Ratio and Proportion

Tip 1

- Ratio and Proportions is one of the easiest concepts in CAT. It is just an extension of high school mathematics.
- Questions from this concept are mostly asked in conjunction with other concepts like similar triangles, mixtures and alligations.
- Hence fundamentals of this concept are important not just from a stand-alone perspective, but also to answer questions from other concepts

Tip 2

- A ratio can be represented as fraction a/b or using the notation a:b. In each of these representation 'a' is called the antecedent and 'b' is called the consequent.

- For a ratio to be defined, the quantitiesof the items should be of same nature. We can not compare the length of the rod to the area of a square.

- However if these quantities are represented in numbers, i.e., length of a rod is a cm and area of a square is b sq.km, we can still define the ratio of these numbers as a:b

Tip 3

Properties of Ratios :

- A ratio need not be positive. However, if we are dealing with quantities of items, their ratios will be positive. In this concept we will consider only positive ratios.

- A ratio remains the same if both antecedent and consequent are multiplied or divided by the same non-zero number, i.e.,

$$\frac{a}{b} = \frac{pa}{pb} = \frac{qa}{qb}\text{, } p, q \neq 0$$

$$\frac{a}{b} = \frac{a/p}{b/p} = \frac{a/q}{b/q}\text{, } p, q \neq 0$$

Tip 4

- Two ratios in their fraction notation can be compared just as we compare real numbers.

$$\frac{a}{b} = \frac{p}{q} \Leftrightarrow aq = bp$$

$$\frac{a}{b} > \frac{p}{q} \Leftrightarrow aq > bp$$

$$\frac{a}{b} < \frac{p}{q} \Leftrightarrow aq < bp$$

- If antecedent > consequent, the ratio is said to be ratio of greater inequality.
- If antecedent < consequent, the ratio is said to be ratio of lesser inequality.
- If the antecedent = consequent, the ratio is said to be ratio of equality

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Tip 5

If a, b, x are positive, then

- If $a > b$, then $\frac{a+x}{b+x} < \frac{a}{b}$
- If $a < b$, then $\frac{a+x}{b+x} > \frac{a}{b}$
- If $a > b$, then $\frac{a-x}{b-x} > \frac{a}{b}$
- If $a < b$, then $\frac{a-x}{b-x} < \frac{a}{b}$
- If $\frac{a}{p} = \frac{b}{q} = \frac{c}{r} = \frac{d}{s} = \ldots$, then $a:b:c:d:\ldots = p:q:r:s:\ldots$

Tip 6

If two ratios a/b and c/d are equal

- $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{b}{a} = \frac{d}{c}$ (Invertendo)
- $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a}{c} = \frac{b}{d}$ (Alternendo)
- $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a+b}{b} = \frac{c+d}{d}$ (Componendo)
- $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a-b}{b} = \frac{c-d}{d}$ (Dividendo)
- $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a+b}{a-b} = \frac{c+d}{c-d}$ (Componendo-Dividendo)
- $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{pa+qb}{ra+sb} = \frac{pc+qd}{rc+sd}$, for all real p, q, r, s such that $pa+qb \neq 0$ and $rc+sd \neq 0$

Tip 7

If a, b, c, d, e, f, p, q, r are constants and are not equal to zero

- $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \ldots$ then each of these ratios is equal to $\frac{a+c+e..}{b+d+f..}$
- $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \ldots$ then each of these ratios is equal to $\frac{pa+qc+re+..}{pb+qd+rf+..}$
- $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \ldots$ then each of these ratios is equal to $\frac{(pna+qnc+rne+..)1/^{n}}{(p^{n}b+q^{n}d+rnf+..)1/^{n}}$.
- Duplicate Ratio of a : b is $a^2 : b^2$
- Sub-duplicate ratio of a : b is $\sqrt{a} : \sqrt{b}$
- Triplicate Ratio of a : b is $a^3 : b^3$
- Sub-triplicate ratio of a : b is $a^{1/3} : b^{1/3}$

Tip 8

Proportions:

A proportion is an equality of ratios. Hence a:b = c:d is a proportion. The first and last terms are called extremes and the other two terms are called means.

If four terms a, b, c, d are said to be proportional, then a:b = c:d. If three terms a, b, c are said to be proportional, then a:b = b:c

Tip 9

Properties of proportions:

If a:b = c:d is a proportion, then

- Product of extremes = product of means i.e., ad = bc
- Denominator addition/subtraction: a:a+b = c:c+d and a:a-b = c:c-d
- a, b, c, d,.... are in continued proportion means, a:b = b:c = c:d = ...
- a:b = b:c then b is called mean proportional and $b^2 = ac$
- The third proportional of two numbers, a and b, is c, such that, a:b = b:c
- 'd' is fourth proportional to numbers a, b, c if a:b = c:d

Tip 10

Variations :

- If x varies directly to y, then x is said to be in directly proportionalwith y and is written as $x \propto y$

  $x = ky$ (where k is direct proportionality constant)
  $x = ky + C$ (If x depends upon some other fixed constant C)

- If x varies inversely to y, then x is said to be in inversely proportional with y and is written as $x \propto \frac{1}{y}$

  $x = k\frac{1}{y}$ (where k is indirect proportionality constant)
  $x = k\frac{1}{y} + C$ (If x depends upon some other fixed constant C)

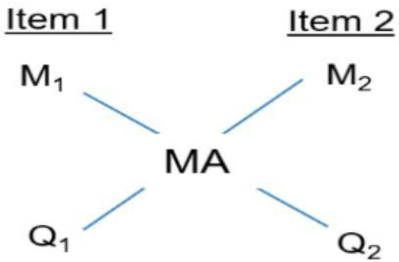
Tip 11

Variations:

- If $x \propto y$ and $y \propto z$ then $x \propto z$
- If $x \propto y$ and $x \propto z$ then $x \propto (y \pm z)$
- If $a \propto b$ and $x \propto y$ then $ax \propto by$

Tip 12(i)

- If M_1 and M_2 are the values and Q_1 and Q_2 are the quantities of item 1 and item 2 respectively, and M_A is the weighted average of the two items, then



$$\frac{Q_1}{Q_2} = \frac{M_2 - M_A}{M_A - M_1}$$

- Weighted average M_A can be calculated by:

$$M_A = \frac{Q_1 M_1 + Q_2 M_2}{Q_1 + Q_2}$$

Tip 12(ii)

- **Successive dilution**: If a container has 'a' litres of liquid A, and if 'b' litres of solution is withdrawn and is replaced with an equal volume of another liquid B and the operation is repeated for 'n' times, then after nth operation,

Final quantity of liquid A in the container = $$\left[\frac{a-b}{a}\right]^n \times a$$