Class XI Session 2024-25
Subject - Applied Mathematics
Sample Question Paper - 5

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - **five sections** A, B, C, D and E. Each section is compulsory. However, there is some internal choice in some questions.
2. Section A has 18 MCQ's and 02 Assertion Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer(VSA) questions of 2 marks each.
4. Section C has 6 Short Answer(SA) questions of 3 marks each.
5. Section D has 4 Long Answer(LA) questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (04 marks each) with sub parts.
7. Internal Choice is provided in 2 questions in Section-B, 2 questions in Section-C, 2 Questions in Section-D. You have to attempt only one alternatives in all such questions.

Section A

1. Bag A contains 3 red and 5 black balls and bag B contains 2 red and 4 black balls. A ball is drawn from one of the bags. The probability that ball drawn is red is **[1]**

   a) $\frac{17}{24}$ b) $\frac{17}{48}$
   c) $\frac{1}{3}$ d) $\frac{3}{8}$

2. The quartile coefficient of skewness is negative, if: **[1]**

   a) $Q_3 + Q_1 >$ Median b) $Q_3 + Q_1 < 2$ Median
   c) $Q_3 + Q_1 > 2$ Median d) $Q_3 + Q_1 = 2$ Median

3. The effective rate which is equivalent to nominal rate of 10% p.a. compounded quarterly is: **[1]**

   a) 10.25% b) 10.38%
   c) 10.53% d) 10.47%

4. If $\log_{12} 27 = a$, then $\log_6 16 =$ **[1]**

   a) $3(\frac{4+a}{4-a})$ b) $\frac{3-a}{3+a}$
   c) $3(\frac{4-a}{4+a})$ d) $4(\frac{3-a}{3+a})$

5. A relation R is defined from {2, 3, 4, 5} to {3, 6, 7, 10} by: x R y $\Leftrightarrow$ x is relatively prime to y. Then domain of R is **[1]**

   a) {2, 3, 4, 5} b) {3, 5}

c) {2, 3, 5}		d) {2, 3, 4}

6. If $\frac{1}{\log_x 10} = \frac{2}{\log_a 10}$ - 2, then x = **[1]**

a) $\frac{a^2}{10}$

b) $\frac{a}{100}$

c) $\frac{a}{2}$

d) $\frac{a^2}{100}$

7. If E_1 and E_2 are two independent events, then P $(E_1 \cap E_2)$ is equal to **[1]**

a) $P(E_1) + \ P\ (E_2)$

b) $P(E_1) + P(E_2) + P(E_1 + \cup E_2)$

c) $P(E_1)P(E_2)$

d) $P(E_1) - \ P\ (E_2)$

8. The equation of two diameters of a circle are x - y = 5 and 2x + y = 4 and the radius of the circle is 5, then the equation of the circle is: **[1]**

a) $x^2 + y^2 + 6x + 4y + 12 = 0$

b) $x^2 + y^2 - 6x + 4y + 12 = 0$

c) $x^2 + y^2 + 6x - 4y - 12 = 0$

d) $x^2 + y^2 - 6x + 4y - 12 = 0$

9. A man can do a piece of work in 5 days, but with the help of his son, he can do it in 3 days. In what time can the son do it alone? **[1]**

a) 8 days

b) $6\frac{1}{2}$ days

c) 7 days

d) $7\frac{1}{2}$ days

10. Let $x_1, x_2, ..., x_n$ be n observations. Let $w_i = lx_i + k$ for i = 1, 2, ..., n, where l and k are constants. If the mean of x_i's is 48 and their standard deviation is 12, the mean of w_i's is 55 and standard deviation of w_i's is 15, the values of l and k should be. **[1]**

a) l = – 1.25, k = 5

b) l = 2.5, k = – 5

c) l = 1.25, k = – 5

d) l = 2.5, k = 5

11. If $\log_{\frac{1}{3}} 27\sqrt{3} = x$, then value of x is **[1]**

a) -7

b) 7

c) $-\frac{7}{2}$

d) $\frac{7}{2}$

12. At what rate percent per annum will a sum of ₹ 12000 become ₹ 13230 in 2 years? **[1]**

a) 5%

b) 6%

c) 6.5%

d) 5.5%

13. There are 4 bus routes between A and B and 3 bus routes between B and C. A man can travel round trip in number of ways by bus from A to C via B. If he does not want to use a bus route more than once, in how many ways can he make the round trip? **[1]**

a) 14

b) 142

c) 72

d) 19

14. Two men hit at a target with probabilities $\frac{1}{2}$ and $\frac{1}{3}$, respectively. What is the probability that exactly one of them hits the target? **[1]**

a) $\frac{1}{6}$

b) $\frac{1}{2}$

c) $\frac{1}{3}$

d) $\frac{2}{3}$

15. 5 boys and 5 girls are sitting in a row randomly. The probability that boys and girls sit alternately is: **[1]**

a) $\frac{5}{126}$
b) $\frac{3}{126}$
c) $\frac{1}{126}$
d) $\frac{4}{126}$

16. The difference between simple interest and compound interest on ₹ 15000 for one yea 8% per annum calculated half-yearly is: **[1]**

a) ₹ 24
b) ₹ 20
c) ₹ 22
d) ₹ 26

17. For the post of 5 teachers, there are 23 applicants. 2 posts are reserved for SC candidates and there are 7 SC candidates among the applicants. In how many ways can the selection be made? **[1]**

a) 3920
b) 11760
c) 4880
d) 5880

18. If (2x + y, y + 3) = (7, 0), then x is **[1]**

a) -3
b) 5
c) 0
d) $\frac{7}{2}$

19. **Assertion (A):** The mean deviation about the mean for the data 4, 7, 8, 9, 10, 12, 13, 17 is 3. **[1]**
Reason (R): The mean deviation about the mean for the data 38, 70, 48, 40, 42, 55, 63, 46, 54, 44 is 8.5.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

20. **Assertion (A):** A sequence is said to definite if it has finite no of terms. **[1]**
Reason (R): The sequence whose nth term if $\frac{2^n}{n}$ if 2, 2, $\frac{8}{3}$, 4...

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

Section B

21. At what time between 2 and 3 O'Clock will the hands of a clock coincide? **[2]**

22. Find the union set of A = {x :x is a natural number and 1 < x $\leq$ 5} and B = {x : x is a natural number and 5 < x $\leq$ 10}. **[2]**

OR

Write down all possible subsets of A = (1, {2, 3}).

23. A and B can do a work in 8 days; B and C can do the same work in 12 days; A, B and C together can finish it in 6 days. In how many days A and C together will do the same work? **[2]**

24. Differentiate the function with respect to x: $\frac{e^x \log x}{x^2}$ **[2]**

OR

Find the derivative of f(x) = x + $\frac{1}{x}$ with respect to x.

25. Convert the decimal number to the binary number: 517 **[2]**

Section C

26. Given a G.P. with a = 729 and 7th term = 64, determine S_7. **[3]**

OR

Find the sum of first 'n' terms of the series 0.7 + 0.77 + 0.777 + ...

27. Find the equation of the line passing through the point (-1, 3) and perpendicular to the line 3x - 4y - 16 = 0 **[3]**

28. Find the domain and the range of the given function: f(x) = $\frac{1}{1-x^2}$ **[3]**

29. Find the present value of ₹25,000 due 10 years hence when the interest of 8% is compounded: **[3]**

i. annually
ii. semi-annually
iii. quarterly
iv. continuously

30. Simplify: **[3]**

$\frac{1}{1+a^{m-n}+a^{m-p}}+\frac{1}{1+a^{n-m}+a^{n-p}}+\frac{1}{1+a^{p-m}+a^{p-n}}$

31. Let A = {1, 2, 4, 5}, B = {2, 3, 5, 6}, C = {4, 5, 6, 7} verify the following identity: **[3]**

A ∪ (B ∩ C) = [(A ∪ B) ∩ (A ∩ C)]

Section D

32. In a factory which manufactures bolts, machines A, B and C manufacture respectively 30%, 50% and 20% of the bolts. Of their outputs 3, 4 and 1 per cent respectively are defective bolts. A bolt is drawn at random from the product and is found to be defective. Find the probability that this is not manufactured by machine B. **[5]**

OR

A die has two faces with number 1, three faces each with number 2 and one face with number 3. If die is ruled once, find

(i) P (3) (ii) P (1 or 2) (iii) P (2)

33. Suppose $f(x)=\begin{cases} a+bx, & x<1 \\ 4, & x=1 \\ b-ax, & x>1 \end{cases}$ and if $\lim_{x\to 1}$ f(x) = f(1), then what are the possible values of a and b? **[5]**

34. Compute the moment coefficient of skewness β_1 for the following distribution: **[5]**

Marks obtained	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	6	12	22	24	16	12	8

OR

Calculate the mean deviation about the mean for the following data:

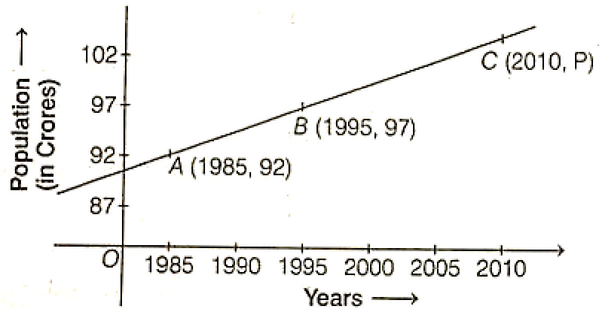
Income per day	0 - 100	100 - 200	200 - 300	300 - 400	400 - 500	500 - 600	600 - 700	700 - 800
Number of persons	4	8	9	10	7	5	4	3

35. Find the equation of a circle whose centre is a point (1, -2) and which passes through the centre of the circle $2x^2 + 2y^2 + 4y = 5$. **[5]**

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Population vs Year graph given below.



(a) In which year the population becomes 110 crores?

(b) Find the equation of line perpendicular to line AB and passing through (1995, 97).

(c) Write the equation of line AB?

OR

Find the slope of line AB.

37. **Read the text carefully and answer the questions:** **[4]**

In class of Statistics, teacher was discussing the concept of Measures of Correlation, in which he was discussing about Karl Pearson's Coefficient of Correlation. During his class, he discussed the following few points on this: This is the best method for finding correlation between two variables provided the relationship between the two variables is linear. This method is also known as product moment correlation coefficient. Pearson's correlation coefficient may be defined as the ratio of covariance between the two variables to the product of the standard deviations of the two variables.

If the two variables are denoted by x and y and of the corresponding bivariates data are (x_i, y_i) for i = 1, 2, 3, ..., n, then the coefficent of correlation between x and y due to Karl Pearson, is given by:

$r = r_{xy}$

or, $r_{xy} = \frac{\text{Cov}(x,y)}{\sqrt{\text{Var}\, x} \cdot \sqrt{\text{Var}\, y}}$

$= \frac{\text{Cov}(x,y)}{\sigma_x \cdot \sigma_y}$

where,

$\text{cov}(x, y) = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{N}$

$= \frac{\Sigma xy}{N} - \bar{x} \cdot \bar{y}$

$\sigma_x = \sqrt{\frac{\Sigma(x-\bar{x})^2}{N}} = \sqrt{\frac{\Sigma x^2}{N} - x^{-2}}$

$\sigma_y = \sqrt{\frac{\Sigma(y-\bar{y})^2}{N}} = \sqrt{\frac{\Sigma y^2}{N} - y^{-2}}$

If x - $\bar{x}$ $-\bar{y}$ are small fractions, we use

$r = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{\Sigma(x-\bar{x})^2}\sqrt{\Sigma(y-\bar{y})^2}}$

If x, y are small numbers, we use

$r = \frac{\Sigma xy - \frac{1}{N}\Sigma x \Sigma y}{\sqrt{\Sigma x^2 - \frac{1}{N}(\Sigma x)^2}\sqrt{\Sigma y^2 - \frac{1}{N}(\Sigma y)^2}}$

If x, y are large numbers, we use assumed mean A and B and u = x - A, v = y - B

$r = \frac{\Sigma uv - \frac{1}{N}\Sigma u \Sigma v}{\sqrt{\Sigma u^2 - \frac{1}{N}(\Sigma u)^2}\sqrt{\Sigma v^2 - \frac{1}{N}(\Sigma v)^2}}$

For example:

Find Karl Pearson’s coefficient of correlation between X and Y for the following:

x	5	4	3	2	1
y	4	2	10	8	6

Following problem was given to students on the same concept:

(a) What is the value Σxy in this data?

(b) What is the value Σx^2?

(c) What is the value of Σy^2?

OR

What is the value of Karl Pearson's Coefficient of Correlation between x and y?

38. **Read the text carefully and answer the questions:** **[4]**

A mobile number is having 10 digits. It is not just a group of numbers strung out at random. All mobile numbers have 3 things in common, a 2-digit Access code (AC), a 3-digit provider code (PC), and a 5 digit subscriber code (SC). AC code and PC code are fixed, then

(a) How many mobile number are possible if number start with 98073 and no other digit can repeat.

(b) How many AC code are possible if both digit in AC code are different and must be greater than 6.

(c) How may mobile number are possible if AC and PC code are fixed and digits can repeat.

(d) How many mobile numbers are possible with AC code 98 and PC code 123 and digit used in AC and PC code will not be used in SC code.

OR

Read the text carefully and answer the questions: **[4]**

Five friends Mohit, Sachin, Rohit, Mohan and kapil were playing in a ground, where they sit in a row in a straight line.

(a) In how many ways these five students can sit in a row?

(b) Total number of sitting arrangements if Mohit and Sachin sit together:

(c) What are the possible arrangements if Rohit and Mohan sits at the extrement positions?

(d) What are the possible orders if Kapil is sitting in the middle?

Solution

Section A

1.

(b) $\frac{17}{48}$

Explanation: As P(red) = P(A) $\cdot$ P($\frac{R}{A}$) + P(B) $\cdot$ P($\frac{R}{B}$)

= $\frac{1}{2} \cdot \frac{3}{8} + \frac{1}{2} \cdot \frac{2}{6}$ = $\frac{3}{16} + \frac{1}{6} = \frac{17}{48}$

2.

(c) $Q_3 + Q_1 > 2$ Median

Explanation: $Q_3 + Q_1 > 2$ Median

3.

(b) 10.38%

Explanation: 10.38%

4.

(d) $4(\frac{3-a}{3+a})$

Explanation: $\log_{12} 27 = a$

$\frac{\log 27}{\log 12} = a$

$\Rightarrow$ 3log3 = a[2log2 + log3]

$\Rightarrow$ 3log3 − alog3 = 2a log2

$\Rightarrow$ log3 = $\frac{2a\log 2}{3-a}$ ...(i)

Now, $\log_6 16 = \frac{\log 16}{\log 6}$

$\Rightarrow \frac{4\log 2}{\log 2+\log 3}$ ...(ii)

$\Rightarrow \frac{4\log 2}{\log 2+\frac{2a\log 2}{3-a}}$ [using (i)]

$\Rightarrow 4(\frac{3-a}{3+a})$

5. **(a)** {2, 3, 4, 5}

Explanation: Relatively prime numbers are those numbers that have only 1 as the common factor.

So, according to this definition we get to know that (2, 3), (2, 7), (3, 7), (3, 10), (4, 3), (4, 7), (5, 3), (5, 6), (5, 7) are relatively prime.

So, R = {(2, 3), (2, 7), (3, 7), (3, 10), (4, 3), (4, 7), (5, 3), (5, 6) ,(5, 7)}.

Therefore, the Domain of R is the values of x or the first element of the ordered pair.

So, Domain = {2, 3, 4, 5}

6.

(d) $\frac{a^2}{100}$

Explanation: $\frac{a^2}{100}$

7.

(c) $P\,(E_1)P(E_2)$

Explanation: We have, $P(E_1 \cap E_2) = P(E_1).\,P(\frac{E_2}{E_1})$

Since E_1 and E_2 are independents, therefore

$P = \left(\frac{E_2}{P(E_1)=P(E_2)}\right)$

$\therefore P(E_1 \cap E_2) = P(E_1).\,P(E_2)$

8.

(d) $x^2 + y^2 - 6x + 4y - 12 = 0$

Explanation: Given that the equation of two diameters of a circle are x - y = 5 and 2x + y = 4 and the radius is 5.

We know that the intersection point of the diameters be a centre
So, solving equation x - y = 5 and 2x + y = 4
for x and y
We get x = 3, y = -2
∴ Centre = (h, k) = (3, -2)
radius = r = 5
∴ The required equation of the circle is $(x - h)^2 + (y - k)^2 = r^2$
⇒ $(x - 3)^2 + (y + 2)^2 = 5^2$
⇒ $x^2 + y^2 - 6x + 4y + 9 + 4 = 25$
⇒ $x^2 + y^2 - 6x + 4y - 12 = 0$

9.
(d) $7\frac{1}{2}$ days
Explanation: Son's 1 day's work = $\left(\frac{1}{3} - \frac{1}{5}\right) = \frac{2}{15}$
∴ The son alone can do the work in $\frac{15}{2} = 7\frac{1}{2}$ days

10.
(c) l = 1.25, k = – 5
Explanation: Given $x_1, x_2, ..., x_n$ be n observations
And Mean of these n observations, $\bar{x}$ = 48
And their standard deviation, $SD_x = 12$.
Another series of n observations is given such that
$w_i = lx_i + k$ for i = 1, 2, ..., n, where l and k are constants
And mean of these n observations, $\bar{w}$ = 55
And their standard deviation, $SD_w = 15$
Applying the given condition for mean we get
$w_i = lx_i + k$
Substituting the corresponding given values of means, we get
55 = l(48) + k …(i)
Now we know
If standard deviation of x series is s, then standard deviation of kx series is ks,
So standard deviation of $x_1, x_2, ..., x_n$ is SD_x,
And hence the standard deviation of $lx_1, lx_2, ..., lx_n$ is lSD_x.
Similarly,
If standard deviation of x series is s, then standard deviation of k+x series is s,
So standard deviation of $lx_1, lx_2, ..., lx_n$ is lSD_x,
And hence the standard deviation of $lx_1+k, lx_2+k, ..., lx_n+k$ is lSD_x.
So applying the given condition for standard deviation, we get
$SD_w = lSD_x$
Substituting the given values, we get
15 = l(12)
⇒ $1 = \frac{15}{12}$ = 1.25
Now substituting the value of l in equation (i), we get
55 = (1.25)(48) + k
55 = 60 + k
⇒ k = 55 - 60 = -5
Hence the values of k and l are -5 and 1.25 respectively

11.
(c) $-\frac{7}{2}$
Explanation: $\log_{\frac{1}{3}} 27\sqrt{3}$ = x

$\log_{\frac{1}{3}} 3^3 \times \sqrt{3}$ = x

$\log_{\frac{1}{2}} 3^{\frac{7}{2}}$ = x

$\log_{\frac{1}{3}} \left(\frac{1}{3}\right)^{-\frac{7}{2}}$ = x $\left[\because a^x = \left(\frac{1}{a}\right)^{-x}\right]$

$\therefore$ x = $-\frac{7}{2}$

12. **(a)** 5%

Explanation: Here, P = ₹ 12000, A = ₹ 13230, n = 2. Let rate be r %.

$\therefore$ 13230 = 12000 $\left(1 + \frac{r}{100}\right)^2$

$\Rightarrow \left(1 + \frac{r}{100}\right)^2 = \frac{13230}{12000} = \frac{441}{400} = \left(\frac{21}{20}\right)^2$

$\Rightarrow 1 + \frac{r}{100} = \frac{21}{20} \Rightarrow \frac{r}{100} = \frac{1}{20} \Rightarrow r = 5\%.$

13.

(c) 72

Explanation: From A to B there are 4 choices and from B to C, there are 3 choices.

And from C to B, only 2 choices are left and from B to A, only 3 choices are left.

So total ways = 4 $\times$ 3 $\times$ 2 $\times$ 3 = 72

14.

(b) $\frac{1}{2}$

Explanation: Let A be the event that Mr. A hit the target and B be the event that Mr. B hit the target

$\therefore$ P(A) = $\frac{1}{2}$ and P(B) = $\frac{1}{3}$

Now, P (exactly one o f them hits the target)

= P(A $\cap \bar{B}$ or $\bar{A} \cap$ B)

= P(A $\cap \bar{B}$) + P($\bar{A} \cap$ B)

= P(A) $\cdot$ P$(\bar{B})$ + P$(\bar{A})$ $\cdot$ P(B)

= $\frac{1}{2} \cdot \frac{2}{3} + \frac{1}{2} \cdot \frac{1}{3} = \frac{3}{6} = \frac{1}{2}$

15.

(c) $\frac{1}{126}$

Explanation: Total number of ways = 10!

Total number of ways in which 5 boys and 5 girls are sitting in a row = 2 $\times$ 5! $\times$ 5!

$\therefore$ Required probability

= $\frac{2 \times 5! \times 5!}{10!} = \frac{2 \times 5! \times 5 \times 4 \times 3 \times 2}{10 \times 9 \times 8 \times 7 \times 6 \times 5!} = \frac{1}{126}$

16. **(a)** ₹ 24

Explanation: For C.I.:

C.I. = 15000 $\left(1 + \frac{4}{100}\right)^2$ - 15000

= 15000 $\left[\frac{26}{25} \times \frac{26}{25} - 1\right] = \frac{15000 \times 51}{25 \times 25}$ = ₹ 1224.

S.I. = $\frac{15000 \times 51}{25 \times 25}$ = ₹ 1200

$\therefore$ Difference between C.I. and S.I. = ₹ 1224 - ₹ 1200 = ₹24

17.

(b) 11760

Explanation: We have to select 2 posts out of 7 SC and 3 posts out of 16.

Required number of ways $= \left({}^7C_2 \times {}^{16}C_3\right) = \left(\frac{7 \times 6}{2} \times \frac{16 \times 15 \times 14}{3 \times 2 \times 1}\right)$ = 11760.

18.

(b) 5

Explanation: 5

19.

(c) A is true but R is false.

Explanation: Assertion Mean of the given series

$\bar{x} = \frac{\text{Sum of terms}}{\text{Number of terms}} = \frac{\sum x_i}{n}$

$= \frac{4+7+8+9+10+12+13+17}{8}$ = 10

xi	**|xi - $\bar{x}$|**
4	|4 - 10| = 6
7	|7 - 10| = 3
8	|8 - 10| = 2
9	|9 - 10| = 1
10	|10 - 10| = 0
12	|12 - 10| = 2
13	|13 - 10| = 3
17	|17 - 10| = 7
$\sum x_i$ = 80	$\sum \lvert x_i - \bar{x} \rvert$ = 24

$\therefore$ Mean deviation about mean

$= \frac{\Sigma|x_i - \bar{x}|}{n} = \frac{24}{8}$ = 3

Reason Mean of the given series

$\bar{x} = \frac{\text{Sum of terms}}{\text{Number of terms}} = \frac{\sum x_i}{n}$

$= \frac{38+70+48+40+42+55}{+63+46+54+44}$ = 50

$\therefore$ Mean deviation about mean

$= \frac{\Sigma|x_i - \bar{x}|}{n}$

$= \frac{84}{10}$ = 8.4

Hence, Assertion is true and Reason is false.

20.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation: Assertion is true.

Reason

Let $t_n = \frac{2^n}{n}$

Putting n = 1, 2, 7, x

$t_1 = 2$, $t_2 = 2$, $t_3 = \frac{8}{3}$, $t_4 = x$

80 the sequence is 2, 2, $\frac{8}{3}$, 4

Reason is also correct but not the correct explanation for Assertion.

Section B

21. At 2 O' clock, the hour hand is at 2 and the minute hand is at 12, i.e. they are 10 minute spaces apart. To be coincident, the minute hand must gain 10 minute spaces. We know that

55 minute spaces are gained by minute hand over the hour hand in 60 minutes

$\therefore$ 10 minute spaces will be gained in $(\frac{60}{55} \times 10)$ minutes = $10\frac{10}{11}$ minutes

Hence, the two hands will coincide in $10\frac{10}{11}$ minutes

22. According to the Question,

A = {x: x is a natural number and $1 < x \leq 5$}

$\Rightarrow$ A = {2, 3, 4, 5}

B = {x : x is a natural number and $5 < x \leq 10$}

$\Rightarrow$ B = {6, 7, 8, 9, 10}

On taking the union of A and B,

$A \cup B$ = {2, 3, 4, 5} $\cup${6, 7, 8, 9, 10} = {2, 3, 4, 5, 6, 7, 8, 9, 10}

Also in set builder form,

$A \cup B$ = { x: x is natural number and $1 < x \leq 10$ }

OR

Here, we have,

A contains two elements, namely 1 and {2, 3}

{2, 3} = B, then A = {1, B}

$\therefore$ P(A) = {ϕ, {1}, {B}, {1, B}}

$\Rightarrow$ P(A) = {ϕ , ,{1}, {{2, 3}}, {1, {2, 3}}}.

23. Suppose A, B and C alone can do the work in n_A, n_B and n_C days respectively.

A and B together can do the work in 8 days.

$\Rightarrow \frac{1}{n_A} + \frac{1}{n_B} = \frac{1}{8}$ ...(i)

B and C together can do the work in 12 days.

$\Rightarrow \frac{1}{n_B} + \frac{1}{n_C} = \frac{1}{12}$ ...(ii)

A, B and C together can do the work in 6 days.

$\Rightarrow \frac{1}{n_A} + \frac{1}{n_B} + \frac{1}{n_C} = \frac{1}{6}$ ...(iii)

Subtracting (i) and (ii) successively from (iii), we get

$\frac{1}{n_C} = \frac{1}{6} - \frac{1}{8}$ and $\frac{1}{n_A} = \frac{1}{6} - \frac{1}{12}$

$\Rightarrow \frac{1}{n_C} = \frac{1}{24}$ and $\frac{1}{n_A} = \frac{1}{12} \Rightarrow \frac{1}{n_A} + \frac{1}{n_C} = \frac{1}{12} + \frac{1}{24} = \frac{1}{8}$

Hence, A and C together can do the work in 8 days

24. Let, y = $\frac{e^x \log x}{x^2}$

Differentiate with respect to x we get,

$\frac{dy}{dx} = \frac{x^2 \frac{d}{dx}(e^x \log x) - (e^x \log x)\frac{d}{dx}x^2}{\left(x^2\right)^2}$ [Using quotient rule]

= $\frac{x^2\left\{e^x \frac{d}{dx}(\log x) + \log x \frac{d}{dx}(e^x)\right\} - e^x \log x \times 2x}{x^4}$ [Using product rule]

= $\frac{x^2\left[\frac{e^x}{x} + e^x \log x\right] - 2xe^x \log x}{x^4}$

= $\frac{\frac{x^2 e^x (1 + x \log x)}{x} - 2xe^x \log x}{x^4}$

= $\frac{xe^x[1 + x \log x - 2 \log x]}{x^4}$

= $\frac{xe^x}{x^3}\left[\frac{1}{x} + \frac{x \log x}{x} - \frac{2 \log x}{x}\right]$

= $e^x x^{-2}$ [$\frac{1}{x}$ + log x - $\frac{2}{x}$ log x]

So, $\frac{d}{dx}\left[\frac{e^x \log x}{x^2}\right]$ = $e^x x^{-2}$ [$\frac{1}{x}$ + log x - $\frac{2}{x}$ log x]

OR

f(x) = x + $\frac{1}{x}$

f'(x) = $\frac{d}{dx}x + \frac{d}{dx}\left(\frac{1}{x}\right)$

= $\frac{d}{dx}x + \frac{d}{dx}x^{-1}$

= 1 + (-1)x^{-1-1}

= 1 - x^{-2}

= 1 - $\frac{1}{x^2}$

25. Given decimal number is 517

2	517	
2	258	1
2	129	0
2	64	1
2	32	0
2	16	0
2	8	0
2	4	0
2	2	0
2	1	0
2	0	1

The required binary number is 1000000101

Section C

26. Let r be the common ratio of G.P. Then

$a_7 = 64 \Rightarrow ar^{7-1} = 64 \Rightarrow 729\ r^6 = 64$

$\Rightarrow \quad r^6 = \frac{64}{729} \Rightarrow r^6 = \left(\frac{2}{3}\right)^6 \text{ or } \left(-\frac{2}{3}\right)^6$

$\Rightarrow \quad r = \frac{2}{3} \text{ or } -\frac{2}{3}$

When r = $\frac{2}{3}, S_7 = \frac{a(1-r^7)}{1-r} = \frac{729\left(1-\left(\frac{2}{3}\right)^7\right)}{1-\frac{2}{3}} \;\; 3 \times 729\left(1 - \left(\frac{2}{3}\right)^7\right)$

= 2187 - 128 = 2059

When r = $\frac{2}{3}, S_7 = \frac{729\left(1-\left(-\frac{2}{3}\right)^7\right)}{1-\left(-\frac{2}{3}\right)}$ = $\frac{3}{5} \times 729\left(1 + \frac{2^7}{3^7}\right)$

= $\frac{1}{5}$(2187 + 128) = $\frac{2315}{5}$ = 463

OR

We have, 0.7 + 0.77 + 0.777 + ... to n terms

7 × 0.1 + 7 × 0.11 + 7 × 0.111 + ... to n terms

= 7{0.1 + 0.11 + 0.111 + ... to n terms}

= $\frac{7}{9}$ {0.9 + 0.99 + 0.999 + ... to n terms}

$= \frac{7}{9}\left\{\frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + \ldots \text{ to } n \text{ terms}\right\}$

$= \frac{7}{9}\left\{\left(1 - \frac{1}{10}\right) + \left(1 - \frac{1}{100}\right) + \left(1 - \frac{1}{1000}\right) + \ldots \text{ to } n \text{ terms}\right\}$

$= \frac{7}{9}\left\{\left(1 - \frac{1}{10}\right) + \left(1 - \frac{1}{10^2}\right) + \left(1 - \frac{1}{10^3}\right) + \ldots\left(1 - \frac{1}{10^n}\right)\right\}$

$= \frac{7}{9}\left\{n - \left(\frac{1}{10} + \frac{1}{10^2} + \frac{1}{10^2} + \ldots + \frac{1}{10^n}\right)\right\}$

$= \frac{7}{9}\left\{n - \frac{1}{10}\frac{\left\{1-\left(\frac{1}{10}\right)^n\right\}}{\left(1-\frac{1}{10}\right)}\right\}$

$= \frac{7}{9}\left\{n - \frac{1}{9}\left(1 - \frac{1}{10^n}\right)\right\}$

$= \frac{7}{81}\left\{9n - 1 + \frac{1}{10^n}\right\}$

27. The given equation is

3x - 4y - 16 = 0

$\Rightarrow y = \frac{3}{4}x$ - 4

∴ Slope of the line, $m_1 = \frac{3}{4}$

∴ Slope of the perpendicular line, $m_2 = -\frac{4}{3}$.

Since the line passes through (-1, 3), so the equation of the line is

$y - y_0 = m(x - x_0)$

$\Rightarrow y - 3 = m_2(x + 1)$

$\Rightarrow y - 3 = -\frac{3}{4}(x + 1)$

$\Rightarrow 3y - 9 = -4x - 4$

$\Rightarrow 4x + 3y - 5 = 0$

28. Given $f(x) = \frac{1}{1-x^2}$

For D_f, f(x) must be a real number $\Rightarrow \frac{1}{1-x^2}$ must be a real number

$\Rightarrow 1 - x^2 \neq 0 \Rightarrow x \neq -1, 1$

$\Rightarrow D_f$ = set of all real numbers except -1, i.e. $D_f = \mathbf{R} - \{-1, 1\}$

For R_f, let $y = \frac{1}{1-x^2}$

$\Rightarrow 1 - x^2 = \frac{1}{y}, y \neq 0 \Rightarrow x^2 = 1 - \frac{1}{y}, y \neq 0$

But $x^2 \geq 0$ for all $x \in D_f \Rightarrow 1 - \frac{1}{y} \geq 0$ but $y^2 > 0, y \neq 0$ (Multiply both sides by y^2, a positive real number)

$\Rightarrow y^2 (1 - \frac{1}{y}) \geq 0 \Rightarrow y (y - 1) \geq 0 \Rightarrow (y - 0) (y - 1) \geq 0$

$\Rightarrow$ either $y \leq 0$ or $y \geq 1$ but $y \neq 0$

$\Rightarrow R_f = (-\infty, 0) \cup [1, \infty)$

29. Let P be the present value of S = 25000.

i. We have, S = 25000, $i = \frac{8}{100} = 0.08$ and n = 10.

$\therefore P = S(1 + i)^{-n}$

$\Rightarrow P = 25000(1 + 0.08)^{-10} = 25000(1.08)^{-10} = 25000 \times 0.46319349 = 11579.83$

Hence, the present value is ₹11,579.83.

ii. We have, S = 25000, $i = \frac{8}{200} = 0.04$ and $n = 10 \times 2 = 20$

$\therefore P = S(1 + i)^{-n}$

$\Rightarrow P = 25000(1 + 0.04)^{-20} = 25000(1.04)^{-20} = 25000 \times 0.45638695 = 11409.67$

Hence, the present value is ₹11,409.67

iii. We have, S = 25000, $i = \frac{8}{400} = 0.02$ and $n = 10 \times 4 = 40$

$\therefore P = S(1 + i)^{-n}$

$\Rightarrow P = 25000(1 + 0.02)^{-40} = 25000(1.02)^{-40} = 25000 \times 0.45289042 = 11322.26$

Hence, the present value is ₹11,322.26.

iv. We have, S = 25000, $r = \frac{8}{100} = 0.08$ and n = 10

$\therefore P = Se^{-r\, n}$

$\Rightarrow P = 25000\, e^{-0.8} = 25000 \times 0.44933 = 11233.25$

Hence, the present value is ₹11,233.25

30. $T_1 = \frac{1}{1+a^{m-n}+a^{m-p}}$

$\Rightarrow T_1 = \frac{1}{1+\frac{a^m}{a^n}+\frac{a^m}{a^p}}$

[Using $\frac{a^m}{a^n} = a^{m-n}$]

$\Rightarrow T_1 = \frac{a^{n+p}}{a^{n+p}+a^{m+p}+a^{m+n}}$

[Using $a^m. a^n = a^{m+n}$]

Similarly, let $T_2 = \frac{1}{1+a^{n-m}+a^{n-p}}$

$\Rightarrow T_2 = \frac{1}{1+\frac{a^n}{a^m}+\frac{a^n}{a^p}}$

$\Rightarrow T_2 = \frac{a^{m+p}}{a^{m+p}+a^{m+n}+a^{n+p}}$

and let $T_3 = \frac{1}{1+a^{p-m}+a^{p-n}}$

$\Rightarrow T_3 = \frac{1}{1+\frac{a^p}{a^m}+\frac{a^p}{a^n}}$

$\Rightarrow T_3 = \frac{a^{m+n}}{a^{m+n}+a^{m+p}+a^{n+p}}$

, $T_1 + T_2 + T_3 = \frac{1}{1+a^{m-n}+a^{m-p}}$

$+ \frac{1}{1+a^{n-m}+a^{n-p}} + \frac{1}{1+a^{p-m}+a^{p-n}}$

$= \frac{a^{n+p}}{a^{n+p}+a^{m+p}+a^{m+n}} + \frac{a^{m+p}}{a^{m+p}+a^{m+n}+a^{n+p}} + \frac{a^{m+n}}{a^{m+n}+a^{m+p}} + a^{n+p}$

$= \frac{a^{n+p}+a^{m+p}+a^{m+n}}{a^{n+p}+a^{m+p}+a^{m+n}}$

= 1

Thus,

$\frac{1}{1+a^{m-n}+a^{m-p}} + \frac{1}{1+a^{n-m}+a^{n-p}} + \frac{1}{1+a^{p-m}+a^{p-n}} = 1$

31. L.H.S. = A ∪ (B ∩ C)

= {1, 2, 4, 5} ∪ [{2, 3, 5, 6} ∩ {4, 5, 6, 7}]

= {1, 2, 4, 5} ∪ {5, 6}

= {1, 2, 4, 5, 6}

R.H.S. = (A ∪ B) ∩ (A ∪ C)

= [{1, 2, 4, 5} ∪ {2, 3, 5, 6}] ∩ [{1, 2, 4, 5} ∪ {4, 5, 6, 7}]

= {1, 2, 3, 4, 5, 6} ∩ {1, 2, 4, 5, 6, 7}

= {1, 2, 4, 5, 6}

∴ L.H.S. = R.H.S. Hence verified

Section D

32. $P(A) = \frac{30}{100} = \frac{3}{10}$, $P(B) = \frac{50}{100} = \frac{5}{10}$, $P(C) = \frac{20}{100} = \frac{2}{10}$

E: bolt is defective

$P(\frac{E}{A}) = \frac{3}{100}$, $P(\frac{E}{B}) = \frac{4}{100}$, $P(\frac{E}{C}) = \frac{1}{100}$

Using Bayes' Theorem probability that defective bolt is not manufactured by machine B.

$1 - P(\frac{E}{B}) = 1 - \frac{P(B) \cdot P(\frac{E}{B})}{P(A) \cdot P(\frac{E}{A}) + P(B) \cdot P(\frac{E}{B}) + P(C) \cdot P(\frac{E}{C})}$

$= 1 - \frac{\frac{5}{10} \times \frac{4}{100}}{\frac{3}{10} \times \frac{3}{100} + \frac{5}{10} \times \frac{4}{100} + \frac{2}{10} \times \frac{1}{100}}$

$= 1 - \frac{20}{9+20+2} = 1 - \frac{20}{31} = \frac{11}{31}$

OR

(i) Out of 6 faces, one face marked with number 3

$\therefore\ P(3) = \frac{1}{6}$

(ii) Out of 6 faces, two faces marked with number 1 and three faces marked with number 2

$\therefore P(1 \text{ or } 2) = \frac{5}{6}$

(iii) Out of 6 faces, three faces marked with number 2

$\therefore\ P(2) = \frac{3}{6} = \frac{1}{2}$

33. We have,

$$f(x) = \begin{cases} a + bx, & x < 1 \\ 4, & x = 1 \\ b - ax, x > 1 \end{cases}$$

Now, LHL $= \lim_{x \to 1^-} f(x)$

$= \lim_{x \to 1^-} (a + bx) = \lim_{h \to 0} [a + b(1 - h)]$ [putting x = 1 - h as x → 1, then h → 0]

= a + b

RHL $= \lim_{x \to 1^+} f(x)$

$= \lim_{x \to 1^+} (b - ax) = \lim_{h \to 0} [b - a(1 + h)]$ [putting x = 1 + h as x → 1, then h → 0]

= b - a

Since, $\lim_{x \to 1}$ f(x) = f(1)

∴ LHL = RHL = f(1)

⇒ a + b = b - a = 4 [∵ f(1) = 4, given]

⇒ a + b = 4 ..(i) and b - a = 4 ..(ii)

On solving (i) and (ii), we get

a = 0, b = 4

34. We construct the following table:

Class	Frequency f_i	Class mark (x_i)	f_ix_i	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$	$(x_i - \bar{x})^3$	$f_i(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^3$
0-10	6	5	30	-30	900	-27000	5400	-162000
10-20	12	15	180	-20	400	-8000	4800	-96000
20-30	22	25	550	-10	100	-1000	2200	-22000
30-40	24	35	840	0	0	0	0	0
40-50	16	45	720	10	100	1000	1600	16000
50-60	12	55	660	20	400	8000	4800	96000
60-70	8	65	520	30	900	27000	7200	216000
	100		3500				26000	48000

Here, $\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{3500}{100}$ = 35

$\therefore \mu_2 = \frac{\Sigma f_i (x_i - \bar{x})^2}{\Sigma f_i} = \frac{26000}{100}$ = 260

$\mu_3 = \frac{\Sigma f_i (x_i - \bar{x})^3}{\Sigma f_i} = \frac{48000}{100}$ = 480

$\therefore \beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{(480)^2}{(260)^3}$ = 0.013

OR

We construct the following table. (5th and 6th columns are filled after calculating the mean.)

Income per day	Number of person f_i	Mid-points x_i	f_ix_i	$\lvert x_i - \bar{x} \rvert$	$f_i \lvert x_i - \bar{x} \rvert$
0 - 100	4	50	200	308	1232
100 - 200	8	150	1200	208	1664
200 - 300	9	250	2250	108	972
300 - 400	10	350	3500	8	80
400 - 500	7	450	3150	92	644
500 - 600	5	550	2750	192	960
600 - 700	4	650	2600	292	1168
700 - 800	3	750	2250	392	1176
Total	50		17900		7896

Here n = Σf_i = 50, $\Sigma f_i x_i$ = 17900

$\therefore$ Mean = $\bar{x} = \frac{1}{n}\Sigma f_i x_i = \frac{17900}{50}$ = 358

M.D. $(\bar{x}) = \frac{1}{n}\Sigma f_i \lvert x_i - \bar{x} \rvert = \frac{7896}{50}$ = 157.92

35. Given circle is $2x^2 + 2y^2 + 4y = 5$

$\Rightarrow x^2 + y^2 + 2y - \frac{5}{2} = 0$

Centre is (0, -1).

radius = $\sqrt{(1-0)^2 + (-2+1)^2} = \sqrt{2}$

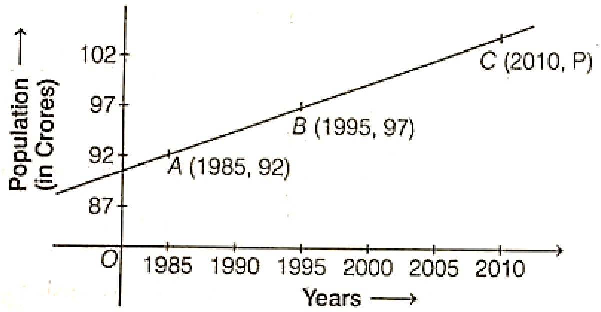
$\therefore$ circle is $(x - 1)^2 + (y + 2)^2 = 2$

$\Rightarrow x^2 + y^2 - 2x + Ay + 3 = 0$

Section E

36. **Read the text carefully and answer the questions:**

Population vs Year graph given below.



(i) Equation of line AB is,

$x - 2y = 1801$

Putting $y = 110$,

$\therefore x = 1801 + 220$

$\Rightarrow x = 2021$

(ii) $\because$ Slope of AB = $\frac{1}{2}$

Slope of the perpendicular of AB = $\frac{-1}{\frac{1}{2}}$ = -2

$\therefore$ Equation of line perpendicular to AB passing through (1995, 97) is

$\Rightarrow y - 97 = -2(x - 1995)$

$\Rightarrow y - 97 = -2x + 3990$

$\Rightarrow 2x + y = 4087$

(iii) Equation of line AB is,

$\Rightarrow y - y_1 = m(x - x_1)$

$\therefore y - 92 = \frac{1}{2}(x - 1985)$

$2y - 184 = x - 1985$

$\Rightarrow x - 2y = 1801$

OR

Slope of line AB joining points A(1985, 92) and B(1995, 97)

$m = \frac{97-92}{1995-1985} = \frac{5}{10} = \frac{1}{2}$

37. **Read the text carefully and answer the questions:**

In class of Statistics, teacher was discussing the concept of Measures of Correlation, in which he was discussing about Karl Pearson's Coefficient of Correlation. During his class, he discussed the following few points on this:

This is the best method for finding correlation between two variables provided the relationship between the two variables is linear. This method is also known as product moment correlation coefficient. Pearson's correlation coefficient may be defined as the ratio of covariance between the two variables to the product of the standard deviations of the two variables.

If the two variables are denoted by x and y and of the corresponding bivariates data are (x_i, y_i) for i = 1, 2, 3, ..., n, then the coefficent of correlation between x and y due to Karl Pearson, is given by:

$r = r_{xy}$

or, $r_{xy} = \frac{\text{Cov}(x,y)}{\sqrt{\text{Var } x} \cdot \sqrt{\text{Var } y}}$

$= \frac{\text{Cov}(x,y)}{\sigma_x \cdot \sigma_y}$

where,

$\text{cov}(x, y) = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{N}$

$= \frac{\Sigma xy}{N} - \bar{x} \cdot \bar{y}$

$\sigma_x = \sqrt{\frac{\Sigma(x-\bar{x})^2}{N}} = \sqrt{\frac{\Sigma x^2}{N} - x^{-2}}$

$\sigma_y = \sqrt{\frac{\Sigma(y-\bar{y})^2}{N}} = \sqrt{\frac{\Sigma y^2}{N} - y^{-2}}$

If x - $\bar{x} - \bar{y}$ are small fractions, we use

$r = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{\Sigma(x-\bar{x})^2}\sqrt{\Sigma(y-\bar{y})^2}}$

If x, y are small numbers, we use

r = $\frac{\Sigma xy - \frac{1}{N}\Sigma x \Sigma y}{\sqrt{\Sigma x^2 - \frac{1}{N}(\Sigma x)^2}\sqrt{\Sigma y^2 - \frac{1}{N}(\Sigma y)^2}}$

If x, y are large numbers, we use assumed mean A and B and u = x - A, v = y - B

r = $\frac{\Sigma uv - \frac{1}{N}\Sigma u \Sigma v}{\sqrt{\Sigma u^2 - \frac{1}{N}(\Sigma u)^2}\sqrt{\Sigma v^2 - \frac{1}{N}(\Sigma v)^2}}$

For example:

Find Karl Pearson's coefficient of correlation between X and Y for the following:

x	5	4	3	2	1
y	4	2	10	8	6

Following problem was given to students on the same concept:

(i)

x	**y**	**xy**
5	4	20
4	2	8
3	10	30
2	8	16
1	6	6
Total		**80**

(ii)

x	**x^2**
5	25
4	16
3	9
2	4
1	1
Σx^2	**55**

(iii)

y	**y^2**
4	16
2	4
10	100
8	64
6	36
Σy^2	220

OR

x	**x^2**	**y**	**y^2**	**xy**
5	25	4	16	20
4	16	2	4	8
3	9	10	100	30
2	4	8	64	16
1	1	6	36	6
$\sum x$ = 15	$\sum x^2$ = 55	$\sum y$ = 30	$\sum y^2$ = 220	$\sum xy$ = 80

$= \dfrac{\Sigma xy - \frac{1}{N}\Sigma x \Sigma y}{\sqrt{\Sigma x^2 - \frac{(\Sigma x)^2}{N}}\sqrt{\Sigma y^2 - \frac{(\Sigma y)^2}{N}}}$

$= \dfrac{80 - \frac{1}{5} \times 15 \times 30}{\sqrt{55 - \frac{(15)^2}{5}}\sqrt{220 - \frac{(30)^2}{5}}}$

$= \dfrac{-10}{\sqrt{10}\sqrt{40}} = \dfrac{-10}{20} = -0.5$

Therefore, Karl Pearson's Coefficient of Correlation between x and y is -0.5.

38. **Read the text carefully and answer the questions:**

A mobile number is having 10 digits. It is not just a group of numbers strung out at random. All mobile numbers have 3 things in common, a 2-digit Access code (AC), a 3-digit provider code (PC), and a 5 digit subscriber code (SC). AC code and PC code are fixed, then

(i) 98073 V IV III II I

The digits which can be used are, 6, 5, 4, 2, 1

Number of ways to fill the 5 places

= 5! = 120

(ii) Digits which can be used in AC code are 7, 8, 9

Total AC code = $^3P_2$ = 3! = 6

(iii) If AC and PC are fixed then only 5 digits is to be filled if digits can repeat then total ways = 100000 = 10^5

(iv) Total ways = $5 \times 5 \times 5 \times 5 \times 5$ = 3125

OR

Read the text carefully and answer the questions:

Five friends Mohit, Sachin, Rohit, Mohan and kapil were playing in a ground, where they sit in a row in a straight line.

(i) Total number of ways = 5! = 120

(ii) Two position are fixed for Mohit and Sachin therefore considering it as one unit, total students left = 3 + 1 = 4

Total possible arrangement = $4! \times 2!$ = 48

(iii) Total possible arrangements = $3! \times 2!$ = 12

(iv) Total possible arrangements = 4! = 24