

Chapter 6

Lines and Angles

Exercise 6.3

Question: 1 In Fig. 6.39, sides QP and RQ of $\triangle PQR$ are produced to points S and T respectively. If $\angle SPR = 135^\circ$ and $\angle PQT = 110^\circ$, find $\angle PRQ$.

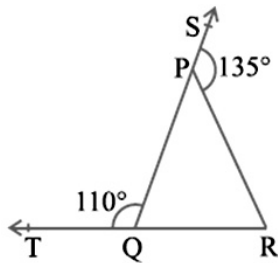


Fig. 6.39

Ans.:

Method 1:

It is given in the question that:

$$\angle SPR = 135^\circ$$

And,

$$\angle PQT = 110^\circ$$

Now, according to the question,

$$\angle SPR + \angle QPR = 180^\circ \text{ (SQ is a straight line)}$$

$$135^\circ + \angle QPR = 180^\circ$$

$$\angle QPR = 45^\circ$$

And,

$$\angle PQT + \angle PQR = 180^\circ \text{ (TR is a straight line)}$$

$$110^\circ + \angle PQR = 180^\circ$$

$$\angle PQR = 70^\circ$$

Now,

$$\angle PQR + \angle QPR + \angle PRQ = 180^\circ \text{ (Sum of the interior angles of the triangle)}$$

$$70^\circ + 45^\circ + \angle PRQ = 180^\circ$$

$$115^\circ + \angle PRQ = 180^\circ$$

$$\angle PRQ = 65^\circ$$

Method 2:

It is given in the question that:

$$\angle SPR = 135^\circ$$

And,

$$\angle PQT = 110^\circ$$

$$\angle PQT + \angle PQR = 180^\circ \text{ (TR is a straight line)}$$

$$110^\circ + \angle PQR = 180^\circ$$

$$\angle PQR = 70^\circ$$

Now, we know that the exterior angle of the triangle equals the sum of interior opposite angles. Therefore, $\angle SPR = \angle PQR + \angle PRQ$

$$135^\circ = 70^\circ + \angle PRQ$$

$$\angle PRQ = 65^\circ$$

Question: 2 In Fig. 6.40, $\angle X = 62^\circ$, $\angle XYZ = 54^\circ$. If YO and ZO are the bisectors of $\angle XYZ$ and $\angle XZY$ respectively of $\triangle XYZ$, find $\angle OZY$ and $\angle YOZ$.

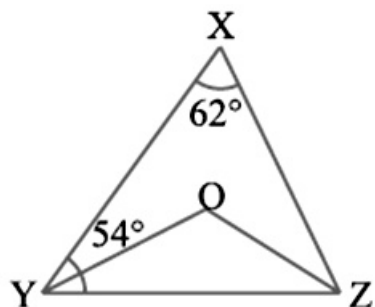


Fig. 6.40

Ans.:

Given: $\angle X = 62^\circ$, $\angle XYZ = 54^\circ$

YO and ZO are the bisectors of $\angle XYZ$ and $\angle XZY$ respectively.

To Find: $\angle OZY$ and $\angle YOZ$.

Now, according to the question,

$$\angle X + \angle XYZ + \angle XZY = 180^\circ \quad (\text{Sum of the interior angles of a triangle} \\ = 180^\circ)$$

$$62^\circ + 54^\circ + \angle XZY = 180^\circ$$

$$116^\circ + \angle XZY = 180^\circ$$

$$\angle XZY = 64^\circ$$

Now,

As ZO is the bisector of $\angle XZY$

$$\angle OZY = \frac{1}{2} \angle XZY$$

$$\angle OZY = 32^\circ$$

And,

As YO is bisector of $\angle XYZ$

$$\angle OYZ = \frac{1}{2} \angle XYZ$$

$$\angle OYZ = 27^\circ$$

Now,

$$\angle OZY + \angle OYZ + \angle O = 180^\circ$$

(Sum of the interior angles of the triangle = 180°)

$$= 32^\circ + 27^\circ + \angle O = 180^\circ$$

$$= 59^\circ + \angle O = 180^\circ$$

$$= \angle O = 121^\circ$$

Question: 3 In Fig. 6.41, if $AB \parallel DE$, $\angle BAC = 35^\circ$ and $\angle CDE = 53^\circ$, find $\angle DCE$.

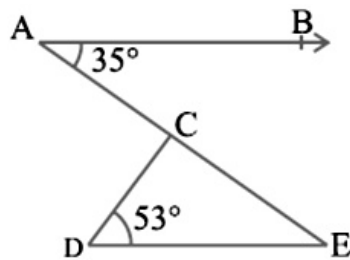


Fig. 6.41

Ans.: Given: AB parallel DE , $\angle BAC = 35^\circ$, $\angle CDE = 53^\circ$

To Find: $\angle DCE$

According to question,

$$\angle BAC = \angle CED \text{ (Alternate interior angles)}$$

Therefore,

$$\angle CED = 35^\circ$$

Now, In $\triangle DEC$,

$\angle DCE + \angle CED + \angle CDE = 180^\circ$ (Sum of the interior angles of the triangle)

$$\angle DCE + 35^\circ + 53^\circ = 180^\circ$$

$$\angle DCE + 88^\circ = 180^\circ$$

$$\angle DCE = 92^\circ$$

Question: 4 In Fig. 6.42, if lines PQ and RS intersect at point T, such that $\angle PRT = 40^\circ$, $\angle RPT = 95^\circ$ and $\angle TSQ = 75^\circ$, find $\angle SQT$.

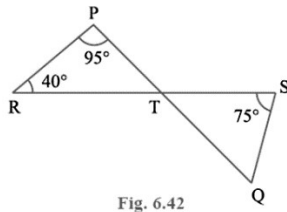


Fig. 6.42

Ans.:

Given:

$$\angle PRT = 40^\circ$$

$$\angle RPT = 95^\circ \text{ and,}$$

$$\angle TSQ = 75^\circ$$

Now according to the question,

$$\angle PRT + \angle RPT + \angle PTR = 180^\circ \text{ (Sum of interior angles of the triangle)}$$

$$40^\circ + 95^\circ + \angle PTR = 180^\circ$$

$$40^\circ + 95^\circ + \angle PTR = 180^\circ$$

$$135^\circ + \angle PTR = 180^\circ$$

$$\angle PTR = 45^\circ$$

$$\angle PTR = \angle STQ = 45^\circ \text{ (Vertically opposite angles)}$$

Now,

$$\angle TSQ + \angle PTR + \angle SQT = 180^\circ \text{ (Sum of the interior angles of the triangle)}$$

$$75^\circ + 45^\circ + \angle SQT = 180^\circ$$

$$120^\circ + \angle SQT = 180^\circ$$

$$\angle SQT = 60^\circ$$

Question: 5 In Fig. 6.43, if $PQ \perp PS$, $PQ \parallel SR$, $\angle SQR = 28^\circ$ and $\angle QRT = 65^\circ$, then find the values of x and y .

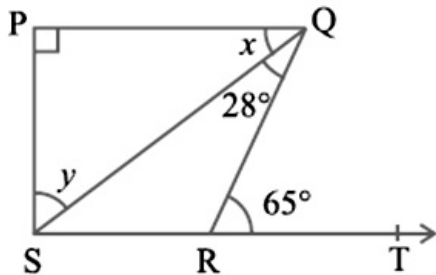


Fig. 6.43

Ans.:

To Find: Values of x and y

Given: PQ is perpendicular to PS , $PQ \parallel SR$

$$\angle SQR = 28^\circ$$

$$\text{And, } \angle QRT = 65^\circ$$

Now according to the question,

$$x + \angle SQR = \angle QRT \text{ (Alternate angles are equal as } QR \text{ is transversal)}$$

$$x + 28^\circ = 65^\circ$$

$$x = 37^\circ$$

Now, in ΔPQS , Sum of interior angles of a triangle = 180°

$$\angle PQS + \angle PSQ + \angle QPS = 180^\circ \text{ Therefore,}$$

$$y + 37^\circ + 90^\circ = 180^\circ$$

$$y = 53^\circ \text{ So } x = 37^\circ \text{ and } y = 53^\circ$$

Question: 6 In Fig. 6.44, the side QR of ΔPQR is produced to a point S. If the bisectors of $\angle PQR$ and $\angle PRS$ meet at point T, then prove that $\angle QTR = \frac{1}{2} \angle QPR$.

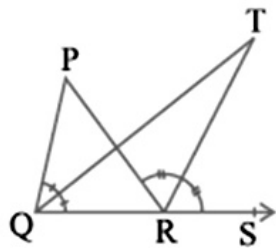


Fig. 6.44

Ans.: To Prove: $\angle QTR = \frac{1}{2} \angle QPR$

Given: Bisectors of $\angle PQR$ and $\angle PRS$ meet at point T

Proof:

In ΔQTR ,

$\angle TRS = \angle TQR + \angle QTR$ (Exterior angle of a triangle equals to the sum of the two opposite interior angles)

$$\angle QTR = \angle TRS - \angle TQR \text{ -----(i)}$$

Similarly in ΔQPR ,

$$\angle SRP = \angle QPR + \angle PQR$$

$$2\angle TRS = \angle QPR + 2\angle TQR$$

($\because \angle TRS$ and $\angle TQR$ are the bisectors of $\angle SRP$ and $\angle PQR$ respectively.)

$$\angle QPR = 2\angle TRS - 2\angle TQR$$

$$\angle TRS - \angle TQR = \frac{1}{2}\angle QPR \dots\dots\dots(ii)$$

From (i) and (ii), we get

$$\angle QTR = \frac{1}{2}\angle QPR$$

Hence, proved.