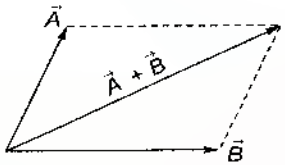
Vector Calculus

Vector Analysis

Vector algebra

1. Addition



$$\vec{A}+\vec{B} = \vec{B}+\vec{A}$$

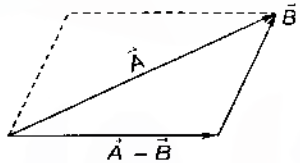
Note:

- It follow commutative law: $\boxed{\vec{A}+\vec{B}=\vec{B}+\vec{A}}$
- It also obey associate law: $\boxed{(\vec{A}+\vec{B})+\vec{C}=\vec{A}+(\vec{B}+\vec{C})}$

2. Subtraction

$$\vec{A}-\vec{B} = \vec{A}+(-\vec{B})$$

–ve sign shows reversal of direction



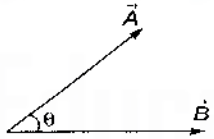
Note:

- It does not follow cumulative law: $\boxed{\vec{A}-\vec{B}\neq\vec{B}-\vec{A}}$

  But follow associative law.

3. Multiplication

(i) Scalar or Dot product



$$\vec{A}\cdot\vec{B} = |A||B|\cos\theta$$

Note:

- $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$
- Dot product of vector is a scalar.

(ii) Vector or cross product

$$\vec{A}\times\vec{B} = |A||B|\sin\theta\hat{i}_n$$

Where, $\hat{i}_n$ = unit vector in the direction of perpendicular to $\vec{A}$ and $\vec{B}$

Note:

- To find direction $\hat{i}_n$, use right hand screw rule.
- Cross product of two vector is always a vector.
- $\vec{A}\times\vec{B} = (\vec{B}\times\vec{A})$
- $(\vec{A}\times\vec{B})\times\vec{C} = \vec{A}\times(\vec{B}\times\vec{C})$

(iii) Scalar triple product

$$\vec{A}\times(\vec{B}\times\vec{C}) = \vec{B}\cdot(\vec{C}\times\vec{A})-\vec{C}\cdot(\vec{A}\times\vec{B})$$

4. A vector multiplied/divided by a scalar

(i) $S\cdot\vec{V}$ = it increases the magnitude by S unit

(ii) $\frac{\vec{V}}{S}$ = it decreases the magnitude by S unit

(iii) $\frac{\vec{A}}{\vec{B}}$ = it does not exist

5. Differentiation of vectors

If $\vec{A} = A_x i_x + A_y i_y + A_z i_z$

$$\boxed{d\vec{A} = dA_x i_x + dA_y i_y + dA_z i_z}$$

For cylindrical and spherical coordinate system:

$$\boxed{d\vec{A} = dA_r\, i_r + dA_\phi\, i_\phi + dA_z\, i_z}$$

and

$$\boxed{d\vec{A} = dA_r\, i_r + dA_\theta\, i_\theta + dA_\phi\, i_\phi}$$

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