7. Polynomial

Let us Work Out 7.1

1. Question

Let us write which are the polynomials in the following algebraic expressions. Let us write the degree of each of the polynomials.

(i) $2x^6 - 4x^5 + 7x^2 + 3$

(ii) $x^{-2} + 2x^{-1} + 4$

(iii) $y^3 - \frac{3}{4}y + \sqrt{7}$

(iv) $\frac{1}{x} - x + 2$

(v) $x^{51} - 1$

(vi) $^3\sqrt{t} + \frac{t}{27}$

(vii) 15

(viii) 0

(ix) $z + \frac{3}{z} + 2$

(x) $y^3 + 4$

(xi) $\frac{1}{\sqrt{2}}x^2 - \sqrt{2}x + 2$

Answer

(i) Since all the exponents (power) of variable(x) are whole no. (i.e., zero or positive integers)

$\Rightarrow 2x^6 - 4x^5 + 7x^2 + 3$ is a polynomial.

(**Note:** The degree is the value of the greatest (highest) exponent of any expression (except the constant) in the polynomial. To find the degree all that you have to do is find the largest exponent in the polynomial).

And, since here the highest power is 6,

⇒ It is a polynomial of degree 6.

(ii) Since all the exponents (power) of variable(x) are not whole no. (i.e., zero or positive integers)

⇒ $x^{-2} + 2x^{-1} + 4$ is not a polynomial.

(iii) Since all the exponents (power) of variable(y) are whole no. (i.e., zero or positive integers)

⇒ $y^3 - \frac{3}{4}y + \sqrt{7}$ is a polynomial.

And, since here the highest power is 3,

⇒ It is a polynomial of degree 3.

(iv) $\frac{1}{x} - x + 2$

$$\Rightarrow \frac{1}{x} - x + 2 = x^{-1} - x + 2$$

Since all the exponents (power) of variable(x) are not whole no.(i.e., zero or positive integers)

⇒ $\frac{1}{x} - x + 2$ is not a polynomial.

(v) Since all the exponents (power) of variable(x) are whole no. (i.e., zero or positive integers)

⇒ $x^{51} - 1$ is a polynomial.

And, since here the highest power is 51,

⇒ $x^{51} - 1$ is a polynomial of degree 51.

(vi) $\sqrt[3]{t} + \frac{t}{27} = t^{\frac{3}{2}} + \frac{t}{27}$

Since all the exponents (power) of t are not whole no.(i.e., zero or positive integers)

⇒ $\sqrt[3]{t} + \frac{t}{27}$ is not a polynomial.

(vii) 15 is a constant polynomial as there is no variable present in that.

And, therefore the highest power is 0

⇒ 15 is a polynomial with degree 0.

(viii) 0 is a constant polynomial (zero polynomial) as there is no variable present in that.

⇒ 0 is a polynomial with degree 0.

(ix) $z + \frac{3}{z} + 2 = z + 3z^{-1} + 2$

Since all the exponents (power) of variable(z) are not whole no.(i.e., zero or positive integers)

⇒ $z + \frac{3}{z} + 2$ is not a polynomial.

(x) Since all the exponents (power) of variable(y) are whole no. (i.e., zero or positive integers)

⇒ y^3 +4 is a polynomial.

And, since here the highest power is 3,

⇒ y^3 +4 is a polynomial of degree 3.

(xi) $\frac{1}{\sqrt{2}}x^2 - \sqrt{2}x + 2$

Since all the exponents (power) of variable(x) are whole no.(i.e., zero or positive integers)

And, since here the highest power is 2

⇒ : $\frac{1}{\sqrt{2}}x^2 - \sqrt{2}x + 2$ is a polynomial of degree 2.

2. Question

In the following polynomials, let us write which are first degree polynomials in one variable, which are second degree polynomials in one variable and which are third degree polynomials in one variable.

(i) 2x + 17

(ii)$x^3 + x^2 + x + 1$

(iii) $-3 + 2y^2 + 5xy$

(iv)$5 - x - x^3$

(v) $\sqrt{2} + t - t^2$

(vi) $\sqrt{5}\ x$

Answer

(i) (**Note:** The degree is the value of the greatest (highest) exponent of any expression (except the constant) in the polynomial. To find the degree all that you have to do is find the largest exponent in the polynomial).

Since, here the highest power of variable(x) is 1,

⇒ $2x + 17$ is a first degree polynomial.

(ii) Since, here the highest power of variable(x) is 3,

⇒ $x^3 + x^2 + x + 1$ is a third degree polynomial.

(iii) Since, here the highest power of variable(x, y) is 2,

⇒ $-3 + 2y^2 + 5xy$ is a second degree polynomial.

(Note: Whenever we have polynomial in two variable (i.e., x and y) and both are in multiplication we see for the degree by adding the powers of both the variable.)

(iv) Since, here the highest power of variable(x) is 3,

⇒ $5 - x - x^3$ is a third degree polynomial.

(v) Since, here the highest power of variable (t) is 2,

⇒ $\sqrt{2} + t + t^2$ is a second degree polynomial.

(vi) Since, here the highest and only power of variable(x) is 1,

⇒ $\sqrt{5}x$ is a first degree polynomial.

3. Question

Let us write the co-efficient of the following polynomials according to the guidelines:

(i) The co-efficient of x^3 in $5x^3 - 13x^2 + 2$

(ii) The co-efficient of x in $x^2 - x + 2$

(iii) The co-efficient of x^2 in $8x - 19$

(iv) The co-efficient of x^0 in $\sqrt{11} - 3\sqrt{11}x + x^2$.

Answer

(i) We can see that with x, the constant written before that is 5,

⇒ The co-efficient of x^3 in $5x^3 - 13x^2 + 2$ is 5.

(ii) We can see that with x, the constant written before that is -1,

⇒ The co-efficient of x in $x^2 - x + 2$ is -1.

(iii) We can see that with x^2, the constant written before that is 0, because there is no x^2 in polynomial.

⇒ The co-efficient of x^2 in $8x - 19$ is 0.

(iv) We can see that with x^0 (i.e., the constant) is $\sqrt{(11)}$

⇒ The co-efficient of x^0 in $\sqrt{(11)} - 3\sqrt{(11)}x + x^2$ is $\sqrt{(11)}$.

4. Question

I write the degree of each of the following polynomials:

(i) $x^4 + 2x^3 + x^2 + x$

(ii) $7x - 5$

(iii) 16

(iv) $2 - y - y^3$

(v) 7t

(vi) $5 - x^2 + x^{19}$

Answer

(i) (**Note:** The degree is the value of the greatest (highest) exponent of any expression (except the constant) in the polynomial. To find the degree all that you have to do is find the largest exponent in the polynomial).

Since, here the highest power of variable is 4,

⇒ Degree of polynomial=4.

(ii) Since, here the highest power of variable is 1,

⇒ Degree of polynomial=1.

(iii) Since, here the highest power of variable is 0, because there exist no variable.

⇒ Degree of polynomial=0.

(iv) Since, here the highest power of variable is 3,

⇒ Degree of polynomial=3.

(v) Since, here the highest power of variable is 1,

⇒ Degree of polynomial=1.

(vi) Since, here the highest power of variable is 19,

⇒ Degree of polynomial=19.

5. Question

I write two separate binomials in one variable whose degrees are 17.

Answer

(**Note:** The degree is the value of the greatest (highest) exponent of any expression (except the constant) in the polynomial. To find the degree all that you have to do is find the largest exponent in the polynomial).

(Binomial means the polynomial will have two terms and degree =17 means that highest power should be 17).

∴ First binomial: $x^{17} - 4x^5$

Second Binomial= $7x^{17} + 27$

6. Question

I write two separate monomials in one variable whose degrees are 4.

Answer

(**Note:** The degree is the value of the greatest (highest) exponent of any expression (except the constant) in the polynomial. To find the degree all that you have to do is find the largest exponent in the polynomial).

(Monomial means the polynomial will have one term and degree =4 means that highest power should be 4).

∴ First binomial: xy^3

Second Binomial=x^4

7. Question

I write two separate trinomials in one variable whose degrees are 3.

Answer

(**Note:** The degree is the value of the greatest (highest) exponent of any expression (except the constant) in the polynomial. To find the degree all that you have to do is find the largest exponent in the polynomial).

(Trinomial means the polynomial will have three terms and degree 3 means that highest power should be 3).

∴ First binomial: $7\,x^3 - 4x - 7$

Second Binomial: x^3+5x^2-x

8. Question

In the following algebraic expressions, which are polynomials in one variable, which are polynomials in two variables and which are not polynomials —Let us write them.

(i) $x^2 + 3x + 2$

(ii) $x^2 + y^2 + a^2$

(iii) $y^2 - 4ax$

(iv) $x + y + 2$

(v) $x^8 + y^4 + x^5y^9$

(vi) $x + \frac{5}{x}$

Answer

(i) Since, here the highest power of variable is 1, which is a whole no.

⇒ $x^2 + 3x + 2$is a polynomial.

But here only one variable, i.e., x is used,

⇒ $x^2 + 3x + 2$ is a polynomial in one variable.

(ii) Since, here the highest power of variable is 2, which is a whole no.

$x^2 + y^2 + a^2$ is a polynomial.

Since, here two variable, i.e., x and y are used,

⇒ $x^2 + y^2 + a^2$ is a polynomial in two variables.

(iii) Since, here the highest power of variable is 2, which is a whole no.

⇒ $y^2 - 4ax$ is a polynomial.

Since, here two variable, i.e., x and y are used,

⇒ $y^2 - 4ax$ is a polynomial in one variable.

(iv) Since, here the highest power of variable is 1, which is a whole no.

⇒ $x + y + 2$ is a polynomial.

But here only two variable, i.e., x and y and are used,

⇒ $x + y + 2$ is a polynomial in one variable.

(v) Since, here the highest power of variable is(5+9=14), which is a whole no.

(Note: 5 for x and 9 for y)

⇒ $x^8 + y^4 + x^5y^9$ is a polynomial.

here only two variable, i.e., x and y is used,

⇒ $x^2 + 3x + 2$ is a polynomial in two variables.

(vi) Since, here the highest power of variable is 1, which is a whole no.

⇒ $x + \frac{5}{x} = x + 5x^{-1}$

And, since the exponent of variable is not zero or whole number,

⇒ $x + \frac{5}{x} =$is not a polynomial.

Let us Work Out 7.2

1. Question

If $f(x) = x^2 + 9x - 6$, then let us write by calculating the values of f(0), f(1) and f(3).

Answer

Formula used/Theory.

Putting the value of ‘x’ in f(x) gives out the value of f(x) on ‘x’

We have,

$f(x) = x^2 + 9x - 6$

When we put

$x = 0$

Then,

$f(0) = 0^2 + 9 \times 0 - 6$

$f(0) = -6$

when we put

$x = 1$

Then,

$f(1) = 1^2 + 9 \times 1 - 6$

$f(1) = 10 - 6$

$f(1) = 4$

When we put

$x = 3$

Then,

$f(3) = 3^2 + 9 \times 3 - 6$

$f(3) = 9+27 - 6$

$f(3) = 30$

Conclusion.

The values of f(0), f(1), f(3) are –6, 4, 30 respectively.

2 A. Question

By calculating the following polynomials f(x) let us write the values of f(1) and f(–1):

$f(x) = 2x^3 + x^2 + x + 4$

Answer

Formula used/Theory.

Putting the value of 'x' in f(x) gives out the value of f(x) on 'x'

We have,

$f(x) = 2x^3 + x^2 + x + 4$

When we put $x = 1$

Then,

$f(1) = 2(1)^3 + (1)^2 + 1 + 4$

$f(1) = 2+1+1+4$

$f(1) = 8$

When we put $x = -1$

Then,

$f(-1) = 2(-1)^3 + (-1)^2 + (-1) + 4$

$f(-1) = (-2) +1-1+4$

$f(-1) = 2$

Conclusion.

The value of f(1), f(-1) are 8, 2 respectively.

2 B. Question

By calculating the following polynomials f(x) let us write the values of f(1) and f(−1):

$f(x) = 3x^4 - 5x^3 + x^2 + 8$

Answer

Formula used/Theory.

Putting the value of 'x' in f(x) gives out the value of f(x) on 'x'

We have,

$f(x) = 3x^4 - 5x^3 + x^2 + 8$

When we put $x = 1$

Then,

$f(1) = 3(1)^4 - 5(1)^3 + (1)^2 + 8$

$f(1) = 3 - 5 + 1 + 8$

$f(1) = 7$

When we put $x = -1$

Then,

$f(-1) = 3(-1)^4 - 5(-1)^3 + (-1)^2 + 8$

$f(-1) = 3 - (-5) + 1 + 8$

$f(-1) = 3 + 5 + 1 + 8$

$f(-1) = 17$

Conclusion.

The value of f(1), f(-1) are 7, 17 respectively.

2 C. Question

By calculating the following polynomials f(x) let us write the values of f(1) and f(−1):

$f(x) = 4 + 3x - x^3 + 5x^6$

Answer

Formula used/Theory.

Putting the value of ‘x’ in f(x) gives out the value of f(x) on ‘x’

We have,

$f(x) = 4 + 3x - x^3 + 5x^6$

When we put x = 1

Then,

$f(1) = 4 + 3 \times 1 - 1 + 5 \times 1$

$f(1) = 4 + 3 - 1 + 5$

$f(1) = 11$

When we put x = -1

Then,

$f(-1) = 4 + 3 \times (-1) - (-1)^3 + 5 \times (-1)^6$

$f(-1) = 4 + 3 \times (-1) - (-1) + 5 \times (-1)$

$f(-1) = 4 - 3 + 1 + 5$

$f(-1) = 7$

Conclusion.

The value of f(1), f(-1) are 11, 7 respectively.

2 D. Question

By calculating the following polynomials f(x) let us write the values of f(1) and f(−1):

$f(x) = 6 + 10x - 7x^2$

Answer

Formula used/Theory.

Putting the value of ‘x’ in f(x) gives out the value of f(x) on ‘x’

We have,

$f(x) = 6 + 10x - 7x^2$

when we put x = 1

then,

$f(1) = 6 + 10 \times (1) - 7(1)^2$

$f(1) = 6 + 10 \times 1 - 7 \times 1$

$f(1) = 6 + 10 - 7$

$f(1) = 9$

when we put $x = -1$

then,

$f(-1) = 6 + 10 \times (-1) - 7(-1)^2$

$f(-1) = 6 + 10 \times (-1) - 7 \times 1$

$f(-1) = 6 - 10 - 7$

$f(-1) = -11$

Conclusion.

The value of f(1), f(-1) are 9, -11 respectively.

3 A. Question

Let us check the following statements—

The zero of the polynomial $P(x) = x - 1$ is 1.

Answer

Formula used/Theory.

There is some value of 'x' for which the P(x) comes to 0, that value of 'x' is said to be zero of polynomial P(x)

We have,

$P(x) = x-1$

And zero of the polynomial P(x) is 1

Then,

Zero of the polynomial P(x)

Means $P(x) = 0$

$x-1 = 0$

$x = 1$

∴ Zero of polynomial P(x) is 1

Conclusion.

Hence, the statement is True

3 B. Question

Let us check the following statements—

The zero of the polynomial P(x) = 3 – x is 3.

Answer

Formula used/Theory.

There is some value of 'x' for which the P(x) comes to 0, that value of 'x' is said to be zero of polynomial P(x)

We have,

P(x) = 3 – x

And zero of the polynomial P(x) is 3

Then,

Zero of the polynomial P(x)

Means P(x) = 0

3 – x = 0

-x = -3

x = 3

∴ Zero of polynomial P(x) is 3

Conclusion.

Hence, the statement is True

3 C. Question

Let us check the following statements—

The zero of the polynomial P(x) = 5x + 1 is $-\frac{1}{5}$.

Answer

Formula used/Theory.

There is some value of 'x' for which the P(x) comes to 0, that value of 'x' is said to be zero of polynomial P(x)

We have,

$P(x) = 5x+1$

And zero of the polynomial P(x) is $-\frac{1}{5}$

Then,

Zero of the polynomial P(x)

Means $P(x) = 0$

$5x+1 = 0$

$5x = -1$

$x = -\frac{1}{5}$

∴ Zero of polynomial P(x) is $-\frac{1}{5}$

Conclusion.

Hence, the statement is True

3 D. Question

Let us check the following statements—

The two zeros of the polynomial $P(x) = x^2 - 9$ are 3 and –3.

Answer

Formula used/Theory.

There is some value of 'x' for which the P(x) comes to 0, that value of 'x' is said to be zero of polynomial P(x)

We have,

$P(x) = x^2 - 9$

And zero of the polynomial P(x) are 3 and -3

Then,

Zero of the polynomial P(x)

Means $P(x) = 0$

$x^2 - 9 = 0$

$x^2 - (3)^2 = 0$

$\Rightarrow a^2 - b^2 = (a+b)(a-b)$

$(x - 3)(x+3) = 0$

$x - 3 = 0$ and $x+3 = 0$

$x = 3$ and $x = -3$

$\therefore$ Zero of polynomial $P(x)$ is 3 and -3

Conclusion.

Hence, the statement is True

3 E. Question

Let us check the following statements—

The two zeros of the polynomial $P(x) = x^2 - 5x$ are 0 and 5.

Answer

Formula used/Theory.

There is some value of 'x' for which the P(x) comes to 0, that value of 'x' is said to be zero of polynomial P(x)

We have,

$P(x) = x^2 - 5x$

And zero of the polynomial P(x) are 0 and 5

Then,

Zero of the polynomial P(x)

Means $P(x) = 0$

$x^2 - 5x = 0$

$x(x- 5) = 0$

$x = 0$ and $(x-5) = 0$

$\Rightarrow x = 0$ and $x = 5$

$\therefore$ Zero of polynomial $P(x)$ is 0 and 5

Conclusion.

Hence, the statement is True

3 F. Question

Let us check the following statements—

The two zeros of the polynomial $P(x) = x^2 - 2x - 8$ are 4 and (−2).

Answer

Formula used/Theory.

There is some value of ‘x’ for which the P(x) comes to 0, that value of ‘x’ is said to be zero of polynomial P(x)

We have,

$P(x) = x^2 - 2x - 8$

And zero of the polynomial P(x) are 4 and -2

Then,

Zero of the polynomial P(x)

Means P(x) = 0

$x^2 - 2x - 8 = 0$

As in quadratic eq

We have to divide (-2x) in 2 parts such that product comes to $(-8x^2)$

Hence the factors are :-

$(-x) \times (8x)$ addition gives $8x - x = 7x$

$(-2x) \times (4x)$ addition gives $4x - 2x = 2x$

$(-4x) \times (2x)$ addition gives $2x - 4x = -2x$

Hence factors are -4x and 2x

$x^2 - 4x + 2x - 8 = 0$

$x(x - 4)+2(x - 4) = 0$

$(x+2)(x - 4) = 0$

$x+2 = 0$ and $x - 4 = 0$

$x = (-2)$ and $x = 4$

∴ Zero of polynomial P(x) is 4 and -2

Conclusion.

Hence, the statement is True

4 A. Question

Let us determine the zeros of the following polynomials—

f(x) = 2 – x

Answer

Formula used/Theory.

There is some value of 'x' for which the f(x) comes to 0, that value of 'x' is said to be zero of polynomial f(x)

We have,

f(x) = 2 – x

Then,

Zero of the polynomial f(x)

Means f(x) = 0

2 – x = 0

-x = -2

x = 2

∴ Zero of polynomial f(x) is 2

Conclusion.

Hence, the zero of the polynomial f(x) is 2

4 B. Question

Let us determine the zeros of the following polynomials—

f(x) = 7x + 2

Answer

Formula used/Theory.

There is some value of 'x' for which the f(x) comes to 0, that value of 'x' is said to be zero of polynomial f(x)

We have,

f(x) = 7x+2

Then,

Zero of the polynomial f(x)

Means f(x) = 0

7x+2 = 0

7x = -2

$x = -\frac{2}{7}$

∴ Zero of polynomial f(x) is $-\frac{2}{7}$

Conclusion.

Hence, the zero of the polynomial f(x) is $-\frac{2}{7}$

4 C. Question

Let us determine the zeros of the following polynomials—

f(x) = x + 9

Answer

Formula used/Theory.

There is some value of 'x' for which the f(x) comes to 0, that value of 'x' is said to be zero of polynomial f(x)

We have,

f(x) = x+9

Then,

Zero of the polynomial f(x)

Means f(x) = 0

x+9 = 0

x = -9

∴ Zero of polynomial f(x) is –9

Conclusion.

Hence, the zero of the polynomial f(x) is –9

4 D. Question

Let us determine the zeros of the following polynomials—

f(x) = 6 – 2x

Answer

Formula used/Theory.

There is some value of ‘x’ for which the f(x) comes to 0, that value of ‘x’ is said to be zero of polynomial f(x)

We have,

f(x) = 6 – 2x

Then,

Zero of the polynomial f(x)

Means f(x) = 0

6 – 2x = 0

-2x = -6

$x = \frac{-6}{-2} = 3$

∴ Zero of polynomial f(x) is 3

Conclusion.

Hence, the zero of the polynomial f(x) is 3

4 E. Question

Let us determine the zeros of the following polynomials—

f(x) = 2x

Answer

Formula used/Theory.

There is some value of ‘x’ for which the f(x) comes to 0, that value of ‘x’ is said to be zero of polynomial f(x)

We have,

f(x) = 2x

Then,

Zero of the polynomial f(x)

Means f(x) = 0

2x = 0

x = 0

∴ Zero of polynomial f(x) is 0

Conclusion.

Hence, the zero of the polynomial f(x) is 0

4 F. Question

Let us determine the zeros of the following polynomials—

$f(x) = ax + b, (a \neq 0)$

Answer

Formula used/Theory.

There is some value of 'x' for which the f(x) comes to 0, that value of 'x' is said to be zero of polynomial f(x)

We have,

$f(x) = ax+b$

Then,

Zero of the polynomial f(x)

Means $f(x) = 0$

$ax+b = 0$

$ax = -b$

$x = \frac{-b}{a}$

∴ Zero of polynomial f(x) is $\frac{-b}{a}$

Conclusion.

Hence, the zero of the polynomial f(x) is $\frac{-b}{a}$

Let us Work Out 7.3

1 A. Question

By applying Remainder Theorem, let us calculate and write the remainder that I shall get in every cases, when $x^3 - 3x^2 + 2x + 5$ is divided by

$x - 2$

Answer

Remainder theorem says that,

f(x) is a polynomial of degree n ($n \geq 1$) and 'a' is any real number. If f(x) is divided by (x – a), then the remainder will be f(a).

Let us solve the following questions on the basis of this remainder theorem.

When $x^3 - 3x^2 + 2x + 5$ is divided by (x – 2).

Let $f(x) = x^3 - 3x^2 + 2x + 5$ …(1)

Now, let's find out the zero of the linear polynomial, (x – 2).

To find zero,

$x - 2 = 0$

$\Rightarrow x = 2$

This means that by remainder theorem, when $x^3 - 3x^2 + 2x + 5$ is divided by (x – 2), the remainder comes out to be f(2).

From equation (1), remainder can be calculated as,

Remainder = f(2)

$\Rightarrow$ Remainder = $(2)^3 - 3(2)^2 + 2(2) + 5$

$\Rightarrow$ Remainder = 8 – 12 + 4 + 5

$\Rightarrow$ Remainder = 5

$\therefore$ the required remainder = 5.

1 B. Question

By applying Remainder Theorem, let us calculate and write the remainder that I shall get in every cases, when $x^3 - 3x^2 + 2x + 5$ is divided by

x + 2

Answer

Remainder theorem says that,

f(x) is a polynomial of degree n ($n \geq 1$) and 'a' is any real number. If f(x) is divided by (x – a), then the remainder will be f(a).

Let us solve the following questions on the basis of this remainder theorem.

When $x^3 - 3x^2 + 2x + 5$ is divided by (x + 2).

Let $f(x) = x^3 - 3x^2 + 2x + 5$ …(1)

Now, let's find out the zero of the linear polynomial, (x + 2).

To find zero,

$x + 2 = 0$

$\Rightarrow x = -2$

This means that by remainder theorem, when $x^3 - 3x^2 + 2x + 5$ is divided by $(x + 2)$, the remainder comes out to be f(-2).

From equation (1), remainder can be calculated as,

Remainder = f(-2)

$\Rightarrow$ Remainder = $(-2)^3 - 3(-2)^2 + 2(-2) + 5$

$\Rightarrow$ Remainder = $-8 - 12 - 4 + 5$

$\Rightarrow$ Remainder = -19

$\therefore$ the required remainder = -19

1 C. Question

By applying Remainder Theorem, let us calculate and write the remainder that I shall get in every cases, when $x^3 - 3x^2 + 2x + 5$ is divided by

$2x - 1$

Answer

Remainder theorem says that,

f(x) is a polynomial of degree n ($n \geq 1$) and 'a' is any real number. If f(x) is divided by $(x - a)$, then the remainder will be f(a).

Let us solve the following questions on the basis of this remainder theorem.

When $x^3 - 3x^2 + 2x + 5$ is divided by $(2x - 1)$.

Let $f(x) = x^3 - 3x^2 + 2x + 5$ …(1)

Now, let's find out the zero of the linear polynomial, $(2x - 1)$.

To find zero,

$2x - 1 = 0$

$\Rightarrow 2x = 1$

$\Rightarrow x = 1/2$

This means that by remainder theorem, when $x^3 - 3x^2 + 2x + 5$ is divided by $(2x - 1)$, the remainder comes out to be f(1/2).

From equation (1), remainder can be calculated as,

Remainder = f(1/2)

⇒ Remainder $= \left(\frac{1}{2}\right)^3 - 3\left(\frac{1}{2}\right)^2 + 2\left(\frac{1}{2}\right) + 5$

⇒ Remainder $= \frac{1}{8} - \frac{3}{4} + 1 + 6$

⇒ Remainder = -19

∴ the required remainder = -19

1 D. Question

By applying Remainder Theorem, let us calculate and write the remainder that I shall get in every cases, when $x^3 - 3x^2 + 2x + 5$ is divided by

2x + 1

Answer

Remainder theorem says that,

f(x) is a polynomial of degree n (n ≥ 1) and 'a' is any real number. If f(x) is divided by (x – a), then the remainder will be f(a).

Let us solve the following questions on the basis of this remainder theorem.

When $x^3 - 3x^2 + 2x + 5$ is divided by (2x + 1).

Let $f(x) = x^3 - 3x^2 + 2x + 5$ ...(1)

Now, let's find out the zero of the linear polynomial, (2x + 1).

To find zero,

2x + 1 = 0

⇒ 2x = -1

⇒ x = -1/2

This means that by remainder theorem, when $x^3 - 3x^2 + 2x + 5$ is divided by (2x + 1), the remainder comes out to be f(-1/2).

From equation (1), remainder can be calculated as,

Remainder = f(-1/2)

⇒ Remainder $= \left(-\frac{1}{2}\right)^3 - 3\left(-\frac{1}{2}\right)^2 + 2\left(-\frac{1}{2}\right) + 5$

⇒ Remainder $= -\frac{1}{8} - \frac{3}{4} - 1 + 5$

⇒ Remainder$= \frac{-1-6}{8} + 4$

$\Rightarrow$ Remainder$= -\frac{7}{8} + 4$

$\Rightarrow$ Remainder$= \frac{-7+32}{8}$

$\Rightarrow$ Remainder$= \frac{25}{8}$

∴ the required remainder = 25/8

2 A. Question

By applying Remainder Theorem, let us calculate and write the remainders, that I shall get when the following polynomials are divided by (x – 1).

$x^3 - 6x^2 + 13x + 60$

Answer

Remainder theorem says that,

f(x) is a polynomial of degree n (n ≥ 1) and 'a' is any real number. If f(x) is divided by (x – a), then the remainder will be f(a).

Let us solve the following questions on the basis of this remainder theorem.

Let $f(x) = x^3 - 6x^2 + 13x + 60$ …(1)

When $x^3 - 6x^2 + 13x + 60$ is divided by (x – 1).

Now, let's find out the zero of the linear polynomial, (x – 1).

To find zero,

x – 1 = 0

$\Rightarrow$ x = 1

This means that by remainder theorem, when $x^3 - 6x^2 + 13x + 60$ is divided by (x – 1), the remainder comes out to be f(1).

From equation (1), remainder can be calculated as,

Remainder = f(1)

$\Rightarrow$ Remainder = $(1)^3 - 6(1)^2 + 13(1) + 60$

$\Rightarrow$ Remainder = 1 – 6 + 13 + 60

$\Rightarrow$ Remainder = -5 + 73

$\Rightarrow$ Remainder = 68

∴ the required remainder = 68

2 B. Question

By applying Remainder Theorem, let us calculate and write the remainders, that I shall get when the following polynomials are divided by (x – 1).

$x^3 - 3x^2 + 4x + 50$

Answer

Remainder theorem says that,

f(x) is a polynomial of degree n (n ≥ 1) and 'a' is any real number. If f(x) is divided by (x – a), then the remainder will be f(a).

Let us solve the following questions on the basis of this remainder theorem.

Let $f(x) = x^3 - 3x^2 + 4x + 50$ …(1)

When $x^3 - 3x^2 + 4x + 50$ is divided by (x – 1).

Now, let's find out the zero of the linear polynomial, (x – 1).

To find zero,

$x - 1 = 0$

$\Rightarrow x = 1$

This means that by remainder theorem, when $x^3 - 3x^2 + 4x + 50$ is divided by (x – 1), the remainder comes out to be f(1).

From equation (1), remainder can be calculated as,

Remainder = f(1)

$\Rightarrow$ Remainder = $(1)^3 - 3(1)^2 + 4(1) + 50$

$\Rightarrow$ Remainder = 1 – 3 + 4 + 50

$\Rightarrow$ Remainder = 1 + 1 + 50

$\Rightarrow$ Remainder = 52

$\therefore$ the required remainder = 52

2 C. Question

By applying Remainder Theorem, let us calculate and write the remainders, that I shall get when the following polynomials are divided by (x – 1).

$4x^3 + 4x^2 - x - 1$

Answer

Remainder theorem says that,

f(x) is a polynomial of degree n ($n \geq 1$) and 'a' is any real number. If f(x) is divided by (x – a), then the remainder will be f(a).

Let us solve the following questions on the basis of this remainder theorem.

Let $f(x) = 4x^3 + 4x^2 - x - 1$ ...(1)

When $4x^3 + 4x^2 - x - 1$ is divided by (x – 1).

Now, let's find out the zero of the linear polynomial, (x – 1).

To find zero,

$x - 1 = 0$

$\Rightarrow x = 1$

This means that by remainder theorem, when $4x^3 + 4x^2 - x - 1$ is divided by (x – 1), the remainder comes out to be f(1).

From equation (1), remainder can be calculated as,

Remainder = f(1)

$\Rightarrow$ Remainder = $4(1)^3 + 4(1)^2 - (1) - 1$

$\Rightarrow$ Remainder = $4 + 4 - 1 - 1$

$\Rightarrow$ Remainder = $8 - 2$

$\Rightarrow$ Remainder = 6

$\therefore$ the required remainder = 6

2 D. Question

By applying Remainder Theorem, let us calculate and write the remainders, that I shall get when the following polynomials are divided by (x – 1).

$11x^3 - 12x^2 - x + 7$

Answer

Remainder theorem says that,

f(x) is a polynomial of degree n ($n \geq 1$) and 'a' is any real number. If f(x) is divided by (x – a), then the remainder will be f(a).

Let us solve the following questions on the basis of this remainder theorem.

Let $f(x) = 11x^3 - 12x^2 - x + 7$ ...(1)

When $11x^3 - 12x^2 - x + 7$ is divided by (x – 1).

Now, let's find out the zero of the linear polynomial, (x – 1).

To find zero,

$x - 1 = 0$

$\Rightarrow x = 1$

This means that by remainder theorem, when $11x^3 - 12x^2 - x + 7$ is divided by $(x - 1)$, the remainder comes out to be f(1).

From equation (1), remainder can be calculated as,

Remainder = f(1)

$\Rightarrow$ Remainder $= 11(1)^3 - 12(1)^2 - (1) + 7$

$\Rightarrow$ Remainder $= 11 - 12 - 1 + 7$

$\Rightarrow$ Remainder $= -1 - 1 + 7$

$\Rightarrow$ Remainder $= -2 + 7$

$\Rightarrow$ Remainder $= 5$

$\therefore$ the required remainder = 5

3 A. Question

Applying Remainder Theorem, let us write the remainders, when,

the polynomial $x^3 - 6x^2 + 9x - 8$ is divided by $(x - 3)$

Answer

Remainder theorem says that,

f(x) is a polynomial of degree n $(n \geq 1)$ and 'a' is any real number. If f(x) is divided by $(x - a)$, then the remainder will be f(a).

Let us solve the following questions on the basis of this remainder theorem.

Let $f(x) = x^3 - 6x^2 + 9x - 8$ ...(1)

When $x^3 - 6x^2 + 9x - 8$ is divided by $(x - 3)$.

Now, let's find out the zero of the linear polynomial, $(x - 3)$.

To find zero,

$x - 3 = 0$

$\Rightarrow x = 3$

This means that by remainder theorem, when $x^3 - 6x^2 + 9x - 8$ is divided by $(x - 3)$, the remainder comes out to be f(3).

From equation (1), remainder can be calculated as,

Remainder = f(3)

⇒ Remainder = $(3)^3 - 6(3)^2 + 9(3) - 8$

⇒ Remainder = 27 – 54 + 27 – 8

⇒ Remainder = -27 + 27 – 8

⇒ Remainder = 0 – 8

⇒ Remainder = -8

∴ the required remainder = -8

3 B. Question

Applying Remainder Theorem, let us write the remainders, when,

the polynomial $x^3 - ax^2 + 2x - a$ is divided by (x – a)

Answer

Remainder theorem says that,

f(x) is a polynomial of degree n (n ≥ 1) and 'a' is any real number. If f(x) is divided by (x – a), then the remainder will be f(a).

Let us solve the following questions on the basis of this remainder theorem.

Let f(x) = $x^3 - ax^2 + 2x - a$ …(1)

When $x^3 - ax^2 + 2x - a$ is divided by (x – a).

Now, let's find out the zero of the linear polynomial, (x – a).

To find zero,

x – a = 0

⇒ x = a

This means that by remainder theorem, when $x^3 - ax^2 + 2x - a$ is divided by (x – a), the remainder comes out to be f(a).

From equation (1), remainder can be calculated as,

Remainder = f(a)

⇒ Remainder = $(a)^3 - a(a)^2 + 2(a) - a$

⇒ Remainder = $a^3 - a^3 + 2a - a$

⇒ Remainder = 2a – a

⇒ Remainder = a

∴ the required remainder = a

4. Question

Applying Remainder Theorem, let us calculate whether the polynomial.

$P(x) = 4x^3 + 4x^2 - x - 1$ is a multiple of $(2x + 1)$ or not.

Answer

Remainder theorem says that,

f(x) is a polynomial of degree n (n ≥ 1) and 'a' is any real number. If f(x) is divided by (x – a), then the remainder will be f(a).

Let us solve the questions on the basis of this theorem.

Here, let $f(x) = 4x^3 + 4x^2 - x - 1$ ...(i)

First, we need to find zero of the linear polynomial, (2x + 1).

To find zero,

$2x + 1 = 0$

$\Rightarrow 2x = -1$

$\Rightarrow x = -1/2$

f(x) will be multiple of (2x + 1) if f(-1/2) = 0.

$\Rightarrow f\left(-\frac{1}{2}\right) = 4\left(-\frac{1}{2}\right)^3 + 4\left(-\frac{1}{2}\right)^2 - \left(-\frac{1}{2}\right) - 1$

$\Rightarrow f\left(-\frac{1}{2}\right) = \left(-4 \times \frac{1}{8}\right) + \left(4 \times \frac{1}{4}\right) + \frac{1}{2} - 1$

$\Rightarrow f\left(-\frac{1}{2}\right) = -\frac{1}{2} + 1 + \frac{1}{2} - 1$

$\Rightarrow f\left(-\frac{1}{2}\right) = 0$

$\Rightarrow P(x) = 4x^3 + 4x^2 - x - 1$ is a multiple of $(2x + 1)$.

5. Question

For what value of a, the divisions of two polynomials $(ax^3 + 3x^2 - 3)$ and $(2x^3 - 5x + a)$ by $(x - 4)$ give the same remainder —let us calculate and write it.

Answer

Let the two polynomials be,

$P(x) = ax^3 + 3x^2 - 3$ …(i)

$Q(x) = 2x^3 - 5x + a$ …(ii)

Now, we understand by the question that,

P(x) and Q(x) divided by (x – 4) gives the same remainder.

We need to find zero of the linear polynomial, (x – 4).

To find zero, put (x – 4) = 0

$\Rightarrow x - 4 = 0$

$\Rightarrow x = 4$

By Remainder theorem that says, f(x) is a polynomial of degree n (n ≥ 1) and 'a' is any real number. If f(x) is divided by (x – a), then the remainder will be f(a).

Here, a = 4.

This means, remainder when P(x) is divided by (x – 4) is P(4).

$\Rightarrow$ Remainder = P(4)

$\Rightarrow$ Remainder = $a(4)^3 + 3(4)^2 - 3$

$\Rightarrow$ Remainder = 64a + 48 – 3

$\Rightarrow$ Remainder = 64a + 45 …(iii)

And remainder when Q(x) is divided by (x – 4) is Q(4).

$\Rightarrow$ Remainder = Q(4)

$\Rightarrow$ Remainder = $2(4)^3 - 5(4) + a$

$\Rightarrow$ Remainder = 128 – 20 + a

$\Rightarrow$ Remainder = 108 + a …(iv)

When P(x) and Q(x) are divided (x – 4) , they leave same remainder.

Comparing equations (iii) and (iv), we have

$64a + 45 = 108 + a$

$\Rightarrow 64a - a = 108 - 45$

$\Rightarrow 63a = 63$

$\Rightarrow a = \frac{63}{63}$

$\Rightarrow a = 1$

Thus, a = 1.

6. Question

The two polynomials $x^3 + 2x^2 - px - 7$ and $x^3 + px^2 - 12x + 6$ are divided by $(x + 1)$ and $(x - 2)$ respectively and if the remainders R_1 and R_2 are obtained and if $2R_1 + R_2 = 6$, then let us calculate the value of p.

Answer

Let the polynomials be:

$A(x) = x^3 + 2x^2 - px - 7$

$B(x) = x^3 + px^2 - 12x + 6$

We will use remainder theorem here.

By Remainder theorem that says, f(x) is a polynomial of degree n (n ≥ 1) and 'a' is any real number. If f(x) is divided by (x – a), then the remainder will be f(a).

When A(x) is divided by (x + 1), it will leave remainder R_1.

Let's also find zero of linear polynomial, (x + 1).

To find zero,

$\Rightarrow x + 1 = 0$

$\Rightarrow x = -1$

The required remainder, $R_1 = A(-1)$.

$\Rightarrow R_1 = (-1)^3 + 2(-1)^2 - p(-1) - 7$

$\Rightarrow R_1 = -1 + 2 + p - 7$

$\Rightarrow R_1 = 1 - 7 + p$

$\Rightarrow R_1 = p - 6$ …(i)

When B(x) is divided by (x – 2), it will leave remainder R_2.

Let's also find zero of linear polynomial, (x – 2).

To find zero,

$\Rightarrow x - 2 = 0$

$\Rightarrow x = 2$

The required remainder, $R_2 = B(2)$

$\Rightarrow R_2 = (2)^3 + p(2)^2 - 12(2) + 6$

$\Rightarrow R_2 = 8 + 4p - 24 + 6$

$\Rightarrow R_2 = 4p + 14 - 24$

$\Rightarrow R_2 = 4p - 10$ ...(ii)

We have, $2R_1 + R_2 = 6$

Substituting values from (i) and (ii), we get

$2(p - 6) + (4p - 10) = 6$

$\Rightarrow 2p - 12 + 4p - 10 = 6$

$\Rightarrow 6p - 22 = 6$

$\Rightarrow 6p = 6 + 22$

$\Rightarrow 6p = 28$

$\Rightarrow p = \frac{28}{6}$

$\Rightarrow p = \frac{14}{3}$

Thus, p = 14/3.

7. Question

If the polynomial $x^4 - 2x^3 + 3x^2 - ax + b$ is divided by $(x - 1)$ and $(x + 1)$ and the remainders are 5 and 19 respectively. But if that polynomial is divided by $x + 2$, then what will be the remainder —Let us calculate.

Answer

We have the polynomial $x^4 - 2x^3 + 3x^2 - ax + b$.

Let it be P(x), such that

$P(x) = x^4 - 2x^3 + 3x^2 - ax + b$ ...(i)

According to the question,

P(x) is divided by (x – 1) and (x + 1), and leaves the remainder 5 and 19 respectively.

We will use remainder theorem here.

By Remainder theorem that says, f(x) is a polynomial of degree n ($n \geq 1$) and 'a' is any real number. If f(x) is divided by (x – a), then the remainder will be f(a).

At first, let's find the zero of the linear polynomial, $(x - 1)$.

To find zero,

$x - 1 = 0$

$\Rightarrow x = 1$

Now, let's find the zero of the linear polynomial, $(x + 1)$.

To find zero,

$x + 1 = 0$

$\Rightarrow x = -1$

From Remainder theorem, we can say

When $P(x)$ is divided by $(x - 1)$, the remainder comes out to be 5.

We can also say that,

The remainder comes out to be $P(1)$.

$\Rightarrow P(1) = 5$

$\Rightarrow (1)^4 - 2(1)^3 + 3(1)^2 - a(1) + b = 5$

$\Rightarrow 1 - 2 + 3 - a + b = 5$

$\Rightarrow b - a + 2 = 5$

$\Rightarrow b - a = 5 - 2$

$\Rightarrow b - a = 3$ …(ii)

And when $P(x)$ is divided by $(x + 1)$, the remainder comes out to be 19.

We can also say that,

The remainder comes out to be $P(-1)$

$\Rightarrow P(-1) = 19$

$\Rightarrow (-1)^4 - 2(-1)^3 + 3(-1)^2 - a(-1) + b = 5$

$\Rightarrow 1 + 2 + 3 + a + b = 5$

$\Rightarrow a + b + 6 = 5$

$\Rightarrow a + b = 5 - 6$

$\Rightarrow a + b = -1$ …(iii)

Solving equations (ii) and (iii), we get

$(b - a) + (a + b) = 3 + (-1)$

$\Rightarrow 2b = 3 - 1$

$\Rightarrow 2b = 2$

$\Rightarrow b = \frac{2}{2}$

$\Rightarrow b = 1$

Putting b = 1 in equation (ii),

$b - a = 3$

$\Rightarrow 1 - a = 3$

$\Rightarrow a = 1 - 3$

$\Rightarrow a = -2$

The values of a = -2 and b = 1.

$\because$ The polynomial = $x^4 - 2x^3 + 3x^2 - ax + b$

$\therefore$ The polynomial = $x^4 - 2x^3 + 3x^2 - (-2)x + 1$

$\Rightarrow$ The polynomial = $x^4 - 2x^3 + 3x^2 + 2x + 1$

So, when polynomial $x^4 - 2x^3 + 3x^2 + 2x + 1$ is divided by $(x + 2)$, the remainder can be calculated by using Remainder theorem.

First, we need to find zero of the linear polynomial, $(x + 2)$.

To find zero,

Put $(x + 2) = 0$

$\Rightarrow x = -2$

So, Required remainder = P(-2)

$\Rightarrow$ Required remainder = $(-2)^4 - 2(-2)^3 + 3(-2)^2 + 2(-2) + 1$

$\Rightarrow$ Required remainder = $16 + 16 + 12 - 4 + 1$

$\Rightarrow$ Required remainder = $32 + 12 - 3$

$\Rightarrow$ Required remainder = $44 - 3$

$\Rightarrow$ Required remainder = 41

Thus, remainder is 41.

8. Question

If $f(x)=\frac{a(x-b)}{a-b}+\frac{b(x-a)}{b-a}$, then let us show that, f(a) + f(b) = f(a + b)

Answer

We have

$$f(x)=\frac{a(x-b)}{a-b}+\frac{b(x-a)}{b-a}$$

Let us simplify it,

$$f(x)=\frac{a(x-b)}{a-b}-\frac{b(x-a)}{a-b}$$

$$\Rightarrow f(x)=\frac{ax-ab-bx+ab}{a-b}$$

$$\Rightarrow f(x)=\frac{(a-b)x}{a-b}$$

⇒ f(x) = x ...(i)

To show, f(a) + f(b) = f(a + b)

Take LHS: f(a) + f(b)

Just replace x by a in equation (i),

f(a) = a

Now, replace x by b in equation (i),

f(b) = b

LHS: f(a) + f(b)

⇒ f(a) + f(b) = a + b ...(ii)

Now, Put (a + b) in equation (i),

f(a + b) = a + b

Replace (a + b) in equation (ii) from f(a + b), we get

f(a) + f(b) = f(a + b)

Thus, shown that f(a) + f(b) = f(a + b).

9. Question

If f(x) = ax + b and f(0) = 3, f(2) = 5, then let us determine the values of a and b.

Answer

We have,

$f(x) = ax + b$ ...(i)

$f(0) = 3$

$f(2) = 5$

Replace x by 0 in equation (i), we get

$f(0) = a(0) + b$

$\Rightarrow 3 = 0 + b$ [$\because f(0) = 3$]

$\Rightarrow 3 = b$

$\Rightarrow b = 3$

Replace x by 2 in equation (i), we get

$f(2) = a(2) + b$

$\Rightarrow 5 = 2a + b$ [$\because f(2) = 5$]

$\Rightarrow 5 = 2a + 3$ [$\because b = 3$]

$\Rightarrow 2a = 5 - 3$

$\Rightarrow 2a = 2$

$\Rightarrow a = 1$

Thus, a = 1 and b = 3.

10. Question

If $f(x) = ax^2 + bx + c$ and $f(0) = 2$, $f(1) = 1$ and $f(4) = 6$, then let us calculate the values of a, b and c.

Answer

We have,

$f(x) = ax^2 + bx + c$ ...(i)

$f(0) = 2$

$f(1) = 1$

$f(4) = 6$

Replace x by 0 in equation (i), we get

$f(0) = a(0)^2 + b(0) + c$

$\Rightarrow 2 = 0 + 0 + c$ [$\because f(0) = 2$]

$\Rightarrow 2 = c$

$\Rightarrow c = 2$ ...(ii)

Now, replace x by 1 in equation (i), we get

$f(1) = a(1)^2 + b(1) + c$

$\Rightarrow 1 = a + b + 2$ [$\because f(1) = 1$ & $c = 2$]

$\Rightarrow a + b = 1 - 2$

$\Rightarrow a + b = -1$ ...(iii)

Finally, replace x by 4 in equation (i), we get

$f(4) = a(4)^2 + b(4) + c$

$\Rightarrow 6 = 16a + 4b + 2$ [$\because f(4) = 6$ & $c = 2$]

$\Rightarrow 16a + 4b = 6 - 2$

$\Rightarrow 16a + 4b = 4$

$\Rightarrow 4(4a + b) = 4$

$\Rightarrow 4a + b = 1$ ...(iv)

Solving equations (iii) & (iv), we get

$$\begin{array}{r} a + b = -1 \\ 4a + b = 1 \\ (-)\ (-)\quad (-) \\ \hline -3a + 0 = -2 \\ \hline \end{array}$$

$\Rightarrow -3a = -2$

$\Rightarrow a = \frac{2}{3}$

Put a = 2/3 in equation (iii), we get

$\frac{2}{3} + b = -1$

$\Rightarrow b = -1 - \frac{2}{3}$

$\Rightarrow b = \frac{-3-2}{3}$

$\Rightarrow b = -\frac{5}{3}$

Thus, a = 2/3, b = -5/3 and c = 2.

11 A. Question

Which of the followings is a polynomial in one variable?

A. $x + \frac{2}{x} + 3$

B. $3\sqrt{x} + \frac{2}{\sqrt{x}} + 5$

C. $\sqrt{2}\, x^2 - \sqrt{3}\, x + 6$

D. $x^{10} + y^5 + 8$

Answer

Let us check for the option (a),

We have $x + \frac{2}{x} + 3$

We can write it as, $x + 2x^{-1} + 3$.

Here, x^{-1} is a polynomial.

⇒ $x + 2x^{-1} + 3$ is not a polynomial.

⇒ $x + \frac{2}{x} + 3$ is not a polynomial.

For option (b),

We have $3\sqrt{x} + \frac{2}{\sqrt{x}} + 5$

We can write it as, $3\sqrt{x} + 2x^{-1/2} + 5$.

Here, $x^{-1/2}$ is not a polynomial.

⇒ $3\sqrt{x} + 2x^{-1/2} + 5$ is not a polynomial.

⇒ $3\sqrt{x} + \frac{2}{\sqrt{x}} + 5$ is not a polynomial.

For option (c),

We have $\sqrt{2}x^2 - \sqrt{3}x + 6$

Term-wise, $\sqrt{2}\, x^2$ is a polynomial

$\sqrt{3}\, x$ is a polynomial

6 is a polynomial

⇒ $\sqrt{2}\,x^2 - \sqrt{3}\,x + 6$ is a polynomial.

For option (d),

We have $x^{10} + y^5 + 8$.

Here, we have two variables, x and y.

Hence, it is clearly not a polynomial of single variable.

Thus, option (d) is correct.

11 B. Question

Which of the following is a polynomial?

A. x – 1

B. $\frac{x-1}{x+1}$

C. $x^2 - \frac{2}{x^2} + 5$

D. $x^2 + \frac{2x^{\frac{3}{2}}}{\sqrt{x^2}} + 6$

Answer

For option (a),

We have x – 1

Look at each term individually, there's no violation of the definition of polynomials.

The exponent in the algebraic expression is non-negative integers, that's why we can say that (x – 1) is a polynomial.

For option (b),

We have $\frac{x-1}{x+1}$

Rearranging the expression, we get

$$\frac{x}{x+1} - \frac{1}{x+1}$$

Even after re-arranging the expression, notice that the variable in both terms is appearing in denominator, which does not satisfy the definition of polynomials as variables cannot appear in denominator in the expression of a polynomial.

For option (c),

We have $x^2 - \frac{2}{x^2} + 5$

Simplifying the expression, we get

$x^2 - 2x^{-2} + 5$

Look at the term $2x^{-2}$.

In polynomials, the exponent in the algebraic expression should be non-negative integers, but in this expression the exponent is negative.

$\Rightarrow x^2 - \frac{2}{x^2} + 5$ is not a polynomial.

For option (d),

We have $x^2 + \frac{2x^{\frac{3}{2}}}{\sqrt{x^2}} + 6$

Simplifying the expression, we get

$$x^2 + \frac{2x^{\frac{3}{2}}}{x} + 6$$

$\Rightarrow x^2 + 2x^{\frac{3}{2}-1} + 6$

$\Rightarrow x^2 + 2x^{\frac{3-2}{2}} + 6$

$\Rightarrow x^2 + 2x^{\frac{1}{2}} + 6$

Observe the rational root in the second term of the expression, but in polynomials, the exponent should be non-negative integers.

$\Rightarrow x^2 + \frac{2x^{\frac{3}{2}}}{\sqrt{x^2}} + 6$ is not a polynomial.

Thus, option (a) is correct.

11 C. Question

Which of the followings is a linear polynomial?

A. $x + x^2$

B. $x + 1$

C. $5x^2 - x + 3$

D. $x = \frac{1}{x}$

Answer

For option (a),

We have $x + x^2$

In this expression, we have 2 terms.

The degree of this expression is 2 (largest exponent of x).

But linear polynomial has a degree of 1, so that it can give exactly one root.

$\Rightarrow x + x^2$ is not a linear polynomial.

For option (b),

We have $x + 1$

In this expression, we have 2 terms.

And the degree of this expression is 1 (largest exponent of x).

This compliments the definition of a polynomial.

$\Rightarrow x + 1$ is a linear polynomial.

For option (c),

We have $5x^2 - x + 3$.

In this expression, we have 3 terms.

And the degree of this expression is 2 (largest exponent of x).

This violates the definition of linear polynomial, as the degree of linear polynomial is always 1.

$\Rightarrow 5x^2 - x + 3$ is not a linear polynomial.

For option (d),

We have $x = \frac{1}{x}$

Let us simplify the expression.

$\Rightarrow x = \frac{1}{x}$

$\Rightarrow x^2 = 1$

$\Rightarrow x^2 - 1 = 0$

This now is an algebraic expression, having 2 terms.

The degree of this expression is 2 (largest exponent of x).

This again doesn't satisfy the criteria of linear polynomial, which says that degree should be 1.

$\Rightarrow x = \frac{1}{x}$ is not a linear polynomial.

Thus, option (b) is correct.

11 D. Question

Which of the followings is a second degree polynomial?

A. $\sqrt{x} - 4$

B. $x^3 + x$

C. $x^3 + 2x + 6$

D. $x^2 + 5x + 6$

Answer

For option (a),

We have $\sqrt{x} - 4$.

Observe the two terms, x has an exponent of 1/2.

But for second degree polynomials, the degree should be 2 (that is, largest exponent of x should be 2)

$\Rightarrow \sqrt{x} - 4$ is not a second degree polynomial.

For option (b),

We have $x^3 + x$.

The degree of this expression is 3 (largest exponent of x is 3).

But for second degree polynomials, the degree should be 2 (that is, largest exponent of x should be 2)

$\Rightarrow x^3 + x$ is not a second degree polynomial.

For option (c),

We have $x^3 + 2x + 6$.

The degree of this expression is also 3 (largest exponent of x is 3).

But for second degree polynomials, the degree should be 2 (that is, largest exponent of x should be 2)

⇒ $x^3 + 2x + 6$ is not a second degree polynomial.

For option (d),

We have $x^2 + 5x + 6$.

The degree of this expression is 2 (largest exponent of x is 2).

This satisfies the criteria of second degree polynomial.

⇒ $x^2 + 5x + 6$ is second degree polynomial.

Thus, option (d) is correct.

11 E. Question

The degree of the polynomial $\sqrt{3}$ is

A. $\frac{1}{2}$

B. 2

C. 1

D. 0

Answer

We have got polynomial, $\sqrt{3}$.

We can re-write it as, $\sqrt{3}\,x^0$ [$\because x^0 = 1$; any variable or number with exponent 0 gives 1 every time]

So, notice the exponent of x. Here, it is 0.

Since, degree of a term is sum of exponents of the variables that appears in it, and thus is a non-negative integer.

⇒ Degree of the polynomial = 0

Thus, option (d) is correct.

12 A. Question

Let us write the zero of the polynomial $p(x) = 2x - 3$.

Answer

A *zero* is root of a *polynomial* function which is a number that, when plugged in for the variable, makes the function equal to *zero*.

To find the zero of a polynomial, just equate the polynomial equals to 0 and find out the value of the variable.

Here, p(x) = 0

⇒ 2x – 3 = 0

⇒ 2x = 3

⇒ $x = \frac{3}{2}$

Thus, zero of the polynomial is 3/2.

12 B. Question

If p(x) = x + 4, let us write the value of p(x) + p(–x).

Answer

We know,

p(x) = x + 4 ...(i)

Now, to find p(-x), just replace x by –x in the above expression. Let us do it.

p(-x) = (-x) + 4

⇒ p(-x) = -x + 4 ...(ii)

Now, add equations (i) and (ii), we get

p(x) + p(-x) = (x + 4) + (-x + 4)

⇒ p(x) + p(-x) = x + 4 – x + 4

⇒ p(x) + p(-x) = 8

Thus, the values of p(x) + p(-x) = 8

12 C. Question

Let us write the remainder, if the polynomial $x^3 + 4x^2 + 4x - 3$ is divided by x.

Answer

If the polynomial $x^3 + 4x^2 + 4x - 3$ is divided by x, we need to find the remainder.

Let $P(x) = x^3 + 4x^2 + 4x - 3$ ...(i)

Next, we need to find the zero of the linear polynomial, x.

To find zero, equate x to 0.

⇒ $x = 0$

So, when P(x) is divided by x, it leaves a remainder P(0).

Replace x by 0 in equation (i), we get

$P(0) = (0)^3 + 4(0)^2 + 4(0) - 3$

⇒ $P(0) = 0 + 0 + 0 - 3$

⇒ $P(0) = -3$

⇒ Required remainder = -3

Thus, -3 is the answer.

12 D. Question

If $(3x - 1)^7 = a_7x^7 + a_6x^6 + a_5x^5 + \ldots\ldots\ldots + a_1x + a_0$, then let us write the value of $a_7 + a_6 + a_5 + \ldots\ldots\ a_0$ (where $a_7, a_6 \ldots\ldots\ldots a_0$ are constants)

Answer

We have been given that,

$(3x - 1)^7 = a_7x^7 + a_6x^6 + a_5x^5 + \ldots + a_1x + a_0$

If we simply put x = 1 in the above equation,

$(3(1) - 1)^7 = a_7(1)^7 + a_6(1)^6 + a_5(1)^5 + \ldots + a_1(1) + a_0$

⇒ $a_7 + a_6 + a_5 + \ldots + a_1 + a_0 = (3 - 1)^7$

⇒ $a_7 + a_6 + a_5 + \ldots + a_1 + a_0 = 2^7$

⇒ $a_7 + a_6 + a_5 + \ldots + a_1 + a_0 = 128$

Thus, value of $a_7 + a_6 + a_5 + \ldots + a_1 + a_0 = 128$.

Let us Work Out 7.4

1 A. Question

Let us calculate and write, which of the following polynomials will have a factor (x + 1)?

$2x^3 + 3x^2 - 1$

Answer

Formula used.

If f(x) is a polynomial with degree n

Then (x – a) is a factor of f(x) if f(a) = 0

1st we find out zero of polynomial x + 1

$x + 1 = 0$

$x = -1$

if f(x) is $2x^3 + 3x^2 - 1$

then putting $x = -1$;

$f(-1) = 2(-1)^3 + 3(-1)^2 - 1$

$= 2 \times (-1) + 3 \times (1) - 1$

$= -2 + 3 - 1$

$= 0$

Conclusion.

$\therefore$ (x + 1) is a factor of $2x^3 + 3x^2 - 1$

1 B. Question

Let us calculate and write, which of the following polynomials will have a factor (x + 1)?

$x^4 + x^3 - x^2 + 4x + 5$

Answer

Formula used.

If f(x) is a polynomial with degree n

Then (x – a) is a factor of f(x) if f(a) = 0

1st we find out zero of polynomial x + 1

$x + 1 = 0$

$x = -1$

if f(x) is $x^4 + x^3 - x^2 + 4x + 5$

then putting $x = -1$;

$f(-1) = (-1)^4 + (-1)^3 - (-1)^2 + 4(-1) + 5$

$= 1 - 1 - 1 - 4 + 5$

= 0

Conclusion.

∴ (x + 1) is a factor of $x^4 + x^3 - x^2 + 4x + 5$

1 C. Question

Let us calculate and write, which of the following polynomials will have a factor (x + 1)?

$7x^3 + x^2 + 7x + 1$

Answer

Formula used.

If f(x) is a polynomial with degree n

Then (x – a) is a factor of f(x) if f(a) = 0

1st we find out zero of polynomial x + 1

x + 1 = 0

x = – 1

if f(x) is $7x^3 + x^2 + 7x + 1$

then putting x = – 1;

$f(-1) = 7(-1)^3 + (-1)^2 + 7(-1) + 1$

= 7 × (– 1) + 1 + 7 × (– 1) + 1

= – 7 + 1 – 7 + 1

= – 12

Conclusion.

∴ (x + 1) is not a factor of $7x^3 + x^2 + 7x + 1$

1 D. Question

Let us calculate and write, which of the following polynomials will have a factor (x + 1)?

$3 + 3x - 5x^3 - 5x^4$

Answer

Formula used.

If f(x) is a polynomial with degree n

Then $(x - a)$ is a factor of $f(x)$ if $f(a) = 0$

1st we find out zero of polynomial $x + 1$

$x + 1 = 0$

$x = -1$

if $f(x)$ is $3 + 3x - 5x^3 - 5x^4$

then putting $x = -1$;

$f(-1) = 3 + 3(-1) - 5(-1)^3 - 5(-1)^4$

$= 3 + 3 \times (-1) - 5 \times (-1) - 5 \times 1$

$= 3 - 3 + 5 - 5$

$= 0$

Conclusion.

$\therefore$ $(x + 1)$ is a factor of $3 + 3x - 5x^3 - 5x^4$

1 E. Question

Let us calculate and write, which of the following polynomials will have a factor $(x + 1)$?

$x^4 + x^2 + x + 1$

Answer

Formula used.

If $f(x)$ is a polynomial with degree n

Then $(x - a)$ is a factor of $f(x)$ if $f(a) = 0$

1st we find out zero of polynomial $x + 1$

$x + 1 = 0$

$x = -1$

if $f(x)$ is $x^4 + x^2 + x + 1$

then putting $x = -1$;

$f(-1) = (-1)^4 + (-1)^2 + (-1) + 1$

$= 1 + 1 - 1 + 1$

$= 2$

Conclusion.

∴ (x + 1) is not a factor of $x^4 + x^2 + x + 1$

1 F. Question

Let us calculate and write, which of the following polynomials will have a factor (x + 1)?

$x^3 + x^2 + x + 1$

Answer

Formula used.

If f(x) is a polynomial with degree n

Then (x – a) is a factor of f(x) if f(a) = 0

1st we find out zero of polynomial x + 1

x + 1 = 0

x = – 1

if f(x) is $x^3 + x^2 + x + 1$

then putting x = – 1;

$f(-1) = (-1)^3 + (-1)^2 + (-1) + 1$

$= -1 + 1 - 1 + 1$

= 0

Conclusion.

∴ (x + 1) is a factor of $x^3 + x^2 + x + 1$

2 A. Question

By using Factor Theorem, let us write whether g(x) is a factor of the following polynomials f(x).

$f(x) = x^4 - x^2$ and g(x) = x + 2

Answer

Formula used.

If f(x) is a polynomial with degree n

Then (x – a) is a factor of f(x) if f(a) = 0

1st we find out zero of polynomial g(x)

$x + 2 = 0$

$x = -2$

if f(x) is $x^4 - x^2$

then putting $x = -2$;

$f(-2) = (-2)^4 + (-2)^2$

$= 16 + 4$

$= 20$

Conclusion.

∴ g(x) is not a factor of f(x)

2 B. Question

By using Factor Theorem, let us write whether g(x) is a factor of the following polynomials f(x).

$f(x) = 2x^3 + 9x^2 - 11x - 30$ and $g(x) = x + 5$

Answer

Formula used.

If f(x) is a polynomial with degree n

Then (x – a) is a factor of f(x) if f(a) = 0

1st we find out zero of polynomial g(x)

$x + 5 = 0$

$x = -5$

if f(x) is $2x^3 + 9x^2 - 11x - 30$

then putting $x = -5$;

$f(-5) = 2(-5)^3 + 9(-5)^2 - 11(-5) - 30$

$= 2 \times (-125) + 9 \times (25) - 11x(-5) - 30$

$= -250 + 225 + 55 - 30$

$= 0$

Conclusion.

∴ g(x) is a factor of f(x)

2 C. Question

By using Factor Theorem, let us write whether g(x) is a factor of the following polynomials f(x).

$f(x) = 2x^3 + 7x^2 - 24x - 45$ and $g(x) = x - 3$

Answer

Formula used.

If f(x) is a polynomial with degree n

Then (x – a) is a factor of f(x) if f(a) = 0

1st we find out zero of polynomial g(x)

$x - 3 = 0$

$x = 3$

if f(x) is $2x^3 + 7x^2 - 24x - 45$

then putting x = 3 ;

$f(3) = 2(3)^3 + 7(3)^2 - 24(3) - 45$

$= 2 \times (27) + 7 \times (9) - 24x(3) - 45$

$= 54 + 63 - 72 - 45$

$= 0$

Conclusion.

∴ g(x) is a factor of f(x)

2 D. Question

By using Factor Theorem, let us write whether g(x) is a factor of the following polynomials f(x).

$f(x) = 3x^3 + x^2 - 20x + 12$ and $g(x) = 3x - 2$

Answer

Formula used.

If f(x) is a polynomial with degree n

Then (x – a) is a factor of f(x) if f(a) = 0

1st we find out zero of polynomial g(x)

$3x - 2 = 0$

$x = \frac{2}{3}$

if f(x) is $3x^3 + x^2 - 20x + 12$

then putting $x = \frac{2}{3}$;

$f(3) = 3(\frac{2}{3})^3 + (\frac{2}{3})^2 - 20(\frac{2}{3}) + 12$

$= \frac{8}{9} + \frac{4}{9} - \frac{40}{3} + 12$

$= \frac{8 + 4 - 120 + 108}{9}$

$= \frac{0}{9} = 0$

Conclusion.

∴ g(x) is a factor of f(x)

3. Question

Let us calculate and write the value of k for which the polynomial $2x^4 + 3x^3 + 2kx^2 + 3x + 6$ is divided by x + 2.

Answer

Formula used.

If f(x) is a polynomial with degree n

Then (x – a) is a factor of f(x) if f(a) = 0

1^{st} we find out zero of polynomial g(x)

$x + 2 = 0$

$x = -2$

if x + 2 is factor of $f(x) = 2x^4 + 3x^3 + 2kx^2 + 3x + 6$

then f(– 2) = 0 ;

$f(-2) = 2(-2)^4 + 3(-2)^3 + 2k(-2)^2 + 3(-2) + 6 = 0$

$= 2 \times 16 - 3 \times 8 + 8 \times k - 6 + 6$

$= 32 - 24 + 8k - 6 + 6$

$8 + 8k = 0$

$8k = -8$

$k = -\frac{8}{8} = -1$

Conclusion.

∴ if (x + 2) is factor of f(x) then k = – 1

4 A. Question

Let us calculate the value of k for which g(x) will be a factor of the following polynomials f(x).

$f(x) = 2x^3 + 9x^2 + x + k$ and $g(x) = x - 1$

Answer

Formula used.

If f(x) is a polynomial with degree n

Then (x – a) is a factor of f(x) if f(a) = 0

1st we find out zero of polynomial g(x)

$x - 1 = 0$

$x = 1$

if x – 1 is factor of $f(x) = 2x^3 + 9x^2 + x + k$

then f(1) = 0 ;

$f(1) = 2(1)^3 + 9(1)^2 + 1 + k = 0$

$= 2 + 9 + 1 + k$

$12 + k = 0$

$k = -12$

Conclusion.

∴ if (x – 1) is factor of f(x) then k = – 12

4 B. Question

Let us calculate the value of k for which g(x) will be a factor of the following polynomials f(x).

$f(x) = kx^2 - 3x + k$ and $g(x) = x - 1$

Answer

Formula used.

If f(x) is a polynomial with degree n

Then (x – a) is a factor of f(x) if f(a) = 0

1st we find out zero of polynomial g(x)

x – 1 = 0

x = 1

if x – 1 is factor of f(x) = $kx^2 - 3x + k$

then f(1) = 0 ;

$f(1) = k(1)^2 - 3(1) + k = 0$

= k – 3 + k

2k – 3 = 0

$k = \frac{3}{2}$

Conclusion.

∴ if (x – 1) is factor of f(x) then $k = -\frac{3}{2}$

4 C. Question

Let us calculate the value of k for which g(x) will be a factor of the following polynomials f(x).

$f(x) = 2x^4 + x^3 - kx^2 - x + 6$ and g(x) = 2x – 3

Answer

Formula used.

If f(x) is a polynomial with degree n

Then (x – a) is a factor of f(x) if f(a) = 0

1st we find out zero of polynomial g(x)

2x – 3 = 0

$x = \frac{3}{2}$

if 2x – 3 is factor of f(x) = $2x^4 + x^3 - kx^2 - x + 6$

then $f(\frac{3}{2}) = 0$;

$f(\frac{3}{2}) = 2(\frac{3}{2})^4 + (\frac{3}{2})^3 - k(\frac{3}{2})^2 - \frac{3}{2} + 6 = 0$

$= \frac{81}{8} + \frac{27}{8} - \frac{9k}{4} - \frac{3}{2} + 6$

$= \frac{81 + 27 - (9 \times 2k) - (3 \times 4) + (6 \times 8)}{8}$

$= \frac{108 - 18k - 12 + 48}{8}$

$= \frac{144 - 18k}{8} = 0$

144 – 18k = 0

$k = \frac{144}{18} = 8$

Conclusion.

∴ if (2x – 3) is factor of f(x) then k = 8

4 D. Question

Let us calculate the value of k for which g(x) will be a factor of the following polynomials f(x).

$f(x) = 2x^3 + kx^2 + 11x + k + 3$ and $g(x) = 2x - 1$

Answer

Formula used.

If f(x) is a polynomial with degree n

Then (x – a) is a factor of f(x) if f(a) = 0

1st we find out zero of polynomial g(x)

2x – 1 = 0

$x = \frac{1}{2}$

if 2x – 1 is factor of $f(x) = 2x^3 + kx^2 + 11x + k + 3$

then $f(\frac{1}{2}) = 0$;

$f(\frac{1}{2}) = 2(\frac{1}{3})^3 + k(\frac{1}{3})^2 + 11(\frac{1}{2}) + k + 3 = 0$

$= \frac{2}{8} + \frac{k}{4} + \frac{11}{2} + k + 3$

$= \frac{2 + (k \times 2) + (11 \times 4) + (k \times 8) + (3 \times 8)}{8}$

$= \frac{2 + 2k + 44 + 8k + 24}{8}$

$= \frac{70 + 10k}{8} = 0$

70 + 10k = 0

$k = \frac{-70}{10} = -7$

Conclusion.

∴ if (2x – 1) is factor of f(x) then k = – 7

5. Question

Let us calculate and write the values of a and b if $x^2 - 4$ is a factor of the polynomial $ax^4 + 2x^3 - 3x^2 + bx - 4$.

Answer

Formula used.

If f(x) is a polynomial with degree n

Then (x – a) is a factor of f(x) if f(a) = 0

$\Rightarrow a^2 - b^2 = (a + b)(a - b)$

1^{st} we find out zero of polynomial g(x)

$x^2 - 4 = 0$

$x^2 - 2^2 = (x + 2)(x - 2) = 0$

x + 2 = 0 and x – 2 = 0

x = – 2 and x = 2

if x + 2 is factor of $f(x) = ax^4 + 2x^3 - 3x^2 + bx - 4$

then f(– 2) = 0 ;

$f(-2) = a(-2)^4 + 2(-2)^3 - 3(-2)^2 + b(-2) - 4$

= 16a – 16 – 12 – 2b – 4

16a – 2b – 32 = 0

16a = 32 + 2b ………eq 1

if x – 2 is factor of $f(x) = ax^4 + 2x^3 - 3x^2 + bx - 4$

then f(2) = 0 ;

$f(2) = a(2)^4 + 2(2)^3 - 3(2)^2 + b(2) - 4$

$= 16a + 16 - 12 + 2b - 4$

$16a + 2b = 0$ ………eq 2

Putting value of 16a from eq 1 into eq 2

$(32 + 2b) + 2b = 0$

$32 + 4b = 0$

$4b = -32$

$b = -\frac{32}{4} = -8$

Putting value of b in eq 1

$16a = 32 + 2 \times (-8)$

$16a = 32 - 16$

$16a = 16$

$a = \frac{16}{16} = 1$

Conclusion.

∴ if $(x^2 - 4)$ is factor of f(x) then $b = -8$ and $a = 1$

6. Question

If (x + 1) and (x + 2) are two factors of the polynomial $x^3 + 3x^2 + 2ax + b$, then let us calculate and write the values of a and b.

Answer

Formula used.

If f(x) is a polynomial with degree n

Then (x – a) is a factor of f(x) if f(a) = 0

1st we find out zero of both polynomial (x + 1)(x + 2)

$x + 1 = 0$ and $x + 2 = 0$

$x = -1$ and $x = -2$

if x + 2 is factor of $f(x) = x^3 + 3x^2 + 2ax + b$

then $f(-2) = 0$;

$f(-2) = (-2)^3 + 3(-2)^2 + 2a(-2) + b$

$= -8 + 12 - 4a + b$

$b - 4a + 4 = 0$

$b = 4a - 4$ ………eq 1

if $x + 1$ is factor of $f(x) = x^3 + 3x^2 + 2ax + b$

then $f(-1) = 0$;

$f(-1) = (-1)^3 + 3(-1)^2 + 2(-1)a + b$

$= -1 + 3 - 2a + b$

$b - 2a + 2 = 0$ ………eq 2

Putting value of b from eq 1 into eq 2

$(4a - 4) - 2a + 2 = 0$

$2a - 2 = 0$

$2a = 2$

$a = \frac{2}{2} = 1$

Putting value of a in eq 1

$b = 4 \times (1) - 4$

$= 0$

Conclusion.

∴ if $(x + 1)$ and $(x + 2)$ is factor of $f(x)$ then $b = 0$ and $a = 1$

7. Question

If the polynomial $ax^3 + bx^2 + x - 6$ is divided by $x - 2$ and remainder is 4, then let us calculate the values of a and b when $x + 2$ is a factor of this polynomial.

Answer

Formula used.

If f(x) is a polynomial with degree n

Then $(x - a)$ is a factor of $f(x)$ if $f(a) = 0$

⇒ Dividend = Divisor × Quotient + Remainder

When $x + 2$ is factor of polynomial

Then

$x + 2 = 0$

$x = -2$

if $x + 2$ is factor of polynomial $f(x) = ax^3 + bx^2 + x - 6$

then $f(-2)$ is 0

$f(-2) = a(-2)^3 + b(-2)^2 + (-2) - 6 = 0$

$-8a + 4b - 2 - 6 = 0$

$4b - 8a = 8$

$4b = 8 + 8a$ ……… eq 1

When $x - 2$ divides polynomial gives remainder 4

Then

Dividend = Divisor × Quotient + Remainder

$ax^3 + bx^2 + x - 6 = (x - 2) \times$ Quotient $+ 4$

$ax^3 + bx^2 + x - 6 - 4 = (x - 2) \times$ Quotient

$ax^3 + bx^2 + x - 10 = (x - 2) \times$ Quotient

$(x - 2) = 0$

$x = 2$

if $(x - 2)$ is factor of polynomial $ax^3 + bx^2 + x - 10$

then $f(2) = 0$

$f(2) = a(2)^3 + b(2)^2 + 2 - 10$

$8a + 4b - 8 = 0$

$8a + 4b = 8$ ………eq 2

Putting value of 4b from eq 1 into eq 2

$8a + (8 + 8a) = 8$

$16a + 8 = 8$

$16a = 8 - 8 = 0$

$a = 0$

Putting value of 'a' in eq 1

$4b = 8 + 8a$

$4b = 8 + 8 \times 0$

$4b = 8$

$b = \frac{8}{4} = 2$

Conclusion.

∴ The value of a and b comes to be 0 and 2 respectively

8. Question

Let us show that if n is any positive integer (even or odd), $x - y$ is a factor of the polynomial $x^n - y^n$.

Answer

Formula used.

If f(x) is a polynomial with degree n

Then $(x - a)$ is a factor of f(x) if $f(a) = 0$

⇒ Dividend = Divisor × Quotient + Remainder

If $(x - y)$ is a factor of $x^n - y^n$

Then

We have to prove

On dividing $x^n - y^n$ by $(x - y)$ Remainder gets 0

When $n = 1$

$(x - y)$ becomes factor of $(x^1 - y^1)$

Hence,

$x - y = 0$

$x = y$

Suppose on dividing $x^n - y^n$ with $x - y$ we get Remainder R

$x^n - y^n = (x - y) \times \text{Quotient} + R$

Putting $x = y$

$y^n - y^n = (y - y) \times \text{Quotient} + R$

$0 = 0 + R$

$R = 0$

∴ For every value n $(x - y)$ is a factor of $x^n - y^n$

Conclusion.

⇒ For every value n $(x – y)$ is a factor of $x^n – y^n$

9. Question

Let us show that if n is any positive odd integer, then $x + y$ is a factor of $x^n + y^n$.

Answer

Let us suppose, if $x^n + y^n$ is divided by $x + y$, the quotient is Q and remainder without x is R.

Dividend = Divisor × Quotient + Remainder

$\therefore x^n + y^n = (x + y) \times Q + R$ ……. [This is an identity]

Since x does not belong to the remainder R, the value of R will not change for any value of x.

So, in the above identity, putting (-y) for x, we get:

$(-y)^n + y^n = (-y + y) \times Q + R$

Now, as n is odd, we get that $(-y)^n = - y^n$

so, we get,

$- y^n + y^n = 0 \times Q + R$

$0 = R$

$\therefore R = 0$

$\therefore$ $(x + y)$ is a factor of the polynomial $x^n + y^n$, when n is an odd positive integer.

10. Question

Let us show that if n be any positive integer (even or odd), the $x – y$ never be a factor of the polynomial $x^n + y^n$.

Answer

Let us suppose, if $x^n + y^n$ is divided by $x - y$, the quotient is Q and remainder without x is R.

Dividend = Divisor × Quotient + Remainder

$\therefore x^n + y^n = (x - y) \times Q + R$ ……. [This is an identity]

Since x does not belong to the remainder R, the value of R will not change for any value of x.

So, in the above identity, putting (y) for x, we get:

$(y)^n + y^n = (y - y) \times Q + R$

$2(y^n) = 0 + R$

$2(y^n) = R$

Since, value of R is not "0", we can say that x – y is not a factor of the polynomial $x^n + y^n$

∴ we can say that (x – y) can never be a factor of the polynomial $x^n + y^n$.

11 A. Question

If the polynomial $x^3 + 6x^2 + 4x + k$ is divisible by (x + 2), then the value of k is

A. –6

B. –7

C. –8

D. –10

Answer

As it is given that, (x + 2) is a factor of the given polynomial, then -2 is the root of the polynomial.

So, as -2 is root of the polynomial and when we put -2 in the polynomial then it will give us "0" as the answer.

Now, we have,

$(-2)^3 + 6(-2)^2 + 4(-2) + k = 0$

$-8 + 6\times4 - 8 + k = 0$

$-16 + 24 + k = 0$

$k + 8 = 0$

$k = -8$

∴ value of k is -8.

Hence, the correct option is (c).

11 B. Question

In the polynomial f(x) if $f\left(-\frac{1}{2}\right) = 0$, then the factor of f(x) will be

A. 2x – 1

B. 2x + 1

C. x – 1

D. x + 1

Answer

In the question, it is given that $f\left(-\frac{1}{2}\right) = 0$, which means that $\frac{-1}{2}$ is a root of the polynomial f(x).

Which means that, [x – ($\frac{-1}{2}$)] =0 is a factor of the polynomial f(x).

[x + ($\frac{1}{2}$)] =0 is a factor.

2x + 1 = 0 is a factor.

Hence the correct option is (b).

11 C. Question

(x – 1) is a factor of the polynomial f(x) but it is not the factor of g(x). So (x – 1) will be a factor of

A. f(x) g(x)

B. –f(x) + g(x)

C. f(x) –g(x)

D. {f(x) + g(x)}g(x)

Answer

We know that (x – 1) is a factor of the polynomial f(x) but it is not the factor of g(x).

Which means that, f(1) = 0 but g(1) ≠ 0.

But for (x - 1) to be a factor of the given options, when we put x=1 in the given functions we should get an answer as “0”.

So, for option (a), let F(x) = f(x) g(x)

To check whether (x – 1) is a factor of the polynomial F(x) we have to put x=1 in F(x) and see whether it gives us the answer as “0” or not.

$F(1) = f(1) \times g(1)$

$F(1) = 0 \times g(1)$

$F(1) = 0$

So, we can say that $F(x) = f(x)\, g(x)$ is the polynomial whose factor is $(x - 1)$.

$\therefore$ $(x - 1)$ will be the factor of $f(x)\, g(x)$.

Hence the correct option is (a).

11 D. Question

$(x + 1)$ is a factor of the polynomial $x^n + 1$ when

A. n is a positive odd integer

B. n is a positive even integer

C. n is a negative integer

D. n is a positive integer

Answer

As we know that $(x + y)$ is a factor of the polynomial $x^n + y^n$, when n is an odd positive integer, so here we can see that $y = 1$, so, we can say that $(x + 1)$ is a factor of the polynomial $x^n + 1$ when n is a positive odd integer.

Hence the correct option is (a).

11 E. Question

If $n^2 - 1$ is a factor of the polynomial $an^4 + bn^3 + cn^2 + dn + e$ then

A. $a + c + e = b + d$

B. $a + b + e = c + d$

C. $a + b + c = d + e$

D. $b + c + d = a + e$

Answer

For the polynomial $an^4 + bn^3 + cn^2 + dn + e$, we have $n^2 - 1$ is a factor.

So, $n^2 - 1 = 0$ is the root of the polynomial.

$(n - 1)(n + 1) = 0$

$n = -1$ or 1 are the roots of the given polynomial.

Now, when $n = 1$, we have:

$a(1)^4 + b(1)^3 + c(1)^2 + d(1) + e = 0$

$a + b + c + d + e = 0$ ……... (1)

Now, when n = -1, we have:

$a(-1)^4 + b(-1)^3 + c(-1)^2 + d(-1) + e = 0$

$a - b + c - d + e = 0$

$a + c + e = d + b$

Hence the correct option is (a).

12 A. Question

Let us calculate and write the value of a for which x + a will be a factor of the polynomial $x^3 + ax^2 - 2x + a - 12$.

Answer

We have the polynomial as $x^3 + ax^2 - 2x + a - 12$.

Given that x + a is the factor of the polynomial, which means that x = -a is the root of the polynomial.

So f(-a) = 0.

$(-a)^3 + a(-a)^2 - 2(-a) + a - 12 = 0$

$-a^3 + a^3 + 2a + a - 12 = 0$

$3a - 12 = 0$

$3a = 12$

$a = 4.$

∴ value of a is 4.

12 B. Question

Let us calculate and write the value of k for which x – 3 will be a factor of the polynomial $k^2x^3 - kx^2 + 3kx - k$.

Answer

We have the polynomial as $k^2x^3 - kx^2 + 3kx - k$.

Given that x - 3 is the factor of the polynomial, which means that x = 3 is the root of the polynomial.

So f(3) = 0.

$k^2(3)^3 - k(3)^2 + 3k(3) - k = 0$

$27k^2 - 9k + 9k - k = 0$

$27k^2 - k = 0$

$k(27k - 1) = 0$

$k = 0$ or $\frac{1}{27}$

$\therefore$ value of k is 0 or $\frac{1}{27}$.

12 C. Question

Let us write the value of $f(x) + f(-x)$ when $f(x) = 2x + 5$.

Answer

We have $f(x) = 2x + 5$

So, $f(-x) = 2(-x) + 5$

Now, $f(x) + f(-x) = 2x + 5 + 2(-x) + 5$

$f(x) + f(-x) = 2x + 5 - 2x + 5$

$f(x) + f(-x) = 10.$

$\therefore$ value of $f(x) + f(-x)$ when $f(x) = 2x + 5$ is 10.

12 D. Question

Both $(x - 2)$ and $\left(x - \frac{1}{2}\right)$ are factors of the polynomial $px^2 + 5x + r$, let us calculate and write the relation between p and r.

Answer

The given polynomial is $px^2 + 5x + r$.

Given that, $(x - 2)$ is the factor of the polynomial, we have $f(2) = 0$.

So, $f(2) = p(2)^2 + 5(2) + r = 0$

$4p + 10 + r = 0$ ……… (1)

Now, given that, $\left(x - \frac{1}{2}\right)$ is the factor of the polynomial, we have $f\left(\frac{1}{2}\right) = 0$

So, $f\left(\frac{1}{2}\right) = p\left(\frac{1}{2}\right)^2 + 5\left(\frac{1}{2}\right) + r = 0$

$$\frac{p}{4} + \frac{5}{2} + r = 0 \quad \ldots \ldots \ldots . (2)$$

From (1), we get r = -4p – 10

So, we can say that the relation between p and r is r = -4p – 10.

∴ the relation between p and r is r = -4p – 10.

12 E. Question

Let us write the zero of the linear polynomial f(x) = 2x + 3.

Answer

The polynomial is f(x) = 2x + 3.

The zero of the polynomial means value of x for which f(x) = 0.

So, f(x) = 2x + 3 = 0

2x + 3 = 0

2x = -3

$$x = \frac{-3}{2}$$

∴ zero of the polynomial f(x) = 2x + 3 is $x = \frac{-3}{2}$.