# **Chapter 5. Factorisation**

## Exercise 5(A)

### **Solution 1:**

$$3a^2 - 9ab = 3a \times a - 3a \times 3b$$
  
=  $3a(a - 3b)$ 

### **Solution 2:**

[Taking (x + y) common from both terms]

$$=(x+y)[2(x+y)^2-6]$$

$$= 2(x+y)[(x+y)^2-3]$$

$$= 2(x + y)(x^2 + y^2 + 2xy - 3)$$

#### **Solution 3:**

Taking (2x - 3y) common from both terms

$$= (2x - 3y)[x^3 - x^2(2x - 3y)]$$

$$=x^{2}(2x-3y)[x-(2x-3y)]$$

$$= x^2(2x - 3y)[x - 2x + 3y]$$

$$= x^2(2x - 3y)[-x + 3y]$$

$$= x^2(2x - 3y)(3y - x)$$

## **Solution 4:**

Taking (2x - 5y) common from both terms

$$= (2x - 5y)[2(3x + 4y) - 6(x - y)]$$

$$=(2x - 5y)(6x + 8y - 6x + 6y)$$

$$=(2x - 5y)(8y + 6y)$$

$$=(2x - 5y)(14y)$$

$$=(2x - 5y)14y$$

#### **Solution 5:**

$$a^3 + a - 3a^2 - 3 = a(a^2 + 1) - 3(a^2 + 1)$$
  
=  $(a^2 + 1)(a - 3)$ .

#### **Solution 6:**

$$16 (a + b)^{2} - 4a - 4b = 16 (a + b)^{2} - 4 (a + b)$$
$$= 4 (a + b) [4 (a + b) - 1]$$
$$= 4 (a + b) (4a + 4b - 1)$$

### **Solution 7:**

$$a^4 - 2a^3 - 4a + 8 = a^3(a - 2) - 4(a - 2)$$
  
=  $(a^3 - 4)(a - 2)$ 

#### **Solution 8:**

$$ab - 2b + a^2 - 2a = b(a - 2) + a(a - 2)$$
  
=  $(a + b)(a - 2)$ 

## **Solution 9:**

$$ab(x^2 + 1) + x(a^2 + b^2) = abx^2 + ab + a^2x + b^2x$$
  
=  $ax(bx + a) + b(bx + a)$   
=  $(ax + b)(bx + a)$ 

### **Solution 10:**

$$a^{2} + b - ab - a = a^{2} - a + b - ab$$

$$= a(a - 1) + b(1 - a)$$

$$= a(a - 1) - b(a - 1)$$

$$= (a - 1)(a - b)$$

## **Solution 11:**

$$(ax + by)^{2} + (bx - ay)^{2} = a^{2}x^{2} + b^{2}y^{2} + 2axby + b^{2}x^{2} + a^{2}y^{2} - 2bxay$$

$$= a^{2}x^{2} + b^{2}y^{2} + b^{2}x^{2} + a^{2}y^{2}$$

$$= x^{2}(a^{2} + b^{2}) + y^{2}(a^{2} + b^{2})$$

$$= (x^{2} + y^{2})(a^{2} + b^{2})$$

## **Solution 12:**

$$a^{2}x^{2} + (ax^{2} + 1)x + a = a^{2}x^{2} + a + (ax^{2} + 1)x$$
$$= a(ax^{2} + 1) + x(ax^{2} + 1)$$
$$= (a + x)(ax^{2} + 1)$$

## **Solution 13:**

$$(2a-b)^2 - 10a + 5b = (2a-b)^2 - 5(2a-b)$$
  
=  $(2a-b)(2a-b-5)$ 

## **Solution 14:**

$$a(a-4)-a+4=a(a-4)-1(a-4)$$
  
=  $(a-4)(a-1)$ 

## **Solution 15:**

$$y^{2} - (a + b)y + ab = y^{2} - ay - by + ab$$
  
=  $y(y - a) - b(y - a)$   
=  $(y - a)(y - b)$ 

## **Solution 16:**

$$a^{2} + \frac{1}{a^{2}} - 2 - 3a + \frac{3}{a} = \left(a - \frac{1}{a}\right)^{2} - 3\left(a - \frac{1}{a}\right)$$
$$= \left(a - \frac{1}{a}\right)\left[\left(a - \frac{1}{a}\right) - 3\right]$$
$$= \left(a - \frac{1}{a}\right)\left[a - \frac{1}{a} - 3\right]$$

#### **Solution 17:**

$$= (x^2 + y^2 + 2xy) + (x + y)$$

[As 
$$(x + y)^2 = x^2 + 2xy + y^2$$
]

$$=(x + y)^2 + (x + y)$$

$$=(x+y)(x+y+1)$$

## **Solution 18:**

$$= a^2 + 4b^2 - 4ab - 3a + 6b$$

$$= a^2 + (2b)^2 - 2 \times a \times (2b) - 3(a - 2b)$$

[As 
$$(a - b)^2 = a^2 - 2ab + b^2$$
]

## **Solution 19:**

$$= m(x-3y)^2 - n(x-3y) + 5(x-3y)$$

[Taking (x - 3y) common from all the three terms]

$$=(x-3y)[m(x-3y)-n+5]$$

$$=(x-3y)(mx-3my-n+5)$$

### **Solution 20:**

$$=(6x-5y)[x-4(6x-5y)]$$

[Taking (6x - 5y) common from the three terms]

$$= (6x - 5y)(x - 24x + 20y)$$

$$= (6x - 5y)(-23x + 20y)$$

$$= (6x - 5y)(20y - 23x)$$

## Exercise 5(B)

## **Solution 1:**

$$a^{2} + 10a + 24 = a^{2} + 6a + 4a + 24$$
  
=  $a(a+6) + 4(a+6)$   
=  $(a+6)(a+4)$ 

## **Solution 2:**

$$a^{2} - 3a - 40 = a^{2} - 8a + 5a - 40$$
  
=  $a(a - 8) + 5(a - 8)$   
=  $(a - 8)(a + 5)$ 

#### **Solution 3:**

$$1-2a-3a^{2} = 1-3a+a-3a^{2}$$
$$= 1(1-3a)+a(1-3a)$$
$$= (1+a)(1-3a)$$

## **Solution 4:**

$$x^{2} - 3ax - 88a^{2} = x^{2} - 11ax + 8ax - 88a^{2}$$
$$= x(x - 11a) + 8a(x - 11a)$$
$$= (x + 8a)(x - 11a)$$

## **Solution 5:**

$$6a^{2} - a - 15 = 6a^{2} - 10a + 9a - 15$$
$$= 2a(3a - 5) + 3(3a - 5)$$
$$= (2a + 3)(3a - 5)$$

## **Solution 6:**

$$24a^{3} + 37a^{2} - 5a = a(24a^{2} + 37a - 5)$$

$$= a(24a^{2} + 40a - 3a - 5)$$

$$= a \times [8a(3a + 5) - 1(3a + 5)]$$

$$= a[(8a - 1)(3a + 5)]$$

$$= a(8a - 1)(3a + 5)$$

## **Solution 7:**

$$a(3a-2)-1=3a^2-2a-1$$

$$=3a^2-3a+a-1$$

$$=3a(a-1)+1(a-1)$$

$$=(3a+1)(a-1)$$

## **Solution 8:**

$$a^{2}b^{2} + 8ab - 9 = a^{2}b^{2} + 9ab - ab - 9$$
  
=  $ab(ab + 9) - 1(ab + 9)$   
=  $(ab + 9)(ab - 1)$ 

## **Solution 9:**

$$3-a(4+7a) = 3-4a-7a^{2}$$

$$= 3-7a+3a-7a^{2}$$

$$= 1(3-7a)+a(3-7a)$$

$$= (3-7a)(a+1)$$

#### **Solution 10:**

$$(2a+b)^2 - 6a - 3b - 4 = (2a+b)^2 - 3(2a+b) - 4$$

Assume that 2a+b=x

Therefore,

Therefore,  

$$(2a+b)^2 - 6a - 3b - 4 = x^2 - 3x - 4$$
  
 $= x^2 - 4x + x - 4$   
 $= 1(x - 4) + x(x - 4)$   
 $= (x + 1)(x - 4)$   
 $= (2a+b+1)(2a+b-4)$   
[resubstitute the value of x]

## **Solution 11:**

Assume that a + b = x;

$$1-2(a+b)-3(a+b)^{2} = 1-2x-3x^{2}$$

$$= 1-3x+x-3x^{2}$$

$$= 1(1-3x)+x(1-3x)$$

$$= (1-3x)(1+x)$$

$$= (1-3(a+b))(1+(a+b))$$

$$= (1-3a-3b)(1+a+b)$$

## **Solution 12:**

$$3a^{2} - 1 - 2a = 3a^{2} - 2a - 1$$

$$= 3a^{2} - 3a + a - 1$$

$$= 3a(a - 1) + 1(a - 1)$$

$$= (3a + 1)(a - 1)$$

## **Solution 13:**

$$x^{2} + 3x + 2 + ax + 2a = x^{2} + 2x + x + 2 + ax + 2a$$
$$= x(x+2) + 1(x+2) + a(x+2)$$
$$= (x+2)(x+a+1)$$

#### **Solution 14:**

Assume that 3x - 2y = aTherefore,  $(3x - 2y)^2 + 3(3x - 2y) - 10 = a^2 + 3a - 10$   $= a^2 + 5a - 2a - 10$ = a(a + 5) - 2(a + 5)

$$= (a+5)(a-2)$$
$$= (3x-2y+5)(3x-2y-2)$$

## **Solution 15:**

$$5 - (3a^2 - 2a)(6 - 3a^2 + 2a) = 5 - (3a^2 - 2a)[6 - (3a^2 - 2a)]$$

Assume that  $3a^2 - 2a = x$ 

Therefore,

$$5 - (3a^{2} - 2a)(6 - 3a^{2} + 2a) = 5 - x(6 - x)$$

$$= 5 - 6x + x^{2}$$

$$= 5 - 5x - x + x^{2}$$

$$= 5(1 - x) - x(1 - x)$$

$$= (5 - x)(1 - x)$$

$$= (x - 5)(x - 1)$$

$$= (3a^{2} - 2a - 5)(3a^{2} - 2a - 1)$$

$$= (3a^{2} - 5a + 3a - 5)(3a^{2} - 3a + a - 1)$$

$$= (3a - 5) + 1(3a - 5)(3a(a - 1) + 1(a - 1))$$

$$= (3a - 5)(a + 1)(3a + 1)(a - 1)$$

#### **Solution 16:**

(i) Given expression:  $x^2 - 3x - 54$ 

Comparing with  $ax^2 + bx + c$ , we get a = 1, b = -3 and c = -54

 $b^2 - 4ac = (-3)^2 - 4(1)(-54) = 9 + 216 = 225$ , which is a perfect square.

 $\therefore x^2 - 3x - 54$  is factorisable.

Now, 
$$x^2 - 3x - 54 = x^2 - 9x + 6x - 54$$
  
=  $x(x - 9) + 6(x - 9)$   
=  $(x - 9)(x + 6)$ 

(ii) Given expression:  $2x^2 - 7x - 15$ 

Comparing with  $ax^2 + bx + c$ , we get a = 2, b = -7 and c = -15

 $b^2 - 4ac = (-7)^2 - 4(2)(-15) = 49 + 120 = 169$ , which is a perfect square.

 $\therefore 2x^2 - 7x - 15$  is factorisable.

Now, 
$$2x^2 - 7x - 15 = 2x^2 - 10x + 3x - 15$$
  
=  $2x(x - 5) + 3(x - 5)$   
=  $(2x + 3)(x - 5)$ 

(iii) Given expression:  $2x^2 + 2x - 75$ 

Comparing with  $ax^2 + bx + c$ , we get a = 2, b = 2 and c = -75

 $b^2 - 4ac = (2)^2 - 4(2)(-75) = 4 + 600 = 604$ , which is not a perfect square.

 $\therefore 2x^2 + 2x - 75$  is not factorisable.

(iv) Given expression:  $3x^2 + 4x - 10$ 

Comparing with  $ax^2 + bx + c$ , we get a = 3, b = 4 and c = -10

:  $b^2 - 4ac = (4)^2 - 4(3)(-10) = 16 + 120 = 136$ , which is not a perfect square.

 $\therefore 3x^2 + 4x - 10$  is not factorisable.

(v) Given expression: x(2x - 1) - 1

Now,  $x(2x-1)-1=2x^2-x-1$ 

Comparing with  $ax^2 + bx + c$ , we get a = 2, b = -1 and c = -1

:  $b^2 - 4ac = (-1)^2 - 4(2)(-1) = 1 + 8 = 9$ , which is a perfect square.

 $\therefore 2x^2 - x - 1$  is factorisable.

Now, 
$$2x^2 - x - 1 = 2x^2 - 2x + x - 1$$
  
=  $2x(x - 1) + 1(x - 1)$   
=  $(2x + 1)(x - 1)$ 

## Exercise 5(C)

#### **Solution 1:**

$$25a^{2} - 9b^{2} = (5a)^{2} - (3b)^{2}$$
$$= (5a - 3b)(5a + 3b) [\because a^{2} - b^{2} = (a + b)(a - b)]$$

### **Solution 2:**

$$a^{2} - (2a + 3b)^{2} = (a)^{2} - (2a + 3b)^{2}$$

$$= (a - 2a - 3b)(a + 2a + 3b) [\because a^{2} - b^{2} = (a + b)(a - b)]$$

$$= (-a - 3b)(3a + 3b)$$

$$= -3(a + 3b)(a + b)$$

## **Solution 3:**

$$a^{2} - 81(b - c)^{2} = (a)^{2} - [9(b - c)]^{2}$$

$$= (a - (9b - 9c))(a + (9b - 9c))[\because a^{2} - b^{2} = (a + b)(a - b)]$$

$$= (a - 9b + 9c)(a + 9b - 9c)$$

#### **Solution 4:**

$$25(2a - b)^{2} - 81b^{2} = [5(2a - b)]^{2} - (9b)^{2}$$

$$= [5(2a - b) - 9b][5(2a - b) + 9b]$$

$$[\because a^{2} - b^{2} = (a + b)(a - b)]$$

$$= [10a - 5b - 9b][10a - 5b + 9b]$$

$$= [10a - 14b][10a + 4b]$$

$$= 2 \times (5a - 7b) \times 2 \times (5a + 2b)$$

$$= 4(5a - 7b)(5a + 2b)$$

#### **Solution 5:**

$$50a^{3} - 2a = 2a(25a^{2} - 1)$$

$$= 2a[(5a)^{2} - 1^{2}]$$

$$= 2a(5a + 1)(5a - 1) \quad [\because a^{2} - b^{2} = (a + b)(a - b)]$$

### **Solution 6:**

$$4a^{2}b - 9b^{3} = b(4a^{2} - 9b^{2})$$

$$= b[(2a)^{2} - (3b)^{2}]$$

$$= b(2a - 3b)(2a + 3b) \quad [\because a^{2} - b^{2} = (a + b)(a - b)]$$

### **Solution 7:**

$$3a^{5} - 108a^{3} = 3a^{3}(a^{2} - 36)$$

$$= 3a^{3}[(a)^{2} - (6)^{2}]$$

$$= 3a^{3}(a - 6)(a + 6) \qquad \therefore a^{2} - b^{2} = (a + b)(a - b)$$

#### **Solution 8:**

$$9(a-2)^{2} - 16(a+2)^{2} = [3(a-2)]^{2} - [4(a+2)]^{2}$$

$$= [3(a-2) - 4(a+2)][3(a-2) + 4(a+2)]$$

$$[\because a^{2} - b^{2} = (a+b)(a-b)]$$

$$= [3a-6-4a-8][3a-6+4a+8]$$

$$= (-a-14)(7a+2)$$

$$= -(a+14)(7a+2)$$

## **Solution 9:**

$$a^{4} - 1 = (a^{2})^{2} - (1)^{2}$$

$$= (a^{2} + 1)(a^{2} - 1) [ \because a^{2} - b^{2} = (a + b)(a - b)]$$

$$= (a^{2} + 1)((a)^{2} - (1)^{2})$$

$$= (a^{2} + 1)(a + 1)(a - 1)$$

## **Solution 10:**

$$a^{3} + 2a^{2} - a - 2 = a^{2}(a+2) - 1(a+2)$$

$$= (a^{2} - 1)(a+2)$$

$$= (a+1)(a-1)(a+2) \quad [\because a^{2} - b^{2} = (a+b)(a-b)]$$

#### **Solution 11:**

$$(a+b)^{3}-a-b = (a+b)^{3}-(a+b)$$

$$= (a+b)[(a+b)^{2}-1]$$

$$= (a+b)[(a+b)^{2}-1^{2}]$$

$$= (a+b)((a+b)+1)((a+b)-1)$$

$$[\because a^{2}-b^{2}=(a+b)(a-b)]$$

$$= (a+b)(a+b+1)(a+b-1)$$

## **Solution 12:**

$$a(a-1)-b(b-1) = a^{2}-a-b^{2}+b$$

$$= a^{2}-b^{2}-a+b$$

$$= (a+b)(a-b)-(a-b)$$

$$[\because a^{2}-b^{2}=(a+b)(a-b)]$$

$$= (a-b)[(a+b)-1]$$

$$= (a-b)[a+b-1]$$

#### **Solution 13:**

$$4a^{2} - (4b^{2} + 4bc + c^{2}) = (2a)^{2} - (2b + c)^{2}$$
$$= [2a - (2b + c)][2a + (2b + c)]$$
$$[\because a^{2} - b^{2} = (a + b)(a - b)]$$
$$= [2a - 2b - c][2a + 2b + c]$$

### **Solution 14:**

$$4a^{2} - 49b^{2} + 2a - 7b = \left[ (2a)^{2} - (7b)^{2} \right] + \left[ 2a - 7b \right]$$
$$= \left[ 2a - 7b \right] \left[ 2a + 7b \right] + \left[ 2a - 7b \right]$$
$$\left[ \because a^{2} - b^{2} = (a + b)(a - b) \right]$$
$$= \left[ 2a - 7b \right] \left[ 2a + 7b + 1 \right]$$

#### **Solution 15:**

$$9a^{2} + 3a - 8b - 64b^{2} = 9a^{2} - 64b^{2} + 3a - 8b$$

$$= (3a)^{2} - (8b)^{2} + 3a - 8b$$

$$= (3a - 8b)(3a + 8b) + (3a - 8b)$$

$$\left[\because a^{2} - b^{2} = (a + b)(a - b)\right]$$

$$= (3a - 8b)(3a + 8b + 1)$$

## **Solution 16:**

$$4a^{2} - 12a + 9 - 49b^{2} = (2a)^{2} - 12a + (3)^{2} - 49b^{2}$$

$$= (2a - 3)^{2} - 49b^{2}$$

$$= (2a - 3)^{2} - (7b)^{2}$$

$$= (2a - 3 - 7b)(2a - 3 + 7b)$$

$$[\because a^{2} - b^{2} = (a + b)(a - b)]$$

## **Solution 17:**

$$4xy - x^{2} - 4y^{2} + z^{2} = z^{2} - (x^{2} + 4y^{2} - 4xy)$$

$$= z^{2} - (x - 2y)^{2}$$

$$= [z - (x - 2y)] [z + (x - 2y)]$$

$$[\because a^{2} - b^{2} = (a + b)(a - b)]$$

$$= [z - x + 2y][z + x - 2y]$$

#### **Solution 18:**

$$a^{2} + b^{2} - c^{2} - d^{2} + 2ab - 2cd$$

$$= (a^{2} + b^{2} + 2ab) - (c^{2} + d^{2} + 2cd)$$

$$= (a + b)^{2} - (c + d)^{2}$$

$$= [(a + b) - (c + d)] [(a + b) + (c + d)] [ \because a^{2} - b^{2} = (a + b)(a - b)]$$

$$= (a + b - c - d)(a + b + c + d)$$

## **Solution 19:**

$$4x^{2} - 12ax - y^{2} - z^{2} - 2yz + 9a^{2}$$

$$= 4x^{2} + 9a^{2} - 12ax - y^{2} - z^{2} - 2yz$$

$$= (2x)^{2} + (3a)^{2} - 12ax - (y^{2} + z^{2} + 2yz)$$

$$= (2x - 3a)^{2} - (y + z)^{2}$$

$$= [(2x - 3a) - (y + z)][(2x - 3a) + (y + z)]$$

$$[\because a^{2} - b^{2} = (a + b)(a - b)]$$

$$= [2x - 3a - y - z][2x - 3a + y + z]$$

## **Solution 20:**

$$(a^{2}-1)(b^{2}-1) + 4ab = a^{2}b^{2} - a^{2} - b^{2} + 1 + 4ab$$

$$= a^{2}b^{2} + 1 + 2ab - a^{2} - b^{2} + 2ab$$

$$= (a^{2}b^{2} + 1 + 2ab) - (a^{2} + b^{2} - 2ab)$$

$$= (ab + 1)^{2} - (a - b)^{2}$$

$$= [(ab + 1) - (a - b)][(ab + 1) + (a - b)]$$

$$[\because a^{2} - b^{2} = (a + b)(a - b)]$$

$$= [ab + 1 - a + b][ab + 1 + a - b]$$

## **Solution 21:**

$$x^{4} + x^{2} + 1 = x^{4} + 2x^{2} + 1 - x^{2}$$

$$= (x^{2})^{2} + 2x^{2} + (1)^{2} - x^{2}$$

$$= (x^{2} + 1)^{2} - (x)^{2}$$

$$[ \because a^{2} - b^{2} = (a + b)(a - b)]$$

$$= (x^{2} + 1 - x)(x^{2} + 1 + x)$$

### **Solution 22:**

$$(a^{2} + b^{2} - 4c^{2})^{2} - 4a^{2}b^{2} = (a^{2} + b^{2} - 4c^{2})^{2} - (2ab)^{2}$$

$$= (a^{2} + b^{2} - 4c^{2} - 2ab)(a^{2} + b^{2} - 4c^{2} + 2ab)$$

$$[\because a^{2} - b^{2} = (a + b)(a - b)]$$

$$= (a^{2} + b^{2} - 2ab - 4c^{2})(a^{2} + b^{2} + 2ab - 4c^{2})$$

$$= ((a - b)^{2} - (2c)^{2})((a + b)^{2} - (2c)^{2})$$

$$= (a - b + 2c)(a - b - 2c)(a + b + 2c)(a + b - 2c)$$

## **Solution 23:**

$$(x^{2} + 4y^{2} - 9z^{2})^{2} - 16x^{2}y^{2}$$

$$= (x^{2} + 4y^{2} - 9z^{2})^{2} - (4xy)^{2}$$

$$= (x^{2} + 4y^{2} - 9z^{2} - 4xy)(x^{2} + 4y^{2} - 9z^{2} + 4xy)$$

$$[\because a^{2} - b^{2} = (a + b)(a - b)]$$

$$= (x^{2} + 4y^{2} - 4xy - 9z^{2})(x^{2} + 4y^{2} + 4xy - 9z^{2})$$

$$= [(x - 2y)^{2} - (3z)^{2}][(x + 2y)^{2} - (3z)^{2}]$$

$$= [(x - 2y) - 3z][(x - 2y) + 3z][(x + 2y) - 3z][(x + 2y) + 3z]$$

$$= [x - 2y - 3z][x - 2y + 3z][x + 2y - 3z][x + 2y + 3z]$$

#### **Solution 24:**

$$(a+b)^2 - a^2 + b^2$$
  
=  $a^2 + 2ab + b^2 - a^2 + b^2$   
=  $2ab + 2b^2$   
=  $2b(a+b)$ 

#### **Solution 25:**

$$a^{2} - b^{2} - (a + b)^{2}$$
  
=  $a^{2} - b^{2} - (a^{2} + 2ab + b^{2})$   
=  $a^{2} - b^{2} - a^{2} - 2ab - b^{2}$   
=  $-2ab - 2b^{2}$   
=  $-2b(a + b)$ 

#### **Solution 26:**

$$9a^{2} - (a^{2} - 4)^{2}$$

$$= (3a)^{2} - (a^{2} - 4)^{2}$$

$$= [3a - (a^{2} - 4)][3a + (a^{2} - 4)]$$

$$= [3a - a^{2} - 4][3a + a^{2} - 4]$$

$$= [-a^{2} + 3a - 4][a^{2} + 3a - 4]$$

$$= [-a^{2} + 4a - a - 4][a^{2} + 4a - a - 4]$$

$$= [a(-a + 4) + 1(-a + 4)][a(a + 4) - 1(a + 4)]$$

$$= [(a + 1)(4 - a)][(a + 4)(a - 1)]$$

$$= (a + 1)(4 - a)(a + 4)(a - 1)$$

## **Solution 27:**

$$x^{2} + \frac{1}{x^{2}} - 11$$

$$= x^{2} + \frac{1}{x^{2}} - 2 - 9$$

$$= x^{2} + \frac{1}{x^{2}} - 2x \times x + \frac{1}{x} - 9$$

$$= \left(x - \frac{1}{x}\right)^{2} - \left(3\right)^{2}$$

$$= \left(x - \frac{1}{x} + 3\right)\left(x - \frac{1}{x} - 3\right)$$

## **Solution 28:**

$$4x^{2} + \frac{1}{4x^{2}} + 1$$

$$= 4x^{2} + \frac{1}{4x^{2}} + 2 - 1$$

$$= 4x^{2} + \frac{1}{4x^{2}} + 2x + 2x + \frac{1}{2x} - 1$$

$$= \left(2x + \frac{1}{2x}\right)^{2} - \left(1\right)^{2}$$

$$= \left(2x + \frac{1}{2x} + 1\right) \left(2x + \frac{1}{2x} - 1\right)$$

#### **Solution 29:**

$$4x^{4} - x^{2} - 12x - 36$$

$$= 4x^{4} - (x^{2} + 12x + 36)$$

$$= (2x^{2})^{2} - (x^{2} + 2x \times x + 6 + 6^{2})$$

$$= (2x^{2})^{2} - (x + 6)^{2}$$

$$= (2x^{2} + x + 6)(2x^{2} - x - 6)$$

$$= (2x^{2} + x + 6)(2x^{2} - 4x + 3x - 6)$$

$$= (2x^{2} + x + 6)[2x(x - 2) + 3(x - 2)]$$

$$= (2x^{2} + x + 6)[(x - 2)(2x + 3)]$$

$$= (2x^{2} + x + 6)(x - 2)(2x + 3)$$

### **Solution 30:**

$$a^{2}(b+c) - (b+c)^{3}$$
= (b+c)[a^{2} - (b+c)^{2}]  
= (b+c)[(a+b+c)(a-b-c)]  
= (b+c)(a+b+c)(a-b-c)

## Exercise 5(D)

#### **Solution 1:**

$$a^{3}-27 = (a)^{3}-(3)^{3}$$

$$= (a-3)[(a)^{2}+a\times3+(3)^{2}] \quad [\because a^{3}-b^{3}=(a-b)(a^{2}+ab+b^{2})]$$

$$= (a-3)[a^{2}+3a+9]$$

#### **Solution 2:**

$$1 - 8a^{3} = (1)^{3} - (2a)^{3}$$

$$= (1 - 2a)[(1)^{2} + 1 \times 2a + (2a)^{2}]$$

$$[\because a^{3} - b^{3} = (a - b)(a^{2} + ab + b^{2})]$$

$$= (1 - 2a)[1 + 2a + 4a^{2}]$$

## **Solution 3:**

$$64 - a^{3}b^{3} = (4)^{3} - ab^{3}$$

$$= (4 - ab)[(4)^{2} + 4 \times ab + (ab)^{2}]$$

$$[\because a^{3} - b^{3} = (a - b)(a^{2} + ab + b^{2})]$$

$$= (4 - ab)[16 + 4ab + a^{2}b^{2}]$$

#### **Solution 4:**

$$a^{6} + 27b^{3} = (a^{2})^{3} + (3b)^{3}$$

$$= (a^{2} + 3b) [(a^{2})^{2} - a^{2} \times 3b + (3b)^{2}]$$

$$[\because a^{3} + b^{3} = (a + b)(a^{2} - ab + b^{2})]$$

$$= (a^{2} + 3b) [a^{4} - 3a^{2}b + 9b^{2}]$$

## **Solution 5:**

$$3x^{7}y - 81x^{4}y^{4} = 3xy (x^{6} - 27x^{3}y^{3})$$

$$= 3xy ((x^{2})^{3} - (3xy)^{3})$$

$$= 3xy (x^{2} - 3xy)[(x^{2})^{2} + x^{2} \times 3xy + (3xy)^{2}]$$

$$[\because a^{3} - b^{3} = (a - b)(a^{2} + ab + b^{2})]$$

$$= 3xy (x^{2} - 3xy)[x^{4} + 3x^{3}y + 9x^{2}y^{2}]$$

$$= 3xy (x (x - 3y)x^{2}[x^{2} + 3xy + 9y^{2}]$$

$$= 3x^{4}y (x - 3y)[x^{2} + 3xy + 9y^{2}]$$

## **Solution 6:**

$$a^{3} - \frac{27}{a^{3}} = (a)^{3} - \left(\frac{3}{a}\right)^{3}$$

$$= \left(a - \frac{3}{a}\right) \left(a^{2} + a \times \frac{3}{a} + \left(\frac{3}{a}\right)^{2}\right)$$

$$\left[\because a^{3} - b^{3} = (a - b)(a^{2} + ab + b^{2})\right]$$

$$= \left(a - \frac{3}{a}\right) \left(a^{2} + 3 + \frac{9}{a^{2}}\right)$$

#### **Solution 7:**

$$a^{3} + 0.064 = (a)^{3} + (0.4)^{3}$$

$$= (a + 0.4) [(a)^{2} - a \times 0.4 + (0.4)^{2}]$$

$$[\because a^{3} + b^{3} = (a + b)(a^{2} - ab + b^{2})]$$

$$= (a + 0.4) [a^{2} - 0.4a + 0.16]$$

#### **Solution 8:**

$$a^{4} - 343a = a(a^{3} - 7^{3})$$

$$= a(a - 7)[(a)^{2} + a \times 7 + (7)^{2}]$$

$$[\because a^{3} - b^{3} = (a - b)(a^{2} + ab + b^{2})]$$

$$= a(a - 7)[a^{2} + 7a + 49]$$

## **Solution 9:**

$$= (x - y)^{3} - (2x)^{3}$$

$$= (x - y - 2x)[(x - y)^{2} + 2x(x - y) + (2x)^{2}]$$
[Using identity  $(a^{3} - b^{3}) = (a - b)(a^{2} + ab + b^{2})$ ]
$$= (-x - y)[x^{2} + y^{2} - 2xy + 2x^{2} - 2xy + 4x^{2}]$$

## **Solution 10:**

 $=-(x + y) [7x^2 - 4xy + y^2]$ 

$$\frac{8a^{3}}{27} - \frac{b^{3}}{8} = \left(\frac{2a}{3}\right)^{3} - \left(\frac{b}{2}\right)^{3}$$

$$= \left(\frac{2a}{3} - \frac{b}{2}\right) \left[\left(\frac{2a}{3}\right)^{2} + \frac{2a}{3} \times \frac{b}{2} + \left(\frac{b}{2}\right)^{2}\right]$$

$$[\because a^{3} - b^{3} = (a - b)(a^{2} = ab + b^{2})]$$

$$= \left(\frac{2a}{3} - \frac{b}{2}\right) \left[\frac{4a^{2}}{9} + \frac{ab}{3} + \frac{b^{2}}{4}\right]$$

#### **Solution 11:**

We know that,

$$a^{3} + b^{3} = (a + b)(a^{2} - ab + b^{2}) \dots (1)$$

$$a^{3} - b^{3} = (a - b)(a^{2} + ab + b^{2}) \dots (2)$$

$$a^{6} - b^{6} = (a^{3})^{2} - (b^{3})^{2}$$

$$= (a^{3} + b^{3})(a^{3} - b^{3})$$

$$= (a + b)(a^{2} - ab + b^{2})(a - b)(a^{2} + ab + b^{2}) \text{ [from (1) and (2)]}$$

$$= (a + b)(a - b)(a^{2} - ab + b^{2})(a^{2} + ab + b^{2})$$

### **Solution 12:**

We know that,

$$a^{3} + b^{3} = (a + b)(a^{2} - ab + b^{2}) \dots (1)$$

$$a^{3} - b^{3} = (a - b)(a^{2} + ab + b^{2}) \dots (2)$$

$$a^{6} - 7a^{3} - 8 = a^{6} - 8a^{3} + a^{3} - 8$$

$$= a^{3}(a^{3} - 8) + 1(a^{3} - 8)$$

$$= (a^{3} + 1)(a^{3} - 8)$$

$$= (a^{3} + 1^{3})(a^{3} - 2^{3})$$

$$= (a + 1)(a^{2} - a + 1)(a - 2)(a^{2} + 2a + 4) \text{ [from (1) and (2)]}$$

$$= (a + 1)(a - 2)(a^{2} - a + 1)(a^{2} + 2a + 4)$$

## **Solution 13:**

We know that,

$$a^{3} - b^{3} = (a - b)(a^{2} + ab + b^{2}) \dots (1)$$

$$a^{3} - 27b^{3} + 2a^{2}b - 6ab^{2}$$

$$= (a)^{3} - (3b)^{3} + 2ab(a - 3b)$$

$$= (a - 3b)[a^{2} + a \times 3b + (3b)^{2}] + 2ab(a - 3b) \quad [from (1)]$$

$$= (a - 3b)[a^{2} + 3ab + 9b^{2}] + 2ab(a - 3b)$$

$$= (a - 3b)[a^{2} + 3ab + 9b^{2} + 2ab]$$

$$= (a - 3b)[a^{2} + 5ab + 9b^{2}]$$

#### **Solution 14:**

We know that,

$$a^{3} - b^{3} = (a - b)(a^{2} + ab + b^{2}) \dots (1)$$

$$8a^{3} - b^{3} - 4ax + 2bx = [(2a)^{3} - (b)^{3}] - 2x(2a - b)$$

$$= (2a - b)[(2a)^{2} + 2a \times b + (b)^{2}] - 2x(2a - b)$$
[from (1)]
$$= (2a - b)[4a^{2} + 2ab + b^{2}] - 2x(2a - b)$$

$$= (2a - b)[4a^{2} + 2ab + b^{2} - 2x]$$

#### **Solution 15:**

We know that,

$$a^{3} - b^{3} = (a - b)(a^{2} + ab + b^{2}) \dots (1)$$

$$a - b - a^{3} + b^{3} = a - b - (a^{3} - b^{3})$$

$$= (a - b) - (a - b)[a^{2} + ab + b^{2}] \text{ [from (1)]}$$

$$= (a - b)[1 - a^{2} - ab - b^{2}]$$

## **Solution 16:**

$$= 2(x^3 + 27y^3 - 2x - 6y)$$

$$= 2\{[(x)^3+(3y)^3]-2(x+3y)\}$$

[Using identity  $(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$ ]

$$=2\{[(x+3y)(x^2-3xy+9y^2)]-2(x+3y)\}$$

$$=2(x+3y)(x^2-3xy+9y^2-2)$$

#### **Solution 17:**

(i) 
$$(13^3 - 5^3)$$

[Using identity  $(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$ ]

$$=(13-5)(13^2+13\times5+5^2)$$

$$=8(169+65+25)$$

Therefore, the number is divisible by 8.

(ii) 
$$(35^3 + 27^3)$$

[Using identity  $(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$ ]

$$=(35+27)(35^2+35\times27+27^2)$$

$$=62 \times (35^2 + 35 \times 27 + 27^2)$$

Therefore, the number is divisible by 62.

## Exercise 5(E)

#### **Solution 1:**

$$x^{2} + \frac{1}{4x^{2}} + 1 - 7x - \frac{7}{2x} = (x)^{2} + \frac{1}{(2x)^{2}} + 2x \times \frac{1}{2x} - 7(x + \frac{1}{2x})$$

$$= (x + \frac{1}{2x})^{2} - 7(x + \frac{1}{2x})$$

$$= (x + \frac{1}{2x})(x + \frac{1}{2x} - 7)$$

$$= (x + \frac{1}{2x})(x - 7 + \frac{1}{2x})$$

## **Solution 2:**

$$9a^{2} + \frac{1}{9a^{2}} - 2 - 12a + \frac{4}{3a} = (3a)^{2} + \frac{1}{(3a)^{2}} - 2 \times 3a \times \frac{1}{3a} - 4(3a - \frac{1}{3a})$$

$$= \left(3a - \frac{1}{3a}\right)^{2} - 4\left(3a - \frac{1}{3a}\right)$$

$$= \left(3a - \frac{1}{3a}\right)\left(\left(3a - \frac{1}{3a}\right) - 4\right)$$

$$= \left(3a - \frac{1}{3a}\right)\left(3a - 4 - \frac{1}{3a}\right)$$

#### **Solution 3:**

$$x^{2} + \frac{a^{2} + 1}{a} \times + 1 = x^{2} + a \times + \frac{1}{a} \times + 1$$
$$= x(x + a) + \frac{1}{a}(x + a)$$
$$= (x + a)\left(x + \frac{1}{a}\right)$$

#### **Solution 4:**

$$x^{4} + y^{4} - 27x^{2}y^{2} = (x^{2})^{2} + (y^{2})^{2} - 2x^{2}y^{2} - 25x^{2}y^{2}$$

$$= (x^{2} - y^{2})^{2} - 25x^{2}y^{2}$$

$$= (x^{2} - y^{2})^{2} - (5xy)^{2} \quad [\because a^{2} - b^{2} = (a + b)(a - b)]$$

$$= [(x^{2} - y^{2}) + 5xy][(x^{2} - y^{2}) - 5xy]$$

$$= [x^{2} + 5xy - y^{2}][x^{2} - 5xy - y^{2}]$$

### **Solution 5:**

$$4x^{4} + 9y^{4} + 11x^{2}y^{2} = (2x^{2})^{2} + (3y^{2})^{2} + 12x^{2}y^{2} - x^{2}y^{2}$$

$$= (2x^{2} + 3y^{2})^{2} - x^{2}y^{2}$$

$$= (2x^{2} + 3y^{2})^{2} - (xy)^{2}$$

$$= (2x^{2} + 3y^{2} - xy)(2x^{2} + 3y^{2} + xy)$$

$$[\because a^{2} - b^{2} = (a + b)(a - b)]$$

## **Solution 6:**

$$x^{2} + \frac{1}{x^{2}} - 3 = x^{2} + \frac{1}{x^{2}} - 2 - 1$$

$$= x^{2} + \frac{1}{x^{2}} - 2 \times x \times \frac{1}{x} - 1$$

$$= \left(x - \frac{1}{x}\right)^{2} - 1$$

$$= \left(x - \frac{1}{x}\right)^{2} - (1)^{2}$$

$$= \left(x - \frac{1}{x} - 1\right) \left(x - \frac{1}{x} + 1\right) \quad [\because a^{2} - b^{2} = (a + b)(a - b)]$$

## **Solution 7:**

$$a-b-4a^{2}+4b^{2} = (a-b)-4(a^{2}-b^{2})$$

$$= (a-b)-4(a-b)(a+b) [:: a^{2}-b^{2} = (a+b)(a-b)]$$

$$= (a-b)[1-4(a+b)]$$

$$= (a-b)[1-4a-4b]$$

### **Solution 8:**

$$(2a-3)^{2} - 2(2a-3)(a-1) + (a-1)^{2}$$

$$= [(2a-3) - (a-1)]^{2}$$

$$= [2a-3-a+1]^{2}$$

$$= (a-2)^{2}$$

#### **Solution 9:**

Let us assume,  $a^2 - 3a = x$ Then the given expression is,

$$(a^{2} - 3a)(a^{2} - 3a + 7) + 10 = x(x + 7) + 10$$

$$= x^{2} + 7x + 10$$

$$= x^{2} + 5x + 2x + 10$$

$$= x(x + 5) + 2(x + 5)$$

$$= (x + 5)(x + 2)$$

$$= (a^{2} - 3a + 5)(a^{2} - 3a + 2)$$
[resubstitute the value of x]
$$= (a^{2} - 3a + 5)(a^{2} - 2a - a + 2)$$

$$= (a^{2} - 3a + 5)(a(a - 2) - 1(a - 2))$$

$$= (a^{2} - 3a + 5)[(a - 1)(a - 2)]$$

#### **Solution 10:**

Let us assume  $a^2 - a = x$ 

Then the given expression is

$$(a^{2}-a)(4a^{2}-4a-5)-6 = x(4x-5)-6$$

$$= 4x^{2}-5x-6$$

$$= 4x(x-2)+3(x-2)$$

$$= (4x+3)(x-2)$$

$$= (4(a^{2}-a)+3)(a^{2}-a-2)$$
[resubstitute the value of x]
$$= (4a^{2}-4a+3)(a^{2}-a-2)$$

$$= (4a^{2}-4a+3)(a^{2}-2a+a-2)$$

$$= (4a^{2}-4a+3)(a(a-2)+1(a-2))$$

$$= (4a^{2}-4a+3)(a-2)(a+1)$$

## **Solution 11:**

$$x^{4} + y^{4} - 3x^{2}y^{2} = x^{4} + y^{4} - 2x^{2}y^{2} - x^{2}y^{2}$$

$$= (x^{2})^{2} + (y^{2})^{2} - 2x^{2}y^{2} - x^{2}y^{2}$$

$$= (x^{2} - y^{2})^{2} - (xy)^{2}$$

$$= (x^{2} - y^{2} - xy)(x^{2} - y^{2} + xy)$$

$$[\because a^{2} - b^{2} = (a + b)(a - b)]$$

### **Solution 12:**

$$5a^{2}-b^{2}-4ab+7a-7b$$

$$= 4a^{2}+a^{2}-b^{2}-4ab+7a-7b$$

$$= a^{2}-b^{2}+4a^{2}-4ab+7a-7b$$

$$= (a^{2}-b^{2})+4a(a-b)+7(a-b)$$

$$= (a-b)(a+b)+4a(a-b)+7(a-b) \quad [::a^{2}-b^{2}=(a+b)(a-b)]$$

$$= (a-b)[(a+b)+4a+7]$$

$$= (a-b)[a+b+4a+7]$$

$$= (a-b)[5a+b+7]$$

## **Solution 13:**

$$12(3x-2y)^2-3x+2y-1=12(3x-2y)^2-(3x-2y)-1$$
  
Let us assume that  $3x-2y=a$ 

Then the given expression is

$$12(3x - 2y)^{2} - 3x + 2y - 1 = 12a^{2} - 3a - 1$$

$$= 12a^{2} - 4a + 3a - 1$$

$$= 4a(3a - 1) + 1(3a - 1)$$

$$= (4a + 1)(3a - 1)$$

$$= (4(3x - 2y) + 1)(3(3x - 2y) - 1)$$
[resubstitute the value of a]
$$= (12x - 8y + 1)(9x - 6y - 1)$$

## **Solution 14:**

$$4(2x-3y)^2-8x+12y-3=4(2x-3y)^2-4(2x-3y)-3$$

Let us assume that 2x - 3y = a

Then the given expression is

$$4(2x-3y)^{2}-8x+12y-3=4a^{2}-4a-3$$

$$=4a^{2}-6a+2a-3$$

$$=2a(2a-3)+1(2a-3)$$

$$=(2a-3)(2a+1)$$

$$=(2(2x-3y)-3)(2(2x-3y)+1)$$

$$=(4x-6y-3)(4x-6y+1)$$

### **Solution 15:**

$$3-5x+5y-12(x-y)^2=3-5(x-y)-12(x-y)^2$$

Let us assume that x - y = a

Then the given expression is

$$3-5x + 5y - 12(x - y)^{2} = 3-5a - 12a^{2}$$

$$= 3-9a + 4a - 12a^{2}$$

$$= 3(1-3a) + 4a(1-3a)$$

$$= (3+4a)(1-3a)$$
[resubstitute the value of a]
$$= (3+4(x-y))(1-3(x-y))$$

$$= (3+4x-4y)(1-3x+3y)$$

## **Solution 16:**

$$9x^{2} + 3x - 8y - 64y^{2}$$

$$= 9x^{2} - 64y^{2} + 3x - 8y$$

$$= [(3x)^{2} - (8y)^{2}] + (3x - 8y)$$

$$= [(3x + 8y)(3x - 8y)] + (3x - 8y)$$

$$= (3x - 8y)(3x + 8y + 1)$$

### **Solution 17:**

$$2\sqrt{3}x^{2} + x - 5\sqrt{3}$$

$$= 2\sqrt{3}x^{2} + 6x - 5x - 5\sqrt{3}$$

$$= 2\sqrt{3}x(x + \sqrt{3}) - 5(x + \sqrt{3})$$

$$= (2\sqrt{3}x - 5)(x + \sqrt{3})$$

## **Solution 18:**

$$\frac{1}{4}(a+b)^{2} - \frac{9}{16}(2a-b)^{2}$$

$$= \frac{1}{4}\left[(a+b)^{2} - \frac{9}{4}(2a-b)^{2}\right]$$

$$= \frac{1}{4}\left[(a+b)^{2} - \left(\frac{3}{2}(2a-b)\right)^{2}\right]$$

$$= \frac{1}{4}\left[(a+b+\frac{3}{2}(2a-b))(a+b-\frac{3}{2}(2a-b))\right]$$

$$= \frac{1}{4}\left[(a+b+3a-\frac{3b}{2})(a+b-3a+\frac{3b}{2})\right]$$

$$= \frac{1}{4}\left[(4a-\frac{b}{2})(\frac{5b}{2}-2a)\right]$$

$$= \frac{1}{4}\left[\frac{8a-b}{2}(\frac{5b-4a}{2})\right]$$

$$= \frac{1}{4}\left[\frac{1}{4}(8a-b)(5b-4a)\right]$$

$$= \frac{1}{16}(8a-b)(5b-4a)$$

## **Solution 19:**

$$2(ab + cd) - a^{2} - b^{2} + c^{2} + d^{2}$$

$$= 2ab + 2cd - a^{2} - b^{2} + c^{2} + d^{2}$$

$$= c^{2} + d^{2} + 2cd - a^{2} - b^{2} + 2ab$$

$$= (c^{2} + d^{2} + 2cd) - (a^{2} + b^{2} - 2ab)$$

$$= (c + d)^{2} - (a - b)^{2}$$

$$= (c + d + a - b)(c + d - a + b)$$

## **Solution 20:**

(i) 
$$(987)^2 - (13)^2$$
  
=  $(987 + 13)(987 - 13)$   
=  $1000 \times 974$   
=  $974000$ 

(ii) 
$$(67.8)^2 - (32.2)^2$$
  
=  $(67.8 + 32.2)(67.8 - 32.2)$   
=  $100 \times 35.6$   
=  $3560$ 

(iii) 
$$\frac{(6.7)^2 - (3.3)^2}{6.7 - 3.3}$$
$$= \frac{(6.7 + 3.3)(6.7 - 3.3)}{(6.7 - 3.3)}$$
$$= 10$$

(iv) 
$$\frac{(18.5)^2 - (6.5)^2}{18.5 + 6.5}$$
$$= \frac{(18.5 + 6.5)(18.5 - 6.5)}{(18.5 + 6.5)}$$
$$= 12$$