Kinetic Theory

OBJECTIVE TYPE QUESTIONS

Multiple Choice Questions (MCQs)

1. If C_P and C_V denoted the specific heats of unit mass of nitrogen at constant pressure and volume respectively, then

(a) $C_P - C_V = \frac{R}{28}$ (b) $C_P - C_V = \frac{R}{7}$

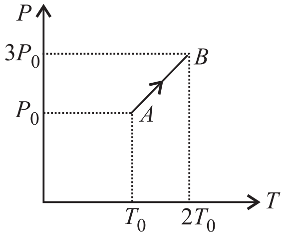
(c) $C_P - C_V = \frac{R}{14}$ (d) $C_P - C_V = R$

2. An ideal gas is compressed isothermally until its pressure is doubled and then allowed to expand adiabatically to regain its original volume ($\gamma = 1.4$ and $2^{-1.4} = 0.38$). The ratio of the final to initial pressure is

(a) 0.76 : 1 (b) 1 : 1

(c) 0.66 : 1 (d) 0.86 : 1

3. Pressure versus temperature graph of an ideal gas is as shown in figure. Density of the gas at point A is ρ_0. Density at point B will be



(a) $\frac{3}{4}\rho_0$

(b) $\frac{3}{2}\rho_0$

(c) $\frac{4}{3}\rho_0$

(d) $2\rho_0$

4. A vessel has 6 g of hydrogen at pressure P and temperature 500 K. A small hole is made in it so that hydrogen leaks out. How much hydrogen leaks out if the final pressure is $\frac{P}{2}$ and temperature falls to 300 K?

(a) 2 g (b) 3 g (c) 4 g (d) 1 g

5. The pressure and density of a diatomic gas $\left(\gamma = \frac{7}{5}\right)$ changes adiabatically from (P, d) to (P', d'). If $\frac{d'}{d} = 32$ then $\frac{P'}{P}$ is

(a) $\frac{1}{128}$ (b) 32 (c) 128 (d) 256

6. The temperature of the gas contained in a closed vessel increases by 1°C when pressure of the gas is increased by 1%. The initial temperature of the gas is

(a) 100 K (b) 100°C (c) 200 K (d) 250°C

7. Two flasks R and S of volume V_1 and V_2 contain same gas at pressure P_1 and P_2 respectively at the same temperature. Pressure of the gas when the flasks R and S are connected by a tube of negligible volume is

(a) $\frac{P_1V_1 + P_2V_2}{V_1 + V_2}$ (b) $\frac{P_1V_1 + P_2V_2}{2(V_1 + V_2)}$

(c) $\frac{P_1V_2 + P_2V_1}{V_1 + V_2}$ (d) $\frac{(P_1 + P_2)(V_1 + V_2)}{(V_1 + 2V_2)}$

8. 1 mole of H_2 gas is contained in a box of volume $V = 1.00$ m³ at $T = 300$ K. The gas is heated to a temperature of $T = 3000$ K and the gas gets converted to a gas of hydrogen atoms. The final pressure would be (considering all gases to be ideal)

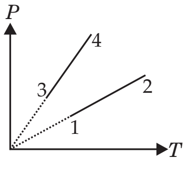
(a) same as the pressure initially.
(b) 2 times the pressure initially.
(c) 10 times the pressure initially.
(d) 20 times the pressure initially.

9. An inflated rubber balloon contains one mole of an ideal gas, has a pressure P, volume V and temperature T. If the temperature rises to 1.1 T, and the volume is increased to 1.05 V, the final pressure will be

(a) 1.1 P (b) P

(c) less than P (d) between P and 1.1

10. Pressure versus temperature graph of an ideal gas of equal number of moles of different volumes are plotted as shown in figure. Choose the correct alternative.



(a) $V_1 = V_2 = V_3 = V_4$

(b) $V_4 > V_3 > V_2 > V_1$

(c) $V_1 = V_2$; $V_3 = V_4$ and $V_2 > V_3$

(d) $V_1 = V_2$, $V_3 = V_4$ and $V_2 < V_3$

11. The temperature of an ideal gas is increased from 27°C to 127°C, then percentage increase in v_{rms} is

(a) 37% (b) 11% (c) 33% (d) 15.5%

12. An ideal gas at a pressure of 1 atmosphere and temperature of 27°C is compressed adiabatically until its pressure becomes 8 times the initial pressure. Then the final temperature is $\left(\text{Given } \gamma = \frac{3}{2}\right)$

(a) 627°C (b) 527°C (c) 427°C (d) 327°C

13. A cylinder contains 10 kg of gas at pressure of 10^7 N m^{-2}, the quantity of gas taken out of the cylinder, if final pressure is 2.5×10^6 N m^{-2} is

(a) 7.5 kg (b) 10.5 kg

(c) 5.2 kg (d) 2.5 kg

14. A vessel is filled with a gas at a pressure of 76 cm of mercury of a certain temperature. The mass of the gas is increased by 50% by introducing more gas in the vessel at the same temperature. The resultant pressure of the gas is

(a) 76 cm of mercury (b) 108 cm of mercury

(c) 112 cm of mercury (d) 114 cm of mercury

15. A gas at absolute temperature 300 K has pressure 4×10^{-10} N/m^2. The number of molecules per cm^3 is of the order of (Boltzmann constant, $k_B = 1.38 \times 10^{-23}$ J/K)

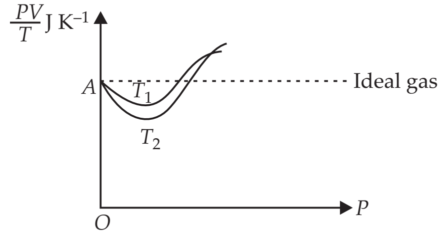
(a) 100 (b) 10^5 (c) 10^8 (d) 10^{11}

16. Two containers of equal volume contain the same gas at pressures P_1 and P_2 and absolute temperatures T_1 and T_2 respectively. On joining the vessels, the gas reaches a common pressure P and a common temperature T. The ratio $\frac{P}{T}$ is

(a) $\frac{P_1}{T_1} + \frac{P_2}{T_2}$ (b) $\frac{1}{2}\left[\frac{P_1}{T_1} + \frac{P_2}{T_2}\right]$

(c) $\frac{P_1T_2 + P_2T_1}{T_1 + T_2}$ (d) $\frac{P_1T_2 - P_2T_1}{T_1 - T_2}$

17. Given is the graph between $\frac{PV}{T}$ and P for 1 g of oxygen gas at two different temperatures T_1 and T_2, as shown in figure. Given, density of oxygen = 1.427 kg m^{-3}. The value of PV/T at the point A and the relation between T_1 and T_2 are respectively



(a) 0.259 J K^{-1} and $T_1 < T_2$

(b) 8.314 J mol^{-1} K^{-1} and $T_1 > T_2$

(c) 0.259 J K^{-1} and $T_1 > T_2$

(d) 4.28 J K^{-1} and $T_1 < T_2$

18. A cubic vessel (with faces horizontal + vertical) contains an ideal gas at NTP. The vessel is being carried by a rocket which is moving at a speed of 500 m s^{-1} in vertical direction. The pressure of the gas inside the vessel as observed by us on the ground

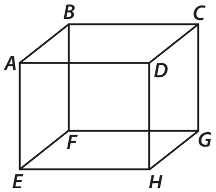
(a) remains the same because 500 m s^{-1} is very much smaller than v_{rms} of the gas.

(b) remains the same because motion of the vessel as a whole does not affect the relative motion of the gas molecules and the walls.

(c) will increase by a factor equal to $[v_{\text{rms}}^2 + (500)^2]/v_{\text{rms}}^2$, where v_{rms} was the original mean square velocity of the gas.

(d) will be different on the top wall and bottom wall of the vessel.

19. 1 mole of an ideal gas is contained in a cubical volume V, $ABCDEFGH$ at 300 K as shown in figure. One face of the cube ($EFGH$) is made up of a material which totally absorbs any gas molecule incident on it. At any given time,



(a) the pressure on $EFGH$ would be zero.

(b) the pressure on all the faces will be equal.

(c) the pressure of $EFGH$ would be double the pressure on $ABCD$.

(d) the pressure on $EFGH$ would be half that on $ABCD$.

20. The average kinetic energy of the molecules of a low density gas at 27°C is (Boltzmann constant = 1.38×10^{-23} J K^{-1})

(a) 3.1×10^{-20} J (b) 3.5×10^{-21} J

(c) 5.3×10^{-18} J (d) 6.21×10^{-21} J

21. At 27°C temperature, the kinetic energy of an ideal gas is E_1. If the temperature is increased to 327°C, then kinetic energy would be

(a) $\frac{E_1}{2}$ (b) $\frac{E_1}{\sqrt{2}}$ (c) $\sqrt{2}E_1$ (d) $2E_1$

22. At what temperature is the root mean square speed of an atom in an argon gas cylinder equal to the rms speed of a helium gas atom at –20°C ? (Atomic mass of Ar 39 u and He = 400 u)

(a) 2.52×10^3 K (b) 2.52×10^2 K

(c) 4.03×10^3 K (d) 4.03×10^2 K

23. One mole of an ideal monoatomic gas at temperature T_0 expands slowly according to the law $\frac{P}{V}$ = constant. If the final temperature is $2T_0$, heat supplied to the gas is

(a) $2RT_0$ (b) RT_0 (c) $\frac{3}{2}RT_0$ (d) $\frac{1}{2}RT_0$

24. The average kinetic energy of a gas molecule at 27°C is 6.21×10^{-21} J. Its average kinetic energy at 227°C will be

(a) 52.2×10^{-21} J (b) 5.22×10^{-21} J

(c) 10.35×10^{-21} J (d) 11.35×10^{-21} J

25. The molecules of a given mass of a gas have root mean square speeds of 100 m s^{-1} at 27°C and 1 atmospheric pressure. The root mean square speeds of the molecules of the gas at 127°C and 2 atmospheric pressure is

(a) $\frac{200}{\sqrt{3}}$ (b) $\frac{100}{\sqrt{3}}$ (c) $\frac{400}{3}$ (d) $\frac{200}{3}$

26. A vessel has 6 g of oxygen at pressure P and temperature 400 K. A small hole is made in it so that oxygen leaks out. How much oxygen leaks out if the final pressure is $\frac{P}{2}$ and temperature 300 K?

(a) 5 g (b) 4 g (c) 2 g (d) 3 g

27. A gaseous mixture consists of 16 g of helium and 16 g of oxygen. The ratio C_P/C_V of the mixture is

(a) 1.4 (b) 1.54

(c) 1.59 (d) 1.62

28. The ratio $\frac{C_P}{C_V} = \gamma$ for a gas. Its molecular weight is M. Its specific heat capacity at constant pressure is

(a) $\frac{R}{\gamma - 1}$ (b) $\frac{\gamma R}{\gamma - 1}$

(c) $\frac{\gamma R}{M(\gamma - 1)}$ (d) $\frac{\gamma RM}{(\gamma - 1)}$

29. One mole of a monatomic gas is mixed with 3 moles of a diatomic gas. What is the molar specific heat of the mixture at constant volume?

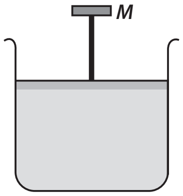
(a) $\frac{5}{4}R$ (b) $\frac{9}{4}R$ (c) $\frac{3}{4}R$ (d) R

30. One kg of a diatomic gas is at a pressure of 8×10^4 N m^{-2}. The density of the gas is 4 kg m^{-3}. What is the energy of the gas due to its thermal motion?

(a) 3×10^4 J (b) 5×10^4 J

(c) 6×10^4 J (d) 7×10^4 J

31. A cylinder containing an ideal gas is in vertical position and has a piston of mass M that is able to move up or down without friction. If the temperature is increased,



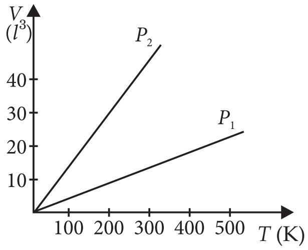
(a) both P and V of the gas will change.

(b) only P will increase according to Charle's law.

(c) V will change but not P.

(d) P will change but not V.

32. Volume versus temperature graphs for a given mass of an ideal gas are shown in figure at two different values of constant pressure. What can be inferred about relation between P_1 and P_2 ?

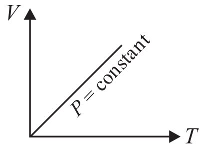


(a) $P_1 > P_2$ (b) $P_1 = P_2$

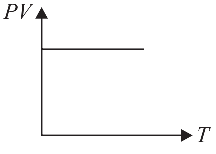
(c) $P_1 < P_2$ (d) data is insufficient

33. Which of the following graphs represent the behaviour of an ideal gas ?

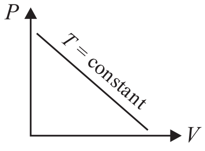
(a)



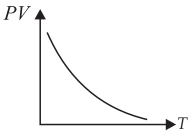
(b)



(c)



(d)



34. If one mole of a monatomic gas $\left(\gamma = \frac{5}{3}\right)$ is mixed with one mole of a diatomic gas $\left(\gamma = \frac{7}{5}\right)$, the value of γ for the mixture is

(a) 1.40 (b) 1.50 (c) 1.53 (d) 3.07

35. A gas mixture consists of 2.0 moles of oxygen and 4.0 moles of neon at temperature T. Neglecting all vibrational modes, calculate the total internal energy of the system. (Oxygen has two rotational modes.)

(a) 11 RT (b) 13 RT

(c) 15 RT (d) 19 RT

36. The heat capacity per mole of water is (R is universal gas constant)

(a) $9R$ (b) $\frac{9}{2}R$ (c) $6R$ (d) $5R$

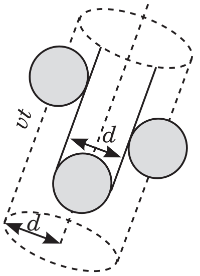
37. Two mole of oxygen is mixed with eight mole of helium. The effective specific heat of the mixture at constant volume is

(a) $1.3R$ (b) $1.4R$ (c) $1.7R$ (d) $1.9R$

Case Based MCQs

Case I : Read the passage given below and answer the following questions from 38 to 42.

Mean Free Path



The average distance travelled between successive collisions of molecules of a gas is called as mean free path (λ).

Let in time interval t, the molecules moves a distance vt and collides with every molecules in the cylindrical volumes, $V = \pi d^2 vt$ and number of molecules in cylindrical volume be N.

$\therefore$ Number of molecules per unit volume be n

$$= \frac{N}{V} \Rightarrow N = n\pi d^2 vt$$

The mean free path is the total distance divided by the number of collisions.

$$\lambda = \frac{vt}{n\pi d^2 vt} = \frac{1}{n\pi d^2}$$

By considering motion of all the molecules the exact expression is

$$\lambda = \frac{1}{\sqrt{2}\, n\pi d^2}$$

38. A vessel contains 60,000 molecules of a gas. Due to a very fine hole in the wall, 10,000 molecules escape from the vessel. Then the mean free path of the molecules of the gas

(a) is increased.

(b) is decreased.

(c) is not changed.

(d) may increase or decrease.

39. The path lengths travelled by a molecule A in 6 collisions are 3, 7, 1, 2, 4, 3 units respectively. The mean free path of the molecule A is

(a) $\frac{13}{6}$ unit (b) $\frac{20}{6}$ unit

(c) $\frac{87}{6}$ unit (d) $\frac{6}{20}$ unit

40. The mean free path of a gas varies with the density of gas according to the following relation

(a) $\lambda \propto \rho$ (b) $\lambda \propto \sqrt{\rho}$

(c) $\lambda \propto \frac{1}{\rho}$ (d) $\lambda \propto \rho^2$

41. Mean free path of a gas molecule is

(a) inversely proportional to number of molecules per unit volume

(b) inversely proportional to diameter of the molecule

(c) directly proportional to the square root of the absolute temperature

(d) directly proportional to the molecular mass

42. The volume of a gas and the number of molecules within that volume for three situations are (i) $2V_0$ and N_0 (ii) $3V_0$ and $3N_0$ (iii) $3V_0$ and $9N_0$. The situations are ranked according to the mean free path (greatest first) as

(a) (i), (ii), (iii) (b) (iii), (ii), (i)

(c) (ii), (iii), (i) (d) (ii), (i), (iii)

Case II : Read the passage given below and answer the following questions from 43 to 46.

Law of Equipartition of Energy

In equilibrium the total energy is equally distributed in all possible energy modes, with each mode having an average energy equal to

(1/2) kT. This is known as law of equipartition energy.

Each translational and rotational degree of freedom contributes (1/2) kT to the energy.

Each vibrational frequency contributes 2 × (1/2) $kT = kT$ energy since vibration has both kinetic and potential modes of energy.

43. According to equipartition law of energy each particle in a system of particles have thermal energy E equal to

(a) $E = k_B T$ (b) $E = \frac{1}{2} k_B T$

(c) $E = 3k_B T$ (d) $E = \frac{3}{2} k_B T$

44. The average energy per molecule of a triatomic gas at room temperature T is

(a) $3kT$ (b) $\frac{1}{2}kT$

(c) $\frac{3}{2}kT$ (d) $\frac{5}{2}kT$

45. The gases carbon-monoxide (CO) and nitrogen are kept at the same temperature. If their kinetic energies are E_1 and E_2 respectively, then

(a) $E_1 = E_2$

(b) $E_1 > E_2$

(c) $E_1 < E_2$

(d) E_1 and E_2 cannot be compared

46. Which one of the following molecules does not possess vibrational energy?

(a) Oxygen (b) Nitrogen

(c) Argon (d) CO_2

Assertion & Reasoning Based MCQs

For question numbers 47-56, two statements are given-one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below.

(a) Both A and R are true and R is the correct explanation of A

(b) Both A and R are true but R is NOT the correct explanation of A

(c) A is true but R is false

(d) A is false and R is also false

47. Assertion (A) : Mean free path of gas molecules varies inversely as density of the gas.

Reason (R) : Mean free path of gas molecules is defined as the average distance travelled by a molecule between two successive collisions.

48. Assertion (A) : The root mean square velocity of molecules of a gas having Maxwellian distribution of velocities is higher than their most probable velocity, at any temperature.

Reason (R) : A very small number of molecules of a gas molecules which posses very large velocities.

49. Assertion (A) : The number of degrees of freedom of a linear triatomic molecules is 7.

Reason (R) : The number of degree of freedom depends on number of particle in the system.

50. Assertion (A) : Air pressure in a car tyre increases during driving.

Reason (R) : Absolute zero degree temperature is not zero energy temperature.

51. Assertion (A) : Absolute zero is not the temperature corresponding to zero energy.

Reason (R) : The temperature at which no molecular motion ceases is called absolute zero temperature.

52. Assertion (A) : The ratio of specific heat of a gas at constant pressure and specific heat at constant volume for a diatomic gas is more than that for a monatomic gas.

Reason (R) : The molecules of a monatomic gas have more degree of freedom than those of a diatomic gas.

53. Assertion (A) : Specific heat of a gas at constant pressure is greater than its specific heat at constant volume.

Reason (R) : At constant pressure, some heat is spent in expansion of the gas.

54. Assertion (A) : The total translational kinetic energy of all the molecules of a given mass of an ideal gas is 1.5 times the product of its pressure and its volume.

Reason (R) : The molecules of a gas collide with each other and the velocities of the molecules change due to collision.

55. **Assertion (A) :** Vibrational energy of diatomic molecule corresponding to each degree of freedom is k_BT.

Reason (R) : For every molecule, vibrational degree of freedom is 2.

56. **Assertion (A) :** An undamped spring-mass system is simplest free vibration system.

Reason (R) : It has three degrees of freedom.

SUBJECTIVE TYPE QUESTIONS

Very Short Answer Type Questions (VSA)

1. The volume of a given mass of a gas at 27°C and 1 atm is 100 cm^3. What will be its volume at 327°C?

2. A gas mixture consists of molecules of types A, B and C with masses $m_A > m_B > m_C$. Rank the three types of molecules in decreasing order of their rms speeds.

3. Plot pressure (P) versus volume (V) graphs for a fixed mass of a gas are drawn at two different temperatures.

4. On reducing the volume of a gas at constant temperature, the pressure of the gas increases. Explain it on the basis of kinetic theory.

5. What would be the effect on the rms velocity of gas molecules if the temperature of the gas is increased by a factor of 4?

6. Two gases are at temperatures of 300 K and 350 K respectively. What is the ratio of the average kinetic energies of their molecules ?

7. Name the two factors on which the degree of freedom of gas depend.

8. Calculate the ratio of the mean free paths of the molecules of two gases having molecular diameters 1 Å and 2 Å. The gases may be considered under identical conditions of temperature, pressure and volume.

9. What is the order of mean free path (λ) of the gas molecules?

10. Find the energy per mole per degree of freedom of an ideal gas.

Short Answer Type Questions (SA-I)

11. For a rigid diatomic molecule, universal gas constant $R = nC_P$ where C_P is the molar specific heat at constant pressure and n is a number. Find the value of n.

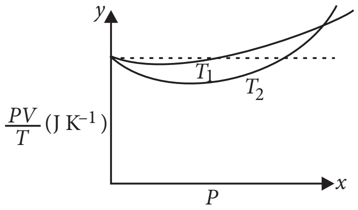
12. Calculate the internal energy of 1g of oxygen gas at STP.

13. A balloon has 5 g mole of helium at 7°C. Calculate,

(a) the number of atoms of helium in the balloon,

(b) the total internal energy of the system.

14. Figure shows plot of PV/T versus P for 1.00×10^{-3} kg of oxygen gas at two different temperatures.



(a) Which is true: $T_1 > T_2$ or $T_1 < T_2$?

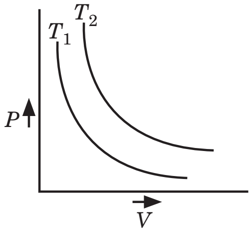
(b) What is the value of PV/T where the curves meet on the y-axis?

15. Two vessels of the same volume are filled with the same gas at the same temperature. If the pressure of the gas in these vessels be in the ratio 1 : 2 then, find

(i) the ratio of the rms speeds of the molecules.

(ii) the ratio of the number of molecules.

16. Isothermal curves for a given mass of gas are shown at two different temperatures T_1 and T_2. State whether $T_1 > T_2$ or $T_2 > T_1$. Justify your answer.



17. If C_P and C_V denote the specific heats of nitrogen per unit mass at constant pressure and constant volume respectively, then what is the value of $C_P - C_V$?

18. Calculate the number of degrees of freedom of molecules of hydrogen in 1 cm^3 of hydrogen gas at NTP.

19. A gas mixture consists of molecules of types A, B and C with masses $m_A > m_B > m_C$. Rank the three types of molecules in decreasing order of (a) average K.E., (b) rms speeds.

20. State the law of equipartition of energy.

Short Answer Type Questions (SA-II)

21. (a) Write the SI unit of R.

(b) Write the values of R in different units, where R is universal gas constant.

22. Calculate specific heat of water using law of equipartition of energy.

23. An insulated container containing monoatomic gas of molar mass m is moving with a velocity v_o . If the container is suddenly stopped, find the change in temperature.

24. From a certain apparatus, the diffusion rate of hydrogen has an average value of 28.7 $cm^3\ s^{-1}$. The diffusion of another gas under the same conditions is measured to have an average rate of 7.2 $cm^3\ s^{-1}$. Identify the gas.

25. Two monatomic ideal gases A and B occupying the same volume V, are at the same temperature T and pressure P. If they are mixed, the resultant mixture has volume V and temperature T. Calculate the pressure of the mixture.

26. Hydrogen is at temperature T and helium is at temperature $2T$. The internal energy of both gases is the same. What is the ratio of number of moles of hydrogen and helium gases?

27. (a) What do you understand by specific heat capacity of water (assume water behaves as a solid)?

(b) If one mole of ideal monatomic gas ($\gamma = 5/3$) is mixed with one mole of diatomic gas ($\gamma = 7/5$). What is the value of γ for the mixtures? (here, γ represents the ratio of specific heat at constant pressure to that at constant volume)

28. Ten small planes are flying at a speed of 150 km h^{-1} in total darkness in an air space that is $20 \times 20 \times 1.5$ km^3 in volume. You are in one of the planes, flying at random within this space with no way of knowing where the other planes are. On the average about how long a time will elapse between near collision with your plane. Assume for this rough computation that a safety region around the plane can be approximated by a sphere of radius 10 m.

29. (a) How much volume does one mole of a gas occupy at NTP?

(b) What are the essential characteristics of an ideal gas?

(c) What are the conditions under which real gases behave nearly as a perfect gas?

30. What do you understand by mean speed, root mean square speed and most probable speed of a gas. The velocities of ten particles in m/s are 0, 2, 3, 4, 4, 4, 5, 5, 6, 9. Calculate

(i) average speed

(ii) rms speed.

31. Explain why

(a) there is no atmosphere on moon.

(b) there is fall in temperature with altitude.

32. A flask contains argon and chlorine in the ratio of 2 : 1 by mass. The temperature of the mixture is 27°C. Obtain the ratio of average kinetic energy per molecule and root mean square speed v_{rms} of the molecules of the two gases. Given : atomic mass of argon = 39.9 u and molecular mass of chlorine = 70.9 u.

33. A closed vessel contains a mixture of two diatomic gases A and B. Molar mass of A is 16 times that of B and mass of gas A, contained in the vessel is 2 times that of B. Find the ratio of

(i) average kinetic energy per molecule of gas A is equal to that of gas B.

(ii) root mean square value of translational velocity of gas B to that of A.

(iii) pressure exerted by gas B to that exerted by gas A.

34. The velocities of ten particles of a sample of gas in m s^{-1} are 8, 1, 4, 4, 4, 4, 7, 7, 6, 5. Find the (i) average speed (ii) rms speed (iii) most probable speed.

35. Estimate the average thermal energy of a helium atom at (i) room temperature (27°C), (ii) the temperature on the surface of the Sun (6000 K), (iii) the temperature of 10 million kelvin (the typical core temperature in case of a star).

36. A metre long narrow bore held horizontally (and closed at one end) contains a 76 cm long mercury thread, which traps a 15 cm column of air. What happens if the tube is held vertically with the open end at the bottom?

Long Answer Type Questions (LA)

37. Derive an expression for pressure of a gas in a container. Using it, relate K.E. with pressure.

38. Using the law of equipartition of energy, determine the values of C_P, C_V and γ for (a) monatomic, (b) diatomic and (c) triatomic gases.

39. What is meant by mean free path of a gas molecule? Derive expression for it. On which factors does the mean free path depends?

40. Consider a rectangular block of wood moving with a velocity v_0 in a gas at temperature T and mass density ρ. Assume the velocity is along x-axis and the area of cross-section of the block perpendicular to v_0 is A. Show that the drag force on the block is $4\rho A v_0\sqrt{\frac{kT}{m}}$ where m is the mass of the gas molecule.

ANSWERS

OBJECTIVE TYPE QUESTIONS

1. **(a)**

2. **(a) :** Let V be the original volume of the gas.

For an isothermal process

PV = constant

$$\therefore \quad P_iV_i = P_fV_f$$

$$V_f = V_i\left(\frac{P_i}{P_f}\right) = V\left(\frac{P_i}{2P_i}\right) = \frac{V}{2}$$

For an adiabatic process

PV^γ = constant

According to the question

$$\therefore \quad (2P_i)\left(\frac{V}{2}\right)^\gamma = P_fV^\gamma$$

$$\frac{P_f}{P_i} = 2(2)^{-\gamma} = 2(2)^{-1.4} = 2\,(0.38) = \frac{0.76}{1}$$

3. **(b) :** According to ideal gas equation

$$P = \frac{\rho RT}{M} \quad \text{or} \quad \rho = \frac{PM}{RT} \quad \text{or} \quad \rho \propto \frac{P}{T}$$

From the graph,

$$\left(\frac{P}{T}\right)_A = \frac{P_0}{T_0} \quad \text{and} \quad \left(\frac{P}{T}\right)_B = \frac{3}{2}\left(\frac{P_0}{T_0}\right)$$

$$\text{or} \quad \left(\frac{P}{T}\right)_B = \frac{3}{2}\left(\frac{P}{T}\right)_A \qquad \therefore \rho_B = \frac{3}{2}\rho_A = \frac{3}{2}\rho_0$$

4. **(d) :** As $PV = \frac{m}{M}RT$

$$\therefore \quad \text{Initially, } PV = \frac{6}{M}R \times 500 \qquad \text{...(i)}$$

and finally, $\frac{P}{2}V = \frac{(6-x)}{M}R \times 300$ (if x g of gas leaks out) ...(ii)

Dividing eqn. (i) by (ii), we get $2 = \frac{6}{6-x} \times \frac{5}{3}$

On solving, we get $x = 1$ g

5. **(c) :** Here, $\gamma = \frac{7}{5}$, $P_1 = P$, $d_1 = d$, $P_2 = P'$, $d_2 = d'$

For an adiabatic process

PV^γ = constant

$$\therefore \quad P_1V_1^\gamma = P_2V_2^\gamma$$

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^\gamma = \left(\frac{d_2}{d_1}\right)^\gamma$$

$$\frac{P'}{P} = \left(\frac{d'}{d}\right)^{7/5} = (32)^{7/5}$$

$$\frac{P'}{P} = (2^5)^{7/5} = 2^7 = 128$$

6. **(a) :** Here, volume of the gas is constant.

$$\therefore \quad \frac{P_2}{P_1} = \frac{T_2}{T_1} \quad \text{or} \quad \frac{P_1 + 0.01P_1}{P_1} = \frac{T_1+1}{T_1} \quad \text{or} \quad T_1 = 100 \text{ K}$$

7. **(a) :** Number of moles of gas in two flasks are

$$n_1 = \frac{P_1V_1}{RT} \quad \text{and} \quad n_2 = \frac{P_2V_2}{RT} \quad \because \quad n = n_1 + n_2$$

$$\therefore \quad P = \frac{(n_1+n_2)RT}{V_1+V_2} = \frac{P_1V_1 + P_2V_2}{V_1+V_2}$$

8. **(d) :** According to gas equation

$PV = nRT$

or $\frac{P_2V_2}{T_2} = \frac{P_1V_1}{T_1}$ or $\frac{P_2}{P_1} = \frac{V_1}{V_2}\cdot\frac{T_2}{T_1}$

Here, $T_2 = 3000$ K, $T_1 = 300$ K

Since H_2 splits into hydrogen atoms, therefore volume become half *i.e.*, $V_2 = \frac{1}{2}V_1$

∴ $\frac{P_2}{P_1} = \frac{V_1}{\frac{1}{2}V_1}\times\frac{3000}{300}$ or $\frac{P_2}{P_1} = 20$

9. (d) : Using $PV = nRT$

or $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

Here, $P_1 = P$, $V_1 = V$, $T_1 = T$

$T_2 = 1.1\ T$ and $V_2 = 1.05\ V$

∴ $P_2 = \frac{P_1V_1T_2}{V_2T_1}$

$$P_2 = \frac{P\times V\times 1.1T}{1.05V\times T} = 1.05\ P$$

10. (c) : As P-T graphs are straight lines, passing through origin, therefore, $P \propto T$. But as $P = \left(\frac{nR}{V}\right)T$, therefore, the slope of straight line $\propto \frac{1}{V}$.

As $(\text{Slope})_{12} < (\text{Slope})_{34}$ ∴ $V_2 > V_3$

As for a given slope, V is constant,

∴ $V_1 = V_2$ and $V_3 = V_4$

11. (d) : $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$

$$\%\text{ increase in } v_{\text{rms}} = \frac{\sqrt{\frac{3RT_2}{M}} - \sqrt{\frac{3RT_1}{M}}}{\sqrt{\frac{3RT_1}{M}}}\times 100$$

$$= \frac{\sqrt{T_2}-\sqrt{T_1}}{\sqrt{T_1}}\times 100 = \frac{\sqrt{400}-\sqrt{300}}{\sqrt{300}}\times 100$$

$$= \frac{20-17.32}{17.32}\times 100 = 15.5\%$$

12. (d) : Here, $P_1 = 1$ atm, $T_1 = 27\ °\text{C} = 27 + 273 = 300$ K

$P_2 = 8P_1$, $T_2 = ?$, $\gamma = \frac{3}{2}$

As changes are adiabatic,

∴ $P_1^{\gamma-1}T_1^{-\gamma} = P_2^{\gamma-1}T_2^{-\gamma}$

$$\left(\frac{T_2}{T_1}\right)^{-\gamma} = \left(\frac{P_1}{P_2}\right)^{\gamma-1}$$

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = 300(8)^{(1.5-1)/1.5} = 300(8)^{1/3}$$

$T_2 = 600\ \text{K} = (600 - 273)\ °\text{C} = 327\ °\text{C}$

13. (a) : $P = \frac{1}{3}\frac{M}{V}\overline{v}^2$; so, $P \propto M$

where M = mass of the gas, V = volume and $\overline{v}^2$ = mean square speed of the gas molecules.

∴ $\frac{P_1}{P_2} = \frac{M_1}{M_2}$ or $M_2 = M_1\frac{P_2}{P_1} = 10\times\frac{2.5\times10^6}{10^7} = 2.5\ \text{kg}$

Quantity of gas taken out $= M_1 - M_2 = 10 - 2.5 = 7.5$ kg

14. (d) : Pressure exerted by a gas.

$$P = \frac{1}{2}\frac{M}{V}v_{av}^2$$

Since the temperature T is kept constant, v_{av}^2 and V are also constant.

∴ $P \propto M$

or $\frac{P_2}{P_1} = \frac{M_2}{M_1}$

According to the question,

$$\therefore\quad \frac{P_2}{76} = \frac{\left[M_1 + \left(\frac{50}{100}\right)M_1\right]}{M_1} = \frac{3}{2}$$

⇒ $P_2 = \frac{3}{2}\times 76 = 114$ cm of mercury.

15. (b) : As $P = nk_BT$, where n = number of molecules per unit volume

$$\therefore\quad n = \frac{P}{k_BT} = \frac{4\times10^{-10}}{1.38\times10^{-23}\times300} \cong 10^{11}\text{ per m}^3 = 10^5\text{ per cm}^3$$

16. (b) : For a closed system, the total mass of gas or the number of moles remains constant.

Let n_1 and n_2 be number of moles of gas in container 1 and container 2 respectively. Then

$P_1V = n_1RT_1$ or $n_1 = \frac{P_1V}{RT_1}$

$P_2V = n_2RT_2$ or $n_2 = \frac{P_2V}{RT_2}$

$P(2V) = (n_1 + n_2)RT$ ...(i)

Substituting the values of n_1 and n_2 in equation (i), we get

$$P(2V) = \left(\frac{P_1V}{RT_1} + \frac{P_2V}{RT_2}\right)RT \quad\text{or}\quad \frac{P}{T} = \frac{1}{2}\left(\frac{P_1}{T_1} + \frac{P_2}{T_2}\right)$$

17. (c) : $PV = nRT = \frac{m}{M}RT$

where m = mass of the gas and $\frac{m}{M} = n$ = number of moles.

$\frac{PV}{T} = nR$ = a constant for all values of P.

That is why, ideally it is a straight line.

$$\therefore\quad \frac{PV}{T} = \frac{1\text{ g}}{32\text{ g mol}^{-1}}\times 8.31\text{ J mol}^{-1}\text{ K}^{-1} = 0.259\text{ J K}^{-1}$$

Also $T_1 > T_2$ (Due to intermolecular attraction, volume will be lesser at lower temperature).

18. **(b) :** As $P = \frac{nRT}{V}$, it (*i.e.*, P) remains unaffected as n, R, T and V.

19. **(d) :** In an ideal gas, when a molecule collides elastically with a wall, the momentum transferred to each molecule will be twice the magnitude of its normal momentum. For the face *EFGH*, it transfers only half of that.

20. **(d) :** Average kinetic energy

$$= \frac{3}{2}k_BT = \frac{3}{2}(1.38\times10^{-23})(300) = 6.21\times10^{-21}\ \text{J}$$

21. **(d) :** Kinetic energy of an ideal gas,

$$E_K = \frac{3}{2}RT \text{ or } E_K \propto T$$

$$\therefore\quad E_{327} = E_{27}\,\frac{(327+273)}{(27+273)} = 2E_{27} = 2E_1$$

22. **(a) :** Let 1 and 2 represent for Argon atom and Helium atom.

rms speed of Argon, $v_{rms_1} = \sqrt{\frac{3RT_1}{M_1}}$

rms speed of Helium, $v_{rms_2} = \sqrt{\frac{3RT_2}{M_2}}$

According to question,

$v_{rms_1} = v_{rms_2}$

$$\therefore\quad \sqrt{\frac{3RT_1}{M_1}} = \sqrt{\frac{3RT_2}{M_2}}$$

$$\frac{T_1}{M_1} = \frac{T_2}{M_2}$$

$$\text{or}\quad T_1 = \frac{T_2}{M_2}\times M_1 = \frac{253}{4}\times 39.9$$

$$= 2.52\times10^3\ \text{K}$$

23. **(a)**

24. **(c) :** Average kinetic energy, $E = \frac{3}{2}k_BT$

where k_B is Boltzmann constant and T is the absolute temperature.

$$\therefore\quad \frac{E_2}{E_1} = \frac{T_2}{T_1} = \frac{273+227}{273+27} = \frac{500}{300}$$

$$\text{or}\quad E_2 = \frac{5}{3}\times6.21\times10^{-21}\ \text{J} = 10.35\times10^{-21}\ \text{J}$$

25. **(a) :** Here, $v_{rms_1} = 100$ m s^{-1},

$T_1 = 27$ °C $= (27 + 273)$ K $= 300$K

$P_1 = 1$ atm $v_{rms_2} = ?$, $T_2 = 127$ °C $= (127 + 273)$ K $= 400$ K

$P_2 = 2$ atm

From, $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2};\ \frac{V_1}{V_2} = \frac{P_2}{P_1}\cdot\frac{T_1}{T_2} = 2\times\frac{300}{400} = \frac{3}{2}$

Again, $P_1 = \frac{1}{3}\frac{M}{V_1}v_{rms_1}^2$ and $P_2 = \frac{1}{3}\frac{M}{V_2}v_{rms_2}^2$

$$\therefore\quad \frac{v_{rms_2}^2}{v_{rms_1}^2}\cdot\frac{V_1}{V_2} = \frac{P_2}{P_1}$$

$$v_{rms_2}^2 = v_{rms_1}^2\times\frac{P_2}{P_1}\times\frac{V_2}{V_1} = (100)^2\times2\times\frac{2}{3}$$

$$v_{rms_2} = \frac{200}{\sqrt{3}}\ \text{m s}^{-1}$$

26. **(c) :** $PV = \frac{m}{M}RT$ (for m grams of gas) ...(i)

$P'V = \frac{m'}{M}RT'$ ...(ii)

Dividing equation (ii) by (i) we get

$$\frac{P'}{P} = \frac{m'}{m}\frac{T'}{T}$$

$$m' = \frac{P'}{P}\times\frac{T}{T'}\times m = \left(\frac{P/2}{P}\right)\times\frac{400}{300}\times6 = 4\ \text{g}$$

∴ Mass of oxygen leaked

$\Delta m = m - m' = 6 - 4 = 2$ g

27. **(d) :** For 16 g of helium, $n_1 = \frac{16}{4} = 4$

For 16 g of oxygen, $n_2 = \frac{16}{32} = \frac{1}{2}$

For mixture of gases,

$$C_V = \frac{n_1C_{V_1} + n_2C_{V_2}}{n_1+n_2}\quad\text{where}\quad C_V = \frac{f}{2}R$$

$$C_P = \frac{n_1C_{P_1} + n_2C_{P_2}}{n_1+n_2}\quad\text{where}\quad C_P = \left(\frac{f}{2}+1\right)R$$

For helium, $f_1 = 3$, $n_1 = 4$. For oxygen, $f_2 = 5$, $n_2 = \frac{1}{2}$

$$\therefore\quad \frac{C_P}{C_V} = \frac{\left(4\times\frac{5}{2}R\right)+\left(\frac{1}{2}\times\frac{7}{2}R\right)}{\left(4\times\frac{3}{2}R\right)+\left(\frac{1}{2}\times\frac{5}{2}R\right)} = \frac{47}{29} = 1.62$$

28. **(c) :** According to Mayer's relation

$$C_P - C_V = R\quad\text{or}\quad 1 - \frac{C_V}{C_P} = \frac{R}{C_P}$$

$$\text{or}\quad 1 - \frac{1}{\gamma} = \frac{R}{C_P}\qquad\left(\because\ \gamma = \frac{C_P}{C_V}\right)$$

$$\text{or}\quad \frac{\gamma-1}{\gamma} = \frac{R}{C_P}\quad\text{or}\quad C_P = \frac{\gamma R}{\gamma-1}$$

$$\text{Specific heat capacity} = \frac{\text{molar heat capacity}}{\text{molecular weight}}$$

$$\text{Specific heat capacity at constant pressure} = \frac{\gamma R}{M(\gamma-1)}$$

29. **(b) :** For monatomic gas, $C_V = \frac{3}{2}R$

For diatomic, $C_V = \frac{5}{2}R$

$$\therefore\quad (C_V)_{\text{mixture}} = \frac{n_1(C_V)_1 + n_2(C_V)_2}{n_1+n_2} = \frac{1\times\left(\frac{3}{2}\right)R + 3\left(\frac{5}{2}\right)R}{1+3} = \frac{9}{4}R$$

30. (b) : Here, $m = 1$ kg, $\rho = 4$ kg m^{-3}, $P = 8 \times 10^4$ N m^{-2}

$$\therefore \quad V = \frac{m}{\rho} = \frac{1\,\text{kg}}{4\,\text{kg m}^{-3}} = \frac{1}{4}\text{m}^3$$

As diatomic gas has 5 degrees of freedom, so the energy of the gas due to its thermal motion (or internal energy) is

$$U = \frac{5}{2}nRT, \quad \text{But } PV = nRT$$

$$\therefore \quad U = \frac{5}{2}PV = \frac{5}{2} \times 8 \times 10^4 \times \frac{1}{4} = 5 \times 10^4 \text{ J}$$

31. (c) : Pressure = force/area

$$= \frac{Mg}{\text{area of the piston}} = \text{constant.}$$

If the temperature is increased, only the volume increases as the piston moves up without friction. Recall $V \propto T$ at constant pressure.

32. (a) : According to Charles' law

$V \propto T$

or $\quad \dfrac{V}{T} = \text{constant} = \dfrac{1}{P}$

As in graph, slope at P_2 is more than slope at P_1,

$\therefore \quad P_1 > P_2$

33. (a)

34. (b) : $$\gamma_{\text{mixture}} = \frac{\dfrac{n_1\gamma_1}{\gamma_1 - 1} + \dfrac{n_2\gamma_2}{\gamma_2 - 1}}{\dfrac{n_1}{\gamma_1 - 1} + \dfrac{n_2}{\gamma_2 - 1}}$$

Here, $n_1 = 1, \ \gamma_1 = \dfrac{5}{3}, \ n_2 = 1, \ \gamma_2 = \dfrac{7}{5}$

$$\therefore \quad \gamma_{\text{mixture}} = \frac{\dfrac{1 \times \frac{5}{3}}{\left[\frac{5}{3} - 1\right]} + \dfrac{1 \times \frac{7}{5}}{\left[\frac{7}{5} - 1\right]}}{\left[\dfrac{1}{\frac{5}{3} - 1}\right] + \left[\dfrac{1}{\frac{7}{5} - 1}\right]} = \frac{3}{2} = 1.5$$

35. (a) : Given, $n_o = 2$ moles, $n_{\text{Ne}} = 4$ moles

Temperature of the gas = T

Number of degrees of freedom in oxygen

$= 3 + 2 = 5$

$\therefore \quad$ Energy per mole $= 5\left(\dfrac{1}{2}RT\right) = \dfrac{5}{2}RT$

Number of degrees of freedom in neon = 3

$\therefore \quad$ Energy per mole $= 3\left(\dfrac{1}{2}RT\right) = \dfrac{3}{2}RT$

$\therefore \quad$ Net energy of gas mixture

$$= n_{\text{O}}\left(\frac{5}{2}RT\right) + n_{\text{Ne}}\left(\frac{3}{2}RT\right)$$

$$= 5\,RT + 6\,RT = 11\,RT$$

36. (a) : We treat water molecules like a solid. For each atom average energy is $3k_BT$. Water molecule has three atoms, two hydrogen and one oxygen. The total energy of one mole of water is

$$U = 3 \times 3k_BT \times N_A = 9RT \qquad \left(\because \ k_B = \frac{R}{N_A}\right)$$

$\therefore \quad$ Heat capacity per mole of water is $C = \dfrac{\Delta Q}{\Delta T} = \dfrac{\Delta U}{\Delta T} = 9R$

37. (c) : For mixture of gases, let specific heat be C_V.

$$\therefore \quad C_V = \frac{n_1(C_V)_1 + n_2(C_V)_2}{n_1 + n_2}$$

Here, $(C_V)_1 = \dfrac{5R}{2}, \ (C_V)_2 = \dfrac{3R}{2}, \ n_1 = 2, \ n_2 = 8$

$$\therefore \quad C_V = \frac{2 \times \frac{5R}{2} + 8 \times \frac{3R}{2}}{2 + 8} = \frac{17R}{10} = 1.7R$$

38. (a)

39. (b) : Mean free path $= \dfrac{3 + 7 + 1 + 2 + 4 + 3}{6} = \dfrac{20}{6}$ unit

40. (c) : As density of gas increase, the molecule become closer to each other. Therefore they are more likely to run into each other, so mean free path decreases.

41. (a) : Mean free path, $\lambda = \dfrac{1}{\sqrt{2}\pi d^2 n}$ where,

n = number of molecules per unit volume

d = diameter of the molecule

As $\quad PV = k_BNT \quad \therefore \quad n = \dfrac{N}{V} = \dfrac{P}{k_BT} \quad \therefore \quad \lambda = \dfrac{k_BT}{\sqrt{2}\pi d^2 P}$

42. (a) : (i) Volume = $2V_0$, number of molecules = N_0

Number of molecules per unit volume $= \dfrac{N_0}{2V_0}$

(ii) Number of molecules per unit volume $= \dfrac{3N_0}{3V_0} = \dfrac{N_0}{V_0}$

(iii) Number of molecules per unit volume $= \dfrac{9N_0}{3V_0} = \dfrac{3N_0}{V_0}$

The presence of the gas is maximum for (3) *i.e.*, the mean free path is minimum.

(i) > (ii) > (iii). Mean free path is inversely proportional to the number of molecules per unit volume for the same gas.

43. (d)

44. (a) : A triatomic (non-linear) gas molecule has 6 degrees of freedom (3 translational, 3 rotational and no vibrational) at room temperature.

According to the law of equipartition of energy, the average energy per molecule of a triatomic gas at room temperature T is

$$\bar{E} = \frac{1}{2}f\,kT = \frac{1}{2} \times 6 \times kT = 3kT$$

45. (a) : Both carbon monoxide (CO) and nitrogen (N_2) are diatomic. Hence they have equal K.E. Both of them have 5 degree of freedom and each has

$$\text{K.E.} = 5\left(\frac{1}{2}K_BT\right)=\frac{5}{2}K_BT \quad \therefore\ E_1 = E_2$$

46. (c) : Monoatomic gases like Argon do not possess vibrational energy.

47. (b) : Mean free path of molecules is given by

$$\lambda = \frac{1}{\sqrt{2}n\pi d^2},$$

where n is number of molecules per unit volume, d is diameter of molecules. From this $n = \frac{N}{V} = \frac{N}{m}\rho$. Therefore $\lambda \propto 1/\rho$, mean free path is inversely proportional to the density of gas molecules.

48. (c) : $v_{\text{rms}} = \sqrt{\frac{3k_BT}{m}}$; $v_m = \sqrt{\frac{2k_BT}{m}}$

$\therefore$ $v_{\text{rms}} > v_m$. Most probable speed is that which is possessed by large number of molecules in the given system. There are other molecules whose speed is greater than this speed and some other whose speed is less than this value. That is why rms (root mean square) speed of all the molecules becomes greater than the most probable speed.

49. (b) : The number of degree of freedom of a dynamical system is $N = 3A - R$, where A is number of particles in the system, R is number of independent relations among the particles. In linear triatomic gas molecule, $A = 3$, $R = 2$ (for linear molecules)

$\therefore$ $N = 3A - R = 9 - 2 = 7$.

50. (b) : During driving, the temperature of air inside the tyre increases due to motion. According to Gay Lussac's law. $P \propto T$. Therefore, air pressure inside the tyre increases. The themodynamic temperature relate to translational energy so particle may have non-zero total energy but zero translational kinetic energy at particular temperature.

51. (a) : Only the energy of translatory motion of molecules is represented by temperature. Others forms of energy such as intermolecular potential energy, energy of molecular vibration, etc., are not represented by temperature. Hence at absolute zero, the translatory motion of molecules ceases but other forms of molecular energy do not become zero. Therefore absolute zero temperature is not the temperature of zero energy.

52. (d) : For a monatomic gas, number of degree of freedom, $n = 3$, and for a diatomic gas $n = 5$.

As, $\frac{C_P}{C_V} = \gamma = 1 + \frac{2}{n}$,

$\therefore$ For monatomic gas, $\frac{C_P}{C_V} = \frac{5}{3} = 1.67$ and

for diatomic gas, $\frac{C_P}{C_V} = \frac{7}{5} = 1.4$,

or, $\left(\frac{C_P}{C_V}\right)_{\text{mono}} > \left(\frac{C_P}{C_V}\right)_{\text{dia}}$.

53. (a) : In case of C_V, volume of the gas is kept constant and heat is required only for raising the temperature of one gram mole of the gas through 1°C or 1 K. No heat, what so ever, is spent in expansion of the gas. In case of C_P, as pressure of the gas is kept constant, the gas would expand on heating. Therefore, some heat is spent in expansion of the gas against external pressure. This is in addition to the amount of heat energy required for raising the temperature of one gram mole of the gas through 1°C or 1 K. Hence specific heat at constant pressure is greater than the specific heat at constant volume.

54. (b)

55. (c) : According to law of equipartition of energy, in thermal equilibrium, at temperature T, each degree of freedom of translational, rotational and vibrational motion contributes an average energy equal to $\frac{1}{2}k_BT$. The vibrational motion has two types of energy associated with the vibrations along the length of the molecule-kinetic energy and potential energy. Thus, it contributes two degrees of freedom. Thus vibrational energy $= 2\times\frac{1}{2}k_BT = k_BT$.

56. (c) : An undamped spring-mass system is the simplest free vibration system. It has one degree of freedom.

SUBJECTIVE TYPE QUESTIONS

1. Here, $T_1 = 27\ °\text{C} = 300\ \text{K}$, $V_1 = 100\ \text{cm}^3$
$T_2 = 327\ °\text{C} = 600\ \text{K}$, $V^2 = ?$

At constant pressure, $V \propto T$ or $\frac{V_1}{V_2} = \frac{T_1}{T_2}$

$\therefore$ $V_2 = \frac{V_1T_2}{T_1} = \frac{100\times 600}{300} = 200\ \text{cm}^3$

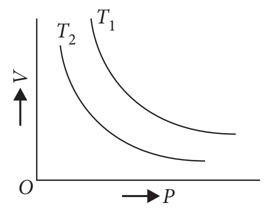
2. Here, $m_A > m_B > m_C$

We know,

$v_{\text{rms}} = \sqrt{\frac{3k_BT}{m}}$; $v_{\text{rms}} \propto \frac{1}{\sqrt{m}}$

$(v_{\text{rms}})_C > (v_{\text{rms}})_B > (v_{\text{rms}})_A$

3.



PV represents the area under the curve. $T_1 \neq T_2$, but $T_1 > T_2$.

4. On reducing the volume, the number of molecules per unit volume increases. Hence, a large number of molecules collide with the walls of the vessel per second and a larger momentum is transferred to the wall per second. This increases the pressure of the gas.

5. $\because\ v_{\text{rms}} \propto \sqrt{T}$. If temperature is increased 4 times then, v_{rms} is doubled.

6. The average kinetic energy is given as

$$E = \frac{3}{2}\ k_B T$$

or $\quad \dfrac{E_1}{E_2} = \dfrac{T_1}{T_2} = \dfrac{300}{350} = 6 : 7$

7. The degree of freedom of gas depend on

(i) N = number of particles in the system

(ii) k = number of independent relations between the particles.

8. As, $\lambda \propto 1/d^2$

Given, $d_1 = 1$Å and $d_2 = 2$Å $\Rightarrow\ \lambda_1 : \lambda_2 = 4 : 1$

9. λ is of the order of a micron.

1 micron = 10^{-6} m

10. According to the law of equipartition of energy, the energy per mole per degree of freedom of an ideal gas

$$= \frac{1}{2}k_B N_A T = \frac{1}{2}RT \qquad \left(\text{as } k_B = \frac{R}{N_A}\right)$$

11. For a rigid diatomic molecule

$$\frac{C_P}{C_V} = \frac{7}{5};\ C_V = \frac{5}{7}C_P,\ C_P - C_V = R$$

$$C_P - \frac{5}{7}C_P = nC_P$$

$$\Rightarrow \quad \frac{2}{7}C_P = nC_P \Rightarrow n = \frac{2}{7} = 0.2857$$

12. Since oxygen is a diatomic a gas, $C_V = \frac{5}{2}R$

U(internal energy per mole) = $\dfrac{5}{2}\ RT$

Internal energy of 1 g oxygen

$$= \frac{1}{32}\left[\frac{5}{2}RT\right] = \left(\frac{5}{64} \times 8.31 \times 273\right)\text{J} = 177.2 \text{ J}$$

13. (a) Here, $n = 5$ gram mole, $T = 7$ °C = 280 K

$\therefore$ Number of atoms of He

$N = n\,N_A = 5 \times 6.023 \times 10^{23} = 30.115 \times 10^{23}$

(b) Average kinetic energy per molecule = $\dfrac{3}{2}k_B T$

Total internal energy = $\left(\dfrac{3}{2}k_B T\right)N$

$$= \frac{3}{2} \times (1.38 \times 10^{-23}) \times 280 \times 30.115 \times 10^{23}$$

$= 1.74 \times 10^4$ J

14. (a) Curve at temperature T_1 is more close to the dotted plot than the curve at the temperature T_2. Since the behaviour of a real gas approaches the perfect gas behaviour, as the temperature is increased therefore $T_1 > T_2$.

(b) If the amount of gas under consideration is 1 mole, then the value of PV/T, where the curves meet PV/T-axis will be R (= 8.31 J mole^{-1} K^{-1}). For oxygen, molecular mass is 32.0 g *i.e.*, 32.0×10^{-3} kg. Since mass of oxygen gas under considerations is 1.0×10^{-3} kg, the value of PV/T, where the curves meet the PV/T-axis is given by

$$\frac{PV}{T} = \frac{8.31}{32.0 \times 10^{-3}} \times 1.0 \times 10^{-3} = 0.26 \text{ J K}^{-1}$$

15. (i) As, $v_{\text{rms}} = \sqrt{\dfrac{3RT}{M}}$ and $\dfrac{v_{\text{rms }1}}{v_{\text{rms }2}} = 1$

($\because$ Gases are at the same temperature)

(ii) $\quad P = \dfrac{1}{3}mn\overline{v^2} \quad \therefore \quad P_1 = \dfrac{1}{3}\dfrac{mn_1\overline{v^2}}{V};\ P_2 = \dfrac{1}{3}\dfrac{mn_2\overline{v^2}}{V}$

Given $\dfrac{P_1}{P_2} = \dfrac{1}{2} \quad \therefore \quad \dfrac{N_1}{N_2} = \dfrac{1}{2}$

16. Since, $T = \dfrac{PV}{nR}, \quad \therefore \quad T \propto PV$

[$\because R$ = constant]

If area under PV is greater for the curve at T_2 than for the curve T_1. Hence, $T_2 > T_1$.

17. Molar heat capacity

= Molar mass × specific heat capacity

So, the molar heat capacities of nitrogen at constant pressure and constant volume will be $28C_P$ and $28C_V$ respectively.

$\therefore \quad 28C_P - 28C_V = R$ or $\quad C_P - C_V = \dfrac{R}{28}$

18. At NTP, volume occupied by 1 g mole of gas

= 22400 cm^3

Number of molecules in 1 cm^3 of H_2

$$= \frac{6.023 \times 10^{23}}{22400} = 2.688 \times 10^{19}$$

Since each diatomic molecule has 5 degrees of freedom.

$\therefore$ Total number of degrees of freedom

$= 5 \times 2.688 \times 10^{19} = 1.344 \times 10^{20}$

19. Here, $m_A > m_B > m_C$

We know,

(i) Average $KE = \dfrac{3}{2}k_B T$

(ii) $v_{\text{rms}} = \sqrt{\dfrac{3k_B T}{m}}$

(a) Average $KE \propto \sqrt{T}$

So, it will be same for all the three types of molecules.

(b) $v_{\text{rms}} \propto \dfrac{1}{\sqrt{m}}$

$(v_{\text{rms}})_C > (v_{\text{rms}})_B > (v_{\text{rms}})_A$

20. In equilibrium the total energy is equally distributed in all possible energy modes, with each mode having an average energy equal to (1/2) kT. This is known as law of equipartition energy.

For mono-atomic of a single molecule is

$$E = \frac{3}{2}k_BT$$

For diatomic, KE of a single molecule is

$$E = \frac{5}{2}kT$$

21. (a) SI unit of R is J mol^{-1} K^{-1}

(b) $R = 8.31\times10^7$ ergs mol^{-1} K^{-1}

$= 8.31$ J mol^{-1} K^{-1} $= 2$ cal mol^{-1} K^{-1}

22. Consider water molecule as a solid made up of 3 atoms (2 hydrogen and 1 oxygen) and each atom if free to vibrate in three dimensions about its mean position.

In 3-dimension, the average energy per atom of water molecule $= 3k_BT$

$\therefore$ Total energy of one molecule of water is

$$= 3 \times (3k_BT) = 9k_BT$$

Total energy of 1 mole of water is

$$U = 3 \times 3k_BT \times N_A = 9RT$$

$\therefore$ Specific heat of water $C_V = \dfrac{dU}{dT} = 9R$

23. Here, m = molar mass of the gas; v_o = initial speed

Mass of gas = $n\,m$ (n = number of moles in the gas)

$\therefore$ Initial kinetic energy of the gas $= \dfrac{1}{2}(nm)v^2$

Final KE of gas = 0

Change in kinetic energy, $\Delta K = \dfrac{1}{2}(nm)v^2$

Let change in temperature of gas = ΔT

Change in internal energy of the gas

$$\Delta U = nC_v\Delta T = n\left(\frac{3}{2}R\right)\Delta T$$

As, $\Delta U = \Delta K$

$$\Rightarrow \quad \frac{3}{2}nR\Delta T = \frac{1}{2}mnv_0^2$$

$$\Rightarrow \quad \Delta T = \frac{mv_0^2}{3R}$$

24. According to Graham's law of diffusion,

$$\frac{r_1}{r_2} = \sqrt{\frac{M_2}{M_1}}$$

where,

r_1 = diffusion rate of hydrogen = 28.7 cm^3 s^{-1}

r_2 = diffusion rate of unknown gas = 7.2 cm^3 s^{-1}

M_1 = molecular mass of hydrogen = $2u$

M_2 = ?

$$\therefore \frac{28.7}{7.2} = \sqrt{\frac{M_2}{2}} \text{ or } M_2 = \left(\frac{28.7}{7.2}\right)^2 \times 2 = 31.78 \approx 32$$

25. As number of moles is conserved

$\therefore$ $n_A + n_B = n_{\text{mix}}$

where n_A, n_B and n_{mix} represent the number of moles of gas A, B and their mixture respectively.

Using ideal gas equation, we get

$$\frac{P_AV_A}{RT_A} + \frac{P_BV_B}{RT_B} = \frac{P_{\text{mixture}}V_{\text{mixture}}}{RT_{\text{mixture}}}$$

$$\frac{PV}{RT} + \frac{PV}{RT} = \frac{P_{\text{mixture}}V}{RT} \Rightarrow \quad P_{\text{mixture}} = 2P$$

26. Internal energy of a gas, $U = nC_VT$

where n is the number of the moles of a gas and C_V is the molar specific heat at constant volume and T is the temperature of the gas.

As hydrogen is a diatomic gas, so $(C_V)_{H_2} = \dfrac{5}{2}R$

As helium is a monatomic gas, so $(C_V)_{\text{He}} = \dfrac{3}{2}R$

According to the question $U_{H_2} = U_{\text{He}}$

$$\therefore\ n_{H_2} \times (C_V)_{H_2} \times T_{H_2} = n_{\text{He}} \times (C_V)_{\text{He}} \times T_{\text{He}}$$

or $\quad n_{H_2} \times \dfrac{5}{2}R \times T = n_{\text{He}} \times \dfrac{3}{2}R \times 2T$ or $\dfrac{n_{H_2}}{n_{\text{He}}} = \dfrac{6}{5}$

27. (a) Consider water molecule as a solid made up of 3 atoms (2 hydrogen and 1 oxygen) and each atom if free to vibrate in three dimensions about its mean position.

In 3-dimension,

[the average energy per atom of water molecule = $3k_BT$]

$\therefore$ Total energy of one molecule of water is

$$= 3 \times (3k_BT) = 9k_BT$$

Total energy of 1 mole of water is

$$U = 3 \times 3k_BT \times N_A = 9RT$$

$\therefore$ Specific heat of water $C_V = \dfrac{dU}{dT} = 9R$

(b) For monatomic gas, $C_V = 3R/2$

For diatomic gas, $C_V' = 5R/2$

Let, n and n' be the number of mole of monoatomic and diatomic gases than, C_V (mixture) $= \dfrac{nC_V + n'C_V'}{n + n'}$

$$C_V = \frac{1\times\frac{3}{2}R + 1\times\frac{5}{2}R}{1+1} = 2R$$

$$\gamma\text{ (mixture)} = 1 + \frac{R}{C_{V(\text{mixture})}} = 1 + \frac{R}{2R} = 1.5$$

28. Here, $v = 150\text{ km h}^{-1}$

$N = 10$

$V = 20 \times 20 \times 1.5\text{ km}^3$.

Diameter of plane, $d = 2\text{ R} = 2 \times 10 = 20\text{ m}$

$= 20 \times 10^{-3}\text{ km}$

$$n = \frac{N}{V} = \frac{10}{20 \times 20 \times 1.5} = 0.0167\text{ km}^{-3}$$

Mean free path of a plane, $\lambda = \dfrac{1}{\sqrt{2}\,\pi d^2 n}$

Time elapse before collision of two planes randomly,

$$t = \frac{\lambda}{v} = \frac{1}{\sqrt{2}\,\pi d^2 nv}$$

$$= \frac{1}{1.414 \times 3.14 \times (20)^2 \times 10^{-6} \times (0.0167) \times (150)}$$

$$= \frac{10^6}{4449.5} = 224.74\text{ h} \approx 225\text{ h}$$

29. (a) 22.4 litre of volume is occupied by one mole of gas at NTP.

(b) A gas which obeys the ideal gas equation, $PV = nRT$, at all temperature and pressures is called an ideal gas. No such a gas does not really exists. An ideal gas is a simple theoretical model of a gas.

Important characteristics of an ideal gas are

(i) The size of the molecule is negligibly small.

(ii) There is no force of attraction or repulsion amongst its molecules.

(c) At high temperature and low pressure the real gas behaves as an ideal gas.

30. Root mean square velocity (v_{rms}) : It is the square root of mean of squares of the velocities of individual molecules of the gas.

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3RT}{M}}$$

Most probable speed (v_{mp}) : The speed which is possessed by maximum number of molecules of the gas is called most probable speed of the gas molecules.

$$v_{\text{mp}} = \sqrt{\frac{2kT}{m}} = \sqrt{\frac{2RT}{M}}$$

Average speed (v_{av}) : It is the arithmetic mean of the speeds of molecules of a gas at given temperature.

$$v_{\text{av}} = \frac{v_1 + v_2 + v_3 + v_4 + v_5 + \ldots.. + v_N}{N}$$

(where N = total number of molecules)

$$v_{\text{av}} = \sqrt{\frac{8kT}{\pi m}} = \sqrt{\frac{8RT}{\pi M}}$$

(i) Average speed

$$v_{av} = \frac{0+2+3+4+4+4+5+5+6+9}{10} = 4.2\text{ m/s}$$

(ii) $$v_{\text{rms}} = \sqrt{\frac{0^2+2^2+3^2+4^2+4^2+4^2+5^2+5^2+6^2 \times 9^2}{10}}$$

$= (22.8)^{1/2} = 4.8\text{ m/s}$

31. (a) Gravitational force of the moon is roughly 1/6th of the gravitational force of earth, so the escape velocity of air molecules on the moon is smaller than that on earth. As the moon is in the proximity of the Earth as seen from the sun so solar irradiance on the moon surface is same as that of the earth.

Escape velocity of the moon is 2.38 km s^{-1}. The rms speed of the constituents of air (O_2, N_2, CO_2 and water vapour) lies between 0.4 km s^{-1} to 0.8 km s^{-1}, which is smaller than 2.38 km s^{-1}. Inspite of this a significant number of molecules have speeds greater than 2.38 km s^{-1} and hence escape easily. Now rest of molecules arrange the speed distribution for the equilibrium temperature. Again a significant number of molecules get their speeds greater than 2.38 km s^{-1} and escape. Due to this continuous process, the moon has lost its atmosphere over a long period of time.

(b) We know

$E = U + K$ = constant

As the molecules move higher their potential energy (U) increase and due to this its kinetic energy (K) decrease and hence fall in temperature. At greater height more volume is available for gas to expand and hence some cooling takes place *i.e.* a fall in temperature.

32. Average kinetic energy per molecule of any ideal gas

$= \dfrac{3}{2} k_B T$. Here, k_B = Boltzmann constant

and T = Temperature.

(i) As argon and chlorine, both have the same temperature in flask, the ratio of average kinetic energy per molecule of two gases = 1 : 1

(ii) The rms speed of gas $v_{\text{rms}} = \sqrt{\dfrac{3RT}{M}}$

$$\frac{(v_{\text{rms}})_{\text{Ar}}}{(v_{\text{rms}})_{\text{Cl}_2}} = \sqrt{\frac{M_{\text{Cl}_2}}{M_{\text{Ar}}}} = \sqrt{\frac{70.9}{39.9}} = 1.33$$

33. (i) K.E./atom $= \dfrac{f}{2} kT$ for both the gases. So ratio is 1.

(ii) $v_{rms_A} = \sqrt{\dfrac{3RT}{16M_0}}$, $v_{rms_B} = \sqrt{\dfrac{3RT}{M_0}}$, $(v_{\text{rms}})_B = 4\,(v_{\text{rms}})_A$.

So ratio is 4 .

(iii) A B

A	B
Molecular wt. = 16 M_0	Molecular wt. = M_0
mass = $2m_0$	mass = m_0
$n_A = \dfrac{n_0}{8}$	$n_B = n_0$

$(P_A) = \dfrac{(n_0 / 8)RT}{V}$, $(P_B) = \dfrac{n_0 RT}{V}$, $(P_B) = 8\ (P_A)$.

So ratio is 8

34. (i) Average speed,

$$v_{av} = \left[\frac{8+1+4+4+4+4+7+7+6+5}{10}\right] = 5 \text{ ms}^{-1}$$

(ii) rms speed, v_{rms}

$$= \left[\frac{(8)^2+(1)^2+(4)^2+(4)^2+(4)^2+(4)^2+(7)^2+(7)^2+(6)^2+(5)^2}{10}\right]^{1/2}$$

$$= \left[\frac{288}{10}\right]^{1/2} = 5.4 \text{ m s}^{-1}$$

(iii) Most probable speed(Velocity possessed by maximum number of molecules)

$v_{mp} = 4$ m/s

35. The average kinetic energy of the gas at a temperature T

$$E = \frac{3}{2}kT$$

where $k = 1.38 \times 10^{-23}$ J K^{-1}

T = Temperature in kelvin

(i) $T = 27°\text{C} = 27 + 273 = 300$ K

$$E = \frac{3}{2} \times 1.38 \times 10^{-23} \times 300 = 6.21 \times 10^{-21} \text{ J}$$

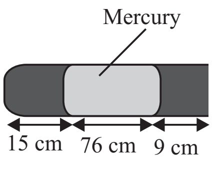
(ii) $T = 6000$ K

$$E = \frac{3}{2} \times 1.38 \times 10^{-23} \times 6000 = 1.242 \times 10^{-19} \text{ J}$$

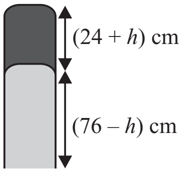
(iii) $T = 10 \times 10^6 \text{ K} = 10^7$ K

$$E = \frac{3}{2} \times 1.38 \times 10^{-23} \times 10^7 = 2.07 \times 10^{-16} \text{ J}$$

36. When the tube is held horizontally, the mercury thread of length 76 cm traps a length of air = 15 cm. A length of 9 cm of the tube will be left at the open end, figure (a). The pressure of air enclosed in tube will be atmospheric pressure. Let area of cross-section of the tube be 1 sq. cm.



(a)



(b)

∴ $P_1 = 76$ cm and $V_1 = 15$ cm^3

When the tube is held vertically, 15 cm air gets another 9 cm of air filled in the right hand side (in the horizontal position) and let h cm of mercury flows out to balance the atmospheric pressure, figure (b) Then the heights of air column and mercury column are $(24 + h)$ cm and $(76 - h)$ cm respectively.

The pressure of air = $76 - (76 - h) = h$ cm of mercury

∴ $V_2 = (24 + h)$ cm^3 and $P_2 = h$ cm.

If we assume that temperature remains constant, then

$$P_1V_1 = P_2V_2$$

or $76 \times 15 = h \times (24 + h)$

or $h_2 + 24h - 1140 = 0$

or $h = \dfrac{-24 \pm \sqrt{(24)^2 + 4 \times 1140}}{2} = 23.8$ cm

or -47.8 cm

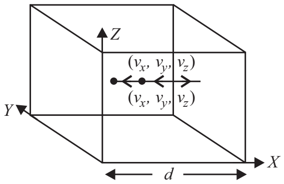
Since h cannot be negative (because more mercury cannot flow into the tube), therefore $h = 23.8$ cm. Thus, in the vertical position of the tube, 23.8 cm of mercury flows out.

37. Consider a cylinder with piston of cross section area A and length L. The cylinder contains ideal gas and we need to find the pressure exerted by gas molecule on the wall of the cylinder. According to assumptions, the collisions are perfectly elastic hence KE is conserved.

Consider the motion of particle only in one direction say (x-axis), velocities in y and z directions remain unaltered, only x-component changes by reversing itself from v_x to $-v_x$.

Now, change in momentum of a particle along x-axis, $\Delta p_x = 2mv_x$.

If length of cylinder is d, the time taken to hit the wall and bouncing back to the position be Δt.



∴ $\Delta t = \dfrac{2d}{v_x}$

Now, force exerted by a single molecule by Newton's 2nd law,

$$F = \frac{\Delta p}{\Delta t} = \frac{2mv_x^2}{2d} = \frac{mv_x^2}{d}$$

For number of molecules, average force is

$$F_{avg} = \frac{Nmv_x^2}{d}$$

Hence pressure exerted, $P = \dfrac{F_{avg}}{A} = \dfrac{Nmv_x^2}{Ad}$

As the particles or gas molecules are in constant random motion. So taking an account of motion in all directions, total speed of particles is given by

$$v^2 = v_x^2 + v_y^2 + v_z^2$$

or $v_x^2 = v_y^2 = v_z^2 = \dfrac{1}{3}v^2$

∴ Pressure becomes, $P = \dfrac{Nmv^2}{3d\,A}$

$\because$ Molecules are not moving with same speed therefore taking an average speed of all molecules as

$$\bar{v}^2 = \frac{v_1^2 + v_2^2 + v_3^2 + \ldots}{N}$$

($\because$ N = total number of particles)

$$\therefore \quad P = \frac{Nm\,\bar{v}^2}{3dA} = \frac{Nm\,v_{avg}^2}{3V} = \frac{1}{3}\rho\bar{v}^2$$

($\because$ $V = d \times A$ = Volume of cylinder)

Mean K.E. of translation per unit volume of the gas is $E = \frac{1}{2}\rho\bar{v}^2$

$$\therefore \quad \frac{P}{E} = \frac{1/3\rho\bar{v}^2}{1/2\rho\bar{v}^2} = \frac{2}{3}$$

$$P = \frac{2}{3}E$$

38. (a) Monoatomic gases : For monoatomic gases, degree of freedom, $f = 3$

Total internal energy = $\frac{3}{2}kT \times N_A = \frac{3}{2}RT$

Total specific heat at constant volume,

$$C_V = \frac{dU}{dT} = \frac{3}{2}R$$

Also for an ideal gas, $C_P - C_V = R$

$$C_P - \frac{3}{2}R = R \quad \text{or} \quad C_P = \frac{5}{2}R$$

Hence, ratio of specific heats,

$$\gamma = \frac{C_P}{C_V} = \frac{5}{3}$$

(b) Diatomic gases : For diatomic gases, degree of freedom is $f = 5$

Total internal energy

$$U = \frac{5}{2}kT \times N_A = \frac{5}{2}RT$$

$$C_V = \frac{dU}{dT} = \frac{5}{2}R$$

$$\therefore \quad C_P - C_V = R \Rightarrow C_P = \frac{7}{2}R$$

Hence, $\gamma = \frac{C_P}{C_V} = \frac{7}{5}$

(c) Triatomic gases (linear) : For triatomic gases, $f = 7$;

$$U = \frac{7}{2}KT \times N_A = \frac{7}{2}RT$$

$$C_V = \frac{dU}{dT} = \frac{7}{2}R$$

$$\therefore \quad C_P - C_V = R \Rightarrow C_P = \frac{9}{2}R;$$

$$\therefore \quad \gamma = \frac{C_P}{C_V} = \frac{9}{7}$$

Triatomic gases (non linear) : Degree of freedom $f = 6$

$$U = 3\,kT\,N_A = 3\,RT;\ C_V = \frac{dU}{dT} = 3\,R$$

$$\therefore \quad C_P - C_V = R \Rightarrow C_P = 4\,R;$$

$$\therefore \quad \gamma = \frac{C_P}{C_V} = \frac{4}{3}$$

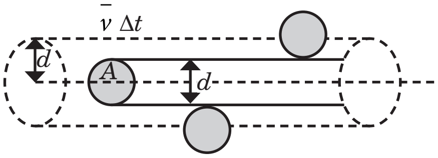
39. The mean free path of a gas molecule is defined as the average distance travelled by a molecule between two successive collisions.

Let us consider the mean free path, we make use of the following assumptions:

(i) Each molecule of the gas is a sphere of diameter d.

(ii) All molecules of the gas except the molecule A under consideration are at rest.

As shown figure, suppose the molecule A has average speed $\bar{v}$. It will collide with all those molecules whose centres lie within a distance d from its path. In time Δt, it will collide with all those molecules in the cylinder of volume $\pi d^2 \bar{v}\,\Delta t$. Let n be the number of molecules per unit volume.



Number of collisions suffered by the molecule A in time Δt

= Volume of the cylinder swept by molecule A in time $\Delta t \times$ number of molecules per unit volume

$= \pi d^2 \bar{v}\,\Delta t \times n$

Mean free path of a gas molecule,

$$\lambda = \frac{\text{Distance covered in time } \Delta t}{\text{Number of collisions suffered in time } \Delta t}$$

$$= \frac{\bar{v}\Delta t}{\pi d^2 \bar{v}\,\Delta t n} = \frac{1}{n\pi d^2}$$

Taking into consideration the motion of all the gas molecules, the mean free path comes out to be

$$\lambda = \frac{1}{\sqrt{2}n\pi d^2}$$

If m is the mass of each gas molecule, then the density of the gas is

$$\rho = mn \quad \text{or} \quad n = \frac{\rho}{m} \quad \therefore \quad \lambda = \frac{m}{\sqrt{2}\pi d^2 \rho}$$

For one mole of a gas,

$$PV = RT \quad \text{or} \quad P = \frac{RT}{V} = \frac{N}{V} \times \frac{R}{N} \times T = nk_BT$$

$$\text{or} \quad n = \frac{P}{k_BT} \quad \therefore \quad \lambda = \frac{k_BT}{\sqrt{2}\pi d^2 P}$$

Dependency of mean free path (λ) :

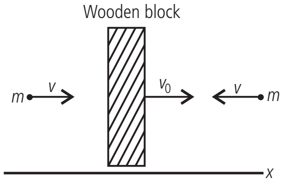
The mean free path of a gas molecule is defined as the average distance travelled by a molecule between two successive collisions.

$\because$ Mean free path,

$$\lambda = \frac{m}{\sqrt{2}\pi d^2\rho} \quad \text{or} \quad \lambda = \frac{k_B T}{\sqrt{2}\pi d^2\rho}$$

So, it (λ) depends on mass (m), diameter (d), density (ρ) and temperature (T) of the given gas.

40.



Speed of wooden block = v_o

Cross-sectional area of block = A

Temperature of the gas = T

Let n be the number of molecules per unit volume.

rms speed of gas molecules = v_{rms}.

Relative speed of the molecule with respect to front face of the block = $v + v_o$

In head on collision, momentum transferred to block per collision = $2m(v + v_o)$

where m is the mass of molecule.

Number of collision in time Δt,

$$= \frac{1}{2}(v + v_o)n\Delta tA.$$

(Factor of $\frac{1}{2}$ appears due to particles moving towards block)

Hence, momentum transferred in time Δt,

$= m(v + v_o)^2 nA\Delta t$

It is from front surface only.

Similarly for back surface, momentum transferred in time $\Delta t = m(v - v_o)^2 nA\Delta t$

Net force $\frac{\Delta p}{\Delta t} = m[(v + v_o)^2 - (v - v_o)^2]nA$

$= mnA(4vv_o)$

$= (4mnAv)v_o = (4\rho Av)v_o \quad (\because mn = \rho)$ ... (i)

Also, $\frac{1}{2}mv^2 = \frac{1}{2}kT$ (v along x-axis)

$\therefore \quad v = \sqrt{\frac{kT}{m}}$

$\therefore$ Net force = $4\rho A\sqrt{\frac{kT}{m}}v_o$. [Using (i)]

This is required drag force.