at the most can be $\frac{99}{4}$ cm.

Thus, the side of the square with largest frame = 24 cm.

OR

The two equations are

$x^2 + 2cx + ab = 0$ ...(i)

and $x^2 - 2(a + b)x + a^2 + b^2 + 2c^2 = 0$ ...(ii)

Let D_1 and D_2 be the discriminants of equations (i) and (ii), respectively. Then,

$D_1 = (2c)^2 - 4 \times 1 \times ab = 4(c^2 - ab)$

$D_2 = (-2(a + b))^2 - 4 \times 1 \times (a^2 + b^2 + 2c^2)$

$= 4(a + b)^2 - 4(a^2 + b^2 + 2c^2)$

$= 4(a^2 + b^2 + 2ab) - 4a^2 - 4b^2 - 8c^2$

$= 8ab - 8c^2$

$= -8(c^2 - ab)$

Since, the roots of equation (i) are real and unequal. Therefore,

$D_1 > 0$

$\Rightarrow \quad 4(c^2 - ab) > 0$

$\Rightarrow \quad c^2 - ab > 0$

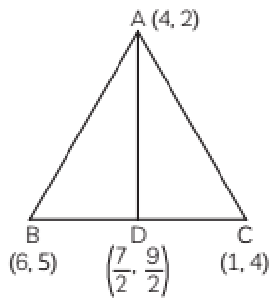
$\Rightarrow \quad -8(c^2 - ab) < 0$

$\Rightarrow \quad D_2 < 0$

$\therefore$ Roots of equation (ii) are not real.

28. Here, D is D $\left(\frac{6+1}{2}, \frac{5+4}{2}\right)$ *i.e.* $\left(\frac{7}{2}, \frac{9}{2}\right)$

The point P divides AD in the ratio 2 : 1.

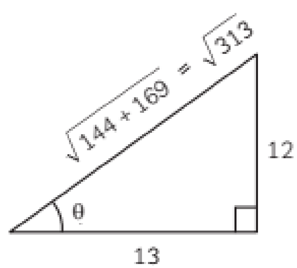


So, $\left(\frac{2 \times \frac{7}{2} + 1 \times 4}{2+1}, \frac{2 \times \frac{9}{2} + 1 \times 2}{2+1}\right)$

i.e., $P\left(\frac{11}{3}, \frac{11}{3}\right)$

29. Given $\tan\theta = \frac{12}{13}$, we have

$\sin\theta = \frac{12}{\sqrt{313}}$ and $\cos\theta = \frac{13}{\sqrt{313}}$



Now,

$$\frac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta} = \frac{2\left(\frac{12}{\sqrt{313}}\right)\left(\frac{13}{\sqrt{313}}\right)}{\left(\frac{13}{\sqrt{313}}\right)^2 - \left(\frac{12}{\sqrt{313}}\right)^2}$$

$$= \frac{312}{313} \times \frac{313}{169-144}$$

$$= \frac{312}{25}$$

30. Total number of marbles in the box = 5 + 8 + 4 = 17 Total number of elementary events = 17

(A) There are 5 red marbles in the box. Favourable number of elementary events = 5

P(getting a red marble) = $\frac{5}{17}$

(B) There are 8 white marbles in the box. Favourable number of elementary events = 8

P(getting a white marble) = $\frac{8}{17}$

(C) There are 5 + 8 = 13 marbles in the box, which are not green.

Favourable number of elementary events = 13

P(not getting a green marble) = $\frac{13}{17}$

OR

We have, Radius of the sphere = 3 cm

$\therefore$ Volume of the sphere = $\frac{4}{3}\pi(3)^3$ cm^3

= 36π cm^3

Radius of the cylindrical vessel = 6 cm

Suppose water level rises by 'h' cm in the cylindrical vessel. Then,

Volume of the cylinder of height 'h' cm and radius 6 cm

= $\pi(6)^2 h$ cm^3, i.e. $36\pi h$ cm^3

Clearly, volume of water displaced by the sphere is equal to the volume of the sphere.

$\Rightarrow \quad 36\pi h = 36\pi$

$\Rightarrow \quad h = 1$ cm

Hence, water level rises by 1 cm.

31. Proof:

$$\text{L.H.S.} = \frac{m^2-1}{m^2+1}$$

$$= \frac{(\sec\theta+\tan\theta)^2-1}{(\sec\theta+\tan\theta)^2+1}$$

$$= \frac{\sec^2\theta+\tan^2\theta+2\sec\theta\tan\theta-1}{\sec^2\theta+\tan^2\theta+2\sec\theta\tan\theta+1}$$

$$= \frac{2\tan^2\theta+2\sec\theta\tan\theta}{2\sec^2\theta+2\sec\theta\tan\theta}$$

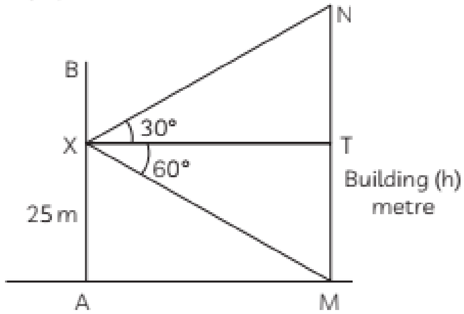
$$= \frac{2\tan^2\theta(\sec\theta+\tan\theta)}{2\sec^2\theta(\tan\theta+\sec\theta)}$$

$$= \frac{\sin\theta}{\cos\theta} \times \cos\theta$$

= $\sin\theta$ = R.H.S. **Hence proved.**

SECTION - D

32. Let the first floor of Qutab Minar AB be at the point X.



Let MN represent the building of height h metre.

Here, XT || AM

In right ΔXTN, $\frac{TN}{XT} = \tan 30° = \frac{1}{\sqrt{3}}$ ...(i)

In right ΔXTM, $\frac{TM}{XT} = \tan 60° = \sqrt{3}$

$\Rightarrow \quad \frac{25}{XT} = \sqrt{3}$

$\Rightarrow \quad XT = \frac{25}{\sqrt{3}}$ m ...(ii)

From (i), using (ii), we have:

$$TN = \frac{25}{\sqrt{3}} \times \frac{1}{\sqrt{3}}$$

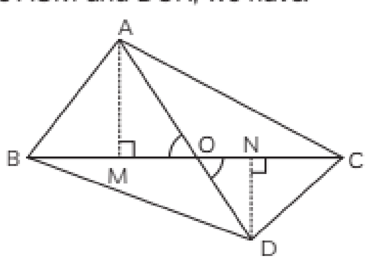
$\Rightarrow \quad TN = \frac{25}{3}$ m

Thus, the height of the building MN = MT + TN

$$= \left(25 + \frac{25}{3}\right)\text{m} = 33\frac{1}{3} \text{ m.}$$

33. Draw AM ⊥ BC and DN ⊥ BC

In Δs AOM and DON, we have:



$\angle M = \angle N$ [Each = 90°]

$\angle AOM = \angle DON$

[Vertically opposite angles]

So, by AA similarity criterion,

$$\Delta AOM \sim \Delta DON$$

$\Rightarrow \quad \frac{AM}{DN} = \frac{AO}{DO}$ ...(i)

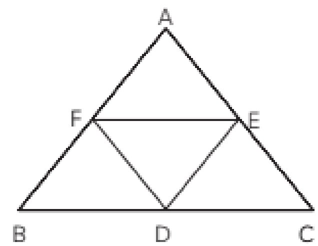
Now, $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{\frac{1}{2} \times AM \times BC}{\frac{1}{2} \times DN \times BC} = \frac{AM}{DN}$

$= \frac{AO}{DO}$ [Using (i)]

OR

Consider a triangle ABC with D, E, F as the mid-points of sides BC, CA, AB respectively.

Since, F and E are mid-points of AB and AC respectively,



So, by mid-point theorem,

FE || BC ⇒ ∠AFE = ∠B

Thus, in ΔAFE and ΔABC, we have:

∠AFE = ∠B and ∠A is common in both.

So, ΔAFE ~ ΔABC

Similarly, ΔFBD ~ ΔABC and ΔEDC ~ ΔABC

Now, we shall show that ΔDEF ~ ΔABC

Clearly, ED || AF and DF || EA

∴ AFDE is a parallelogram.

⇒ ∠EDF = ∠A

Similarly, BDEF is a parallelogram.

∴ ∠DEF = ∠B

Thus, in Δs DEF and ABC, we have:

∠EDF = ∠A, ∠DEF = ∠B

So, by AA similarity criterion, ΔDEF ~ ΔABC

Thus, each one of the triangles AFE, FBD, EDC and DEF is similar to triangle ABC.

34. Let 'l' metres be the length of the piece, with a total cost of ₹ 35.

So, cost of 1 metre long piece = ₹ $\left(\frac{35}{l}\right)$

Also, "$l + 4$" metres long piece costs ₹ 35.

So, cost of 1 metre long piece = ₹ $\left(\frac{35}{l + 4}\right)$

As per the question;

$$\frac{35}{l+4} + 1 = \frac{35}{l}$$

$$\Rightarrow \quad \frac{35 + l + 4}{l+4} = \frac{35}{l}$$

$$\Rightarrow \quad 39l + l^2 = 35l + 140$$

$$\Rightarrow \quad l^2 + 4l - 140 = 0$$

$$\Rightarrow \quad l^2 + 14l - 10l - 140 = 0$$

$$\Rightarrow \quad l(l + 14) - 10(l + 14) = 0$$

$$\Rightarrow \quad (l + 14)(l - 10) = 0$$

$$\Rightarrow \quad l - 10 = 0$$

$$[\because (l + 14) \neq 0]$$

Thus, 10 metres is the length of the piece.

35. Given PQ = 16 cm

PO= 10 cm

To Find: TP

$$PR = RQ = \frac{16}{2} = 8 \text{ cm}$$

[Perpendicular from the centre bisects the chord]

In ΔOPR

$$OR = \sqrt{OP^2 - PR^2}$$

$$= \sqrt{10^2 - 8^2}$$

$$= \sqrt{100 - 64}$$

$$= \sqrt{36} = 6 \text{ cm}$$

Let ∠POR be θ

In ΔPOR, $\tan\theta = \frac{PR}{RO} = \frac{8}{6}$

$$\tan\theta = \frac{4}{3}$$

We know, OP ⊥ TP (Point of contact of a tangent is perpendicular to the line from the centre)

In ΔOTP, $\tan\theta = \frac{OP}{TP} \Rightarrow \frac{4}{3} = \frac{10}{TP}$

$$TP = \frac{10 \times 3}{4} = \frac{15}{2} = 7.5 \text{ cm}$$

OR

The given frequency distribution in "exclusive" form is:

Marks	Frequency
0.5-9.5	4
9.5-19.5	6
19.5-29.5	12
29.5-39.5	6
39.5-49.5	7
49.5-59.5	5

Calculation of Mode:

Here, class with highest frequency is 19.5-29.5. So, modal class = 19.5-29.5

Here modal class is 19.5-29.5

So, $l = 19.5$, $f_1 = 12$, $cf = 6$, $f_0 = 6$, $f_2 = 6$ and $h = 10$

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$\text{Mode} = 19.5 + \frac{12 - 6}{24 - 6 - 6} \times 10$$

$$= 19.5 + \frac{60}{12}$$

$$= 19.5 + 5$$

$$= 24.5$$

Calculation of Mean:

Let the assumed mean(A) be 34.5.

Class interval	Mid-value (x_i)	f_i	$d_i = f_i - A$ A = 34.5	$f_i\, d_i$
0.5–9.5	4.5	4	–30	–120
9.5–19.5	14.5	6	–20	–120
19.5–29.5	24.5	12	–10	–120
29.5–39.5	34.5	6	0	0
39.5–49.5	44.5	7	10	70
49.5–59.5	54.5	5	20	100
		$\Sigma f_i = 40$		$\Sigma f_i\, d = -190$

Then, Mean = $A + \frac{\Sigma f_i d_i}{\Sigma f_i}$

Thus, Mean = $34.5 + \frac{(-190)}{40}$

$= 34.5 + \frac{(-19)}{4}$

$= 34.5 - 4.75$

$= 29.75$

SECTION - E

36. (A)

Time	*f*	*c.f.*
70-80	4	4
80-90	10	14
90-100	14	28
100-110	20	48
110-120	24	72
120-130	8	80
Total	80	

Then, $\frac{N}{2} = 40$

Cumulative frequency just greater than 40 is 48, which lies in the class interval 100-110.

So, median class is 100-110.

OR

Time	f_i	x_i	f_ix_i
70-80	4	75	300
80-90	10	85	850
90-100	14	95	1330
100-110	20	105	2100
110-120	24	115	2760
120-130	8	125	1000
Total	80		8340

Then, Mean = $A + \frac{\Sigma f_i d_i}{\Sigma f_i}$

$\therefore$ Mean, $\bar{x} = \frac{8340}{80} = 104.25$

(B) P(takes less than 90 minutes),

$= \frac{4 + 10}{80}$

$= \frac{14}{80} = \frac{7}{40}$

(C) Required probability,

$= \frac{4}{80} \times \frac{8}{80}$

$= \frac{1}{20} \times \frac{1}{10}$

$= \frac{1}{200}$

37. (A) Scale factor = $\frac{AC}{AE}$

$= \frac{AC}{AC + CE} = \frac{8}{8 + 4}$

$= \frac{8}{12} = \frac{2}{3}$

(B) Since, $\Delta EBC \sim \Delta EAF$

$\frac{EC}{EA} = \frac{BC}{AF}$

$\Rightarrow \frac{4}{12} = \frac{3.6}{AF}$

$\Rightarrow AF = 3.6 \times 3$

$= 10.8$ cm

(C) In ΔABC, using Pythagoras theorem, we have

$AB^2 = BC^2 + AB^2$

$AB^2 = 64 - 12.96$

$= 51.04$

$\Rightarrow AB = 7.15$

$\therefore$ Area of $\Delta ABC = \frac{1}{2} \times BC \times AB$

$= \frac{1}{2} \times 3.6 \times 7.15$

$= 12.87$

OR

Since, with centre E enlargement as done to ΔEBC and ΔEFA.

$\therefore \Delta EBC \sim \Delta EFF$

$\frac{ar(\Delta EBC)}{ar(\Delta EFA)} = \frac{EC^2}{AC^2}$

$\Rightarrow \frac{42}{ar(\Delta EFA)} = \frac{4^2}{12^2}$

$\Rightarrow ar(\Delta EFA) = \frac{12 \times 12}{4 \times 4} \times 4.2$

$= 3 \times 3 \times 4.2$

$= 37.8$ sq. cm

38. (A) Length of OB = $\sqrt{OA^2 - AB^2}$

(using Pythagoras theorem in ΔOBA)

$= \sqrt{36 - 9}$

$= \sqrt{27}$

$= 5.19 \cong 5.2$ cm

(B) Length of BE = 3 cm

$\therefore$ Length of OE = OB + BE = 5.2 + 3

= 8.2 cm

(C) Volume of a biscuit, after cooking

= Volume of cylinder = $\pi r^2 h$

$= \frac{22}{7} \times 3 \times 3 \times 0.7$

$= 22 \times 0.9$

= 19.8 cu cm

Air trapped = Volume of a biscuit after cooking – Volume of a biscuit before cooking

= 19.8 – 18

= 1.8 cu. cm

Caution

➥ *When the biscuit is placed in a right circular cylinder then, the air trapped while cooking the biscuit is equal to the difference between the volume of biscuits after cooking and before cooking.*

OR

Since, $\Delta OAB = \Delta ODE$

$\frac{OB}{OE} = \frac{AB}{DE}$

$\Rightarrow \frac{5.2}{8.2} = \frac{3}{DE}$

$\therefore$ DE = 4.73

$\therefore$ DF = 2 × DE = 9.4 cm