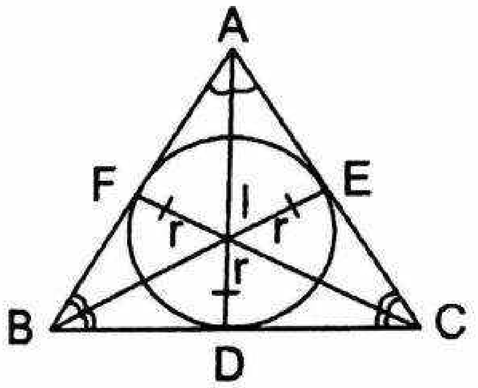
CHAPTER 12

CENTRES OF A TRIANGLES

Centres of a Triangle

(i) **In-centre**
(ii) **circum-centre**
(iii) **Centroid O**
(vi) **rtho-centre**

In-Centre of Triangle -The point of intersection of all the three angle bisectors of a triangle is called its in-centre (I).



The distance between In-centre and its all three sides is always equal and called inradius (r).
ID = IE = IF = r (inradius)

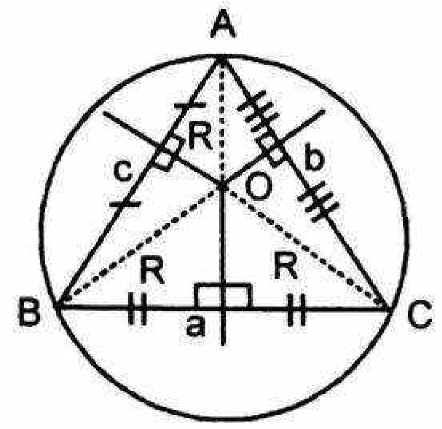
$$\text{Inradius (r)} = \frac{\text{Area of a } \Delta \text{ (A)}}{semiperimeter(s)} = \frac{A}{S}$$

$$\angle BIC = 90 + \frac{1}{2}\angle A$$

$$\angle AIC = 90 + \frac{1}{2}\angle B$$

$$\angle AIB = 90 + \frac{1}{2}\angle C$$

Circum-centre of Triangle -The point of intersection of the ⊥ bisectors of three sides of a triangle is called its circumcentre.



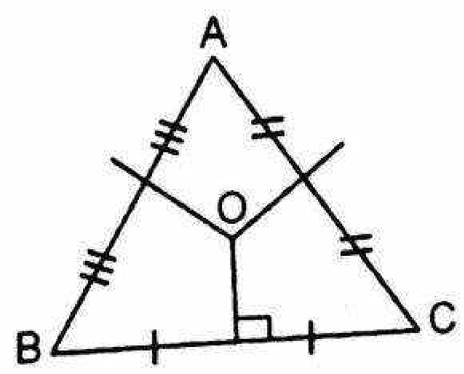
The distance between Circum-centre and its all three vertices are always equal and called circumradius.
OA = OB = OC = R (circumradius)

$$\text{Circumradius (R)} = \frac{a.b.c}{4A}$$

$\angle BOC = 2.\angle A$
$\angle AOC = 2.\angle B$
$\angle AOB = 2.\angle C$

NOTE - The circum-centre in

(i) Acute-angled Triangle - Lies inside the Triangle.



OA = OB = OC = (R)

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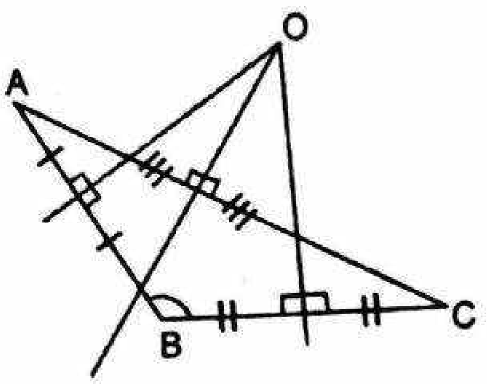
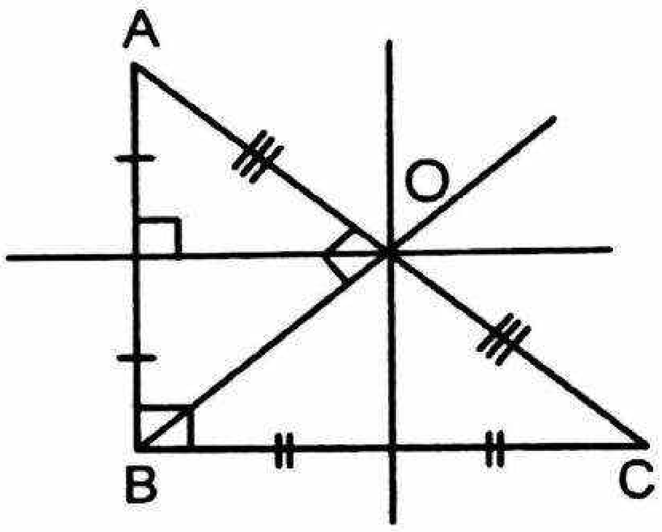
(ii) Obtuse-angled Triangle - Lies infront of the obtuse - angle and outside of the triangle.



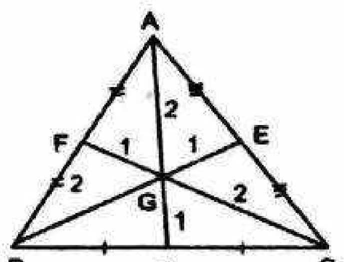
OA = OB = OC (R)

(iii) Right-angled Triangle - Lies on the triangle, at the midpoint of Hypotenuse.

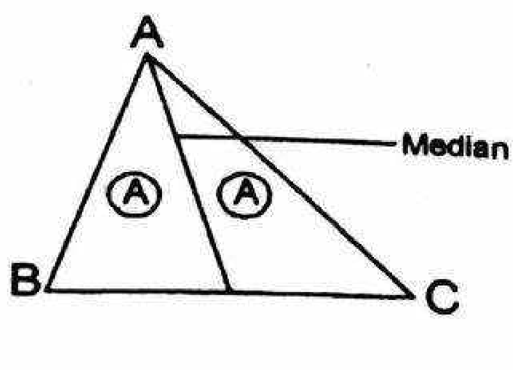


OA = OB = OC (R)

- **Centroid** - It is the point of intersection of all the three medians. It is denoted by G.



- **Medians** - A line segment joining the mid-point of the side of the side with opposite vertex.

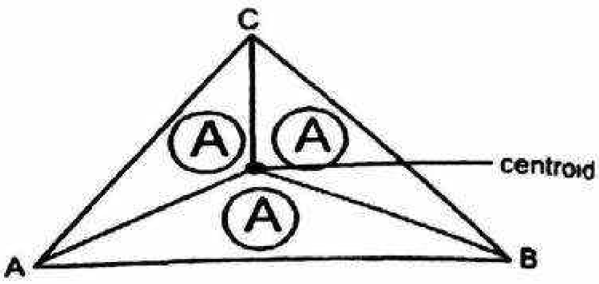


The centroid (G) divides a median in the ratio of 2 : 1.

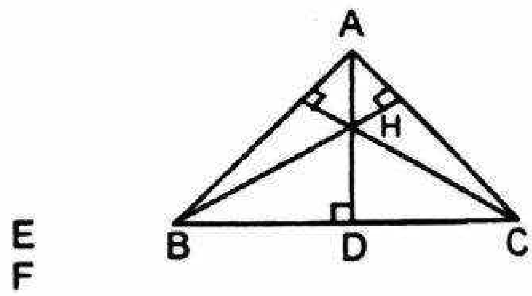
AG ; GD = BG : GE = CG : GF = 2 : 1

A median divides Area of a Δ in exactly two parts

A centroid divides Area of a Δ in extactly three parts.



- **Orhtocentre** - It is the point of intersection of all the three altitudes.
- **Altitude** - The altitude of a triangle is a line segment perpendicularly drawn from vertex to the side opposite to it. The side on which ⊥ is drown is called its base.



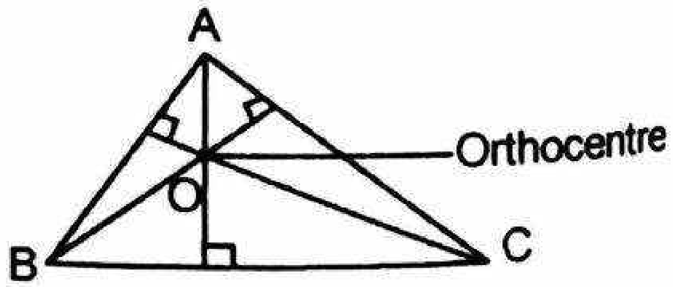
$\angle BHC = 180 - \angle A$

$\angle AHB = 180 - \angle C$

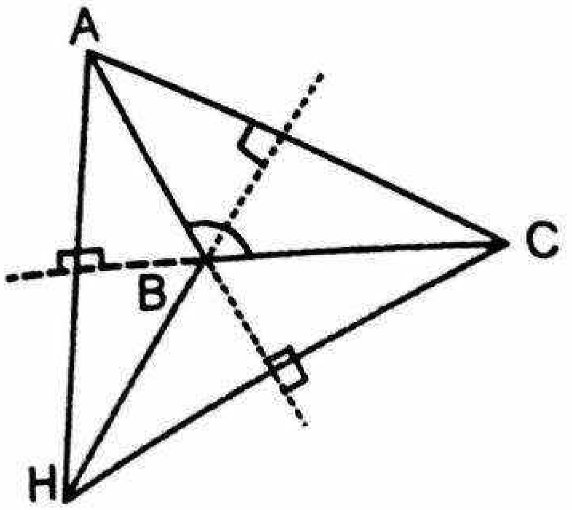
$\angle AHC = 180° - \angle B$

Note - The Orthocentre in -

(i) Acute-angled Triangle - lies inside the triangle.

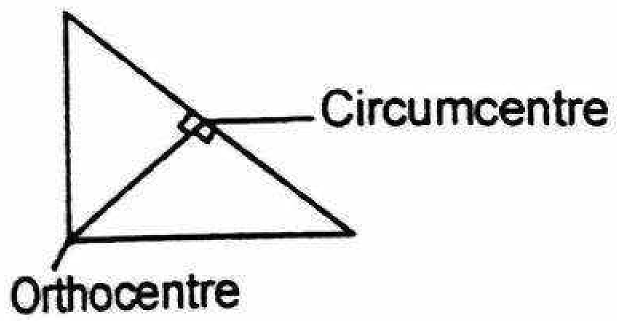


(ii) Obtuse-angled Triangle - lies outside of the triangle, on the back-side of the obtuse angle.

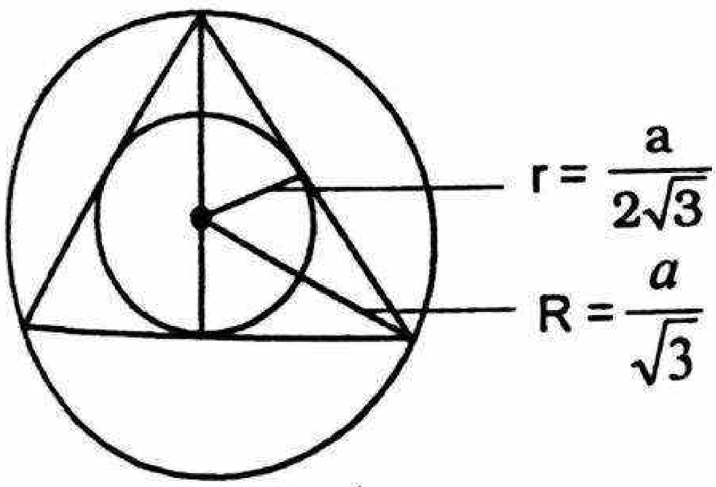


Orthocentre and Circumcentre lie opposite to each other in obtuse angle triangle.

(iii) **Right-angled Triagnle** - lies on the triangle, at the right angle.



Note - For equilateral Triangle,
All centres i.e. Incentre, Circum centre, centroid and Orthocentre are lies at the same point.

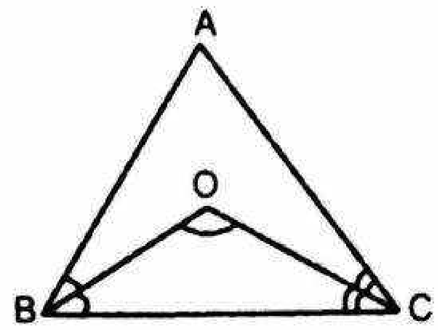


Area of equilateral $\Delta = \frac{\sqrt{3}}{4}a^2$

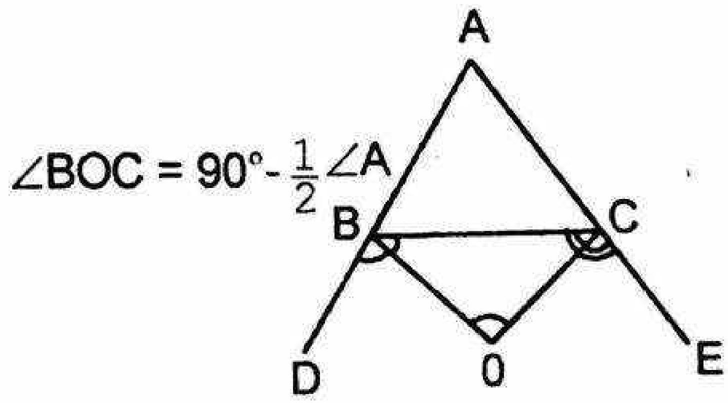
Altitude $= \frac{\sqrt{3}}{2}a$

Some useful results

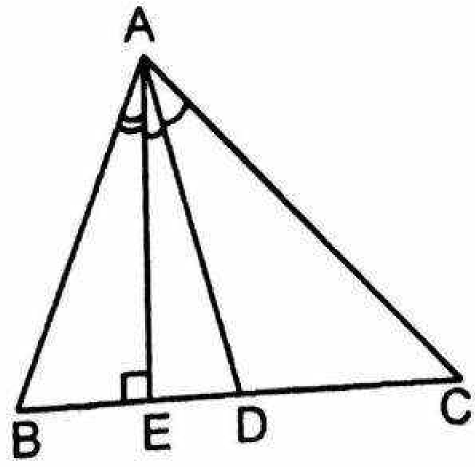
1. In a ΔABC, if the bisectors of $\angle B$ and $\angle C$ meet at O then $\angle BOC = 90° + \frac{1}{2}\angle A$



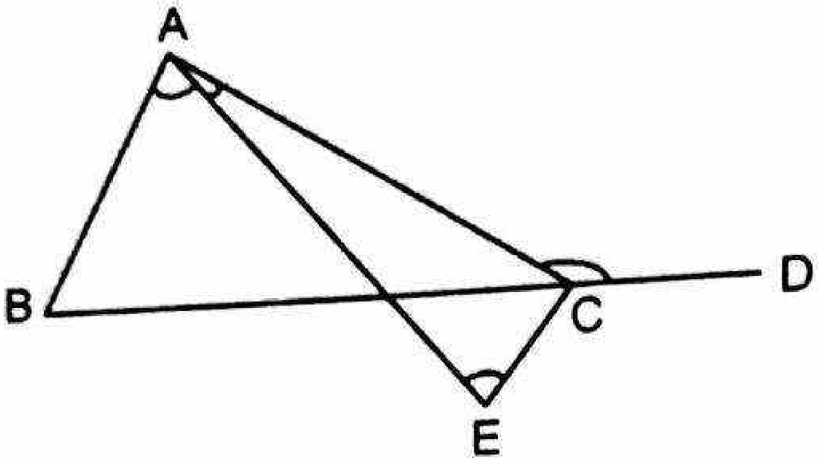
2. In a ΔABC, if sides AB and AC are produced to D and E respectively and the bisectiors of $\angle DBC$ and $\angle ECB$ intersect at O, then



3. In a ΔABC, if AD is the angle bisector of $\angle BAC$ and $AE \perp BC$, then $\angle DAE = \frac{1}{2}(\angle ABC - \angle ACB)$

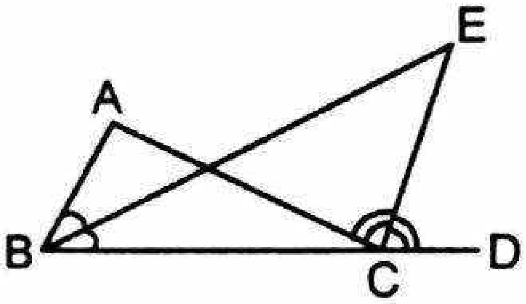


4. In a ΔABC, if BC is produced to D and AE is the angle bisector of $\angle A$, then



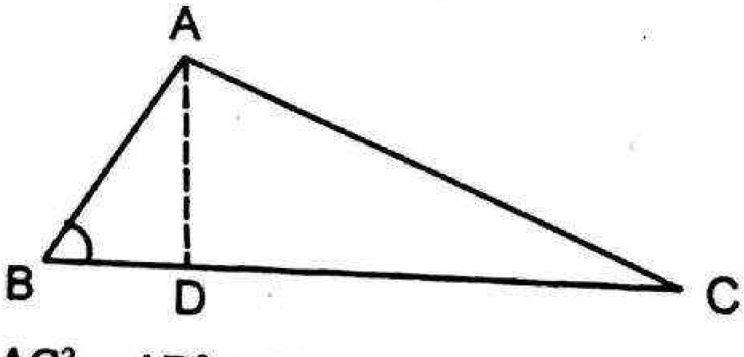
$\angle ABC + \angle ACD = 2\angle AEC$

5. In a Δ ABC, if side BC is produced to D and bisectors of $\angle ABC$ and $\angle ACD$ meet at E, then



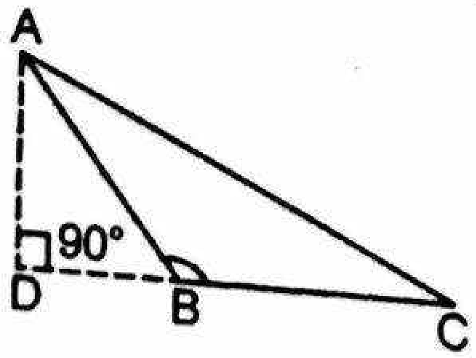
$\angle BEC = \frac{1}{2}\angle BAC$

6. In an acute angle Δ ABC, AD is a perpendicular dropped on the opposite side of $\angle A$, then



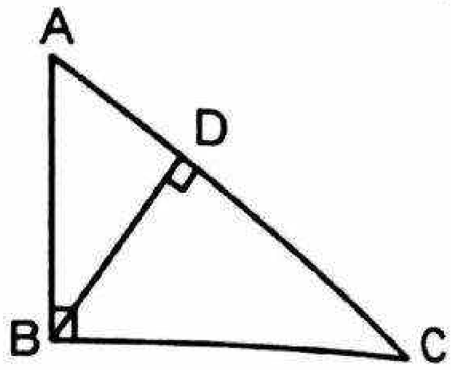
$AC^2 = AB^2 + BC^2 - 2\,BD.\,BC$ ($\angle B < 90°$)

7. In an obtuse angle Δ ABC, AD is perpendicular dropped on BC, BC is produced to D to meet AD, then



$AC^2 = AB^2 + BC^2 + 2BD.BC$ ($\angle B >$ 90°)

8. In a right angle Δ ABC, $\angle B = 90°$ and AC is hypotenuse. The perpendicular BD is dropped on hypotenuse AC from right angle vertex B, then



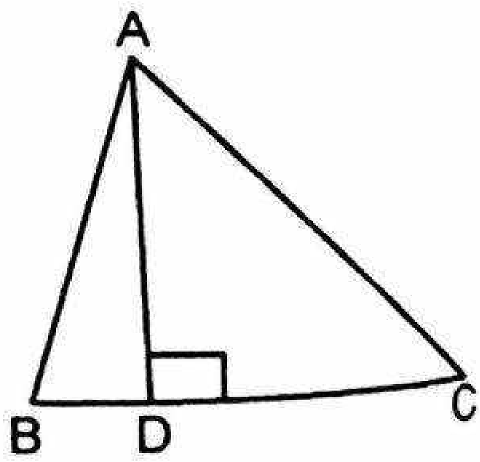
(a) $BD = \frac{AB \times BC}{AC}$

(b) $AD = \frac{AB^2}{AC}$

(c) $CD = \frac{BC^2}{AC}$

(d) $\frac{1}{BD^2} = \frac{1}{AB^2} + \frac{1}{BC^2}$

9. **Area of triangle (General formula)**

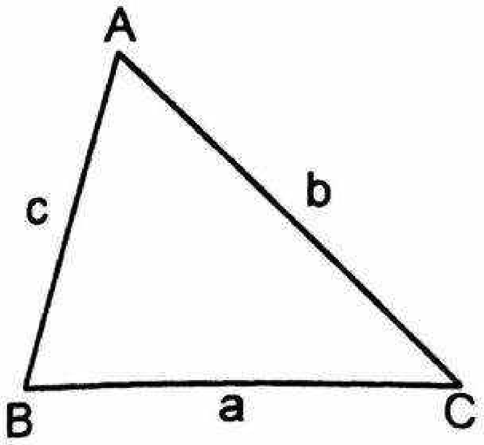


$A(\Delta) = \frac{1}{2} \times base \times height$

$A(\Delta) = \frac{1}{2} \times BC \times AD$

10. **Area of triangle =**

$$\sqrt{s(s-a)(s-b)(s-c)}$$



Also, $A(\Delta) = r \times s = \dfrac{abc}{4R}$

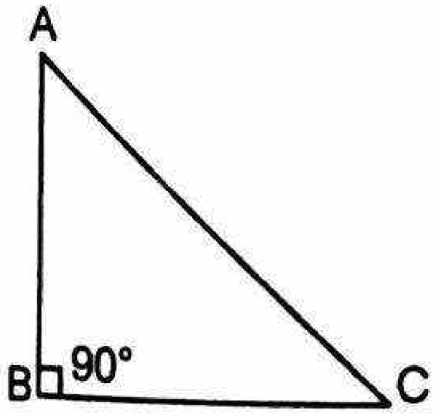
where, a, b and c are the sides of the triangle r ⊥ inradius

R ⊥ circumradius

S ⊥ semiperimeter = $\dfrac{a+b+c}{2}$

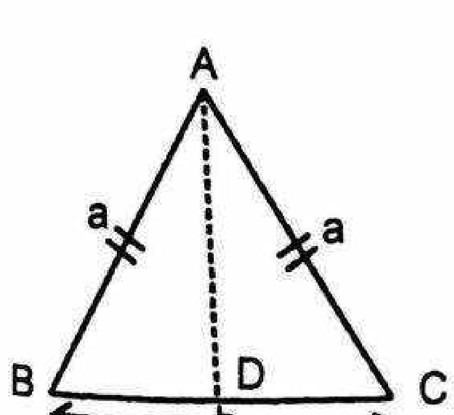
11. **Area of right angled triangle**

$$= \frac{1}{2} \times base \times height$$



$= \dfrac{1}{2} \times BC \times AB$ (as per the figure)

12. **Area of an isosceles triangle**



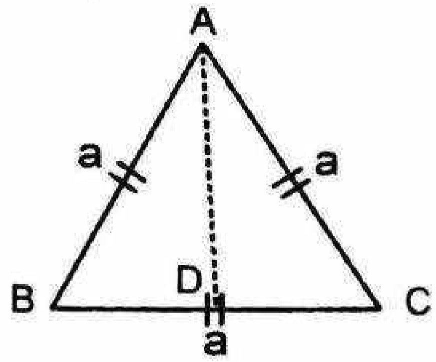
$$= \frac{b}{4}\sqrt{4a^2 - b^2}$$

AB = AC and $\angle B = \angle C$

$\Delta ABD \cong \Delta ACD$

13. **Area of an equilateral triangle**

$$= \frac{\sqrt{3}}{4}a^2$$

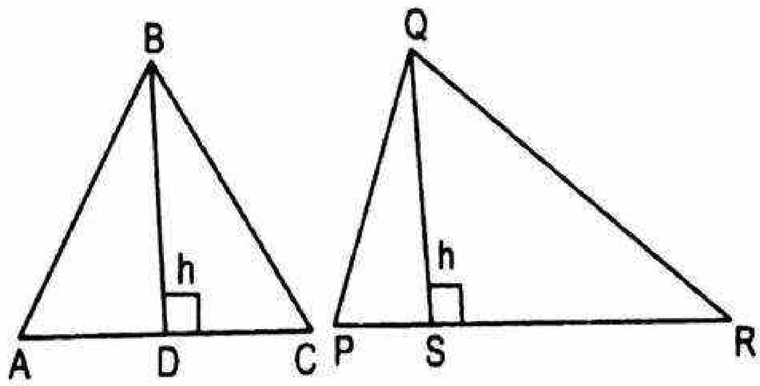


Note - (AD ⊥ Angle bisector, median, altitude and perpendicular bisector.)

14. For the given perimeter of a triangle, the area of equilateral triangle is maximum.

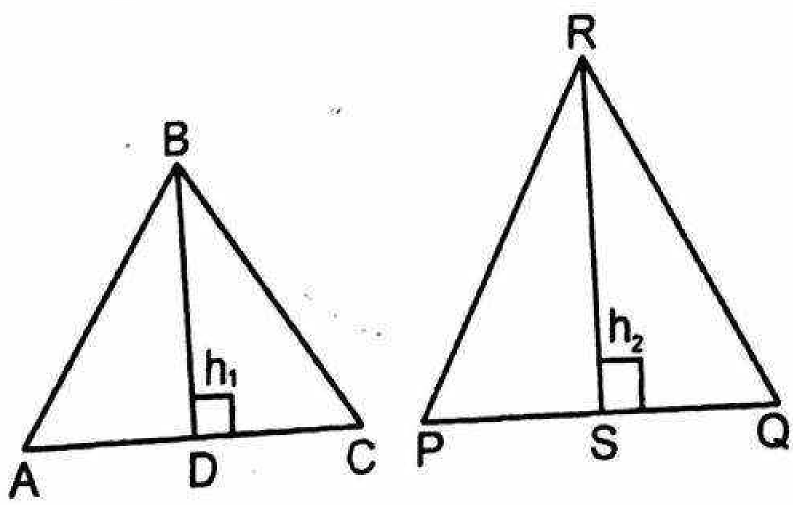
For the given area of a triangle, the perimeter of equilateral triangle is minimum.

15. The ratio of areas of two triangles of equal heights is equal to the ratio of their corresponding bases. i.e.,



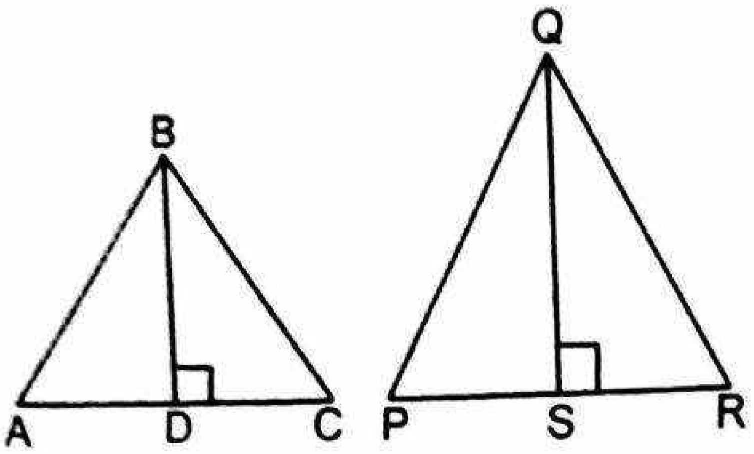
$$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{AC}{PR}$$

16. The ratio of areas of triangles of equal bases is equal to the ratio of their heights.



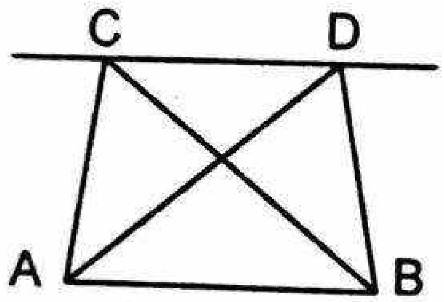
i.e. $\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{BD}{RS}$

17. The ratio of the areas of two triangles is equal to the ratio of products of base and its corresponding height i.e.,

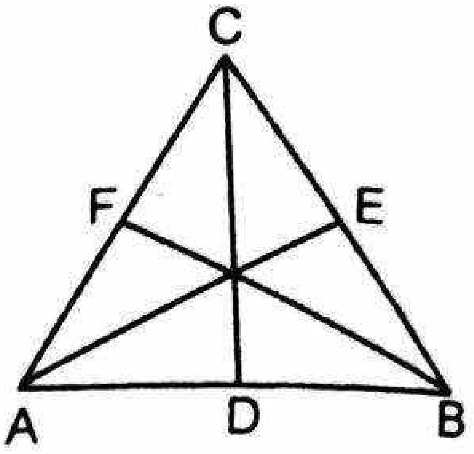


$$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{AC \times BD}{PR \times QS}$$

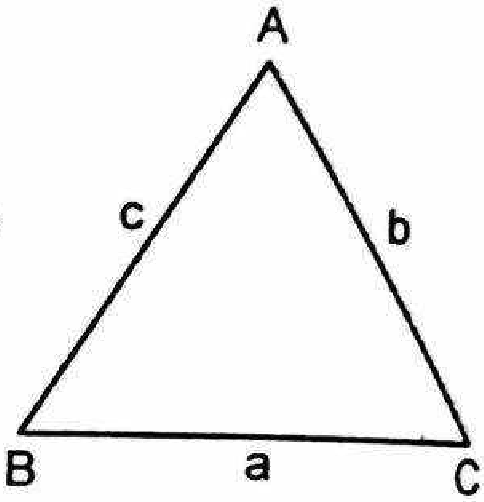
18. If the two triangles have the same base and lie between the same parallel lines (as shown in figure),then the area of two triangles will be equal.
i.e. $A(\Delta ABC) = A(\Delta ADB)$



19. In a tiangle AE, CD and BF are the medians then
$3(AB^2 + BC^2 + AC^2) = 4(CD^2 + BF^2 + AE^2)$

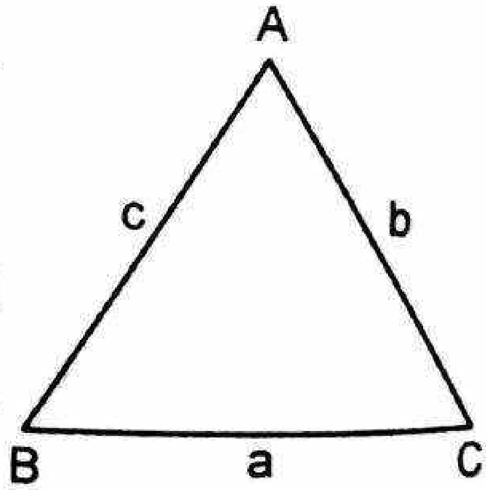


20. The sum of any two sides of a triangle is always greater than the third side :



i.e. $a + b > c$
$a + b > a$ and
$c + a > b$

21. The difference of any two sides of a triangle is always less than the third side i.e.



$a \sim b < c$
$a \sim b < a$ and
$c \sim a < b$

Exercise

LEVEL - 1

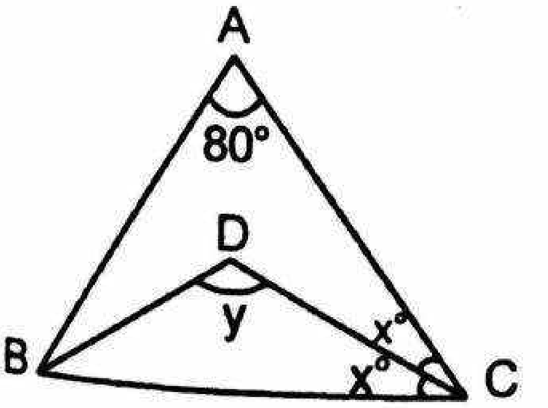
1. If any two sides of a triangle are produced beyond its base and the exterior angles thus obtained are bisected, then these bisectors will include :
(a) half the sum of the base angles
(b) sum of the base angles
(c) half the difference of the base angles
(d) difference of the base angles

2. If I is the in-centre of ΔABC and $\angle A = 60°$, then the value of $\angle BIC$ is:
(a) 100° (b) 120°
(c) 150° (d) 110°

3. In an obtuse-angled triangle ABC, $\angle A$ is the obtuse angle and O is the orthocentre. If $\angle BOC = 54°$, then $\angle BAC$ is :
(a) 108° (b) 126°
(c) 136° (d) 116°

4. ABC is an equilateral triangle. If a,b, and c denotes the lengths of perpendiculars from A,B and C respectively on the opposite sides then :
(a) $a \neq b \neq c$ (b) a = b = c
(c) a = b = 2c (d) a - b = c

5. In the given figure, $\angle A = 80°$, $\angle B = 60°$, $\angle C = 2x$ and $\angle BDC = y°$, BD and CD bisect angles B and C respectively. The value of x and y respectively are :



(a) 15° and 70° (b) 10° and 160°
(c) 20° and 130° (d) 20° and 125°

6. The sides of a right triangle containing the right angle measure 3 cm and 4 cm. The radius of the incircle of the triangle is :
(a) 3.5 cm (b) 1.75 cm
(c) 1 cm (d) 0.875 cm

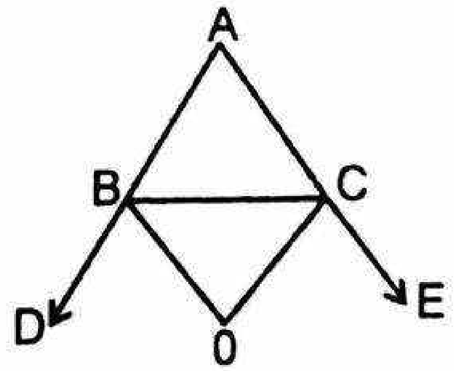
7. If the sides of a right-triangle are x, $x + 1$ and $x - 1$, then the hypotenuse is:
(a) 5 (b) 1.75 cm
(c) 1 (d) 0

8. The circum-centre of a triangle is always the point of intersection of the:
(a) Medians (b) bisectors
(c) Perpendiculars
(d) Perpendiculars dropped from the vertices on the opposite sides of the triangle.

9. The radius of circum-circle of an equilateral triangle of side 12cm is :
(a) $(4/3)\sqrt{3}$ (b) $4\sqrt{2}$
(c) $4\sqrt{3}$ (d) 4

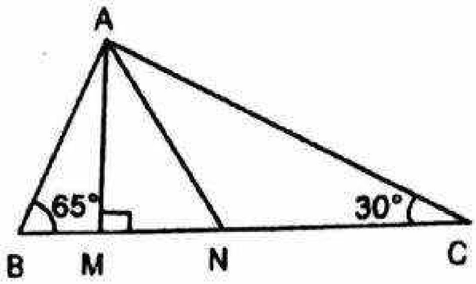
10. In ΔABC, $\angle B$ is a right angle, AC = 6cm, and D is the mid-point of AC. The length of BD is :
(a) 4 cm (b) $\sqrt{6}$ cm
(c) 3 cm (d) 3.5 cm

11. In the given figure, BO and Co are the bisector of $\angle CBD$ and $\angle BCE$ respectively and $\angle A = 40°$, then $\angle BOC$ is equal to :



(a) 60° (b) 65°
(c) 75° (d) 70°

12. In the given figure, AM ⊥ BC and AN is the bisector of ∠A. What si the measure of ∠MAN ?



(a) 17.5° (b) 15.5°
(c) 16° (d) 20°

13. If the bisector of an angle of Δ bisects the opposite side, then Δ is :
(a) Scalaene (b) Isosceles
(c) Right triangle
(d) None of these.

14. In the given figure, ∠BAD = ∠CAD, AB = 4cm, AC = 5.2cm, BD = 3cm.

Find BC :

17. In Δ ABC, the bisectors of ∠B and ∠C intersect each-other at a point O, then ∠BOC is equal to :

(a) $90° - \frac{1}{2}\angle A$

(b) $90° + \frac{1}{2}\angle A$

(c) $120° + \frac{1}{2}\angle A$

(d) $120° - \frac{1}{2}\angle A$

18. In Δ ABC, the sides AB and AC are produced to P and Q respectively. The bisectors of ∠PBC and ∠QCB intersect at a point O, th...

21. The sides of a triangle are 41cm, 40cm and 9cm respectively, the triangle is :
(a) acute (b) obtuse
(c) right
(d) can't be determined

22. The sides of a triangle are 13cm, 7cm and 9cm respectively, the triangle is:
(a) acute (b) obtuse
(c) right
(d) can't be determined

23. The three sides of a triangle are given which one of the following is not a right angle :
(a) 16, 63, 65 (b) 20,21,29
(c) 56,90,106 (d) 36,35,74

24. If a triangle, the circumcentre, incentre, centroid and othocentre coincide, then the triangle is :
(a) Isosceles
(b) Right-angled
(c) Equilateral
(d) Acute angled

25. O is the incentre of $\triangle$ ABC and $\angle$BOC = 130°. Find $\angle$BAC :
(a) 80° (b) 40°
(c) 150° (d) 50°

26. The internal bisector of $\angle$ABC and $\angle$ACB of $\triangle$ ABC meet each-other at O. If $\angle$BOC = 120°, then $\angle$BAC is equal to :
(a) 80° (b) 50°
(c) 60° (d) 90°

27. O is the incentre of $\triangle$ABC and $\angle$A = 30°,then $\angle$BOC is :
(a) 100° (b) 105°
(c) 110° (d) 90°

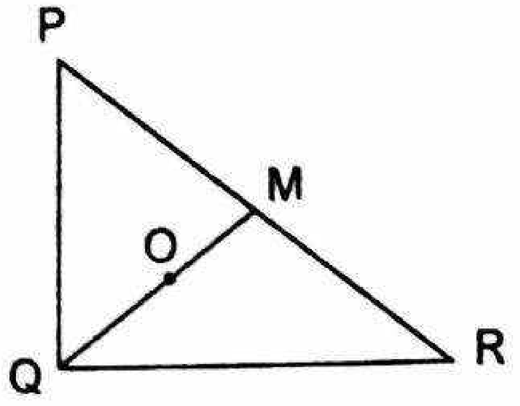
28. In $\triangle$ABC, AD is the internal bisector of $\angle$A, meeting the side BC at D. If BD = 5cm, BC = 7.5cm, then AB : AC is :
(a) 2 : 1 (b) 1 : 2
(c) 4 : 5 (d) 3 : 5

29. The external bisector of $\angle$B and $\angle$C of $\triangle$ABC meet at point P. If $\angle$BAC = 80°, then $\angle$BPC is :
(a) 50° (b) 40°
(c) 80° (d) 100°

30. PQR is a triangle such that PQ = 9cm and PR = 4cm the side QR is :
(a) equal to 5
(b) greater than 5
(c) less than 5
(d) None of these.

LEVEL - 2

1. If in the given figure, $\angle PQR = 90°$, O is the centroid of ΔPQR, PQ = 5cm and QR = 12cm, then OQ is equal to:



(a) $3\frac{1}{2}$ (b) $4\frac{1}{3}$

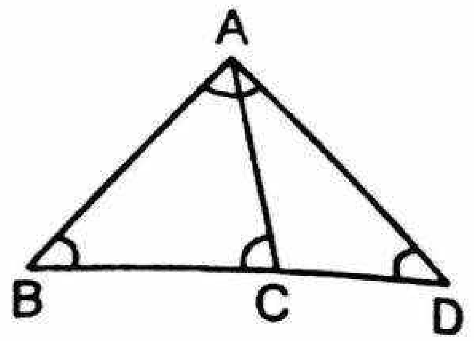
(c) $4\frac{1}{2}$ (d) $5\frac{1}{3}$

2. If P and Q are the mid- points of the sides CA and CB respectively of a triangle ABC, right-angled at C, then the value of $4(AQ^2 + BP^2)$ is equal to :
(a) $4BC^2$ (b) $2AC^2$
(c) $2BC^2$ (d) $5AB^2$

3. The medians AD, BE CF of a triangle ABC intersect in G. Which of the following istrue for any ΔABC ?
(a) GB + GC = 2GA
(b) GB + GC < GA
(c) GB + GC > GA
(d) GB + GC = GA

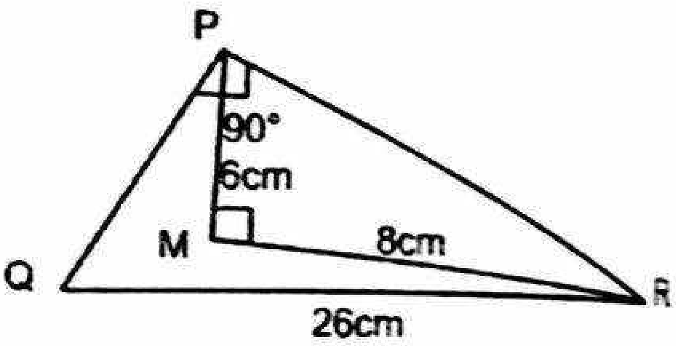
4. If a,b and c are the sides of a triangle and $a^2+b^2 + c^2 = ab + bc + ca$, then the triangle is :
(a) Equilateral (b) Isosceles
(c) Right-angled
(d) Obtuse-angle

5. In the given figure, $\angle B = \angle C = 55°$ and $\angle D = 25°$ then :



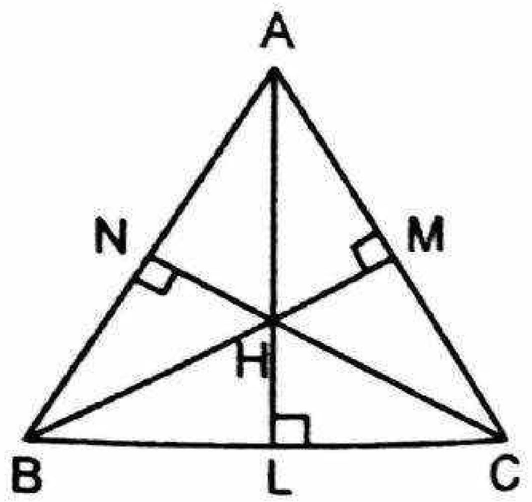
(a) BC < CA < CD
(b) BC > CA > CD
(c) BC < CA, CA > CD
(d) BC > CA, CA < CD

6. In the given figure $\angle QPR = 90°$, QR = 26 cm, PM = 6 cm, MR = 8cm and $\angle PMR = 90°$, find the area of ΔPQR



(a) 180 cm² (b) 240 cm²
(c) 120 cm² (d) 150 cm²

7. If H is the orthocentre of ΔABC, then the orthocentre of ΔHBC is (figure given) :



(a) N (b) A
(c) L (d) M

8. The point in the plane of a triangle which is at equal perpendicular distance from the sides of the triangle is :
(a) circumcentre (b) centroid
(c) incentre (d) orthocentre

9. The length of the two sides forming the right angle of a right angled triangle are 6cm and 8cm. The length of its circum-radius :

(a) 5 cm (b) 7 cm
(c) 6 cm (d) 10 cm

10. The equidistant point from the vertices of a tirangle is called its :

(a) centroid (b) incentre
(c) circumcentre (d) orthocentre

11. Internal bisectorsof $\angle B$ and $\angle C$ of a ΔABC intersect at O, if $\angle BOC = 102°$, then the value of $\angle BAC$ is :

(a) 12° (b) 24°
(c) 48° (d) 60°

12. If G is the centroid and AD be a median with length 12cm of ΔABC, then the value of AG is :

(a) 4 cm (b) 6 cm
(c) 10 cm (d) 8 cm

13. In a right-angled triangle ABC, AB = 2.5 cm, cos B = 0.5, $\angle ACB = 90°$. The length of side AC, in cm is:

(a) $5\sqrt{3}$ (b) $\frac{5}{2}\sqrt{3}$
(c) $\frac{5}{4}\sqrt{3}$ (d) $\frac{5}{16}\sqrt{3}$

14. The in-radius of an equilateral triangle is of length 3cm, Then the length of each of its medians is :

(a) 12 cm (b) $\frac{9}{2}$ cm
(c) 4cm (d) 9 cm

15. Two medians AD and BE of ΔABC intersect at G at right angles. If AD = 9cm and BE = 6cm, then the length of BD, in cm is :

(a) 10 (b) 6
(c) 5 (d) 3

16. In ΔABC, $\angle BAC = 90°$ adn AB $= \frac{1}{2}$ BC. Then the measure of $\angle ACB$ is :

(a) 60° (b) 30°
(c) 45° (d) 15°

17. In ΔABC, AD is the median and AD $= \frac{1}{2}$ BC. If $\angle BAD = 30$ is, then $\angle ACB$ is :

(a) 90° (b) 45°
(c) 30° (d) 60°

18. In ΔABC, $\angle B = 60°$, $\angle C = 40°$, If AD bisects $\angle BAC$ and $AE \perp BC$, then $\angle EAD$ is :

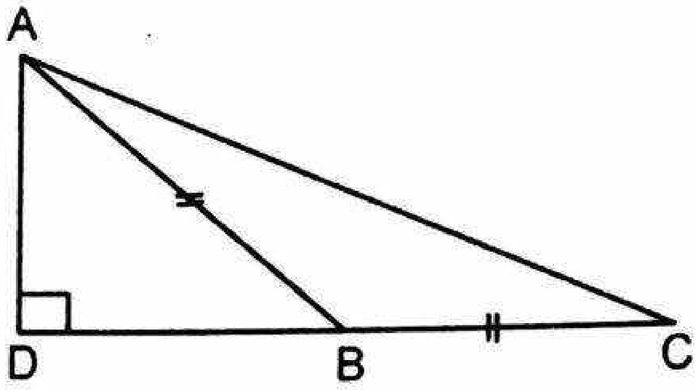
(a) 10° (b) 20°
(c) 40° (d) 80°

19. In ΔABC, $\angle B = 90°$, $\angle C = 45°$ and D is the mid-point of AC. If AC = $4\sqrt{2}$ units, then BD is :

(a) $2\sqrt{2}$ units (b) $4\sqrt{2}$
(c) $\frac{5}{2}$ units (d) 2 units

LEVEL - 3

1. In the given figure, AB = BC and $\angle BAC = 15°$, AB = 10cm. Find the area of ΔABC :



(a) 50 cm^2 (b) 40 cm^2
(c) 25 cm^2 (d) 32 cm^2

2. In ΔABC, G is the centroid, AB = 15cm, BC = 18cm, and AC = 25cm. Find GD, where D is th mid-point of BC :

(a) $\frac{1}{2}\sqrt{86}$ (b) $\frac{1}{3}\sqrt{86}$

(c) $\frac{7}{3}\sqrt{86}$ cm (d) $\frac{2}{3}\sqrt{86}$ cm

3. In a right angled triangle ABC, $\angle B$ is the right angle and AC = $2\sqrt{5}$ cm, If AB - BC = 2cm then the value of$(\cos^2A - \cos^2C)$ is :

(a) $\frac{2}{5}$ (b) $\frac{3}{5}$

(c) $\frac{6}{5}$ (d) $\frac{3}{10}$

4. If G is the centroid of ΔABC and AG = BC, then $\angle BGC$ is :

(a) 75° (b) 45°
(c) 90° (d) 60°

5. By decreasing 15° each angle of a triangle the ratios of their angles are 2 : 3 : 5, the radian measure for greatest angle is :

(a) $\frac{11\pi}{24}$ (b) $\frac{\pi}{12}$

(c) $\frac{\pi}{24}$ (d) $\frac{5\pi}{24}$

6. Two sides of a triangle are of length 4cm and 10cm. If the length of the third side is 'a' cm, then :

(a) a > 5 (b) $6 \leq a \leq 12$
(c) a < 6 (d) 6 < a <14

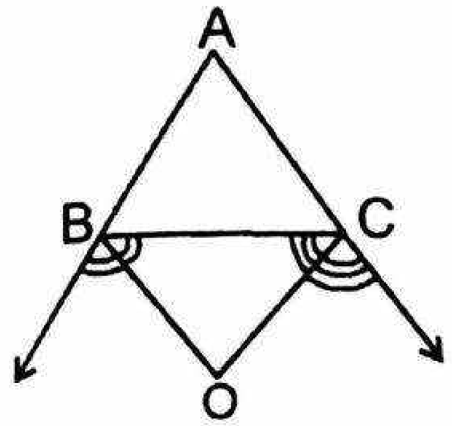
7. O is any point on the bisector of the acute angle $\angle ABC$. From O a line parallel to CB meets AB at P. Then ΔBPO is :

(a) right angled isosceles triangle
(b) isosceles but not a right angled
(c) equilateral triangle
(d) None of these

Hints and Solutions:

LEVEL-1

1.(a) $\angle BOC = 90^\circ - \frac{1}{2}\angle A$



$\angle A + \angle B + \angle C = 180^\circ$

$\Rightarrow \frac{1}{2}\angle A = 90^\circ - \frac{1}{2}(\angle B + \angle C)$

$\Rightarrow \frac{1}{2}(\angle B + \angle C) = 90^\circ - \frac{1}{2}\angle A$

$\therefore \angle BOC = \frac{1}{2}(\angle B + \angle C)$

2.(b) $\angle BIC = 90^\circ + \frac{A}{2} = 90^\circ + 30^\circ = 120^\circ$

3.(b) $\angle BAC = 180^\circ - \angle BOC = 180^\circ - 54^\circ$
$= 126^\circ$

5.(c) $\angle A + \angle B + \angle C = 180^\circ$

$\Rightarrow \angle C = 180^\circ - (\angle A + \angle B)$

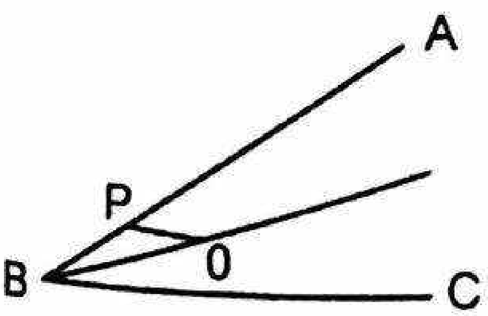
$\Rightarrow \angle C = 180^\circ - (80^\circ + 60^\circ) = 40^\circ$

$\Rightarrow 2x = 40 \Rightarrow x = 20^\circ \quad (\because C = 2x)$

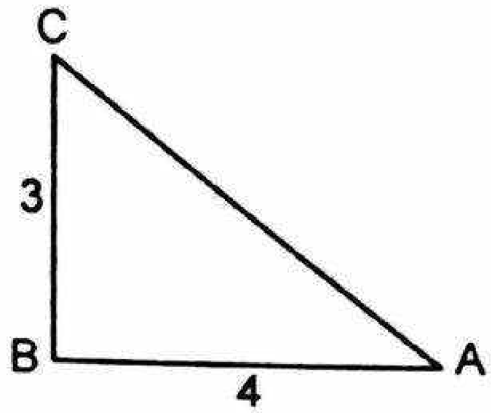
$y = 90^\circ + \frac{1}{2}\angle A = 90^\circ + \frac{1}{2}(80)$

$= 130^\circ$

6.(c)



a. scalane
b. isosceles
c. equilateral
d. right angled and isosceles



$AC = \sqrt{3^2 + 4^2} = 5cm$

$A = \text{area of } \Delta ABC = \frac{1}{2} \times 4 \times 3 = 6cm$

$s = \frac{3+4+5}{2} = 6cm$

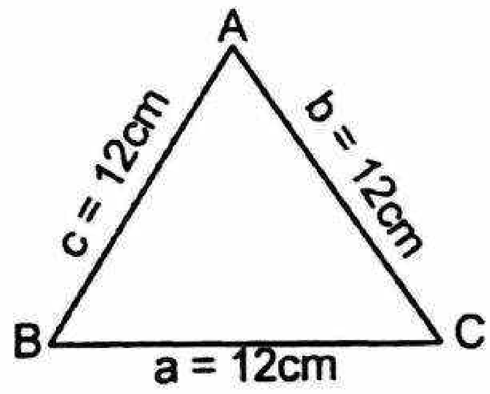
$\therefore r = A/S = 6/6 = 1cm$

7.(a) largest side = hypotenuse = $(x + 1)$

$\therefore (x + 1)^2 = x^2 + (x - 1)^2 \Rightarrow x = 4$

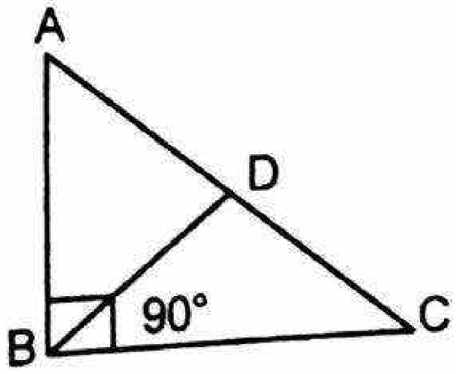
$\therefore$ hypotenuse = $x + 1 = 5$

9.(c)



$R = \frac{a}{\sqrt{3}} = \frac{12}{\sqrt{3}} = 4\sqrt{3}$

10.(c) In the right-angled triangle the length of median to the hypotenuse is half the length of the hypot enuse.



Hence, BD $= \frac{1}{2}$ AC = 3cm

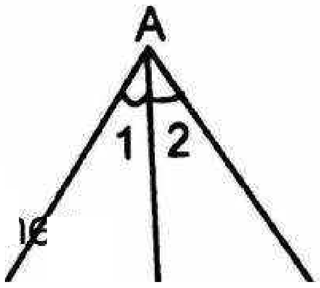
11.(d) $\angle BOC$

$= 90° - \frac{1}{2}\angle A = 90° - \frac{1}{2}(40) = 70°$

12.(a) $\angle MAN = \frac{1}{2}(\angle B - \angle C)$

$= \frac{1}{2}(65° - 30°) = \frac{1}{2}(35°) = 17.5°$

13.(b)

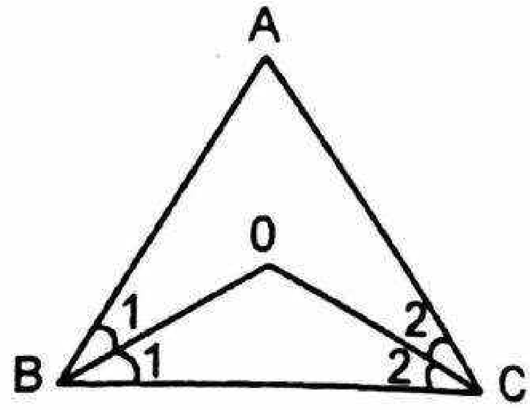


$AC = \sqrt{15^2 - 9^2} = \sqrt{144} = 12cm$

$BC = \sqrt{15^2 - 12^2} = \sqrt{81} = 9cm$

width of the street = AC + BC = 12 + 9 = 21 m

17.(b)



$A + B + C = 180°$

$\Rightarrow A + 2\angle 1 + 2\angle 2 = 180°$

$\Rightarrow \angle 1 + \angle 2 = \frac{1}{2}(180° - A) = 90°$.

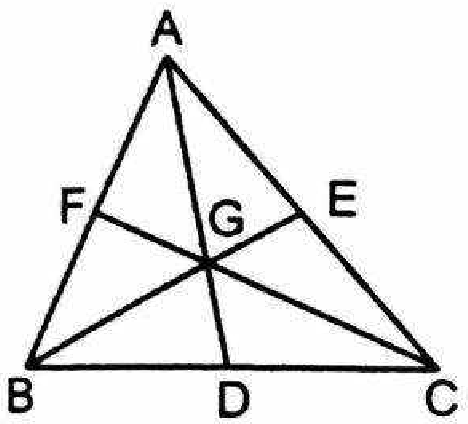
$= \frac{1}{2}[\angle B + \angle C) = \frac{1}{2}(180° - \angle A]$

$= 90° - \frac{1}{2}\angle A$

19.(d) We know that the centroid of a triangle divides each median in the ratio of 2 : 1

$\therefore$ BG : BE = 2 : 3

$\Rightarrow BE = \frac{3}{2}BG = \frac{3}{2} \times 6 = 9$ cm.



21.(c) $(41)^2 = (40)^2 + (9)^2 \Rightarrow$ right angle triangle

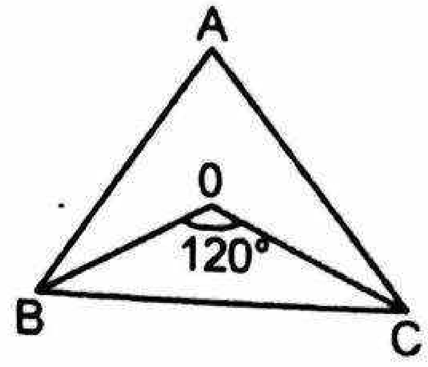
22.(b) $(13)^2 > (7)^2 + (9)^2 \Rightarrow$ obtuse triangle

23.(d) Apply pythagorus theorem or sum of two sides $\ngtr$ third side i.e. $36 + 35 \ngtr 74$

25.(a) $\angle BOC = 90° + \frac{1}{2}(\angle BAC)$

$\Rightarrow \angle BAC = (130 - 90) \times 2 = 80°$

26.(c)



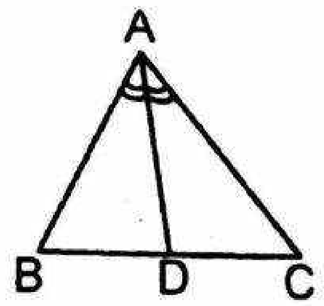
$\angle BOC = 90° + \frac{1}{2}(\angle BAC)$

$\Rightarrow \angle BAC = (120° - 90°) \times 2 = 60°$

27.(b) $\angle BOC = 90° + \frac{1}{2}(\angle A) = 105°$

28.(a) $\frac{AB}{AC} = \frac{BD}{DC} = \frac{5}{7.5-5} = \frac{50}{25} = \frac{2}{1}$

= 2 : 1



29.(a) $\angle BPC = 90° - \frac{A}{2}$

$= 90° - 40°$

$= 50°$

30.(b) QR > PQ - PR

$\Rightarrow$ QR > 9 - 4 $\Rightarrow$ QR > 5

LEVEL-2

1.(b) By Pythagoras theorem,
PR =

$\sqrt{PQ^2 + QR^2} = \sqrt{5^2 + 12^2} = 13cm$

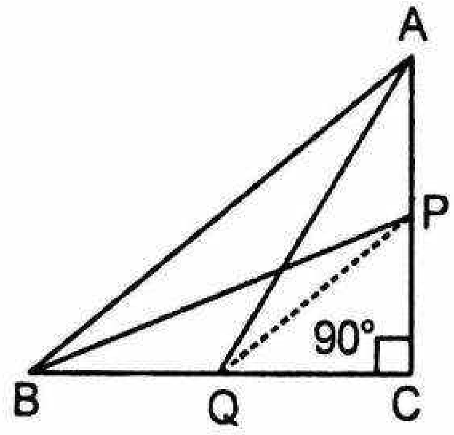
$\because$ O is the centroid $\Rightarrow$ QM is median and M is the mid-point of PR .

$\therefore$ QM = PM = $\frac{13}{2}$

$\because$ centroid divides median in ratio 2 : 1

$\therefore$ OQ = $\frac{2}{3}$QM = $\frac{2}{3} \times \frac{13}{2} = \frac{13}{3} = 4\frac{1}{3}cm$

2.(d)



$AQ^2 = AC^2 + QC^2$
$BP^2 = BC^2 + CP^2$
$AQ^2 + BP^2 = (AC^2 + BC^2) + (QC^2 + CP^2)$

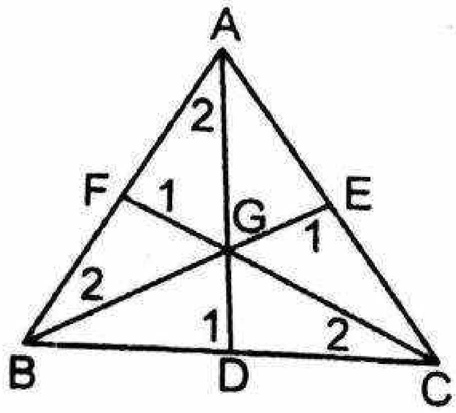
$= AB^2 + \left(\frac{BC}{2}\right)^2 + \left(\frac{AC}{2}\right)^2$

$= AB^2 + \frac{1}{4}(BC^2 + AC^2)$

$= AB^2 + \frac{1}{4}AB^2 = \frac{5}{4}AB^2$

$\Rightarrow 4(AQ^2 + BP^2) = 5\ AB^2$

3.(c)



4.(a) $a^2 + b^2 + c^2 = ab + bc + ca$ (given)

$\Rightarrow a^2 + b^2 + c^2 - ab - bc - ca = 0$

$\Rightarrow 2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca = 0$

$\Rightarrow (a - b)^2 + (b - c)^2 + (c - a)^2 = 0$

$\therefore$ sum of perfect square is 0, $\therefore$ all of them are 0.

$\Rightarrow a - b = 0 = b - c = c - a$

$\Rightarrow a = b = c$

$\therefore$ triangle is equilateral.

5.(d) $\angle B = \angle C \Rightarrow AB = BC$

$\angle CAD = 30°$

$\therefore \angle CAD > \angle CDA \Rightarrow CD > AC$

(In a triangle, greater angle has longer side opposite to it)

$\angle BAC = 180° - 110° = 70° > \angle ABC$

$\Rightarrow$ BC > AB and BC > AC

$\therefore$ BC > CA and CA < CD.

6.(c) PR= $\sqrt{PM^2 + MR^2} = \sqrt{36+64} = 10cm$

PQ = $\sqrt{QR^2 - PR^2} = \sqrt{26^2 - 10^2} = 24cm$

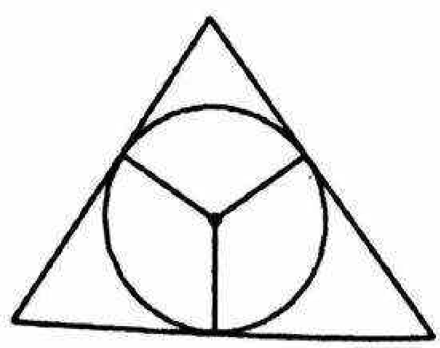
$\therefore$ ar (ΔPQR) = $\frac{1}{2}$(PR) (PQ)

$= \frac{1}{2} \times 10 \times 24 = 120cm^2$

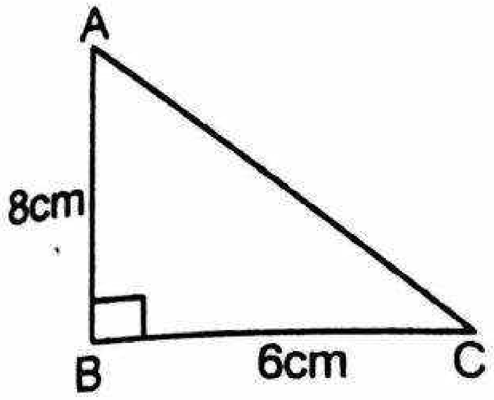
7.(b) In ΔABC, $HL \perp BC$ and $BN \perp CH$

Thus, the two altitudes HI and BN of ΔHBC, intersect at A.

8.(c)



9.(a)



AC = $\sqrt{6^2+8^2} = 10cm.$

$\therefore$ circum radius = $\frac{10.}{2}$ = 5cm.

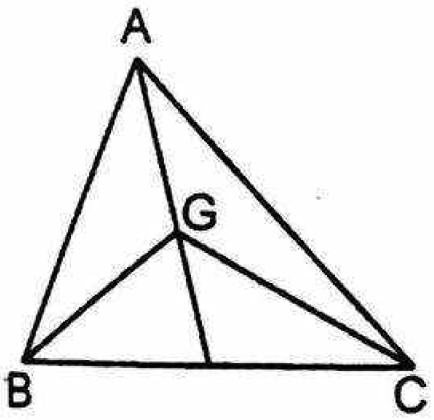
i.e. mid-point of hypotenuse.

10.(c) The right bisector of sides meet at a point called 'circumcentre'.

11.(b) $\angle BOC = 90° + \frac{1}{2}(\angle BAC)$

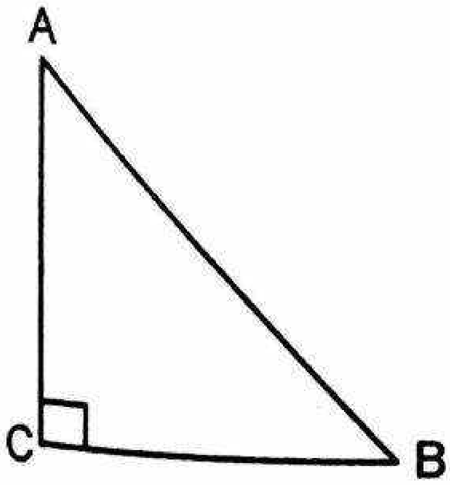
$\Rightarrow \angle BAC = (102° - 90°) \times 2 = 24°$

12.(d)



AG = $\frac{2}{3}$ AD = $\frac{2}{3} \times 12$ = 8cm.

13.(c)



Cos B = 0.5 = $\frac{1}{2}$

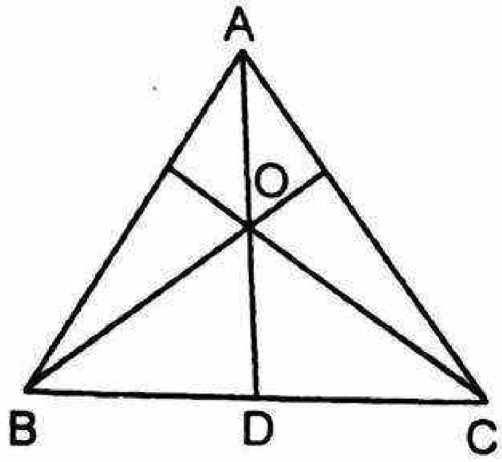
$\therefore$ Sin B =

$$\sqrt{1-\cos^2 B} = \sqrt{1-\frac{1}{4}} = \frac{\sqrt{3}}{2}$$

$\therefore$ Sin B = $\frac{AC}{BC} \Rightarrow$ AC = AB sin B

$$= 2.5 \times \frac{\sqrt{3}}{2} = \frac{5\sqrt{3}}{4}$$

14.(d)

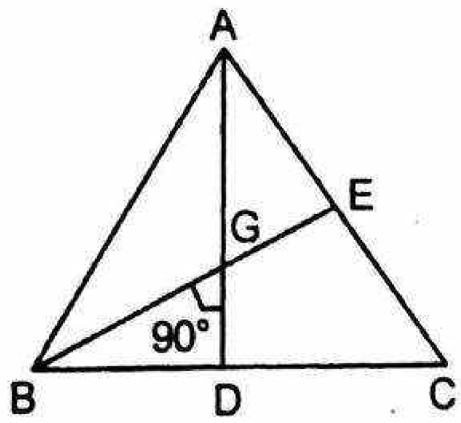


In equilateral triangle centroid, incentre, orthocentre concide at the same point.

$\therefore$ in-radius = OD = 3 cm

$\therefore$ $\frac{AD}{3}$ = 3cm $\Rightarrow AD = 9$cm = median.

15.(c)

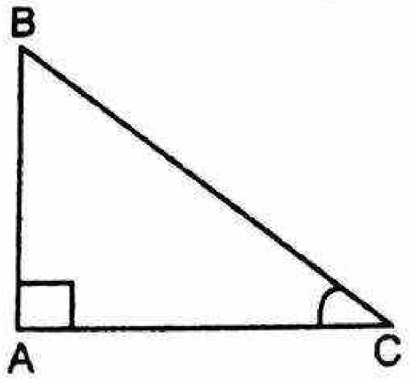


AD = 9cm $\Rightarrow$ GD = $\frac{1}{3} \times 9$ = 3cm

BE = 6cm $\Rightarrow$ BG = $\frac{2}{3} \times 6$ = 4cm

$\therefore$ BD = $\sqrt{3^2+4^2}$ = 5cm.

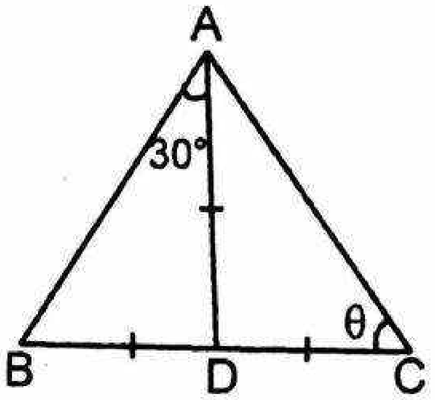
16.(b)



$\sin C = \frac{AB}{BC} = \frac{1}{2}$

$\Rightarrow \angle C = 30° \Rightarrow \angle ACB = 30°$

17.(d)



BD = CD = AD

$\because$ $\angle BAD = 30°$

Now in $\triangle ABD$

$\angle ABD = 30°$ ($\because$ BD = AD)

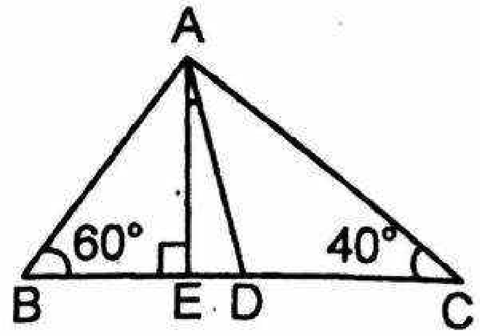
$\because$ AD = DC $\therefore \angle DAC = \angle DCA =$ θ (let)

Now, $\angle A + \angle B + \angle C = 180°$

$\Rightarrow 30° + \theta + 30° + \theta = 180°$

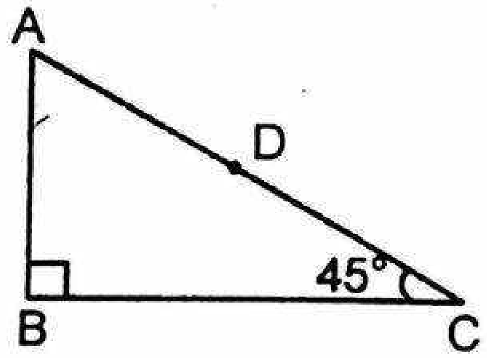
$\Rightarrow 2\theta = 120° \Rightarrow \theta = \angle ACB = 60°$

18.(a)



$\angle EAD = \frac{1}{2}(60° - 40°) = 10°$

19.(a)



BD = AD = CD (mid-point of hypotenuse is circumcentre.)

$\therefore$ $BD = \frac{1}{2}(4\sqrt{2}) = 2\sqrt{2}$ units.

LEVEL-3

1.(c) $\angle BAC = 15°$

$\angle BCA = 15°$ ($\because AB = BC$)

$\therefore \angle ABC = 180° - (15°+15°) = 150°$

$\therefore \angle ABD = 30°$

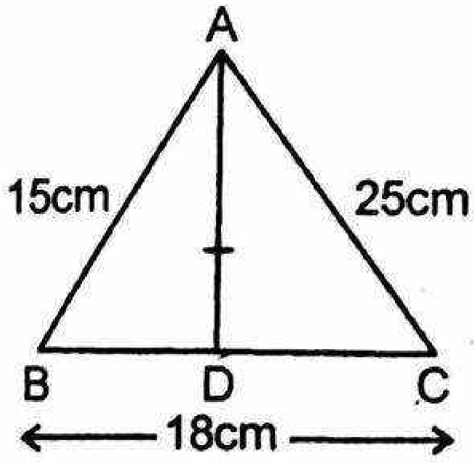
$\therefore \sin 30° =$

$$\frac{AD}{AB} \Rightarrow \frac{1}{2} = \frac{AD}{10} \Rightarrow AD = 5cm.$$

$\therefore$ Area of ΔABC =

$$\frac{1}{2} \times BC \times AD = \frac{1}{2} \times 10 \times 5 = 25cm^2$$

2.(d)



$\angle AB^2 + AC^2 = 2(AD^2 + BD^2)$

$\Rightarrow 225 + 625 = 2(AD^2 + 81)$

$\Rightarrow AD^2 = 344$

$AD = 2\sqrt{86}$ and $GD = \frac{1}{3}AD$

$\Rightarrow GD = \frac{2}{3}\sqrt{86} cm$

3.(b) Let $BC = x$ $\therefore AB = x + 2$

$AC^2 = AB^2 + BC^2$

$\Rightarrow (2\sqrt{5})^2 = (x+2)^2 + x^2$

$\Rightarrow 20 = x^2 + 4 + 4x + x^2$

$\Rightarrow 2x^2 + 4x - 16 = 0$

$\Rightarrow x^2 + 2x - 8 = 0$

$\Rightarrow (x - 2)(x + 4) = 0$

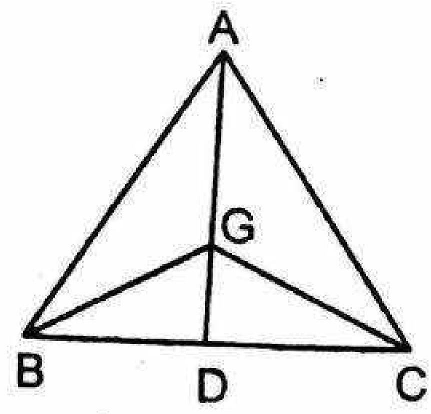
$\Rightarrow x = 2 = BC$

$\therefore AB = 2 + 2 = 4cm$

$\therefore \cos^2 A - \cos^2 C =$

$$\left(\frac{AB}{AC}\right)^2 - \left(\frac{BC}{AC}\right)^2 = \frac{16}{20} - \frac{4}{20} = \frac{12}{20} = \frac{3}{5}$$

4.(c)



$AG = BC = 2x$ (let)

$\therefore GD = x$ ($\because$ centroid divides median in 2 : 1)

Now in ΔBDG, $BD = GD = x$

$\therefore \angle DBG = \angle BGD = \theta_1$ (let)

Similarly in ΔDGC, $CD = GD = x$

$\therefore \angle DCG = \angle DGC = \theta_2$ (let)

$\therefore \angle BGC = \theta_1 + \theta_2$

Now in $\Delta BGC = \theta_1 + \theta_2 + (\theta_1 + \theta_2) = 180°$

$\Rightarrow \theta_1 + \theta_2 = 90°$

$\Rightarrow \angle BGC = 90°$

5.(a) $2x + 3x + 5x = 180° - 45° = 135°$

$$\Rightarrow x = \frac{135}{10} = \frac{27}{2}$$

$\therefore$ largest angle $= 5x + 15° =$

$$\left(5 \times \frac{27}{2}\right)° + 15° = \frac{165°}{2}$$

$\because 180° = \pi$ radian

$$\therefore \frac{165°}{2} = \frac{\pi}{180} \times \frac{165}{2} = \frac{11\pi}{24} radian$$

6.(d) The sum of any two sides of a triangle is greater than third side and their difference is less than third side .

$\therefore$ $a > 10 - 4 \Rightarrow a > 6$

Again $a < 10 + 4 \Rightarrow$ $a < 14$

$\therefore$ $6 < a < 14$

7.(b) $\because$ $OP \parallel BC$

$\Rightarrow$ $\angle POB = \angle OBC$

$\Rightarrow$ $\angle PBO = \angle POB$ $\because$ OB is the bisector of $\angle B$.

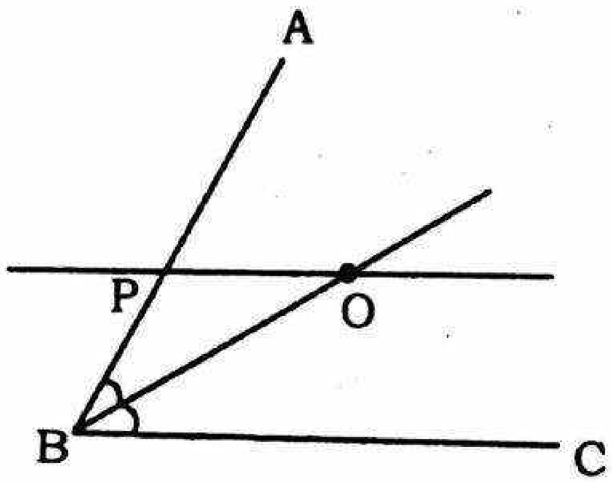
$\therefore$ $PB = PO$

$\because$ $\angle ABC$ is an acute angle

or $\angle ABC < 90°$

$\therefore$ $\frac{1}{2}\angle ABC < 45°$

$\Rightarrow$ $\angle POB = \angle PBO < 45°$



$\therefore$ $\angle BPO > 90°$

Hence, Δ PBO is isosceles Δ but not a right-angled triangle.

Answer Key

LEVEL - 1

1. (a)	2. (b)	3. (b)
4. (b)	5. (c)	6. (c)
7. (a)	8. (b)	9. (c)
10. (c)	11. (d)	12. (a)
13. (b)	14. (a)	15. (d)
16. (c)	17. (b)	18. (a)
19. (d)	20. (d)	21. (c)
22. (b)	23. (d)	24. (c)
25. (a)	26. (c)	27. (b)
28. (a)	29. (a)	30. (b)

LEVEL - 2

1. (b)	2. (d)	3. (c)
4. (a)	5. (d)	6. (c)
7. (b)	8. (c)	9. (a)
10. (c)	11. (b)	12. (d)
13. (c)	14. (d)	15. (c)
16. (b)	17. (d)	18. (a)
19. (a)		

LEVEL - 3

1. (c)	2. (d)	3. (b)
4. (c)	5. (a)	6. (d)
7. (b)		