Linear Programming

Short Answer Type Questions

1. Determine the maximum value of $Z = 11x + 7y$ subject to the constraints:
$2x + y \le 6, x \le 2, x \ge 0, y \ge 0$.

Sol. We have to maximize $Z = 11x + 7y$ ...(i)

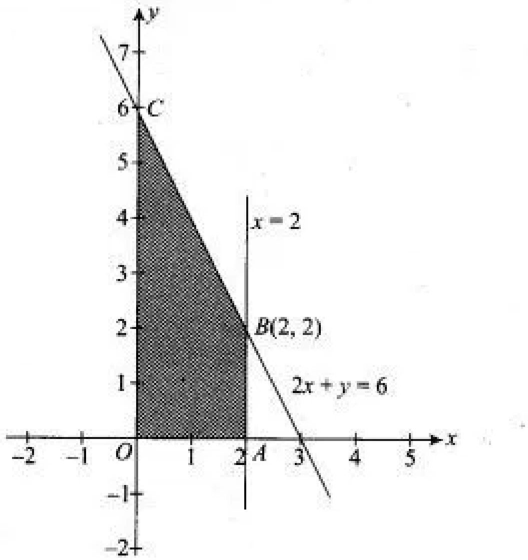
Subject to the constrains

$2x + y \le 6$ ...(ii)

$x \le 2$ ...(iii)

$x \ge 0, y \ge 0$ ...(iv)

These inequalities are plotted as shown in the following figure.

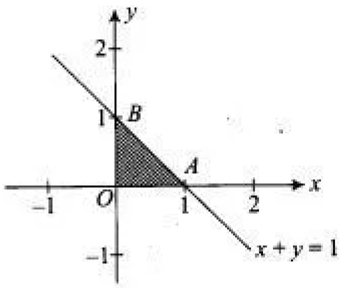


The shaded region shown in the figure as $OABC$ is bounded and the corner points are $O(0, 0)$, $A(2, 0)$, $B(2, 2)$ and $C(0, 6)$.

Corner points	Corresponding value of Z
(0, 0)	0
(2, 0)	22
(2, 2)	36
(0, 6)	**42 (maximum)**

Hence, maximum value of Z is 42 at (0, 6).

2. Maximize $Z = 3x + 4y$, subject to the constraints: $x + y \le 1, x \ge 0, y \ge 0$.



Sol. Maximize $Z = 3x + 4y$, Subject to the constraints

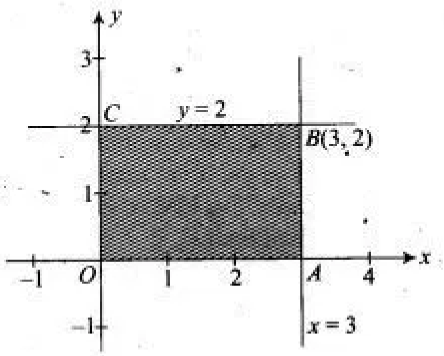
$$x + y \le 1, x \ge 0, y \ge 0.$$

These inequalities are plotted as shown in the adjacent figure.
The shaded region shown in the figure as OAB is bounded corner points are $O(0, 0)$, $A(1, 0)$ and $B(0, 1)$.

Corner points	Corresponding value of Z
(0, 0)	0
(1, 0)	3
(0, 1)	**4 (maximum)**

Hence, the maximum value of Z is 4 at (0, 1)

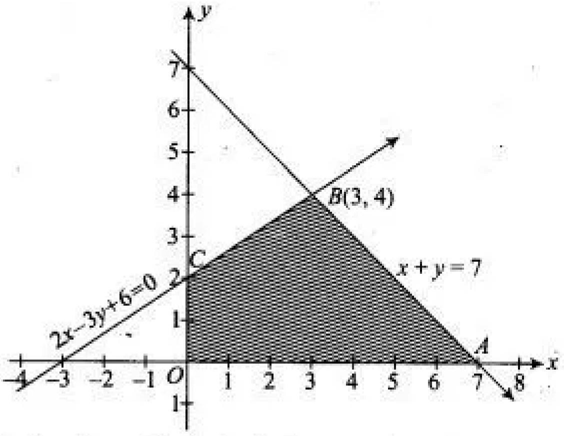
3. Maximize the function $Z = 11x + 7y$, subject to the constraints: $x \le 3, y \le 2, x \ge 0, y \ge 0$.



Sol. We have to maximize $Z = 11x + 7y$, subject to the constraints $x \le 3, y \le 2, x \ge 0, y \ge 0$.
These inequalities are plotted as shown in the adjacent figure.
The shaded region as shown in the figure as $OABC$ is bounded and the corner points are $O(0, 0)$, $A(3, 0)$, $B(3, 2)$ and $C(0, 2)$ respectively.

Corner points	Corresponding value of Z
(0, 0)	0
(3, 0)	33
(3, 2)	**47 (maximum)**
(0, 2)	14

4. Minimize Z = 13x- 15y subject to the constraints: x+y≤ 7,2x-3y + 6 ≥ 0, x ≥ 0, y ≥ 0.
Sol. We have to Minimize Z = 13x – 15y subject to the constraints x + y ≤7, 2x – 3y + 6 ≥ 0, x ≥ 0, y ≥ 0. These inequalities are plotted as shown in the following figure.

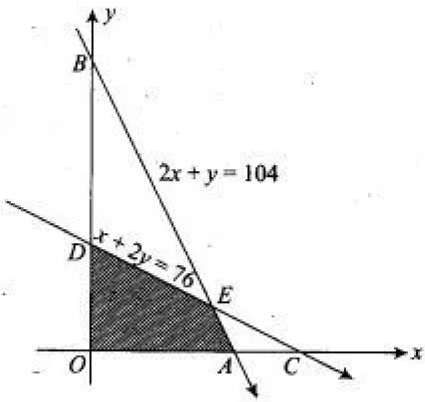


The shaded region as shown in the figure as $OABC$ is bounded and the corner points are O(0, 0), A(7, 0), B(3, 4) and C(0, 7) respectively.

Corner points	Corresponding value of Z
(0, 0)	0
(7, 0)	91
(3, 4)	−21
(0, 2)	**−30 (Minimum)**

Hence, the minimum value of Z is (−30) at (0, 2)

5. Determine the maximum value of $Z = 3x + 4y$ if the feasible region (shaded) for a LPP is shown in the adjacent figure.



Sol. Lines $x + 2y = 76$ and $2x + y = 104$ intersect at E(44, 16)

From the graph, corner points are O(0, 0), A(52, 0), E(44, 16) and D(0, 38).

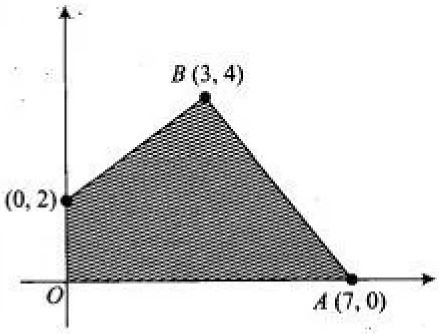
Also, given region is bounded.

Here, $Z = 3x + 4y$

Corner points	Corresponding value of Z
(0, 0)	0
(52, 0)	156
(44, 16)	**196 (maximum)**
(0, 38)	152

Hence, Z is maximum at (44, 16).

6. Feasible region (shaded) for a LPP is shown in the adjacent figure. Maximize $Z = 5x + 7y$.

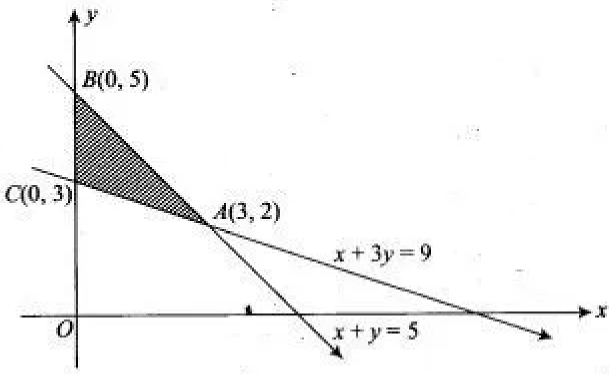


Sol. The shaded region is bounded with corner points as $O(0, 0)$, $A(7, 0)$, $B(3, 4)$ and $C(0, 2)$.
Also, $Z = 5x + 7y$.

Corner points	Corresponding value of Z
(0, 0)	0
(7, 0)	35
(3, 4)	**43 (Maximum)**
(0, 2)	14

Hence, the maximum value of Z occurs at (3, 4)

7. The feasible region for a LPP is shown in the figure. Find the minimum value of $Z = 11x + 7y$.



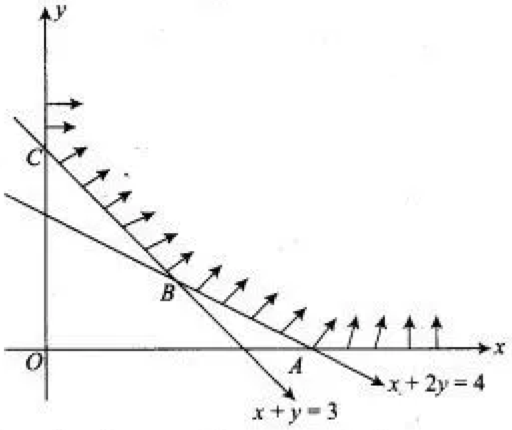
Sol. Lines $x + y = 5$ and $x + 3y = 9$ intersect at (3, 2).
From the figure, the feasible region is bounded with corner points as $C(0, 3)$, $A(3, 2)$ and $B(0, 5)$.
Also $Z = 11x + 7y$

Corner points	Corresponding value of Z
(0, 3)	**21 (Minimum)**
(3, 2)	47
(0, 5)	35

Hence, the minimum values of Z is 21 at (0, 3)

8. Refer to Exercise 7 above. Find the maximum value of Z.
Sol. Z is maximum at (3,2) and its maximum value is 47.

9. The feasible region for a LPP is shown in the following figure. Evaluate Z = 4x+y at each of the comer points of this region. Find the minimum value of Z, if it exists.

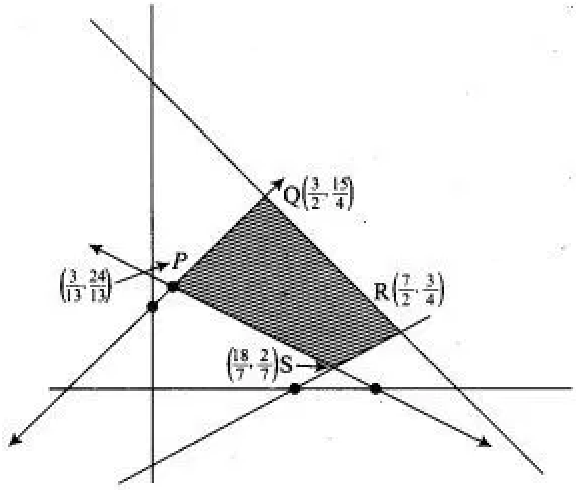


Sol. Lines $x + 2y = 4$ and $x + y = 3$ intersect at (2, 1)
From the figure it is unbounded shaded region with the corner points $A(4, 0)$, $B(2, 1)$ and $C(0, 3)$
Also, we have $Z = 4x + y$.

Corner points	Corresponding value of Z
(4, 0)	16
(2, 1)	9
(0, 3)	**3 ← maximum**

Now, we see that 3 is the smallest value of Z the comer point (0, 3). Note that here we see that, the region is unbounded, therefore 3 may not be the minimum value of Z. To decide this issue, we graph the inequality 4x + y < 3 and check whether the resulting open half plane has no point in common with feasible region otherwise, Z has no minimum value.
From the shown graph above, it is clear that there is no point in common with feasible region and hence Z has minimum value 3 at (0, 3).

10. In the following figure, the feasible region (shaded) for a LPP is shown. Determine the maximum and minimum value of Z = x + 2y



Sol. From the figure we have bounded region with corner points as

$P\left(\frac{3}{13},\frac{24}{13}\right), Q\left(\frac{3}{2},\frac{15}{4}\right), R\left(\frac{7}{2},\frac{3}{4}\right), S\left(\frac{18}{7},\frac{2}{7}\right)Q.$

Also $Z = x + 2y$.

Corner points	Corresponding value of Z
$\left(\frac{3}{13},\frac{24}{13}\right)$	$\frac{51}{13}=3\frac{12}{13}$
$\left(\frac{18}{7},\frac{2}{7}\right)$	$\frac{22}{7}=3\frac{1}{7}$ **(Minimum)**
$\left(\frac{7}{2},\frac{3}{4}\right)$	$\frac{20}{4}=5$
$\left(\frac{3}{2},\frac{15}{4}\right)$	$\frac{36}{4}=9$ **(Maximum)**

Hence, the maximum and minimum value of Z are 9 and $3\frac{1}{7}$, respectively

11. A manufacturer of electronic circuits has a stock of 200 resistors, 120 transistors and 150 capacitors and is required to produce two types of circuits A and B. Type A requires 20 resistors, 10 transistors and 10 capacitors. Type B requires 10 resistors, 20 transistors and 30 capacitors. If the profit on type A circuit is Rs 50 and that on type B circuit is Rs 60, formulate this problem as a LPP so that the manufacturer can maximise his profit.

Sol. Let the manufacture produces x units of type A circuits and y units of type B circuits. Form the given information, we have following corresponding constraint table.

	Type A (x)	Type B (y)	Maximum stock
Resistors	20	10	200
Transistors	10	20	120
Capacitors	10	30	150
Profit	₹50	₹60	

Thus, profit $Z = 50x + 60y$

Now, we have the following mathematical model for the given problem.

Maximize $Z = 50x + 60y$ ...(i)

Subject to the constraints.

$20x + 10y \le 200$ [resistors constraint]

$\Rightarrow$ $2x + y \le 20$ ...(ii)

and $10x + 20y \le 120$ [transistor constraint]

$\Rightarrow$ $x + 2y \le 12$ ...(iii)

and $10x + 30y \le 150$ [capacitor constant]

$\Rightarrow$ $x + 3y \le 15$ ...(iv)

and $x \ge 0, y \ge 0$ [non–negative constant] ...(v)

So, maximize $Z = 50x + 60y$, subject to $2x + y \le 20$, $x + 2y \le 12$, $x + 3y \le 15$, $x \ge 0$, $y \ge 0$.

12. A firm has to transport 1200 packages using large vans which can carry 200 packages each and small vans which can take 80 packages each. The cost for engaging each large van is Rs 400 and each small van is Rs 200. Not more than Rs 3000 is to be spent on the job and the number of large vans can not exceed the number of small vans. Formulate this problem as a LPP given that the objective is to minimise cost.

Sol. Let the firm has x number of large vans and y number of small vans.

From the given information, we have following corresponding constraint table.

	Large van (x)	Small vans (y)	Maximum/Minimum
Package	200	80	1200
Cost	400	200	3000

Thus, objective function for minimum cost is $Z = 400x + 200y$.

Subject to constraints

$200x + 80y \ge 1200$

$\Rightarrow$ $5x + 2y \ge 30$ ...(i)

and $400x + 200y \le 3000$

$\Rightarrow$ $2x + y \le 15$ ...(ii)

and $x \le y$ ...(iii)

and $x \ge 0, y \ge 0$ ...(iv)

Thus, required LPP to minimize cost is minimize $Z = 400x + 200y$, subject to

$5x + 2y \ge 30$

$2x + y \le 15$

$x \le y$

$x \ge 0, y \ge 0$

13. A company manufactures two types of screws A and B. All the screws have to pass through a threading machine and a slotting machine. A box of Type A screws requires 2 minutes on the threading machine and 3 minutes on the slotting machine. A box of type B screws requires 8 minutes of threading on the threading machine and 2 minutes on the slotting machine. In a week, each machine is available for 60 hours. On selling these screws, the company gets a profit of Rs 100 per box on type A screws and Rs 170 per box on type B screws. Formulate this problem as a LPP given that the objective is to maximise profit.

Sol. Let the company manufactures x boxes of type A screws and y boxes of type B screws.

From the given information, we have following corresponding constraint table.

	Type A (x)	Type B (y)	Maximum time available on each machine in a week
Time required for screws on threading machine	2	8	60×60 (min)
Time required for screws on slotting machine	3	2	60×60 (min)
Profit	**₹100**	**₹170**	

Thus, the objective function for maximum profit is $Z = 100x + 170y$.
Subject to constraints

$2x + 8y \le 60 \times 60$ [time constraint for threading machine]

$\Rightarrow$ $x + 4y \le 1800$ ...(i)

and $3x + 2y \le 60 \times 60$ [time constraint for slotting machine]

$\Rightarrow$ $3x + 2y \le 3600$ ...(ii)

Also, $x \ge 0, y \ge 0$ [non-negative constraints] ...(iii)

$\therefore$ Required LLP is,
Maximize $Z = 100x + 170y$, subject to constraints

$x + 4y \le 1800$, $3x + 2y \le 3600$, $x \ge 0$, $y \ge 0$

14. A company manufactures two types of sweaters: type A and type B. It costs Rs 360 to make a type A sweater and Rs 120 to make a type B sweater. The company can make at most 300 sweaters and spend at most Rs 72000 a day. The number of sweaters of type B cannot exceed the number of sweaters of type A by more than 100. The company makes a profit of Rs 200 for each sweater of type A and ?120 for every sweater of type B. Formulate this problem as a LPP to maximise the profit to the company.

Sol. Let the company manufactures x number of type A sweaters and y number of type B.
The company spend at most Rs 72000 a day.
$\therefore$ $360x + 120y \le 72000$
=> $3x+y \le 600$ ...(i)
Also, company can make at most 300 sweaters.
$\therefore$ $x+y \le 300$ ...(ii)
Also, the number of sweaters of type B cannot exceed the number of sweaters of type A by more than 100 i.e., $y-x \le 100$
The company makes a profit of Rs 200 for each sweater of type A and Rs 120 for every sweater of type B
So, the objective function for maximum profit is $Z = 200x + 120y$ subject to constraints.
$3x+y \le 600$
$x+y \le 300$
$x-y \ge -100$
$x \ge 0$, $y \ge 0$

15. A,man rides his motorcycle at the speed of 50 km/hour. He has to spend Rs 2 per km on petrol. If he rides it at a faster speed of 80 km/hour, the petrol cost increases to Rs 3 per km. He has atmost Rs120 to spend on petrol and one hour's time. He wishes to find the maximum distance that he can travel. Express this problem as a linear programming problem.

Sol. Let the man rides to his motorcycle to a distance x km at the speed of 50 km/h and to a distance y km at the speed of 80 km/h.
Therefore, cost on petrol is $2x + 3y$.
Since, he has to spend Rs120 atmost on petrol.
$\therefore$ $2x + 3y \le 120$ ...(i)

Also, he has at most one hour's time.

$\therefore \quad \frac{x}{50} + \frac{y}{80} \le 1$

$\Rightarrow \quad 8x + 5y \le 400 \quad \ldots(ii)$

Also, $x \ge 0, y \ge 0$

So, the objective function for maximize $Z = x + y$ subject to constraints.

$$2x + 3y \le 120, 8x + 5y \le 400, x \ge 0, y \ge 0$$

Long Answer Type Questions

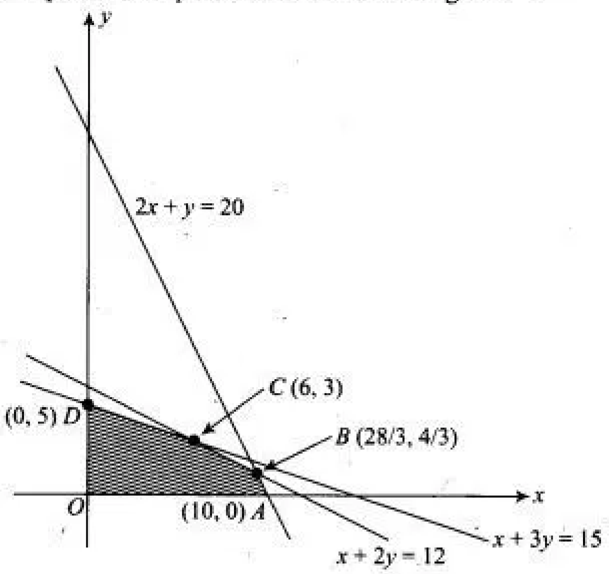
16. Refer to Exercise 11. How many of circuits of Type A and of Type B, should be produced by the manufacturer so as to maximize his profit? Determine the maximum profit.

Sol. Referring to solution 11, we have

Maximize $Z = 50x + 60y$, subject to

$$2x + y \le 20, x + 2y \le 12, x + 3y \le 15, x \ge 0, y \ge 0$$

These inequalities are plotted as shown in the figure.



From the figure shaded region is bounded with the corner points $O(0, 0)$, $A(10, 0)$, $B(28/3, 4/3)$, $C(6, 3)$ and $D(0,5)$.

Corner Points	**Corresponding value of $Z = 50x + 60y$**
(0, 0)	0
(10, 0)	500
$\left(\frac{28}{3}, \frac{4}{3}\right)$	$\frac{1400}{3} + \frac{240}{3} = \frac{1640}{3} = 546.66$ **(Maximum)**
(6, 3)	480
(0, 5)	300

Since, the manufacture is required to produce two type of circuits A and B and it is clear that parts of resistor, transistor and capacitor cannot be in fraction, so the required maximum profit is 480 where circuits of type A is 6 and circuits of type B is 3.

17. Refer to Exercise 12. What will be the minimum cost?

Sol. Referring to solution 12, we have minimize

$Z = 400x + 200y$, subject to

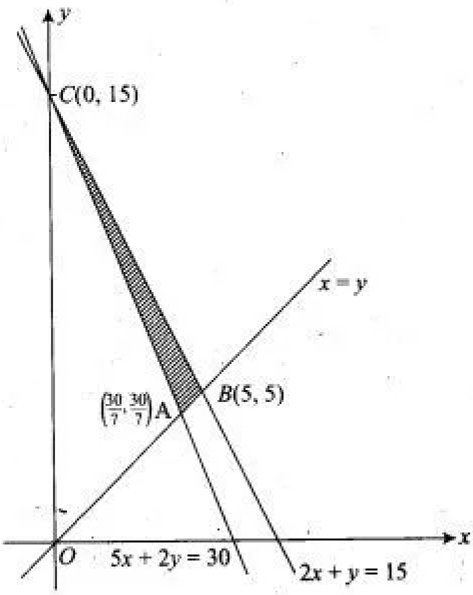
$5x + 2y \geq 30$

$2x + y \leq 15$

$x \leq y$

$x \geq 0, y \geq 0$

These inequalities are plotted as shown in the adjacent figure.

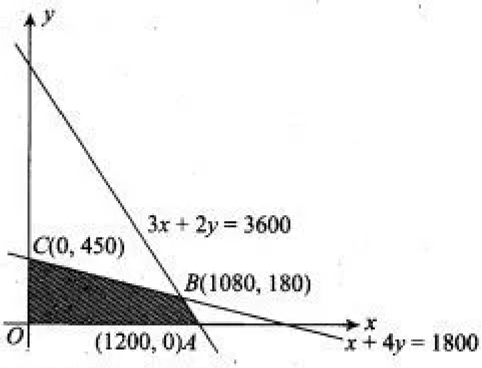


From the figure shaded region is bounded with the corner points $A(30/7, 30/7)$, $B(5, 5)$ and $C(0, 15)$.

Corner Points	**Corresponding value of $Z = 400x + 200y$**
(0, 15)	3000
(5, 5)	3000
$\left(\frac{30}{7}, \frac{30}{7}\right)$	$400 \times \frac{30}{7} + 200 \times \frac{30}{7} = \frac{18000}{7} = 2571.43$ **(Minimum)**

Hence, the minimum cost is ₹2571.43.

18. Refer to Exercise 13. Solve the linear programming problem and determine the maximum profit to the manufacturer.



Sol. Referring to solution 13, we have

Maximize $Z = 100x + 170y$, subject to constraints

$x + 4y \leq 1800$,

$3x + 2y \leq 3600, x \geq 0, y \geq 0$

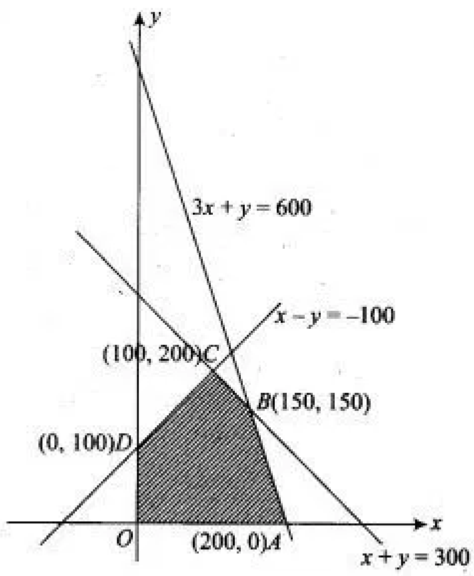
These inequalities are plotted as shown in the figure.

From the figure shaded region is bounded with the corner points $A(1200, 0)$, $B(1080, 18)$ and $C(0, 450)$.

Corner Points	Corresponding value of $Z = 100x + 170y$
(0, 0)	0
(1200, 0)	120000
(1080, 180)	138600 **(Maximum)**
(0, 450)	46500

Hence, the maximum profit to the manufacture is 138600.

19. Refer to Exercise 14. How many sweaters of each type should the company make in a day to get a maximum profit? What is the maximum profit?



Sol. Referring to solution 14, we have maximize $Z = 200x + 120y$, subject to constraints

$3x + y \leq 600$

$x + y \leq 300$

$x - y \geq -100$

$x \geq 0, y \geq 0$

These inequalities are plotted as shown in the figure.

From the figure shaded region is bounded with the corner points $O(0, 0)$, $A(200, 0)$, $B(150, 150)$, $C(100, 200)$ and $D(0, 100)$

Corner Points	Corresponding value of $Z = 200x + 120y$
(0, 0)	0
(200, 0)	40000
(150, 150)	48000 **(Maximum)**
(100, 200)	44000
(0, 100)	12000

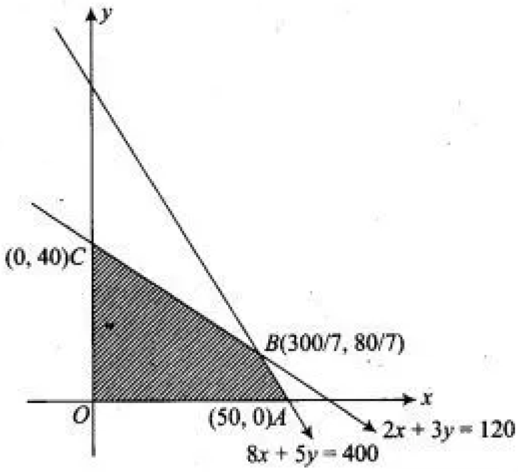
Hence, 150 sweaters of each type made by company and maximum profit = ₹48000.

20. Refer to Exercise 15. Determine the maximum distance that the man can travel.

Sol. Referring to solution 15, we have to maximize $Z = x + y$ subject to constraints.

$2x + 3y \le 120$, $8x + 5y \le 400$, $x \ge 0$, $y \ge 0$

These inequalities are plotted as shown in the following figure.



From the figure shaded region is bounded with the corner points $O(0, 0)$, $A(50, 0)$, $B(300/7, 80/7)$, $C(0, 40)$.

Corner Points	Corresponding value of $Z = x + y$
(0, 0)	0
(50, 0)	50
$\left(\frac{300}{7}, \frac{80}{7}\right)$	$\frac{380}{7} = 54\frac{2}{7}$ km **(Maximum)**
(0, 40)	40

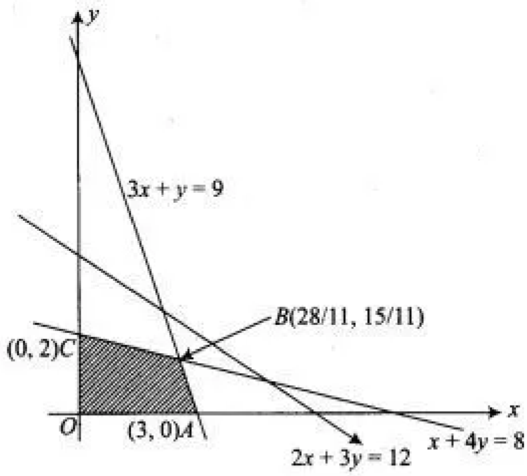
Hence, the maximum distance that the man can travel is $54\frac{2}{7}$ km.

21. Maximize $Z = x + y$ subject to $x + 4y \le 8$, $2x + 3y \le 12$, $3x + y \le 9$, $x \ge 0$, $y \ge 0$.

Sol. We have to maximize $Z = x + y$ under constraints

$x + 4y \le 8$,
$2x + 3y \le 12$,
$3x + y \le 9$,
$x \ge 0, y \ge 0$

These inequalities are plotted as shown in the figure.



From the figure feasible region is bounded with corner points $O(0, 0)$, $A(3, 0)$, $B\left(\frac{28}{11}, \frac{15}{11}\right)$ and $C(0, 2)$.

Corner Points	Value of $Z = x + y$
(0, 0)	0
(3, 0)	3
$\left(\frac{28}{11}, \frac{15}{11}\right)$	$\frac{43}{11} = 3\frac{10}{11}$ **(Maximum)**
(0, 2)	2

Hence, the maximum value is $3\frac{10}{11}$.

22. A manufacturer produces two Models of bikes'-Model X and Model Y. Model X takes a 6 man-hours to make per unit, while Model Y takes 10 man-hours per unit. There is a total of 450 man-hour available per week. Handling and Marketing costs are Rs 2000 and Rs 1000 per unit for Models X and Y respectively. The total funds available for these purposes are Rs 80,000 per

week. Profits per unit for Models X and Y are Rs 1000 and Rs 500, respectively. How many bikes of each model should the manufacturer produce so as to yield a maximum profit? Find the maximum profit.

Sol. Let the manufacture produces x number of models X and y number of model Y bikes.

Model X takes a 6 man–hours to make per unit and model Y takes a 10 man–hours to make per unit.

There is total of 450 man–hour available per week.

$\therefore \quad 6x + 10y \leq 450$

$\Rightarrow \quad 3x + 5y \leq 225 \quad \ldots(i)$

For models X and Y, handing and marketing costs are ₹2000 and ₹1000, respectively, total funds available for these purposes are ₹80000 per week.

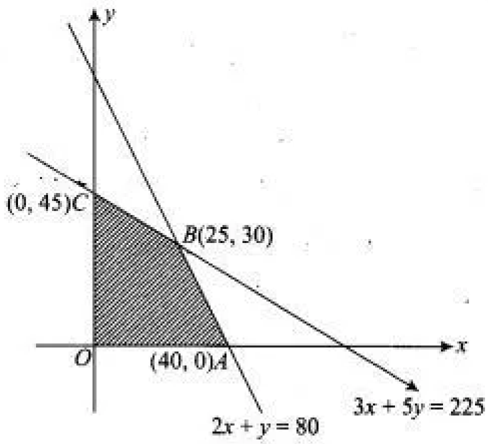
$\therefore \quad 2000x + 1000y \leq 80000$

$\Rightarrow \quad 2x + y \leq 80 \quad \ldots(ii)$

Also $x \geq 0, y \geq 0$

The profits per unit for models X and Y are ₹1000 and ₹500, respectively.

So,we have to maximize $Z = 1000x + 500y$ subject to $3x + 5y \leq 225$, $2x + y \leq 80$, $x \geq 0$, $y \geq 0$



These inequalities are plotted as shown in the figure.

Corner Points	Value of $Z = 1000x + 500y$
(0, 0)	0
(40, 0)	40000 **(Maximum)**
(25, 30)	40000 **(Maximum)**
(0, 45)	22500

So, for maximum profit manufacture must produces 25 number of models X and 30 number of model Y bikes.

23. In order to supplement daily diet, a person wishes to take some X and some Y tablets. The contents of iron, calcium and vitamins in X and Y (in milligrams per tablet) are given as below:

Tablets	Iron	Calcium	Vitamin
X	6	3	2
Y	2	3	4

The person needs at least 18 milligrams of iron, 21 milligrams of calcium and 16 milligram of vitamins. The price of each tablet of X and Y is Rs 2 and Rs 1 respectively. How many tablets of each should the person take in order to satisfy the above requirement at the minimum cost?

Sol. Let the person takes x units of table X and y units of table Y.

So, from the given tabulated information, we have

$6x + 2y \geq 18$

$\Rightarrow \quad 3x + y \geq 9 \quad \ldots(i)$

$3x + 3y \geq 21$

$\Rightarrow \quad x + y \geq 7 \quad \ldots(ii)$

and $\quad 2x + 4y \geq 16$

$\Rightarrow \quad x + 2y \geq 8 \quad \ldots(iii)$

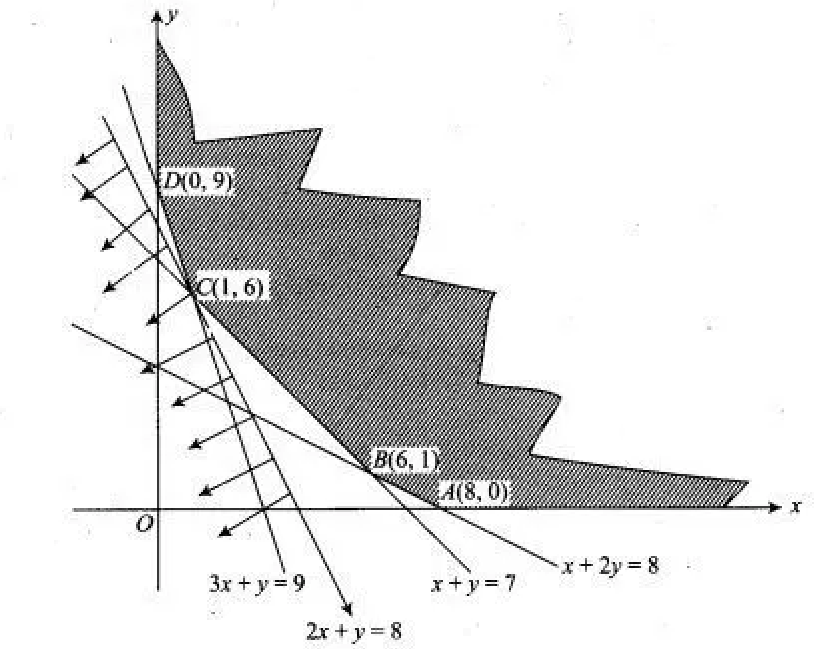
Also, $x \geq 0, y \geq 0 \quad \ldots(iv)$

The price of each tablet of X and Y is ₹2 and ₹1, respectively.

So, we have to minimize $Z = 2x + y$, subject to the constraints

$3x + y \geq 9, x + y \geq 7, x + 2y \geq 8, x \geq 0, y \geq 0$

These inequalities are plotted as shown in the following figure.



From the figure, we can see the feasible region is unbounded region, with comer points as A(%, 0), B(6,1), C(I, 6), and D(0, 9)

Corner Points	Value of $Z = 2x + y$
(8, 0)	16
(6, 1)	13
(1, 6)	8 **(Minimum)**
(0, 9)	9

Thus, the minimum value of Z is '8' occurring at B(1, 6). Since the feasible region is unbounded, '8' may not be the minimum value of Z. To decide this, we plot the inequality 2x+y < 8 and check whether the resulting open half has points common with feasible region or not If it has common point, then 8 will not be the minimum value of Z, otherwise 8 will be the minimum value of Z. Thus, from the graph it is clear that, it has no common point.

Therefore, Z= 2x+y-has 8 as minimum value subject to the given constrains. Hence, the person should take 1 unit of X tablet and 6 units of Y tablets to satisfy the given requirements and at the minimum cost of Rs 8.

24. A company makes 3 models of calculators: A, B and C at factory I and factory II. The company has orders for at least 6400 calculators of model A, 4000 calculator of model B and 4800 calculator of model C. At factory I, 50 calculators of model A, 50 of model B and 30 of model C are made every day; at factory II, 40 calculators of model A, 20 of model B and 40 of model C are made everyday. It costs Rs 12000 and Rs 15000 each day to operate factory I and II, respectively. Find the number of days each factory should operate to minimize the operating costs and still meet the demand.

Sol. Let the factory I operate for x days and the factor II operate for y days.

At factory I, 50 calculators of model A and at factory II, 40 calculators of model A are made everyday.

Also, company has ordered for atleast 6400 calculators of model A.

$\therefore \quad 50x + 40y \geq 6400 \Rightarrow 5x + 4y \geq 640 \quad \ldots(i)$

At factory I, 50 calculators of model B and at factory II, 20 calculators of modal B are made everyday.

Since, the company has ordered atleast 4000 calculators of model B.

$\therefore \quad 50x + 20y \geq 4000 \Rightarrow 5x + 2y \geq 400 \quad \ldots(ii)$

Also at factory I, 30 calculators of model C and at factory II, 40 calculators of modal C are made everyday.

Since, the company has ordered atleast 4800 calculators of model C.

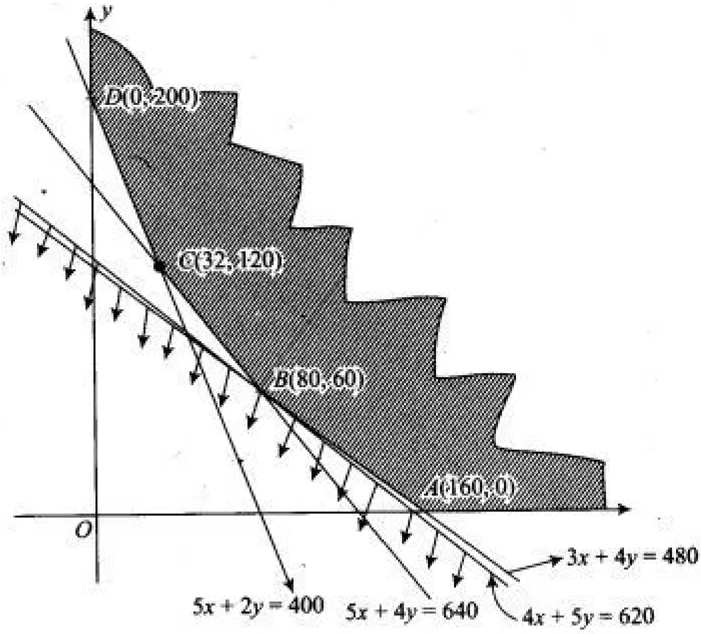
$\therefore \quad 30x + 40y \geq 4800 \Rightarrow 3x + 4y \geq 480 \quad \ldots(iii)$

Also, $x \geq 0, y \geq 0 \quad \ldots(iv)$

It costs ₹12000 and ₹15000 each day to operate factories I and II, respectively.

$\therefore$ We have to minimize $Z = 12000x + 15000y$, subject to the constraints

$$5x + 4y \geq 640$$
$$5x + 2y \geq 400$$
$$3x + 4y \geq 480$$
$$x \geq 0, y \geq 0$$



Thus, from the graph, the feasible region is unbounded with corner points as $A(160, 0)$, $B(80, 60)$, $C(32, 120)$ and $D(0, 200)$

Corner Points	Value of $Z = 12000x + 15000y$
(160, 0)	1920000
(80, 60)	1860000 **(Minimum)**
(32, 120)	2184000
(0, 200)	3000000

Thus the minimum value of Z may be 1860000.

Now, for deciding this, we plot the inequality $12000x + 15000y < 1860000$ or $4x + 5y < 620$ and check whether the resulting open half plane has points in common with feasible region or not.

Thus, as shown in the figure, it has no common points so, $Z = 12000x + 15000y$ has minimum value 186000

So, number of days factor I should be operated is 80 and number of days factory II should be operated is 60 for the minimum cost and satisfying the given constraints.

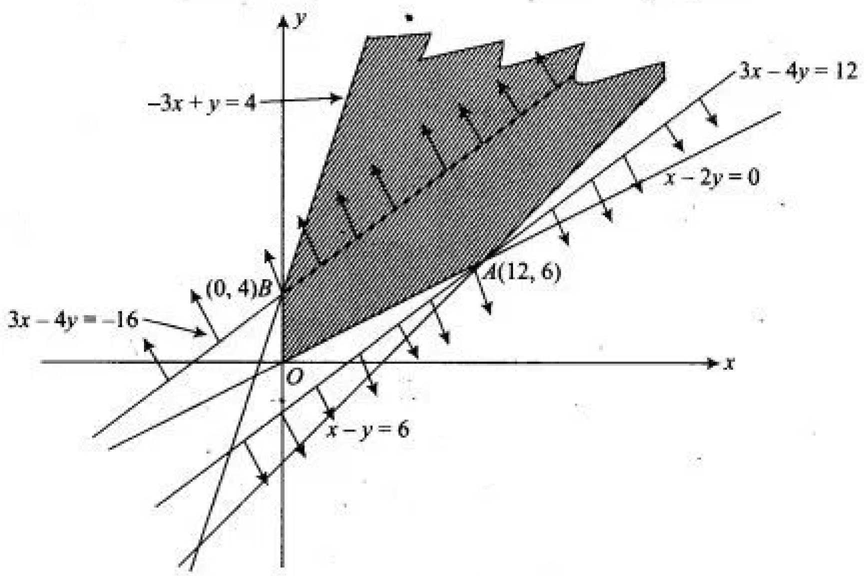
25. Maximize and Minimize $Z = 3x - 4y$ subject to

$$x - 2y \leq 0$$
$$-3x + y \leq 4$$
$$x - y \leq 6$$
$$x, y \geq 0$$

Sol. We have to maximize and minimize $Z = 3x - 4y$ subject to constraints

$$x - 2y \leq 0, -3x + y \leq 4, x - y \leq 6, x, y \geq 0$$

These inequalities are plotted as shown in the following figure.



From the figure we can see that the feasible region is unbounded with corner points $O(0, 0)$, $A(12, 6)$ and $B(0, 4)$.

Corner Points	Corresponding value of $Z = 3x - 4y$
(0, 0)	0
(0, 4)	−16
(12, 6)	12

For given unbounded region the minimum value of Z may or may not be −16. So, for deciding this, we plot the inequality $3x - 4y < -16$ and check whether the resulting open half plane has common points with feasible region or no. Thus, from the figure it has common points with feasible region, so it does not have any minimum value.

Also, similarly for maximum value, we plot the inequality $3x - 4y > 12$ and see that resulting open half plane has no common points with the feasible region and hence maximum value 12 exist for $Z = 3x - 4y$.

Objective Type Questions

26. The comer points of the feasible region determined by the system of linear constraints are (0,0), (0,40), (20,40), (60,20), (60,0). The objective function is Z=4x + 3y.

Compare the quantity in Column A and Column B

Column A	Column B
Maximum of Z	**325**

(a) The quantity in column A is greater

(b) The quantity in column B is greater

(c) The two quantities are equal

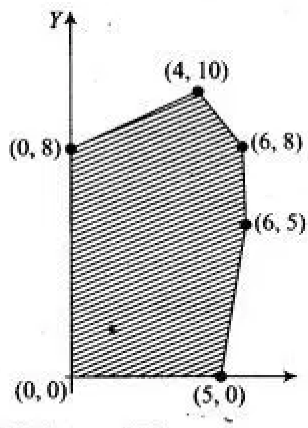
(d) The relationship can not be determined on the basis of the information supplied .

Sol. (b)

Corner Points	Corresponding value of $Z = 4x + 3y$
(0, 0)	0
(0, 40)	120
(20, 40)	200
(60, 20)	300 **(Maximum)**
(60, 0)	240

Hence, maximum value of $Z = 300 < 325$
So, the quantity in column B is greater.

27. The feasible solution for a LPP is shown in the following figure. Let $Z = 3x - 4y$ be the



objective function. Minimum of Z occurs at
(a) (0, 0) (b) (0, 8) (c) (5, 0) (d) (4, 10)

Sol. **(b)**

Corner Points	Corresponding value of $Z = 3x - 4y$
(0, 0)	0
(5, 0)	15 **(Maximum)**
(6, 5)	–2
(6, 8)	–14
(4, 10)	–28
(0, 8)	–32 **(Minimum)**

Hence, the minimum of Z occurs at (0, 8) and its minimum value is (–32)

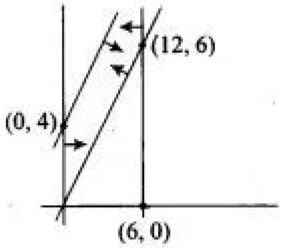
28. Refer to Exercise 27. Maximum of Z occurs at
(a) (5,0) (b) (6,5) (c) (6, 8) (d) (4, 10)
Sol. (a) Refer to solution 27, maximum of Z occurs at (5, 0)
29. Refer to Exercise 27. (Maximum value of Z + Minimum value of Z) is equal to
(a) 13 (b) 1 (c) -13
Sol. (d) Refer to solution 27,

maximum value of Z + minimum value of Z = 15-32 = -17

30. The feasible region for an LPP is shown in the adjacent figure. Let $F = 3x - 4y$ be the objective function. Maximum value of F is.

(a) 0 (b) 8

(c) 12 (d) −18



Sol. **(c)** The feasible region as shown in the figure, has objective function $F = 3x - 4y$.

Corner Points	Corresponding value of $F = 3x - 4y$
(0, 0)	0
(12, 6)	12 **(Maximum)**
(0, 4)	−16 **(Minimum)**

Hence, the maximum value of F is 12.

31. Refer to Exercise 30. Minimum value of F is
(a) 0 (b) -16 . (c) 12 (d) does not exist
Sol. (b) Referring to solution 30, minimum value of F is -16 at (0,4).

32. Comer points of the feasible region for an LPP are (0, 2), (3, 0), (6,0), (6, 8) and (0,5).
Let F = 4x + 6y be the objective function.
The Minimum value of F occurs at
(a) (0,2) only
(b) (3,0) only
(c) the mid point of the line segment joining the points (0,2) and (3,0) only
(d) any point on the line segment joining the points (0,2) and (3, 0).
Sol. (d)

Corner Points	Corresponding value of $F = 4x + 6y$
(0, 2)	12 **(Minimum)**
(3, 0)	12 **(Minimum)**
(6, 0)	24
(6, 8)	72 **(Maximum)**
(0, 5)	30

33. Refer to Exercise 32, Maximum of F – Minimum of F=
(a) 60 (b) 48 (c) 42 (d) 18
Sol. (a) Referring to the solution 32, maximum,of F-minimum of F= 72- 12 = 60

34. Comer points of the feasible region determined by the system of linear constraints are (0, 3), (1, 1) and (3, 0). Let Z = px + qy, where p,q>0. Condition on p and q so that the minimum of Z occurs at (3, 0) and (1,1) is
(a) p = 2q (b)P=q\2 (c)p = 3q (d) p = q
Sol. (b)

Corner Points	Corresponding value of $X = px + qy; p, q > 0$
(0, 3)	$3q$
(1, 1)	$p + q$
(3, 0)	$3p$

So, condition of p and q, so that the minimum of Z occurs at (3, 0) and (1, 1) is

$$p + q = 3p \Rightarrow 2p = q$$

$$\therefore \quad p = \frac{q}{2}$$

Fill in the Blanks Type Questions

35. In a LPP, the linear inequalities or restrictions on the variables are called————-.

Sol. In a LPP, the linear inequalities or restrictions on the variables are called linear constraints.

36. In a LPP, the objective function is always————.

Sol. In a LPP, objective function is always linear.

37. If the feasible region for a LPP is————, then the optimal value of the objective function Z= axH-fiy may or may not exist.

Sol. If the feasible region for a LPP is unbounded, then the optimal value of the objective function Z = ax + by may or may not exist.

38. In a LPP if the objective function Z=ax+ by has the same maximum value on two comer points of the feasible region, then every point on the line segment joining these two points give the same————value.

Sol. In a LPP, if the objective function Z = ax + by has the same maximum value on two comer points of the feasible region, then every point on the line segment joining these two points gives the same maximum value.

39. A feasible region of a system of linear inequalities is said to be————if it can be enclosed within a circle.

Sol. A feasible region of a system of linear inequalities is said to be bounded, if it can be enclosed within a circle.

40. A comer point of a feasible region is a point in the region which is the———— of two boundary lines.

Sol. A comer point of a feasible region is a point in the region which is the intersection of two boundary lines.

41. The feasible region for an LPP is always a————polygon.

Sol. The feasible region for an LPP is always a convex polygon.

True/False Type Questions

42. If the feasible region for a LPP is unbounded, maximum or minimum of the objective function Z = ax+ by may or may not exist.

Sol. True

43. Maximum value of the objective function Z = ax+ by in a LPP always occurs at only one comer point of the feasible region. ,

Sol. False

44. In a LPP, the minimum value of the objective function Z = ax+ by is always 0 if origin is one of the comer point of the feasible region.

Sol. False

45. In a LPP, the maximum value of the objective function Z = ax+ by is always finite.

Sol. True