Polynomials

7) If α and β are zeroes of a quadratic polynomial $4x^2 + 4x + 1$, then form a quadratic polynomial whose zeroes are 2α and 2β.

2014/2016 (3 marks)

α and β are zeroes of polynomial $4x^2 + 4x + 1$.

So, $\alpha+\beta=-\frac{4}{4}=-1$......(1) and $\alpha\beta=\frac{1}{4}$....(2)

Now, $2\alpha+2\beta=2(\alpha+\beta)=2(-1)$ $=-2$ [From (1)]

And $2\alpha\times2\beta=4\alpha\beta=4\times\frac{1}{4}=1$ [From (2)]

So, the required polynomial is $x^2 - (-2)x + 1 = x^2 + 2x + 1$.

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8) If one zero of the polynomial $2x^2 - 5x - (2k+1)$ is twice the other, find both the zeroes of the polynomial and the value of the k.

2015/2016 (3 marks)

Let one zero of $2x^2 - 5x - (2k+1)$ be α.

So the other zero is 2α.

So we have: $\alpha+2\alpha=-\frac{-5}{2}=\frac{5}{2}\Rightarrow3\alpha=\frac{5}{2}\Rightarrow\alpha=\frac{5}{6}$.

So zeroes are $\frac{5}{6}\,and\,\frac{5}{3}$

Now product of zeroes $=\frac{5}{6}\times\frac{5}{3}=\frac{c}{a}$

$$\Rightarrow\frac{25}{18}=\frac{-(2k+1)}{2}\Rightarrow\frac{25}{9}=-(2k+1)$$
$$\Rightarrow25=-18k-9$$
$$\Rightarrow18k=-34\Rightarrow k=\frac{-34}{18}=-\frac{17}{9}$$

Hence the zeroes are $\frac{5}{6}\,and\,\frac{5}{3}$ and the value of k is $-\frac{17}{9}$.

9) What should be added in the polynomial $x^3 - 2x^2 - 3x - 4$ so it is completely divisible by $x^2 - x$? 2015/2016 (3marks)

We divide $x^2 - 2x^2 - 3x - 4$ by $x^2 - x$.

$$x^2 - x \,\big|\, x^3 - 2x^2 - 3x - 4 \,\big|\, x - 1$$

$$x^3 - x^2$$

$$- \quad +$$

$$-x^2 - 3x - 4$$

$$-x^2 + x$$

$$+ \quad -$$

$$-4x - 4$$

Hence remainder is $-4x-4$ so $-(-4x-4)$ must be added for the polynomial becoming divisible by $x^2 - x$.

10) Find all the zeroes of the polynomial $2x^4 - 9x^3 + 5x^2 + 3x - 1$, if two of its zeroes are $2+\sqrt{3} and 2-\sqrt{3}$

2014/2015 (4 marks)

$2+\sqrt{3}$ and $2-\sqrt{3}$ are zeroes of the polynomial, i.e, $x^2 - 4x + 4 - 3$ or $x^2 - 4x + 1$ is a factor of the given polynomial. Now, we have:

$$x^2 - 4x + 1 \,\big|\, 2x^4 - 9x^3 + 5x^2 + 3x - 1 \,\big|\, 2x^2 - x - 1$$

$$2x^4 - 8x^3 + 2x^2$$

$$- \quad + \quad -$$

$$-x^3 + 3x^2 + 3x - 1$$

$$-x^3 + 4x^2 - x$$

$$+ \quad - \quad +$$

$$-x^2 + 4x - 1$$

$$-x^2 + 4x - 1$$

$$+ \quad - \quad +$$

$$0$$

Hence , we have:

$$2x^4 - 9x^3 + 5x^2 + 3x - 1 = (x^2 - 4x + 1)(2x^2 - x - 1)$$

Now, $2x^2 - x - 1 = 2x^2 - 2x + x - 1$

$= 2x(x-1) + 1(x-1)$

$= (x-1)(2x+1)$

Now, $x - 1 = 0 \Rightarrow x = 1$ and $2x + 1 = 0 \Rightarrow x = -\frac{1}{2}$.

Thus all the zeroes of the given polynomial are

$$2+\sqrt{3}, 2-\sqrt{3}, 1 \text{ and } -\frac{1}{2}.$$

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11) If α and β are zeroes of the polynomial $2x^2 - 7x + 5$, then find the value of $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$.

2015/2016(3 marks)

α and β are zeroes of $2x^2-7x+5$.

So, $\alpha + \beta = \frac{-(-7)}{2} = \frac{7}{2}$ and $\alpha\beta = \frac{5}{2}$

$$\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3 + \beta^3}{\alpha\beta} = \frac{(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)}{\alpha\beta}$$

Now, $= \frac{\left(\frac{7}{2}\right)^3 - 3 \times \frac{5}{2} \times \frac{7}{2}}{\frac{5}{2}} = \frac{\frac{343}{8} - \frac{105}{4}}{\frac{5}{2}}$

$= \frac{(343 - 210)}{8} \times \frac{2}{5} = \frac{133}{20}$
