CHAPTER 14

QUADRATIC EQUATIONS

QUADRATIC POLYNOMIALS

An expression in the form of $ax^2 + bx + c$, where a, b, c are real numbers but $a \neq 0$, is called a quadratic polynomial. For examples $2x^2 - 5x + 3$, $-x^2 + 2x$, $3x^2 - 7$, $\sqrt{2}x^2 + 7x + 2$, etc.

QUADRATIC EQUATIONS

A quadratic expression when equated to zero is called a quadratic equation. Hence an equation in the form of $ax^2 + bx + c = 0$, where a, b, c are real numbers and $a \neq 0$, is called a quadratic equation. For examples,

$$2x^2 - 5x + 3 = 0, \ -x^2 + 2x = 0,$$

$$3x^2 - 7 = 0 \text{ and } \sqrt{2}x^2 + 7x + 2 = 0, \text{etc.}$$

Example 1: Which of the following is not a quadratic equation?

(a) $x^2 - 2x + 2(3 - x) = 0$
(b) $x(x + 1) + 1 = (x - 2)(x - 5)$
(c) $(2x - 1)(x - 3) = (x + 5)(x - 1)$
(d) $x^3 - 4x^2 - x + 1 = (x - 2)^3$

Solution: (b) Hint: $x(x + 1) + 1 = (x - 2)(x - 5)$

$\Rightarrow x^2 + x + 1 = x^2 - 7x + 10$

$\Rightarrow 8x - 9 = 0$, which is not a quadratic equation.

DISCRIMINANT (D)

For the quadratic equation $ax^2 + bx + c = 0$,

$$D = b^2 - 4ac$$

Here, D is the symbol of discriminant.

Roots or Solution of a Quadratic Equation

(i) If $D > 0$, then the quadratic equation $ax^2 + bx + c = 0$ has two distinct roots given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} \text{ and } \beta = \frac{-b - \sqrt{D}}{2a}$$

Here α and β are symbols of roots of the quadratic equation.

(ii) If $D = 0$, then the quadratic equation $ax^2 + bx + c = 0$ has two equal roots given by

$$\alpha = \beta = -\frac{b}{2a}$$

(iii) If $D < 0$, then the quadratic equation $ax^2 + bx + c = 0$ has two roots in the form of $a + ib$ and $a - ib$, where a and b are real numbers and $i = \sqrt{-1}$. The roots in the form of $a + ib$ and $a - ib$ are known as imaginary roots. Imaginary roots $a + ib$ and $a - ib$ are also known as complex conjugate roots.

Note that imaginary roots means roots are not real numbers.

Note that if

$$ax^2 + bx + c = a(x - \alpha)(x - \beta),$$

Then α and β satisfy the equation $ax^2 + bx + c = 0$ and hence α and β are the roots of

$$ax^2 + bx + c = 0.$$

Example 2: If $ax^2 + bx + c = 0$ has equal roots, then $c =$

Solution: $ax^2 + bx + c = 0$ has equal roots if disc. $b^2 - 4ac = 0$

$\Rightarrow \quad b^2 = 4ac$

$\Rightarrow \quad c = \dfrac{b^2}{4a}$

Example 3: Find the condition that the quadratic equations $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ may have a common root.

Solution: Let α be a common root of the given equations.

Then $\alpha^2 + a\alpha + b = 0$ and $\alpha^2 + b\alpha + a = 0$

By the method of cross-multiplication, we get

$$\frac{\alpha^2}{a^2 - b^2} = \frac{\alpha}{b - a} = \frac{1}{b - a}$$

This gives $\alpha^2 = \dfrac{a^2 - b^2}{b - a} = -(a + b)$ and $\alpha = 1$

$\Rightarrow (1)^2 = -(a + b) \Rightarrow 1 = -a - b$

$\Rightarrow a + b + 1 = 0$ is the required condition.

Example 4: Solve $\sqrt{4x^2 + 4x + 1} < 3 - x$

Solution: $\sqrt{4x^2 + 4x + 1} < 3 - x \Rightarrow \sqrt{(2x + 1)^2} < 3 - x$

$\Rightarrow \quad \pm(2x + 1) < 3 - x$

$\Rightarrow \quad 2x + 1 < 3 - x$ or $-(2x + 1) < 3 - x$

$\Rightarrow \quad 3x < 2$ or $2x + 1 > x - 3$

$\Rightarrow \quad x < \dfrac{2}{3}$ or $x > -4$

Hence, $-4 < x < \dfrac{2}{3}$

Example 5: Find the solution set of the equation

$$x + 5 - \frac{8}{x + 5} = 7$$

Solution: $x + 5 - \dfrac{8}{x + 5} = 7$

Multiply both sides by $(x + 5)$, we get,

$(x + 5)^2 - 8 = 7(x + 5)$

i.e., $(x + 5)^2 - 7(x + 5) - 8 = 0$

Put $u = x + 5$

The equation reduces to $u^2 - 7u - 8 = 0$

i.e., $(u - 8)(u + 1) = 0$

$\therefore$ $u = 8$ or $u = -1$

$u = x + 5$

i.e., $x + 5 = 8 \Rightarrow x = 8 - 5 = 3$

$x + 5 = -1 \Rightarrow x = -1 - 5 = -6$

$\therefore$ roots are $x = 3$ and $x = -6$.

The solution set = $\{-6, 3\}$.

Example 6: The real roots of the equation $x^{2/3} + x^{1/3} - 2 = 0$ are

(a) 1, 8 **(b) –1, – 8**

(c) –1, 8 **(d) 1, – 8**

Solution: **(d)** The given equation is $x^{2/3} + x^{1/3} - 2 = 0$

Put $x^{1/3} = y$, then $y^2 + y - 2 = 0$

$\Rightarrow$ $(y - 1)(y + 2) = 0$

$\Rightarrow$ $y = 1$ or $y = -2$

$\Rightarrow$ $x^{1/3} = 1$ or $x^{1/3} = -2$

$\therefore$ $x = (1)^3$ or $x = (-2)^3 = -8$

Hence, the real roots of the given equations are 1, –8.

Example 7: If $x^2 + 4x + k = 0$ has real roots, then

(a) $k \geq 4$ **(b) $k \leq 4$**

(c) $k \leq 0$ **(d) $k \geq 0$**

Solution: **(b)** Since $x^2 + 4x + k = 0$ has real roots.

$\therefore$ Disc. $(4)^2 - 4k \geq 0$

$\Rightarrow$ $16 - 4k \geq 0$

$\Rightarrow$ $4k \leq 16$

$\Rightarrow$ $k \leq 4$

PROPERTIES OF QUADRATIC EQUATIONS AND THEIR ROOTS

(i) If D is a perfect square then roots are rational otherwise irrational.

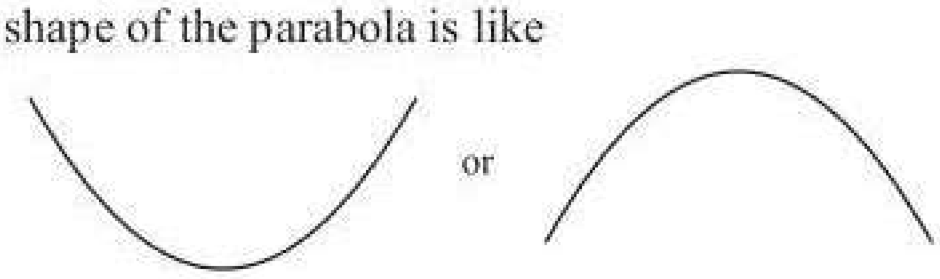
(ii) If $p + \sqrt{q}$ is one root of a quadratic equation, then their conjugate $p - \sqrt{q}$ must be the other root and vice-versa, where p is rational and $\sqrt{q}$ is a surd.

(iii) If a quadratic equation in x has more than two roots, then it is an identity in x.

GRAPH OF A QUADRATIC EXPRESSION

Graph of $y = ax^2 + bx + c$; where a, b, c are real numbers but $a \neq 0$ is always a parabola.

The shape of the parabola is like



Vertically upward open parabola — Vertically downward open parabola

If $a > 0$, then parabola is vertically upward open and if $a < 0$, then parabola is vertically downward open.

Axis of a parabola is a vertical line which divides it in two halves.

A point on the parabola where the graph turn down to up or up to down is called vertex of the parabola. Coordinate of the vertex of the parabola is always $\left(-\frac{b}{2a}, \frac{4ac - b^2}{4a}\right)$.

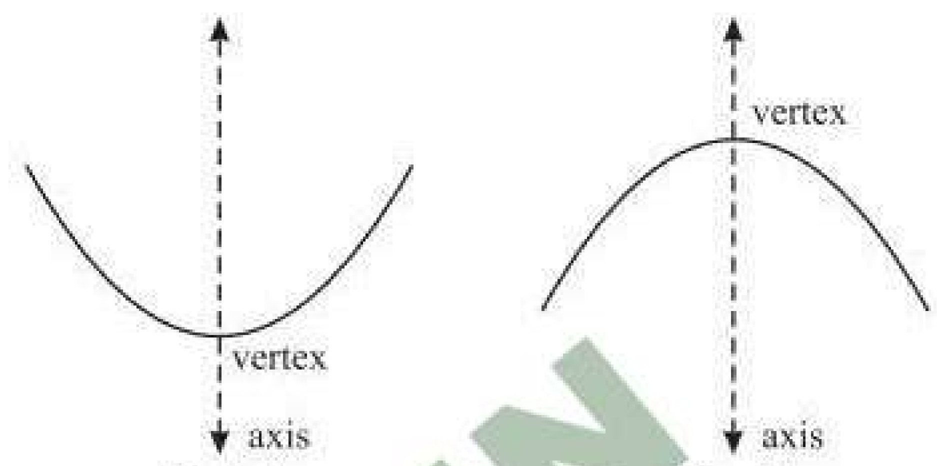


Fig. (a) Fig. (b)

Vertex is either the lowest point on the parabola as in the fig (a) or the highest point on the parabola as in the fig (b).

GEOMETRICAL MEANING OF ROOTS OR SOLUTIONS OF A QUADRATIC EQUATION

x-coordinate of the points where the graph of the quadratic expression $y = ax^2 + bx + c$ intersects or touches the x-axis are called roots of the quadratic equation $ax^2 + bx + c = 0$. If the parabola intersects the x-axis at two distinct points, then there are two different real roots of the quadratic equation.

If the parabola only touches the x-axis at a point, then the quadratic equation has two real equal roots.

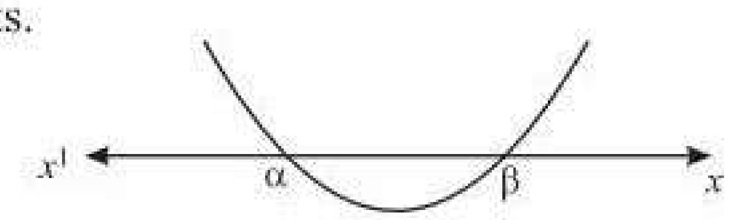
If the parabola does not touch or intersect the x-axis then there are two different imaginary roots. The imaginary roots means roots are not real numbers.

SIGN OF A QUADRATIC EXPRESSION

Let $y = ax^2 + bx + c$, where $ax^2 + bx + c$ is a quadratic expression.

Case-I: If $a > 0$ and

(i) $D > 0$, the graph of $y = ax^2 + bx + c$ is a vertically upward open parabola which intersects the x-axis at two different points.



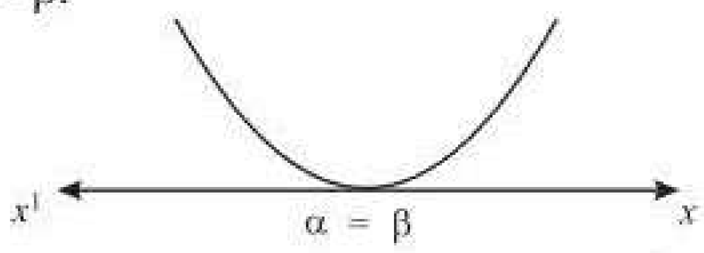
In the figure, α and β are the values of the x-coordinates of the points where the parabola intersects the x-axis. Hence α and β are two roots of the quadratic equation $ax^2 + bx + c = 0$, such that $\alpha < \beta$.

It is clear from the graph,

(a) $ax^2 + bx + c$ will be positive for all real values of x which are less than α or greater than β i.e., $ax^2 + bx + c > 0$ for $x \in (-\infty, \alpha) \cup (\beta, \infty)$

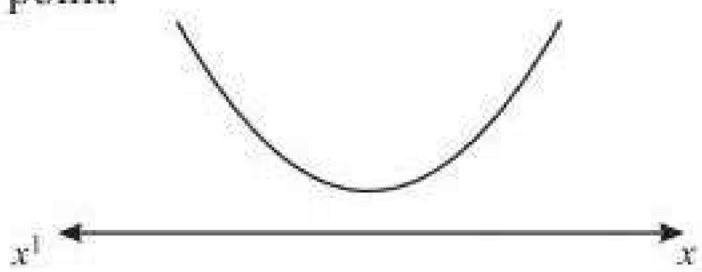
(b) $ax^2 + bx + c$ will be negative for all real values of x which lie between α and β i.e., $ax^2 + bx + c < 0$ for $x \in (\alpha, \beta)$

(ii) $D = 0$, the graph of $y = ax^2 + bx + c$ is vertically upward open parabola which touches the x-axis at only one point. Hence both roots α and β of $ax^2 + bx + c = 0$ are the same i.e. $\alpha = \beta$.



It is clear from the graph, the value of $ax^2 + bx + c$ will be positive for all real values of x except $x = \alpha$ or β.

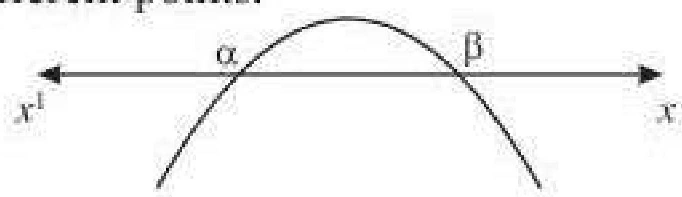
(iii) $D < 0$, then graph of $y = ax^2 + bx + c$ is vertically upward open parabola, which does not intersect or touch the x-axis at any point.



It is clear from graph, the value of $ax^2 + bx + c$ will be positive for all values of x i.e., $ax^2 + bx + c > 0$ for $x \in (-\infty, \infty)$.

Case-II: If $a < 0$ and

(i) $D > 0$, then graph of $y = ax^2 + bx + c$ is the vertically downward open parabola, which intersects the x-axis at two different points.

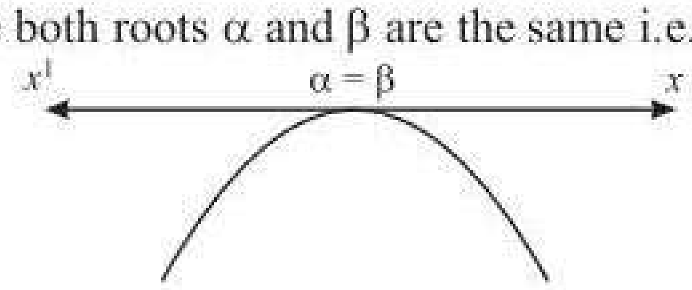


In the figure α and β are the value of the x-coordinates of the points where the graph intersects the x-axis. Hence, α and β are two roots of the quadratic equation $ax^2 + bx + c = 0$, such that $\alpha < \beta$.

(a) $ax^2 + bx + c$ will be positive for all real values of x which are greater than α but less than β i.e., $ax^2 + bx + c > 0$ for $x \in (\alpha, \beta)$

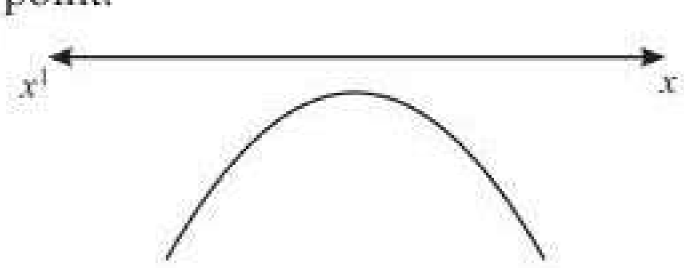
(b) $ax^2 + bx + c$ will be negative for all real values of x which i.e. $ax^2 + bx + c < 0$ for $x \in (-\infty, \alpha) \cup (\beta, \infty)$

(ii) $D = 0$, the graph of $y = ax^2 + bx + c$ is vertically downward open parabola which touches the x-axis at only one point. Hence both roots α and β are the same i.e. $\alpha = \beta$.



It is clear from the graph, the value of $ax^2 + bx + c$ will be negative for all real values of x except $x = \alpha$ or β.

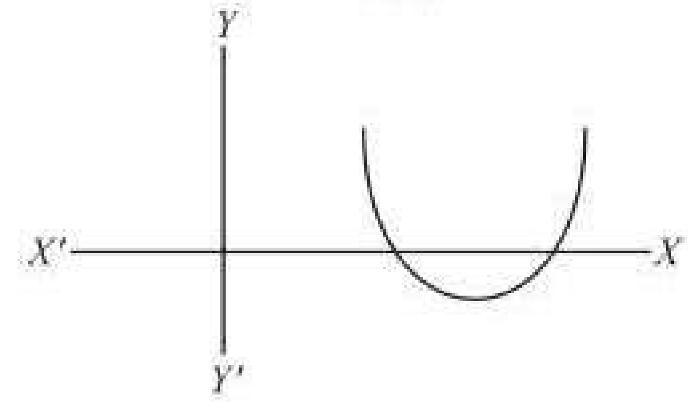
(iii) $D < 0$, then graph of $y = ax^2 + bx + c$ is vertically downward open parabola which does not intersect or touch the x-axis at any point.



It is clear from the graph, the value of $ax^2 + bx + c$ will be negative for all real values of x i.e., $ax^2 + bx + c < 0$ for $x \in (-\infty, \infty)$.

Example 8: For the below figure of $ax^2 + bx + c = 0$

(a) $a < 0$ **(b) $b > 0$**
(c) $D > 0$ **(d) None of these**



***Solution*: (c)**

SUM AND PRODUCT OF ROOTS

If α and β are the roots of a quadratic equation $ax^2 + bx + c = 0$, Then,

$$\text{Sum of roots}, \quad \alpha + \beta = -\frac{b}{a} = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$$

$$\text{Product of roots}, \quad \alpha\beta = \frac{c}{a} = \frac{\text{constant term}}{\text{coefficient of } x^2}$$

Example 9: Find the sum and product of roots of $-2x^2 + 3x - 5 = 0$.

Solution: Sum of roots $= -\frac{b}{a} = -\frac{3}{-2} = \frac{3}{2}$

Product of roots $= \frac{c}{a} = \frac{-5}{-2} = \frac{5}{2}$

FORMATION OF AN EQUATION WITH GIVEN ROOTS

If α and β are the roots of a quadratic equation, then the quadratic equation will be

$$x^2 - (\alpha + \beta)\,x + \alpha.\beta = 0$$

i.e., x^2 – (Sum of the roots) x + Product of the roots = 0

Example 10: If α and β are the roots of the equation $3x^2 - x + 4 = 0$, then find the quadratic equation whose roots are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

Solution: $\alpha + \beta = -\frac{-1}{3} = \frac{1}{3}, \alpha.\beta = \frac{4}{3}$

Now, $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta}$

$$= \frac{\frac{1}{3}}{\frac{4}{3}} = \frac{1}{4}$$

$$\frac{1}{\alpha}\cdot\frac{1}{\beta} = \frac{1}{\alpha.\beta} = \frac{1}{\frac{4}{3}} = \frac{3}{4}$$

Hence required quadratic equation,

$$x^2 - \frac{1}{4}x + \frac{3}{4} = 0$$

$$\Rightarrow \quad 4x^2 - x + 3 = 0$$

Example 11: If α, β are the roots of $x^2 + ax + b = 0$, find the equation for which $\alpha^2 + \beta^2$ and $\alpha^{-2} + \beta^{-2}$ are the roots.

Solution: $\alpha + \beta = -\alpha, \alpha\beta = b$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = a^2 - 2b$

$$\alpha^{-2} + \beta^{-2} = \frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{\alpha^2\beta^2} = \frac{a^2 - 2b}{b^2}$$

Required equation,

$$x^2 - x\left\{\left(a^2 - 2b\right) + \frac{a^2 - 2b}{b^2}\right\} + \left(a^2 - 2b\right)\frac{\left(a^2 - 2b\right)}{b^2} = 0$$

$$\Rightarrow x^2 - x + \left\{\frac{b^2\left(a^2 - 2b\right) + a^2 - 2b}{b^2}\right\} + \frac{\left(a^2 - 2b\right)\left(a^2 - 2b\right)}{b^2} = 0$$

$$\Rightarrow b^2x^2 - x\{b^2(a^2 - 2b) + a^2 - 2b\} + (a^2 - 2b)^2 = 0$$

Example 12: Form the quadratic equations for the given roots.

$$\frac{3 + \sqrt{5}}{4}, \frac{3 - \sqrt{5}}{4}$$

Solution: Here, $S = \dfrac{3 + \sqrt{5}}{4} + \dfrac{3 - \sqrt{5}}{4} = \dfrac{6}{4} = \dfrac{3}{2}$

and $\quad P = \left(\dfrac{3 + \sqrt{5}}{4}\right)\left(\dfrac{3 - \sqrt{5}}{4}\right) = \dfrac{9 - 5}{16} = \dfrac{1}{4}$

$\therefore$ The required equation is $x^2 - Sx + P = 0$

i.e., $\quad x^2 - \dfrac{3}{2}x + \dfrac{1}{4} = 0 \Rightarrow 4x^2 - 6x + 1 = 0$

GREATEST AND LEAST VALUE OF A QUADRATIC EXPRESSION

(i) If $a > 0$, then least value of the quadratic expression

$ax^2 + bx + c$ is $-\dfrac{D}{4a} = \dfrac{4ac - b^2}{4a}$ at $x = -\dfrac{b}{2a}$

Note that there is no greatest value of the quadratic expression $ax^2 + bx + c$ if $a > 0$.

(ii) If $a < 0$, then the greatest value of the quadratic expression

$ax^2 + bx + c$ is $-\dfrac{D}{4a} = \dfrac{4ac - b^2}{4a}$ at $x = -\dfrac{b}{2a}$

Note that there is no least value of the quadratic expression $ax^2 + bx + c$.

CUBIC EQUATIONS

An equation in the form of $ax^3 + bx^2 + cx + d = 0$, where a, b, c, d are real numbers but $a \neq 0$, is called a cubic equation. For example,

$2x^3 - 4x^2 + 3x + 5 = 0, -x^3 - 4x + 7 = 0, 5x^3 = 0,$

$x^3 - 5 = 0, x^3 + 3x^2 = 0$, etc.

Any cubic equation has three roots. If α, β and γ are three roots of a cubic equation $ax^3 + bx^2 + cx + d = 0$, then

(i) $\alpha + \beta + \gamma = -\dfrac{b}{a} = -\dfrac{\text{coefficient of } x^2}{\text{coefficient of } x^3}$

(ii) $\alpha.\beta + \beta.\gamma + \gamma.\alpha = \dfrac{c}{a} = \dfrac{\text{coefficient of } x}{\text{coefficient of } x^3}$

(iii) $\alpha.\beta.\gamma = -\dfrac{d}{a} = -\dfrac{\text{constant term}}{\text{coefficient of } x^3}$

Example 13: If α, β, γ are the roots of the equation $2x^3 - 3x^2 + 6x + 1 = 0$, then $\alpha^2 + \beta^2 + \gamma^2$ is equal to

(a) –15/4 **(b) 15/4**

(c) 9/4 **(d) 4**

Solution: **(a)** Given equation $2x^3 - 3x^2 + 6x + 1 = 0$,

$$\alpha + \beta + \gamma = \frac{3}{2}, \ \alpha\beta\gamma = \frac{-1}{2}, \ \sum\alpha\beta = 3$$

$$(\alpha^2 + \beta^2 + \gamma^2) = (\alpha + \beta + \gamma)^2 - 2(\sum\alpha\beta)$$

$$= \left(\frac{3}{2}\right)^2 - 2.3 = \frac{9}{4} - 6 = \frac{-15}{4}$$

Remember

(i) Value of $\sqrt{P + \sqrt{P + \sqrt{P + ...\infty}}} = \dfrac{\sqrt{4P + 1} + 1}{2}$

(ii) Value of $\sqrt{P - \sqrt{P - \sqrt{P - ....\infty}}} = \dfrac{\sqrt{4P + 1} - 1}{2}$

(iii) Value of $\sqrt{P.\sqrt{P.\sqrt{P. ....\infty}}} = P$

(iv) Value of $\sqrt{P\sqrt{P\sqrt{P\sqrt{P\sqrt{P}}}}} = P^{\left(2^n - 1\right) \div 2^n}$

[Where n = no. of times P repeated].

Note: If factors of P are n & (n + 1) type then value of $\sqrt{P + \sqrt{P + \sqrt{P + ...\infty}}} = (n + 1)$ and $\sqrt{P - \sqrt{P - \sqrt{P - ....\infty}}} = n.$

EXERCISE

1. Solve $x - \frac{1}{x} = 1\frac{1}{2}$
   (a) $-\frac{1}{2}, 2$ (b) $\frac{1}{2}, 2$
   (c) $\frac{1}{2}, \frac{2}{3}$ (d) None of these
2. Father's age is 4 less than five times the age of his son and the product of their ages is 288. Find the father's age.
   (a) 40 years (b) 36 years
   (c) 26 years (d) 42 years
3. The sum of a rational number and its reciprocal is $\frac{13}{6}$, find the number.
   (a) $\frac{2}{3}$ or $\frac{3}{2}$ (b) $\frac{3}{4}$ or $\frac{4}{3}$
   (c) $\frac{2}{5}$ or $\frac{5}{2}$ (d) None of these
4. Minimum value of $x^2 + \frac{1}{x^2+1} - 3$ is
   (a) 0 (b) – 1
   (c) – 3 (d) – 2
5. If the roots, x_1 and x_2, of the quadratic equation $x^2 - 2x + c = 0$ also satisfy the equation $7x_2 - 4x_1 = 47$, then which of the following is true?
   (a) $c = -15$ (b) $x_1 = -5, x_2 = 3$
   (c) $x_1 = 4.5, x_2 = -2.5$ (d) None of these
6. For what value of k, are the roots of the quadratic equation $(k+1)x^2 - 2(k-1)x + 1 = 0$ real and equal?
   (a) $k = 0$ only (b) $k = -3$ only
   (c) $k = 0$ or $k = 3$ (d) $k = 0$ or $k = -3$
7. If α, β are the roots of the equation $2x^2 - 3x - 6 = 0$, find the equation whose roots are $\alpha^2 + 2$ and $\beta^2 + 2$.
   (a) $4x^2 + 49x + 118 = 0$ (b) $4x^2 - 49x + 118 = 0$
   (c) $4x^2 - 49x - 118 = 0$ (d) $4x^2 + 49x - 118 = 0$
8. If α and β are the roots of the equation $x^2 - 2x + 4 = 0$, then what is the value of $\alpha^3 + \beta^3$?
   (a) 16 (b) – 16
   (c) 8 (d) – 8
9. If r and s are roots of $x^2 + px + q = 0$, then what is the value of $\frac{1}{r^2} + \frac{1}{s^2}$?
   (a) $p^2 - 4q$ (b) $\frac{p^2 - 4q}{2}$
   (c) $\frac{p^2 - 4q}{q^2}$ (d) $\frac{p^2 - 2q}{q^2}$
10. Sum of the areas of two squares is 468 m^2. If the difference of their perimeters is 24 m, find the sides of the two squares.
    (a) 9m, 6m (b) 18m, 12m
    (c) 18m, 6m (d) 9m, 12m
11. If the roots of $x^2 - kx + 1 = 0$ are non-real, then
    (a) $-3 < k < 3$ (b) $-2 < k < 2$
    (c) $k > 2$ (d) $k < -2$
12. The sum of two numbers p and q is 18 and the sum of their reciprocals is $\frac{1}{4}$. Then the numbers are
    (a) 10, 8 (b) 12, 6
    (c) 9, 9 (d) 14, 4
13. Two numbers are such that the square of greater number is 504 less than 8 times the square of the other. If the numbers are in the ratio 3 : 4. Find the number.
    (a) 15 and 20 (b) 6 and 8
    (c) 12 and 16 (d) 9 and 12
14. The equation $x + \sqrt{x-2} = 4$ has
    (a) two real roots and one imaginary root
    (b) one real and one imaginary root
    (c) two imaginary roots
    (d) one real root
15. The equation $\sqrt{x+10} - \frac{6}{\sqrt{x+10}} = 5$ has
    (a) an extraneous root between –5 and –1
    (b) an extraneous root between –10 and –6
    (c) two extraneous roots
    (d) a real root between 20 and 25
    [An extraneous root means a root which does not satisfy the equation.]
16. If $\log_{10}(x^2 - 3x + 6) = 1$, then the value of x is
    (a) 10 or 2 (b) 4 or –2
    (c) 4 only (d) 4 or –1
17. Two numbers whose sum is 6 and the absolute value of whose difference is 8 are the roots of the equation
    (a) $x^2 - 6x + 7 = 0$ (b) $x^2 - 6x - 7 = 0$
    (c) $x^2 + 6x - 8 = 0$ (d) $x^2 - 6x + 8 = 0$

18. The roots of the equation $x^2 + 2\sqrt{3}x + 3 = 0$ are
(a) real and equal (b) rational and equal
(c) rational and unequal (d) imaginary

19. The roots of the equation $ax^2 + bx + c = 0$ will be reciprocal if
(a) $a = b$ (b) $a = bc$
(c) $c = a$ (d) $b = c$

20. The discriminant of $ax^2 - 2\sqrt{2}x + c = 0$ with a, c are real constants is zero. The roots must be
(a) equal and integral (b) rational and equal
(c) real and equal (d) imaginary

21. If the product of roots of the equation $x^2 - 3(2a+4)x + a^2 + 18a + 81 = 0$ is unity, then a can take the values as
(a) 3, – 6 (b) 10, – 8
(c) – 10, – 8 (d) – 10, – 6

22. If the roots of the equation $(a^2 + b^2)x^2 - 2(ac + bd)x + (c^2 + d^2) = 0$ are equal, then which of the following is true?
(a) $ab = cd$ (b) $ad = bc$
(c) $ad = \sqrt{bc}$ (d) $ab = \sqrt{cd}$

23. If α and β are the roots of the quadratic equation $ax^2 + bx + c = 0$, then the value of $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$ is
(a) $\frac{3bc - a^3}{b^2c}$ (b) $\frac{3abc - b^3}{a^2c}$
(c) $\frac{3abc - b^2}{a^3c}$ (d) $\frac{ab - b^2c}{2b^2c}$

24. If a, b are the two roots of a quadratic equation such that $a + b = 24$ and $a - b = 8$, then the quadratic equation having a and b as its roots is
(a) $x^2 + 2x + 8 = 0$ (b) $x^2 - 4x + 8 = 0$
(c) $x^2 - 24x + 128 = 0$ (d) $2x^2 + 8x + 9 = 0$

25. If roots of an equation $ax^2 + bx + c = 0$ are positive, then which one of the following is correct?
(a) Signs of a and c should be like
(b) Signs of b and c should be like
(c) Signs of a and b should be like
(d) None of the above

26. If α and β are the roots of the equation $x^2 + 6x + 1 = 0$, then what is $|\alpha - \beta|$ equal to?
(a) 6 (b) $3\sqrt{2}$
(c) $4\sqrt{2}$ (d) 12

27. A natural number when increased by 12, equals 160 times its reciprocal. Find the number.
(a) 3 (b) 5
(c) 8 (d) 16

28. Which is not true?
(a) Every quadratic polynomial can have at most two zeros.
(b) Some quadratic polynomials do not have any zero. [*i.e.* real zero]
(c) Some quadratic polynomials may have only one zero. [*i.e.* one real zero]
(d) Every quadratic polynomial which has two zeros.

29. If $6 \le x \le 8$, then which one of the following is correct?
(a) $(x-6)(x-8) \ge 0$ (b) $(x-6)(x-8) > 0$
(c) $(x-6)(x-8) \le 0$ (d) $(x-6)(x-8) < 0$

30. What are the roots of the quadratic equation
$$a^2b^2x^2 - (a^2 + b^2)x + 1 = 0?$$
(a) $\frac{1}{a^2}, \frac{1}{b^2}$ (b) $-\frac{1}{a^2}, -\frac{1}{b^2}$
(c) $\frac{1}{a^2}, -\frac{1}{b^2}$ (d) $-\frac{1}{a^2}, \frac{1}{b^2}$

31. Two students A and B solve an equation of the form $x^2 + px + q = 0$. A starts with a wrong value of p and obtains the roots as 2 and 6. B starts with a wrong value of q and gets the roots as 2 and –9. What are the correct roots of the equation?
(a) 3 and –4 (b) – 3 and –4
(c) –3 and 4 (d) 3 and 4

32. If one of the roots of quadratic equation $7x^2 - 50x + k = 0$ is 7, then what is the value of k?
(a) 7 (b) 1
(c) $\frac{50}{7}$ (d) $\frac{7}{50}$

33. The quadratic equation whose roots are 3 and –1, is
(a) $x^2 - 4x + 3 = 0$ (b) $x^2 - 2x - 3 = 0$
(c) $x^2 + 2x - 3 = 0$ (d) $x^2 + 4x + 3 = 0$

34. If one root of the equation $\frac{x^2}{a} + \frac{x}{b} + \frac{1}{c} = 0$ is reciprocal of the other, then which one of the following is correct?
(a) a = b (b) b = c
(c) ac = 1 (d) a = c

35. If one of the roots of the equation $x^2 - bx + c = 0$ is the square of the other, then which of the following option is correct?
(a) $b^3 = 3bc + c^2 + c$ (b) $c^3 = 3bc + b^2 + b$
(c) $3bc = c^3 + b^2 + b$ (d) $3bc = c^3 + b^3 + b^2$

36. Consider the following statements in respect of the quadratic equation $ax^2 + bx + b = 0$, where $a \neq 0$.
   I. The product of the roots is equal to the sum of the roots.
   II. The roots of the equation are always unequal and real.
   Which of the statements given above is/are correct?
   (a) Only I (b) Only II
   (c) Both I and II (d) Neither I nor II

37. If $x^2 = 6+\sqrt{6+\sqrt{6+\sqrt{6+...\infty}}}$, then what is one of the values of x equal to?
   (a) 6 (b) 5
   (c) 4 (d) 3

38. If α and β are the roots of the equation $x^2 - x - 1 = 0$, then what is $\frac{\alpha^2+\beta^2}{(\alpha^2-\beta^2)(\alpha-\beta)}$ equal to?
   (a) $\frac{2}{5}$ (b) $\frac{3}{5}$
   (c) $\frac{4}{5}$ (d) 4

39. In solving a problem, one student makes a mistake in the coefficient of the first degree term and obtains –9 and –1 for the roots. Another student makes a mistake in the constant term of the equation and obtains 8 and 2 for the roots. The correct equation was
   (a) $x^2 + 10x + 9 = 0$ (b) $x^2 - 10x + 16 = 0$
   (c) $x^2 - 10x + 9 = 0$ (d) None of these

40. If m and n are the roots of the equation $ax^2 + bx + c = 0$, then the equation whose roots are $\frac{(m^2+1)}{m}$ and $\frac{(n^2+1)}{n}$ is
   (a) $acx^2 + (ab + bc)x + b^2 + (a - c)^2 = 0$
   (b) $acx^2 + (ab - bc)x + b^2 + (a - c)^2 = 0$
   (c) $acx^2 - (ab - bc)x + b^2 - (a - c)^2 = 0$
   (d) $acx^2 - (ab + bc)x + b^2 - (a - c)^2 = 0$

41. The value of $x^2 - 4x + 11$ can never be less than
   (a) 7 (b) 8
   (c) 11 (d) 22

42. If the roots of the equation
   $(a^2 - bc)x^2 + 2(b^2 - ac)x + (c^2 - ab) = 0$
   are equal, where $b \neq 0$, then which one of the following is correct?
   (a) $a + b + c = abc$ (b) $a^2 + b^2 + c^2 = 0$
   (c) $a^3 + b^3 + c^3 = 0$ (d) $a^3 + b^3 + c^3 = 3abc$

43. If m and n are the roots of the equation $x^2 + ax + b = 0$ and m^2, n^2 are the roots of the equation $x^2 - cx + d = 0$, then which of the following is / are correct ?
   1. $2b - a^2 = c$ 2. $b^2 = d$
   Select the correct answer using the codes given below :
   (a) Only 1 (b) Only 2
   (c) Both 1 and 2 (d) Neither 1 nor 2

44. For which value of k does the pair of equations $x^2 - y^2 = 0$ and $(x - k)^2 + y^2 = 1$ yield a unique positive solution of x? **(CDS)**
   (a) 2 (b) 0
   (c) $\sqrt{2}$ (d) $-\sqrt{2}$

45. If a, b and c satisfy the equation $x^3 - 3x^2 + 2x + 1 = 0$ then what is the value of $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$? **(CDS)**
   (a) $-\frac{1}{2}$ (b) 2
   (c) –2 (d) $\frac{1}{2}$

46. If the roots of the quadratic equation $x^2 - 4x - \log_{10} N = 0$ are all real, then the minimum value of N is **(CDS)**
   (a) $\frac{1}{100}$ (b) $\frac{1}{1000}$
   (c) $\frac{1}{10000}$ (d) 10000

47. The difference of maximum values of the expressions $(6 + 5x - x^2)$ and $(y - 6 - y^2)$ for any real values of x and y is **(CDS)**
   (a) 16 (b) 17
   (c) 18 (d) 19

48. If the roots of the equation $lx^2 + mx + m = 0$ are in the ratio $p : q$, then **(CDS)**
   $$\sqrt{\frac{p}{q}}+\sqrt{\frac{q}{p}}+\sqrt{\frac{m}{l}}$$
   is equal to
   (a) 0 (b) 1
   (c) 2 (d) 3

49. If $\sqrt{3x^2 - 7x - 30} - \sqrt{2x^2 - 7x - 5} = x - 5$ **(CDS)**
   has α and β as its roots, then the value of αβ is
   (a) –15 (b) –5
   (c) 0 (d) 5

50. If the equations $x^2 - px + q = 0$ and $x^2 + qx - p = 0$ have a common root, then which one of the following is correct? **(CDS)**
(a) $p - q = 0$ (b) $p + q - 2 = 0$
(c) $p + q - 1 = 0$ (d) $p - q - 1 = 0$

51. If $x = 2^{\frac{1}{3}} + 2^{-\frac{1}{3}}$, then the value of $2x^3 - 6x - 5$ is euqal to **(CDS)**
(a) 0 (b) 1
(c) 2 (d) 3

52. If the linear factors of $ax^2 - (a^2 + 1)x + a$ are p and q then $p + q$ is equal to **(CDS)**
(a) (x – 1) (a + 1) (b) (x + 1) (a + 1)
(c) (x – 1) (a – 1) (d) (x + 1) (a – 1)

53. If $x = \frac{\sqrt{a+2b}+\sqrt{a-2b}}{\sqrt{a+2b}-\sqrt{a-2b}}$
then $bx^2 - ax + b$ is equal to (given that $b \neq 0$) **(CDS)**
(a) 0 (b) 1
(c) ab (d) $2ab$

54. If the sum of the roots of $ax^2 + bx + c = 0$ is equal to the sum of the squares of their reciprocais, then which one of the following relations is correct?
(a) $ab^2 + bc^2 = 2a^2c$ (b) $ac^2 + bc^2 = 2b^2a$
(c) $ab^2 + bc^2 = a^2c$ (d) $a^2 + b^2 + c^2 = 1$

55. Consider the following statements in respect of the expression

$$S_n = \frac{n(n+1)}{2}$$

where n is an integer. **(CDS)**
1. There are exactly two values of n for which $S_n = 861$.
2. $S_n = S_{(n+1)}$ and hence for any integer m, we have two values of n for which $S_n = m$.

Which of the above statement is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

56. Under what condition on p and q, one of the roots of the equation $x^2 + px + q = 0$ is the square of the other? **(CDS)**
(a) $1 + q + q^2 = 3pq$ (b) $1 + p + p^2 = 3pq$
(c) $p^3 + q + q^2 = 3pq$ (d) $q^3 + p + p^2 = 3pq$

57. The solution of the inequation **(CDS)**

$$1 + \frac{1}{x} - \frac{1}{x^2} \geq 0$$

is (given that $x \neq 0$)
(a) $x > 0$
(b) $x < 0$
(c) $\frac{-1-\sqrt{5}}{2} \leq x \leq \frac{-1+\sqrt{5}}{2}$
(d) $x \leq \frac{-1-\sqrt{5}}{2}$ or $x \geq \frac{-1+\sqrt{5}}{2}$

58. If $\sqrt{\frac{x}{y}} = \frac{10}{3} - \sqrt{\frac{y}{x}}$ and x – y = 8, then the value of xy is equal to **(CDS)**
(a) 36 (b) 24
(c) 16 (d) 9

HINTS & SOLUTIONS

1. (a) $x-\frac{1}{x}=1\frac{1}{2} \Rightarrow \frac{x^2-1}{x}=\frac{3}{2}$

$\Rightarrow 2(x^2-1)=3x \Rightarrow 2x^2-2=3x$
$\Rightarrow 2x^2-3x-2=0$
$\Rightarrow 2x^2-4x+x-2=0$
$\Rightarrow 2x(x-2)+1(x-2)=0$
$\Rightarrow (2x+1)(x-2)=0$
Either $2x+1=0$ or $x-2=0$
$\Rightarrow 2x=-1$ or $x=2$

$\Rightarrow x=\frac{-1}{2}$ or $x=2$

$\therefore x=\frac{-1}{2}$, 2 are solutions.

2. (b) Let the son's age be x years.
So, father's age = $5x-4$ years.
$\therefore x(5x-4)=288$
$\Rightarrow 5x^2-4x-288=0 \Rightarrow 5x^2-40x+36x-288=0$
$\Rightarrow 5x(x-8)+36(x-8)=0$
$\Rightarrow (5x+36)(x-8)=0$

Either $x-8=0$ or $5x+36=0 \Rightarrow x=8$ or $x=\frac{-36}{5}$

x cannot be negative; therefore, $x=8$ is the solution.
$\therefore$ Son's age = 8 years and Father's age = $5x-4$ = 36 years.

3. (a) Let the number be x.

Then, $x+\frac{1}{x}=\frac{13}{6} \Rightarrow \frac{x^2+1}{x}=\frac{13}{6} \Rightarrow 6x^2-13x+6=0$
$\Rightarrow 6x^2-9x-4x+6=0 \Rightarrow (3x-2)(2x-3)=0$

$\Rightarrow x=\frac{2}{3}$ or $x=\frac{3}{2}$.

Hence, the required number is $\frac{2}{3}$ or $\frac{3}{2}$.

4. (d) For Minimum value $x^2 \geq 0$

$=0+\frac{1}{1}-3=-2$

5. (a) $7x_2-4x_1=47$
$x_1+x_2=2$
Solving after $11x_2=55$
$x_2=5$ & $x_1=-3$
$\therefore c=-15$

6. (c) Since, the roots of the equation $(k+1)x^2-2(k-1)x+1=0$ are real and equal.
$\therefore \{-2(k-1)\}^2-4(k+1)=0$
$(\because b^2-4ac=0)$
$\Rightarrow 4(k^2-2k+1)-4(k+1)=0$
$\Rightarrow k^2-2k+1-(k+1)=0$
$\Rightarrow k^2-3k=0$
$\Rightarrow k=0, k=3$

7. (b) Since, α, β are root of the equation $2x^2-3x-6=0$

$\therefore \alpha+\beta=\frac{3}{2}$ and $\alpha\beta=-3$

$\Rightarrow \alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$=\frac{9}{4}+6=\frac{33}{4}$

Now, $(\alpha^2+2)+(\beta^2+2)=(\alpha^2+\beta^2)+4$

$=\frac{33}{4}+4=\frac{49}{4}$

and $(\alpha^2+2)(\beta^2+2)=\alpha^2\beta^2+2(\alpha^2+\beta^2)+4$

$=(-3)^2+2\left(\frac{33}{4}\right)+4=\frac{59}{2}$

So, the equation whose roots are α^2+2 and β^2+2 is
$x^2-x\{(\alpha^2+2)+(\beta^2+2)\}+(\alpha^2+2)(\beta^2+2)=0$

$\Rightarrow x^2-\frac{49}{4}x+\frac{59}{2}=0$

$\Rightarrow 4x^2-49x+118=0$

8. (b) Use $\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$

9. (d) $\frac{1}{r^2}+\frac{1}{s^2}=\frac{s^2+r^2}{(rs)^2}$

$=\frac{(s+r)^2-2sr}{(rs)^2}=\frac{p^2-2q}{q^2}$

10. (b) Let first square has side x,
$\therefore$ Area = x^2, Perimeter = $4x$ and let second square has side y,
$\therefore$ Area = y^2, Perimeter = $4y$
Let $x>y$ so that $4x>4y$
Given, $x^2+y^2=468$ ...(1)
and $4x-4y=24 \Rightarrow x-y=6 \Rightarrow y=x-6$ ...(2)
Using (2) in (1), we get $x^2+(x-6)^2=468$
$\Rightarrow x^2+x^2-12x+36=468 \Rightarrow 2x^2-12x-432=0$

$\Rightarrow x^2-6x-216=0 \Rightarrow x=\frac{6\pm\sqrt{36+864}}{2}$

$=\frac{6\pm\sqrt{900}}{2}$

$= \frac{6 \pm 30}{2} = \frac{36}{2}, \frac{-24}{2} = 18, -12$

But x being length cannot be negative $\therefore x = 18$

put $x = 18$ in equ. (2), we get $y = x - 6 = 18 - 6 = 12$

$\therefore$ sides of the two squares $= x, y = 18$ m, 12 m

11. (b) Since the roots of $x^2 - kx + 4 = 0$ are non-real.

$\therefore$ Disc., $(-k^2) - 4 < 0 \Rightarrow k^2 - 4 < 0$

$\Rightarrow k^2 < 4 \Rightarrow |k| < 2 \Rightarrow -2 < k < 2$

12. (b) $p + q = 18$ ...(1)

and $\frac{1}{p} + \frac{1}{q} = \frac{1}{4}$ ...(2) (Given)

i.e $\frac{p+q}{pq} = \frac{1}{4} \Rightarrow \frac{18}{pq} = \frac{1}{4}$

$\Rightarrow pq = 72$ ...(3)

From (1) and (3), $p(18 - p) = 72$

$\Rightarrow p^2 - 18p + 72 = 0 \Rightarrow (p-6)(p-12) = 0$

$\Rightarrow p = 6, 12$ when $p = 6, q = 12$; when $p = 12, q = 6$

Hence the numbers are 12, 6.

13. (d) $x^2 + 504 = 8(y^2)$

$3x = 4y$

$x = \frac{4y}{3}$

$\left(\frac{4y}{3}\right)^2 + 504 = 8y^2$

$\frac{16y^2}{9} + 504 = 8y^2$

$16y^2 + 9 \times 504 = 72y^2$

$72y^2 - 16y^2 = 4536$

$56y^2 = 4536$

$y^2 = 81$

$y = 9$

$x = \frac{4 \times 9}{3} = 12$

14. (d) $x + \sqrt{x-2} = 4$

$\sqrt{x-2} = 4 - x$

Squaring on the both sides

$x - 2 = 16 + x^2 - 8x$

$x^2 - 9x + 18 = 0$

$(x-6)(x-3) = 0$

$x = 6$ or 3

But by checking, only $x = 3$ satisfies the equation.

15. (b) $\sqrt{x+10} - \frac{6}{\sqrt{x+10}} = 5$

$x + 10 - 6 = 5\sqrt{x+10}$

$x + 4 = 5\sqrt{x+10}$

Squaring on both sides,

$x^2 + 8x + 16 = 25x + 250$

$x^2 - 17x - 234 = 0$

$x^2 - 26x + 9x - 234 = 0$

$x(x-26) + 9(x-26) = 0$

$(x-26)(x+9) = 0$

$x = 26$ (or) -9

Here $x = -9$ is not satisfying. So it is extraneous.

16. (d) $\log_{10}(x^2 - 3x + 6) = 1$

$x^2 - 3x + 6 = 10^1$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$x = 4$ or -1

17. (b) Let α and β are the roots

$\alpha + \beta = 6$

$\alpha - \beta = 8$

$2\alpha = 14$

$\alpha = 7$

$\beta = -1$

$\alpha + \beta = 6, \alpha\beta = -7$

The quadratic equation is $x^2 - 6x - 7 = 0$

18. (a) $b^2 - 4ac = (2\sqrt{3})^2 - 4(1)(3) = 0$. So the roots are real and equal.

19. (c) Since roots are reciprocal,

product of the roots $= 1 \Rightarrow \frac{c}{a} = 1$

$\Rightarrow c = a$.

20. (c) $ax^2 - 2\sqrt{2}x + c = 0$

$(2\sqrt{2})^2 - 4ac = 0$

$4ac = 8$

$ac = 2$

$c = \frac{2}{a}$

Let α, β be the roots.

$\alpha + \beta \frac{2\sqrt{2}}{a}, \alpha\beta = \frac{c}{a} = \frac{2}{a^2}$

$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

$= \frac{8}{a^2} - \frac{8}{a^2} = 0$

$\alpha = \beta$

So, $\alpha = \beta = \frac{\sqrt{2}}{a}$

Hence the roots are real and equal.

21. (c) The product of the roots is given by: $(a^2 + 18a + 81)/1$.

Since product is unity we get: $a^2 + 18a + 81 = 1$

Thus, $a^2 + 18a + 80 = 0$

Solving, we get $a = -10$ and $a = -8$.

22. (b) Solve this by assuming each option to be true and then check whether the given expression has equal roots for the option under check.
Thus, if we check for option (b).
$ad = bc$.
We assume $a = 6$, $d = 4$ $b = 12$ $c = 2$ (any set of values that satisfies $ad = bc$)
Then $(a^2 + b^2)x^2 - 2(ac + bc)x + (c^2 + d^2) = 0$
$180x^2 - 120x + 20 = 0$
We can see that this has equal roots. Thus, option (b) is a possible answer. The same way if we check for a, c and d we see that none of them gives us equal roots and can be rejected.

23. (b) Here, $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

Thus, $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3 + \beta^3}{\alpha\beta}$

$= \frac{(\alpha+\beta)\ (\alpha^2 - \alpha\beta + \beta^2)}{\alpha\beta}$ ...(1)

Now, $(\alpha^2 + \beta^2 - \alpha\beta) = [(\alpha+\beta)^2 - 2\alpha\beta - \alpha\beta]$

$= [(\alpha+\beta)^2 - 3\alpha\beta]$
Hence (1) becomes

$$\Rightarrow \frac{(\alpha+\beta)[(\alpha+\beta)^2 - 3\alpha\beta)]}{\alpha\beta} = \frac{\frac{-b}{a}\left[\frac{b^2}{a^2} - \frac{3c}{a}\right]}{\frac{c}{a}}$$

$$= \frac{-b}{c}\left[\frac{b^2 - 3ac}{a^2}\right] = \frac{3abc - b^3}{a^2c}$$

24. (c) $a + b = 24$ and $a - b = 8$
$\Rightarrow a = 16$ and $b = 8 \Rightarrow ab = 16 \times 8 = 128$
A quadratic equation with roots a and b is
$x^2 - (a + b)x + ab = 0$ or $x^2 - 24x + 128 = 0$

25. (a) If roots of an equation $ax^2 + bx + c = 0$ are positive, then signs of a and c should be like.

26. (c) $\because$ α and β are the roots of the equation $x^2 + 6x + 1 = 0$
$\therefore$ $\alpha + \beta = -6$ and $\alpha\beta = 1$
Now, $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$
$= (-6)^2 - 4$
$= 36 - 4 = 32$
$\Rightarrow |\alpha - \beta| = \sqrt{32} = 4\sqrt{2}$

27. (c) Let the natural number be $= x$.

By the given condition: $x + 12 = \frac{160}{x} (x \neq 0)$

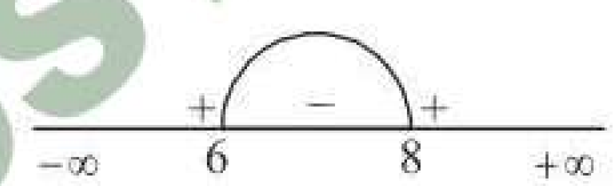
$\Rightarrow x^2 + 12x - 160 = 0 \Rightarrow x = -\frac{12 \pm \sqrt{144 + 640}}{2}$

$= -\frac{12 \pm \sqrt{784}}{2} = \frac{-12 \pm 28}{2} = -\frac{40}{2}$ or $\frac{16}{2}$

$= -20$ or 8. But x is a natural number $\therefore x = 8$.

28. (d) (a) is clearly true.
(b) $x^2 + 1$ is a quadratic polynomial which has no real value of x for which $x^2 + 1$ is zero.
$[\therefore x^2 \geq 0 \Rightarrow x^2 + 1 > 0$ for all real $x]$ $\therefore$ (b) is true.
(c) The quadratic polynomial
$x^2 - 2x + 1 = (x - 1)^2$ has only one zero i.e. 1
$\therefore$ (c) is true.
$[\because (x - 1)^2 > 0$ at $x \neq 1$ and for $x = 1, (x - 1)^2 = 0]$
(d) is false
$[\because$ of (b), (c)$]$
Hence (d) holds.

29. (c) $6 \leq x \leq 8$
i.e.,



or $x \in [6, 8]$
$\Rightarrow (x - 6)(x - 8) \leq 0$

30. (a) Let roots of equation are α and β.

$\therefore$ $\alpha + \beta = \frac{a^2 + b^2}{a^2b^2}$ and $\alpha\beta = \frac{1}{a^2b^2}$

We know that

$$\alpha - \beta = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta} = \sqrt{\left(\frac{a^2 + b^2}{a^2b^2}\right)^2 - \frac{4}{a^2b^2}}$$

$$\Rightarrow \alpha - \beta = \sqrt{\frac{(a^2 - b^2)^2}{(a^2b^2)^2}} = \frac{a^2 - b^2}{a^2b^2}$$

On solving, we get $\alpha = \frac{1}{b^2}$ and $\beta = \frac{1}{a^2}$.

31. (b) Let, the roots of the quadratic equation $x^2 + px + q = 0$ is (α, β).
Given that, A starts with a wrong value of p and obtains the roots as 2 and 6. But this time q is correct.
i.e., Product of roots
$q = \alpha \cdot \beta = 6 \times 2 = 12$ ...(i)
and B starts with a wrong value of q and gets the roots as 2 and – 9. But this time p is correct.
i.e., Sum of roots $p = \alpha + \beta = -9 + 2 = -7$ ... (ii)
$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$
$= (-7)^2 - 4.12 = 49 - 48 = 1$
[From equations (i) and (ii)]
$\Rightarrow \alpha - \beta = 1$ ... (iii)

From equations (ii) and (iii),
$\alpha = -3$ and $\beta = -4$
which is correct roots.

32. (a) Quadratic equation $7x^2 - 50x + k = 0$
Here, a = 7, b = –50 and c = k

Since, $\alpha + \beta = \frac{-b}{a}$

$\therefore \alpha + \beta = \frac{50}{7} \Rightarrow 7 + \beta = \frac{50}{7}$

$\Rightarrow \beta = \frac{1}{7}$ $(\because \alpha = 7, \text{ given})$

and $\alpha\beta = \frac{c}{a}$ or $7 \times \frac{1}{7} = \frac{k}{7} \Rightarrow k = 7$

33. (b) Here, given roots of equation are 3 and –1.
So, expression can be written as
$(x - 3)(x + 1) = 0$
$= x^2 - 3x + x - 3 = 0$
$= x^2 - 2x - 3 = 0$

34. (d) Given equation,

$\frac{x^2}{a} + \frac{x}{b} + \frac{1}{c} = 0$

$bcx^2 + acx + ab = 0$

Let roots are α and $\frac{1}{\alpha}$

Product of roots $= \alpha \cdot \frac{1}{\alpha} = \frac{ab}{bc}$

$\Rightarrow 1 = \frac{a}{c}$

$\Rightarrow \boxed{\therefore c = a}$

35. (a) According to question
Let one roots of equation is α then others roots of equation is a^2.

$\therefore$ Sum of roots $= \alpha + \alpha^2 = -\frac{(-b)}{1}$

$\Rightarrow \alpha(\alpha + 1) = b$ ... (i)

Product of roots $= \alpha \cdot \alpha^2 = \frac{c}{1}$

$\Rightarrow \alpha^3 = c \Rightarrow \alpha = c^{1/3}$ ... (ii)

From equations (i) and (ii),

$c^{\frac{1}{3}}\left(c^{\frac{1}{3}} + 1\right) = b$ ... (iii)

On cubing both sides, we get

$c\left(c^{\frac{1}{3}} + 1\right)^3 = b^3$

$\Rightarrow c\left\{c + 1 + 3c^{\frac{1}{3}}\left(c^{\frac{1}{3}} + 1\right)\right\} = b^3$

$\Rightarrow c(c + 1 + 3b) = b^3$ [from equation (iii)]

$\Rightarrow b^3 = 3bc + c^2 + c$

36. (d) $ax^2 + bx + b = 0$

$\Rightarrow x^2 + \frac{b}{a}x + \frac{b}{a} = 0$

$\therefore$ Sum of roots, $\alpha + \beta = \frac{-b}{a}$

and Products of roots, $\alpha\beta = \frac{b}{a}$

Hence, product of roots is not equal to the sum of roots, so Statement I not correct.
Now, for roots to be real and uequal.
$\therefore$ Determinant, $D > 0$
$\Rightarrow b^2 - 4ac > 0$
$\Rightarrow b^2 - 4a\,(b) > 0$
$\Rightarrow b^2 - 4ab > 0$
$\Rightarrow b^2 > 4ab$
$\therefore b > 4a$
So, if $b > 4a$, then roots are unequal and real, so Statement II is not always true it will depend on values of a and b.

37. (d) $x^2 = 6 + \sqrt{6 + \sqrt{6 + \sqrt{6 + \ldots \infty}}}$

$x^2 = 6 + \sqrt{x^2}$
$\Rightarrow x^2 = 6 + x$
$\Rightarrow x^2 - x - 6 = 0$
$\Rightarrow x^2 + 2x - 3x - 6 = 0$
$\Rightarrow x(x + 2) - 3(x + 2) = 0$
$\Rightarrow (x - 3)(x + 2) = 0$
$\therefore x = 3$

Alternate Method :
Given,

$x^2 = 6 + \sqrt{6 + \sqrt{6 + \sqrt{6 + \ldots \infty}}}$

$x = \sqrt{6 + \sqrt{6 + \sqrt{6 + \sqrt{6 + \ldots \infty}}}}$

Here, factor of 6 = 2 and 3
Sign in expression is positive.
So that x = 3.

38. (b) $\alpha + \beta = \frac{-(-1)}{1} = 1$

$\alpha.\beta = \frac{-1}{1} = -1$

Now, $(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$
$\Rightarrow \alpha^2 + \beta^2 = (1)^2 - 2 \times (-1) = 3$

and $\alpha^2-\beta^2=\sqrt{(\alpha^2+\beta^2)^2-4\alpha^2\beta^2}$

$=\sqrt{9-4(-1)^2}=\sqrt{5}$

$\alpha-\beta=\sqrt{(\alpha-\beta)^2-4\alpha\beta}$

$=\sqrt{1-4(-1)}=\sqrt{5}$

Now, $\dfrac{\alpha^2+\beta^2}{(\alpha^2-\beta^2)(\alpha-\beta)}=\dfrac{3}{\sqrt{5}.\sqrt{5}}=\dfrac{3}{5}$

39. (c) When mistake is done in first degree term the roots of the equation are – 9 and – 1.

∴ Equation is $(x+1)(x+9)=x^2+10x+9$ ...(i)

When mistake is done in constant term, the roots of equation are 8 and 2.

∴ Equation is $(x-2)(x-8)=x^2-10x+16$ ... (ii)

∴ Required equation from equations (i) and (ii), we get $x^2-10x+9$

40. (a) For the given equation $ax^2+bx+c=0$, m and n are the roots

∴ Sum of roots $=m+n=-\dfrac{b}{a}$

and Product of roots $=mn=\dfrac{c}{a}$

Sum of roots & of required equation $=\dfrac{m^2+1}{m}$ and $\dfrac{n^2+1}{n}$

$=\dfrac{m^2+1}{m}+\dfrac{n^2+1}{n}=\dfrac{m^2n+n+mn^2+m}{mn}$

$=\dfrac{mn(m+n)+(m+n)}{mn}=\dfrac{(m+n)(mn+1)}{mn}$

$=\dfrac{-\frac{b}{a}\left(\frac{c}{a}+1\right)}{\frac{c}{a}}=\dfrac{-b(a+c)}{ac}$

Put $m+n=-\dfrac{b}{a}$ and $mn=\dfrac{c}{a}$

∴ Product of roots $=\dfrac{m^2+1}{m}\times\dfrac{n^2+1}{n}=\dfrac{(m^2+1)(n^2+1)}{mn}$

$=\dfrac{m^2n^2+n^2+m^2+1}{mn}$

$=\dfrac{(mn)^2+(m+n)^2-2mn+1}{mn}$

$=\dfrac{\left(\frac{c}{a}\right)^2+\left(\frac{-b}{a}\right)^2-2\left(\frac{c}{a}\right)+1}{\frac{c}{a}}$

$=\dfrac{c^2+b^2-2ac+a^2}{ac}=\dfrac{b^2+(a-c)^2}{ac}$

We know that, quadratic equation is of the form x^2 – (Sum of roots) x + Product of roots = 0

$\Rightarrow x^2-\left(\dfrac{-b(a+c)}{ac}\right)x+\left(\dfrac{b^2+(a-c)^2}{ac}\right)=0$

$\Rightarrow acx^2+b(a+c)x+b^2+(a-c)^2=0$

$\Rightarrow acx^2+(ab+bc)x+b^2+(a-c)^2=0$

41. (a) $x^2-4x+11$

This can be written as

$=(x-2)^2+7$

Here, $(x-2)^2\geq 0$

So, given function can be not be less than 7.

42. (d) Given equation

$(a^2-bc)x^2+2(b^2-ac)x+(c^2-ab)=0$

The given roots are equal, then D must be zero.

∴ D = 0

i.e., $[2(b^2-ac)]^2-4(a^2-bc)(c^2-ab)=0$

$\Rightarrow 4(b^4+a^2c^2-2ab^2c)-4(a^2c^2-bc^3-a^3b+ab^2c)=0$

$\Rightarrow 4b^4+4a^2c^2-8ab^2c-4a^2c^2+4bc^3+4a^3b-4ab^2c=0$

$\Rightarrow 4b^4-12ab^2c+4bc^3+4a^3b=0$

$\Rightarrow b^3+c^3+a^3-3abc=0$

$\therefore a^3+b^3+c^3=3abc$

43. (b) Here m and n are the roots of the equation $x^2+ax+b=0$.

$m+n=-a$ ...(i)

$mn=b$ ...(ii)

Also, m^2 and n^2 are the roots of the equation of $x^2-cx+d=0$.

$m^2+n^2=c$ ...(iii)

$m^2n^2=d$ ...(iv)

By squaring Eq. (i) both sides, we get

$m^2+n^2+2mn=a^2$ [from Eqs. (i) and (ii)]

$\Rightarrow c+2b=a^2\Rightarrow c=a^2-2b$

$\Rightarrow 2b-a^2=-c$

Therefore, Statement 1 is incorrect.

2. From Eq. (ii)

$m^2n^2=b^2\Rightarrow b^2=d$

Therefore , Statement 2 is correct.

44. (a) $x^2-y^2=0\Rightarrow x^2=y^2$

$(x-k)^2+y^2=1$

$x^2+k^2-2kx+y^2-1=0$

Here we put $y^2=x^2$

$x^2+k^2-2kx+x^2-1=0$

$2x^2-2kx+k^2-1=0$

$$x=\frac{2k\pm\sqrt{(-2k)^2-4(2)(2k^2-1)}}{2\times 2}$$

$$=\frac{2k\pm\sqrt{4k^2-8k^2+8}}{4}$$

$$=\frac{2k\pm\sqrt{8-4k^2}}{4}$$

$$=\frac{2k\pm 2\sqrt{2-k^2}}{4}$$

$$=\frac{k\pm\sqrt{2-k^2}}{4}$$

For unique positive solution we put k = 2

$$x=\frac{2}{2}=1.$$

45. (c) $x^3-3x^2+2x+1=0$

as a, b and c are the roots-

$\Rightarrow ab+bc+ca=2$ ----------(i)

$abc=-1$ ----------(ii)

Dividing eq (i) by eq (ii)-

$$\frac{(ab+bc+ca)}{abc}=\frac{2}{-1}$$

$$\frac{ab}{abc}+\frac{bc}{abc}+\frac{ca}{abc}=-2$$

$$\frac{1}{c}+\frac{1}{a}+\frac{1}{b}=-2$$

$$\boxed{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=-2}$$

46. (c) $x^2-4x-\log_{10}N=0$

if roots are real, $b^2-4ac\geq 0$

$\Rightarrow (-4)^2+4.\log_{10}N\geq 0$

$\Rightarrow 16+4\log_{10}N\geq 0$

$\Rightarrow 16\log_{10}10+4\log_{10}N\geq 0$

$\Rightarrow \log_{10}(10)^{16}+\log_{10}N^4\geq 0$

$\Rightarrow \log_{10}[(10)^{16}.N^4]\geq\log_{10}1$

$\Rightarrow (10)^{16}N^4\geq 1$

$$\Rightarrow N^4\geq\left(\frac{1}{10}\right)^{16\div 4}$$

$$\Rightarrow N\geq\left(\frac{1}{10}\right)^4$$

$$\Rightarrow N\geq\frac{1}{10000}$$

So, minimum value of N is $\frac{1}{10000}$

47. (c) $6+5x-x^2-\left(x-\frac{5}{2}\right)^2+\frac{25}{4}+6$

$$=\frac{49}{4}-\left(x-\frac{5}{2}\right)^2$$

So, maximum value of $6+5x-x^2$ is $\frac{49}{4}$.

$$y-6-y^2=-\left(y-\frac{1}{2}\right)^2+\frac{1}{4}-6$$

$$=-\frac{23}{4}-\left(4-\frac{1}{2}\right)^2$$

So maximum value of $y-6-y^2$ is $-\frac{23}{4}$.

Difference between the maximum values

$$=\frac{49}{4}-\left(-\frac{23}{4}\right)$$

$$=\frac{49+23}{4}=\frac{72}{4}=18$$

48. (a) Let α, β be the roots of the equation $lx^2+mx+m=0$

Given $\frac{\alpha}{\beta}=\frac{p}{q}$ ...(1)

Now $\alpha+\beta$ (sum of roots) $=\frac{-m}{l}$

and $\alpha\beta$ (product of roots) $=\frac{m}{l}$

Consider $\sqrt{\frac{p}{q}}+\sqrt{\frac{q}{p}}+\sqrt{\frac{m}{l}}$

Using (1)

$$=\sqrt{\frac{\alpha}{\beta}}+\sqrt{\frac{\beta}{\alpha}}+\sqrt{\frac{m}{l}}$$

$$=\frac{\alpha+\beta}{\sqrt{\alpha\beta}}+\sqrt{\frac{m}{l}}$$

$$=\frac{-\frac{m}{l}}{\sqrt{\frac{m}{l}}}+\sqrt{\frac{m}{l}}=-\sqrt{\frac{m}{l}}+\sqrt{\frac{m}{l}}=0$$

49. (a) $\sqrt{3x^2-7x-30}-\sqrt{2x^2-7x-5}=x-5$

$$\Rightarrow\sqrt{3x^2-7x-30}-(x-5)=\sqrt{2x^2-7x-5}$$

Squaring on both sides, we get

$\Rightarrow (3x^2-7x-30)+(x-5)^2-2(x-5)\sqrt{3x^2-7x-30}$
$= 2x^2-7x-5$

$\Rightarrow 3x^2-7x-30+x^2+25-10x-2(x-5)\sqrt{3x^2-7x-30}$
$= 2x^2-7x-5$

$\Rightarrow 3x^2-30+x^2+25-10x-2x^2+5=2(x-5)\sqrt{3x^2-1x-30}$

$\Rightarrow 2x^2-10x=2(x-5)\sqrt{3x^2-7x-30}$

$\Rightarrow 2x(x-5)=2(x-5)\sqrt{3x^2-7x-30}$

$\Rightarrow x=\sqrt{3x^2-7x-30}$

Again squaring on both sides, we get

$\Rightarrow x^2=3x^2-7x-30$

$\Rightarrow 2x^2-7x-30=0$

Let α and β be roots of this equation

$\Rightarrow \alpha\beta=-\frac{30}{2}$

$=-15$

50. (a) Let α be common root of both the equations

$x^2-px+q=0$ and $x^2+qx-p=0$

so $\alpha^2-p\alpha+q=0$ ...(i)

and $\alpha^2+q\alpha-p=0$ ...(ii)

From (i) we get $\alpha^2=p\alpha-q$

Putting in (ii), we get

$\alpha^2+q\alpha-p=0$

$\Rightarrow p\alpha-q+q\alpha-p=0$

$\Rightarrow (p+q)\alpha-(p+q)=0$

$\Rightarrow (p+q)\alpha=p+q$

$\Rightarrow \alpha=1$

From (i) we get $1^2-p.1+q=0$

$\Rightarrow -p+q=0$

$\Rightarrow p-q=0$

51. (a) Given $x=2^{\frac{1}{3}}+2^{-\frac{1}{3}}$

$x=2^{\frac{1}{3}}+\frac{1}{2^{\frac{1}{3}}}$

Cubing on both sides, we get

$$x^3=\left(2^{\frac{1}{3}}\right)^3+\left(\frac{1}{2^{\frac{1}{3}}}\right)^3+3.2^{\frac{1}{3}}.\frac{1}{2^{\frac{1}{3}}}\left(2^{\frac{1}{3}}+\frac{1}{2^{\frac{1}{3}}}\right)$$

$\Rightarrow x^3=2+\frac{1}{2}+3(x)$

$\Rightarrow x^3=\frac{4+1+3x\times 2}{2}$

$\Rightarrow 2x^3=5+6x$

$\Rightarrow 2x^3-6x-5=0$

52. (a) Consider $ax^2-(a^2+1)x+a$ ...(1)

$\Rightarrow ax^2-a^2x-x+a$

$\Rightarrow ax(x-a)-1(x-a)$

$=(x-a)(ax-1)$

Given p and q are two linear factors of (1)

$\therefore$ $p=x+a$ and $q=ax-1$

$\Rightarrow p+q=x-a+ax-1$

$=x(a+1)-1(a+1)$

$=(x-1)(a+1)$

53. (a) $\frac{x}{1}=\frac{\sqrt{a+2b}+\sqrt{a-2b}}{\sqrt{a+2b}-\sqrt{a-2b}}$

Applying componendo and dividendo, we get

$\frac{x+1}{x-1}=\frac{2\sqrt{a+2b}}{2\sqrt{a-2b}}$

$\Rightarrow \frac{x+1}{x-1}=\frac{\sqrt{a+2b}}{\sqrt{a-2b}}$

Squaring on both sides, we get

$\frac{(x+1)^2}{(x-1)^2}=\frac{a+2b}{a-2b}$

Again applying componendo and dividendo, we get

$\frac{x^2+1+2x+x^2+1-2x}{x^2+1+2x-x^2-1+2x}=\frac{2a}{4ab}=\frac{a}{2b}$

$\Rightarrow \frac{2(x^2+1)}{4x}=\frac{a}{2b}$

$\Rightarrow \frac{x^2+1}{x}=\frac{a}{b}$

$\Rightarrow bx^2+b=ax$

$\Rightarrow bx^2-ax+b=0$

54. (a) Let α and β be two roots of the given equation

$ax^2+bx+c=0$

Then $\alpha+\beta=-\frac{b}{a}$ & $\alpha\beta=\frac{c}{a}$

According to question,

$\alpha+\beta=\frac{1}{\alpha^2}+\frac{1}{\beta^2}$

$$=\frac{\alpha^2+\beta^2+2\alpha\beta-2\alpha\beta}{\alpha^2\beta^2}=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha^2\beta^2}$$

Substitute the value of $\alpha+\beta$ and $\alpha\beta$, we get

$$-\frac{b}{a}=\frac{\left(-\frac{b}{a}\right)^2-\frac{2c}{a}}{\frac{c^2}{a^2}}$$

$$\Rightarrow -\frac{b}{a}=\frac{b^2-2ac}{a^2}\times\frac{a^2}{c^2}=\frac{b^2-2ac}{c^2}$$

$\Rightarrow -bc^2=a(b^2-2ac)$

$\Rightarrow ab^2+bc^2-2a^2c=0$

55. (c) Given $S_n=\frac{n(n+1)}{2}$

Also $S_n=861$

$\Rightarrow 861=\frac{n(n+1)}{2}$

$n^2+n-1722=0$

$n=41,-42$

(1) is true (i.e., there are exactly two values of n for which $S_n=861$)

Given $S_n=S_n+1$

$\Rightarrow \frac{n(n+1)}{2}=\frac{(n+1)(n+2)}{2}$

$n=n+2$

$2\neq 0$ (this is not possible)

$\Rightarrow$ 2 is true.

56. (c) Let α,β be two roots of the equation $x^2+px+q=0$

According to question

$\beta=\alpha^2$

Sum of roots $=\alpha+\alpha^2=-p$ ...(1)

Product of roots $=\alpha^3=q$ ...(2)

Dividing (1) by (2) we get

$$\frac{\alpha(\alpha+1)}{\alpha^3}=\frac{-p}{q}$$

$$\frac{\alpha+1}{\alpha^2}=-\frac{p}{q}$$

Cubing on both sides, we get

$q^3(\alpha+1)^3=(-p)^3(\alpha^2)^3$

$q^3(\alpha^3+1+3\alpha^2+3\alpha)=-p^3\alpha^6\ \begin{bmatrix}\alpha^3=q\\ \alpha^6=q^2\end{bmatrix}$

$q^3[q+1+3(\alpha^2+\alpha)]=-p^3q^2$

$q[q+1+3(-p)]=-p^3$

$q^2+q-3pq=-p^3$

$\Rightarrow p^3+q^2+q=3pq$

$\therefore$ Option (c) is correct

57. (d) $1+\frac{1}{x}-\frac{1}{x^2}\geq 0$

$x^2+x-1\geq 0$

$$\Rightarrow \left(x-\frac{(-1-\sqrt{5})}{2}\right)\left(x-\frac{(-1+\sqrt{5})}{2}\right)=0$$

$\Rightarrow x\leq\frac{-1-\sqrt{5}}{2}$

and $x\geq\frac{-1+\sqrt{5}}{2}$

58. (d) $\sqrt{\frac{x}{y}}=\frac{10}{3}-\sqrt{\frac{y}{x}}$

$$\sqrt{\frac{x}{y}}=\frac{10}{3}-\frac{1}{\sqrt{\frac{y}{x}}}$$

Let $\sqrt{\frac{x}{y}}=z$

$\therefore\ z=\frac{10}{3}-\frac{1}{z}\Rightarrow 3z^2-10z+3=0$

$\Rightarrow z=3$ or $\frac{1}{3}$

$\Rightarrow \sqrt{\frac{x}{y}}=3$ or $\sqrt{\frac{x}{y}}=\frac{1}{3}$

$\Rightarrow x=9y$ ...(i) or $9x=y$ ...(ii)

But $x-y=8$ ...(iii)

$\therefore\ x>y$

Hence eq. (ii) is rejected.

From eq. (i) and (iii),

$9y-y=8,\Rightarrow y=1$ and $x=9$

$\therefore\ xy=9\times 1=9$