Trigonometric Functions

Question 1.
The value of sin 15 + cos 15 is
(a) 1
(b) 1/2
(c) √3/2
(d) √3

Answer: (c) √3/2
Given, sin 15 + cos 15
= sin 15 + cos(90 – 15)
= sin 15 + sin 15
= 2 × sin 45 × cos 30
= 2 × (1/√2) × (√3/2)
= √3/2

Question 2.
The value of tan A/2 – cot A/2 + 2cot A is
(a) 0
(b) 1
(c) -1
(d) None of these

Answer: (a) 0
Given, tan A/2 – cot A/2 + 2cot A
= {sin(A/2)/cos(A/2)} – {cos(A/2)/sin(A/2)} + 2cotA
= {sin² (A/2) – cos² (A/2)}/{cos(A/2) × sin(A/2)} + 2cotA
= -{cos² (A/2) – sin² (A/2)}/{cos(A/2) × sin(A/2)} + 2cotA
= -{cosA}/{cos(A/2) × sin(A/2)} + 2cotA (since cos² A – sin² A = cos²A)
= -{cos(2A/2)}/{cos(A/2) × sin(A/2)} + 2cotA
= -{2 × cosA}/{2 × cos(A/2) × sin(A/2)} + 2cotA
= -{2cosA}/{sin(2A/2)} + 2cotA
= {-(2cosA)/(sinA)} + 2cotA (since sin2A = 2 × sinA × cosA)

= -2cotA + 2cotA
= 0

Question 3.
The value of $4 \times \sin x \times \sin(x + \pi/3) \times \sin(x + 2\pi/3)$ is
(a) sin x
(b) sin 2x
(c) sin 3x
(d) sin 4x

Answer: (c) sin 3x
Given, $4 \times \sin x \times \sin(x + \pi/3) \times \sin(x + 2\pi/3)$
$= 4 \times \sin x \times \{\sin x \times \cos \pi/3 + \cos x \times \sin \pi/3\} \times \{\sin x \times \cos 2\pi/3 + \cos x \times \sin 2\pi/3\}$
$= 4 \times \sin x \times \{(\sin x)/2 + (\sqrt{3} \times \cos x)/2\} \times \{-(\sin x)/2 + (\sqrt{3} \times \cos x)/2\}$
$= 4 \times \sin x \times \{-(\sin^2 x)/4 + (3 \times \cos^2 x)/4\}$
$= \sin x \times \{-\sin^2 x + 3 \times \cos^2 x\}$
$= \sin x \times \{-\sin^2 x + 3 \times (1 - \sin^2 x)\}$
$= \sin x \times \{-\sin^2 x + 3 - 3 \times \sin^2 x\}$
$= \sin x \times \{3 - 4 \times \sin^2 x\}$
$= 3 \times \sin x - 4\sin^3 x$
$= \sin 3x$
So, $4 \times \sin x \times \sin(x + \pi/3) \times \sin(x + 2\pi/3) = \sin 3x$

Question 4.
If $\tan x = (\cos 9 + \sin 9)/(\cos 9 - \sin 9)$, then x =
(a) 45
(b) 54
(c) 36
(d) None of these

Answer: (b) 54
Given, $\tan x = (\cos 9 + \sin 9)/(\cos 9 - \sin 9)$
$\Rightarrow \tan x = \{\cos 9(1 + \sin 9/\cos 9)\}/\{\cos 9(1 - \sin 9/\cos 9)\}$
$\Rightarrow \tan x = (1 + \tan 9)\}/(1 - \tan 9)$
$\Rightarrow \tan x = (\tan 45 + \tan 9)\}/(1 - \tan 45 \times \tan 9)$ {since $\tan 45 = 1$}
$\Rightarrow \tan x = \tan(45 + 9)$ {Apply tan(A + B) formula}
$\Rightarrow \tan x = \tan(54)$
$\Rightarrow x = 54$

Question 5.
In a triangle ABC, sin A – cos B = cos C, then angle B is
(a) π/2
(b) π/3
(c) π/4
(d) π/6

Answer: (a) π/2
Given, sin A – cos B = sin C
⇒ sin A = cos B + sin C
⇒ 2 × sin (A/2) × cos (A/2) = 2 × cos {(B + C)/2} × cos {(B – C)/2}
⇒ 2 × sin (A/2) × cos (A/2) = 2 × cos {π/2 – A/2} × cos {(B – C)/2}
⇒ 2 × sin (A/2) × cos (A/2) = 2 × sin (A/2) × cos {(B – C)/2}
⇒ cos (A/2) = cos {(B – C)/2}
⇒ A/2 = (B – C)/2
⇒ A = B – C
⇒ B = A + C
⇒ B = π – B {Since A + B + C = π}
⇒ 2B = π
⇒ B = π/2

Question 6.
The value of cos 420° is
(a) 0
(b) 1
(c) 1/2
(d) √3/2

Answer: (c) 1/2
cos 420° = cos(360° + 60°) = cos 60° = 1/2

Question 7.
If in a triangle ABC, tan A + tan B + tan C = 6 then the value of cot A × cot B × cot C is
(a) 1/2
(b) 1/3
(c) 1/4
(d) 1/6

Answer: (d) 1/6
Given tanA + tanB + tanC = 6

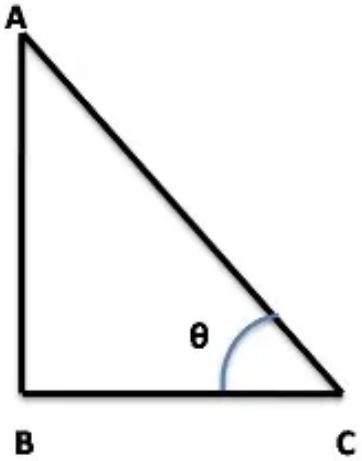
Now tan(A + B + C) = {(tanA + tanB + tanC) – tanA × tanB × tanC}/{1 – (tanA × tanB + tanB × tanC + tanA × tanC)}
We know that,
A + B + C = π
⇒ tan(A + B + C) = tan π
⇒ tan(A + B + C) = 0
Now
0 = {(tanA + tanB + tanC) – tanA × tanB × tanC}/{1 – (tanA × tanB + tanB × tanC + tanA × tanC)}
⇒ tanA + tanB + tanC – tanA × tanB × tanC = 0
⇒ tanA + tanB + tanC = tanA × tanB × tanC
⇒ tanA × tanB × tanC = 6
⇒ (1/cotA) × (1/cotB) × (1/cotC) = 6
⇒ 1/(cot A × cot B × cot C) = 6
⇒ cot A × cot B × cot C = 1/6

Question 8.
If a × cos x + b × cos x = c, then the value of (a × sin x – b × cos x)² is
(a) a² + b² + c²
(b) a² – b² – c²
(c) a² – b² + c²
(d) a² + b² – c²

Answer: (d) a² + b² – c²
We have
(a × cos x + b × sin x)² + (a × sin x – b × cos x)² = a² + b²
⇒ c² + (a × sin x – b × cos x)² = a² + b²
⇒ (a × sin x – b × cos x)² = a² + b² – c²

Question 9.
When the length of the shadow of a pole is equal to the height of the pole, then the elevation of source of light is
(a) 30°
(b) 60°
(c) 75°
(d) 45°

Answer: (d) 45°



Let AB is the length of the pole and BC is the shadow of the pole.
Given AB = BC
Now from triangle ABC,
tan θ = AB/BC
⇒ tan θ = 1
⇒ θ = 45°
So, the elevation of source of light is 45°

Question 10.
In any triangle ABC, if cos A/a = cos B/b = cos C/c and the side a = 2, then the area of the triangle is
(a) √3
(b) √3/4
(c) √3/2
(d) 1/√3

Answer: (a) √3
Given cos A/a = cos B/b = cos C/c
= cos A/k × sin A = cos B/k × sin B = cos C/k × sin C {since sin A/a = sin B/b = sin C/c = k}
= cot A = cot B = cot C
⇒ A = B = C = 60
So, triangle ABS is equilateral.
Now area of the triangle = (√3/4) × a^2 = (√3/4) × 2^2 = (√3/4) × 4 = √3

Question 11.
The least values of $\cos^2 \theta + \sec^2 \theta$ is
(a) 0
(b) 1

(c) 2
(d) more than 2

Answer: (c) 2
If $a \times \cos^2 \theta + b \times \sec^2 \theta$ is given,
then the least value = $2\sqrt{ab}$
Now given, $\cos^2 \theta + \sec^2 \theta$
Here, a = 1, b = 1
Now, least value = $2\sqrt{(1 \times 1)} = 2 \times 1 = 2$

Question 12.
The equation $(\cos p - 1) x^2 + \cos p \times x + \sin p = 0$, where x is a variable, has real roots. Then the interval of p may be any one of the following:
(a) $(0, \pi)$
(b) $(-\pi/2, \pi/2)$
(c) $(0, \pi)$
(d) $(-\pi, 0)$

Answer: (a) $(0, \pi)$
The equation $(\cos p - 1)$
$x^2 + \cos p \times x + \sin p = 0$, where x is a variable, has real roots.
Now, for real roots,
Discriminant ≥ 0
$\Rightarrow \cos^2 p - 4(\cos p - 1)\sin p \geq 0$
$\Rightarrow (\cos p - 2\sin p)^2 - 4\sin^2 p + 4\sin p \geq 0$
$\Rightarrow (\cos p - 2\sin p)^2 + 4\sin p(1 - \sin p) \geq 0$ ……..1
Now, $1 - \sin p \geq 0$
$\Rightarrow$ For all real p such that $0 < p < \pi$
So that $4\sin p(1 - \sin p) \geq 0$
So, $p \in (0, \pi)$

Question 13.
The value of $(\sec 8A - 1)/(\sec 4A - 1)$ is
(a) 0
(b) 1
(c) $\tan 8A/\tan 2A$
(d) $\tan 2A/\tan 8A$

Answer: (c) $\tan 8A/\tan 2A$
Given, $(\sec 8A - 1)/(\sec 4A - 1)$
$= (1/\cos 8A - 1)/(1/\cos 4A - 1)$

= {(1 – cos 8A)/cos 8A}/{(1 – cos 4A)/cos 4A}
= {(1 – cos 8A) × cos 4A}/{(1 – cos 4A) × cos 8A}
= (2sin² 4A × cos 4A}/{2sin² 2A × cos 8A} {since cos 2A = 1 – 2sin² A}
= (2sin 4A × sin 4A × cos 4A}/{2sin 2A × sin 2A × cos 8A}
= (sin 8A × sin 4A}/{2sin 2A × sin 2A × cos 8A} {since sin 2A = 2×sin A × cos A}
= (sin 8A × 2sin 2A × cos 2A}/{2sin 2A × sin 2A × cos 8A}
= (sin 8A × cos 2A}/{sin 2A × cos 8A}
= (sin 8A/cos 8A)/(sin 2A/cos 2A)
= tan 8A/tan 2A
So, (sec 8A – 1)/(sec 4A – 1) = tan 8A/tan 2A

Question 14.
The value of (sin7x + sin5x) /(cos7x + cos5x) + (sin9x + sin3x) / (cos9x + cos3x) is
(a) tan6x
(b) 2 tan6x
(c) 3 tan6x
(d) 4 tan6x

Answer: (b) 2 tan6x
Given, (sin7x + sin5x) /(cos7x + cos5x) + (sin9x + sin3x) / (cos9x + cos3x)
⇒ [{2×sin(7x + 5x)/2 × cos(7x – 5x)/2}/{2 × cos(7x + 5x)/2 × cos(7x – 5x)/2}] +
[{2×sin(9x + 3x)/2 × cos(9x – 3x)/2}/{2 × cos(9x + 3x)/2 × cos(9x – 3x)/2}]
⇒ [{2 × sin6x × cosx}/{2 × cos6x × cosx}] + [{2 × sin6x × cosx}/{2 × cos6x × cosx}]
⇒ (sin6x/cos6x) + (sin6x/cos6x)
⇒ tan6x + tan6x
⇒ 2 tan6x

Question 15.
If x > 0 then the value of f(x) = -3 × cos√(3 + x + x²) lie in the interval
(a) [-1, 1]
(b) [-2, 2]
(c) [-3, 3]
(d) None of these

Answer: (c) [-3, 3]
Given x > 0 then 3 + x + x² > 0
Now, -1 ≤ cos√(3 + x + x²) ≤ 1 {Since -1 ≤ cosx ≤ 1}
⇒ 3 ≥ -3 × cos√(3 + x + x²) ≥ -3 {Multiply by -3}
⇒ -3 ≤ f(x) ≤ 3
⇒ f(x) ∈ [-3, 3]

Question 16.
The value of cos 4A – cos 4B is
(a) (cos A – cos B) × (cos A + cos B) × (cos A – sin B) × (cos A + sin B)
(b) 2(cos A – cos B) × (cos A + cos B) × (cos A – sin B) × (cos A + sin B)
(c) 4(cos A – cos B) × (cos A + cos B) × (cos A – sin B) × (cos A + sin B)
(d) 8(cos A – cos B) × (cos A + cos B) × (cos A – sin B) × (cos A + sin B)

Answer: (d) 8(cos A – cos B) × (cos A + cos B) × (cos A – sin B) × (cos A + sin B)
Given, cos 4A – cos 4B
= $2\cos^2 2A - 1 - (2\cos 2\ 2B - 1)$ {since $2\cos^2 x - 1 = \cos 2x$}
= $2\cos^2 2A - 1 - 2\cos^2 2B + 1$
= $2\cos^2 2A - 2\cos^2 2B$
= $2(\cos^2 2A - \cos^2 2B)$
= $2(\cos 2A - \cos 2B) \times (\cos 2A + \cos 2B)$
= $2\{2\cos^2 A - 1 - (2\cos^2 B - 1)\} \times \{2\cos^2 A - 1 + 1 - 2\sin^2 B\}$ {since $1 - 2\sin^2 x = \cos 2x$}
= $2\{2\cos^2 A - 1 - 2\cos^2 B + 1\} \times \{2\cos^2 A - 1 + 1 - 2\sin^2 B\}$
= $2\{2\cos^2 A - 2\cos^2 B\} \times \{2\cos^2 A - 2\sin^2 B\}$
= $2 \times 2 \times 2\{\cos^2 A - \cos^2 B\} \times \{\cos^2 A - \sin^2 B\}$
= 8(cos A – cos B) × (cos A + cos B) × (cos A – sin B) × (cos A + sin B)
So, cos 4A – cos 4B = 8(cos A – cos B) × (cos A + cos B) × (cos A – sin B) × (cosA + sin B)

Question 17.
The value of cos 420° is
(a) 0
(b) 1
(c) 1/2
(d) √3/2

Answer: (c) 1/2
cos 420° = cos(360° + 60°) = cos 60° = 1/2

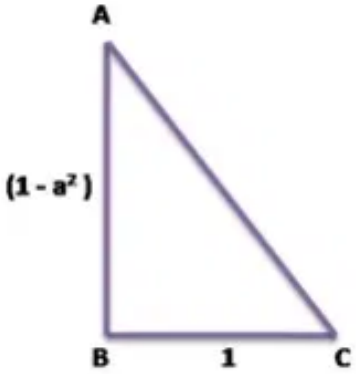
Question 18.
In a ΔABC, (b + c) cos A + (c + a) cos B + (a + b) cos C is equal to
(a) a + b + c
(b) 0
(c) none of these
(d) Rr

Answer: (a) a + b + c
Given (b + c) cos A + (c + a) cos B + (a + b) cos C
= b × cos A + c × cos A + c × cos B + a × cos B + a × cos C + b × cos C

= (b × cos C + c × cos B) + (c × cos A + a × cos C) + (b × cos A + a × cos B)
= a + b + c {since b × cos C + c × cos B = a, c × cos A + a × cos C = b, b × cos A + a × cos B = c}

Question 19.
$\tan^2 \theta = 1 - a^2$ then the value of $\sec \theta + \tan^3 \theta \times \text{cosec } \theta$ is
(a) $(2 - a^2)$
(b) $(2 - a^2)^{1/2}$
(c) $(2 - a^2)^{3/2}$
(d) None of these

Answer: (c) $(2 - a^2)^{3/2}$
Given, $\tan^2 \theta = 1 - a^2$
$\Rightarrow \tan \theta = \sqrt{(1 - a^2)}$



From the figure and apply Pythagorus theorem,
$AC^2 = AB^2 + BC^2$
$\Rightarrow AC^2 = \{\sqrt{(1 - a^2)}\}^2 + 12$
$\Rightarrow AC^2 = 1 - a^2 + 1$
$\Rightarrow AC^2 = 2 - a^2$
$\Rightarrow AC = \sqrt{(2 - a^2)}$
Now, $\sec \theta = \sqrt{(2 - a^2)}$
$\text{cosec } \theta = \sqrt{(2 - a^2)}/\sqrt{(1 - a^2)}$
and $\tan \theta = \sqrt{(1 - a^2)}$
Given, $\sec \theta + \tan^3 \theta \times \text{cosec } \theta$
$= \sqrt{(2 - a^2)} + \{(1 - a^2)^{3/2} \times \sqrt{(2 - a^2)}/\sqrt{(1 - a^2)}\}$
$= \sqrt{(2 - a^2)} + \{(1 - a^2) \times (1 - a^2) \times \sqrt{(2 - a^2)}/\sqrt{(1 - a^2)}\}$
$= \sqrt{(2 - a^2)} + (1 - a^2) \times \sqrt{(2 - a^2)}$
$= \sqrt{(2 - a^2)} \times (1 + 1 - a^2)$
$= \sqrt{(2 - a^2)} \times (2 - a^2)$
$= (2 - a^2)3/2$
So, $\sec \theta + \tan^3 \theta \times \text{cosec } \theta = (2 - a^2)^{3/2}$

Question 20.
The value of $\cos(\pi/7) \times \cos(2\pi/7) \times \cos(4\pi/7)$ is
(a) -1/2
(b) -1/4
(c) -1/6
(d) -1/8

Answer: (d) -1/8

We know that $\cos A \times \cos 2A \times \cos 2^2 A \times \ldots\ldots\ldots\ldots \times \cos 2^{n-1} A = \sin(2^n A)/\{2^n \times \sin A\}$ ……1

Given, $\cos(\pi/7) \times \cos(2\pi/7) \times \cos(4\pi/7)$

$= \cos(\pi/7) \times \cos(2\pi/7) \times \cos(2^2 \pi/7)$

$= [\sin(2^3 \times \pi/7)]/\{2^3 \times \sin(\pi/7)\}$ ……..from equation 1

$= [\sin(8\pi/7)]/\{8 \times \sin(\pi/7)\}$

$= [\sin(\pi + \pi/7)]/\{8 \times \sin(\pi/7)\}$

$= -\sin(\pi/7)/\{8 \times \sin(\pi/7)\}$

$= -1/8$

So, $\cos(\pi/7) \times \cos(2\pi/7) \times \cos(4\pi/7) = -1/8$