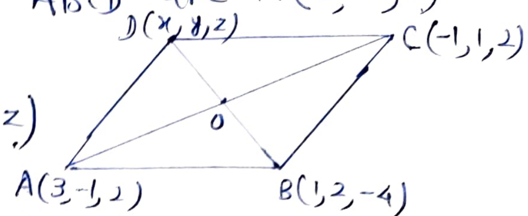
Q.No1. Three vertices of a parallelogram ABCD are $A(3,-1,2)$ $B(1,2,-4)$ and $C(-1,1,2)$. Find the coordinates of fourth vertex.

Sol Three vertices of parallelogram ABCD are $A(3,-1,2)$ $B(1,2,-4)$ and $C(-1,1,2)$



Let fourth vertex D be (x, y, z)

Since ABCD is a llgm.

$\therefore$ Mid point of both the diagonals will be same

ie Mid-point of AC = Mid-point BD.

$$\therefore \left(\frac{3-1}{2}, \frac{-1+1}{2}, \frac{2+2}{2}\right) = \left(\frac{1+x}{2}, \frac{2+y}{2}, \frac{-4+z}{2}\right)$$

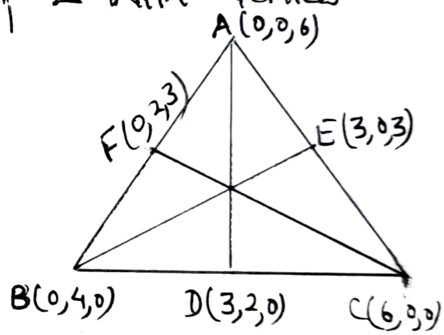
$$\Rightarrow (1, 0, 2) = \left(\frac{1+x}{2}, \frac{2+y}{2}, \frac{-4+z}{2}\right)$$

$$\Rightarrow 1+x = 2 \quad ; \quad 2+y = 0 \quad ; \quad -4+z = 4.$$

$$\Rightarrow x = 1 \quad ; \quad y = -2 \quad ; \quad z = 8$$

$\therefore$ D is $(1, -2, 8)$

Q.No.2: Find the lengths of the medians of Δ with vertices $A(0,0,6)$; $B(0,4,0)$; $C(6,0,0)$



Sol: Vertices of Δ ABC are

$A(0,0,6)$; $B(0,4,0)$; $C(6,0,0)$

Let D, E, F be mid points of BC, CD and AB respectively.

$\therefore$ D is $\left(\frac{0+6}{2}, \frac{4+0}{2}, \frac{0+0}{2}\right)$ ie $(3,2,0)$

E is $\left(\frac{6+0}{2}, \frac{0+0}{2}, \frac{0+6}{2}\right)$ ie $(3,0,3)$

F is $\left(\frac{0+0}{2}, \frac{0+4}{2}, \frac{6+0}{2}\right)$ ie $(0,2,3)$

$\therefore$ Length of Median AD $= \sqrt{(3-0)^2+(2-0)^2+(0-6)^2} = \sqrt{9+4+36} = \sqrt{49} = 7$

Median BE $= \sqrt{(3-0)^2+(0-4)^2+(3-0)^2} = \sqrt{9+16+9} = \sqrt{34}$

Median CF $= \sqrt{(0-6)^2+(2-0)^2+(3-0)^2} = \sqrt{36+4+9} = \sqrt{49} = 7.$

$\therefore$ Lengths of medians are $7, \sqrt{34}, 7$.

Q No 3: If the origin is the centroid of the ΔPQR with vertices $P(2a, 2, 6)$; $Q(-4, 3b, -10)$ and $R(8, 14, 2c)$, then find values of a, b, c.

Sol. Since origin $O(0,0,0)$ is the centroid of ΔPQR with vertices $P(2a, 2, 6)$; $Q(-4, 3b, -10)$ and $R(8, 14, 2c)$

$\therefore \dfrac{2a-4+8}{3} = 0$; $\dfrac{2+3b+14}{3} = 0$; $\dfrac{6-10+2c}{3} = 0$ [Using Centroid formula

ie $2a + 4 = 0$; $3b+16 = 0$; $2c - 4 = 0$

$\therefore 2a = -4$; $3b = -16$; $2c = 4$

$\therefore a = -2$; $b = -\dfrac{16}{3}$; $c = 2.$

Q No 4. Find the coordinates of a point on y-axis which are at a distance of $5\sqrt{2}$ from the point $P(3, -2, 5)$

Sol: Let $R(0, y, 0)$ be point on y-axis

$\therefore$ Distance of $R(0, y, 0)$ from $P(3, -2, 5)$ is $5\sqrt{2}$

ie $RP = 5\sqrt{2}$

$\Rightarrow RP^2 = 50.$

$\Rightarrow (3-0)^2 + (-2-y)^2 + (5-0)^2 = 50$

$\therefore 9 + y^2 + 4y + 4 + 25 = 50$

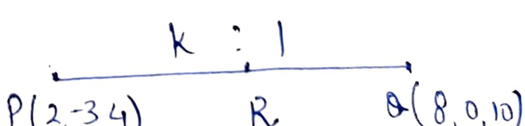
$\therefore y^2 + 4y - 12 = 0 \Rightarrow (y-2)(y+6) = 0$

$\Rightarrow y = 2, -6$

$\therefore$ Required point is $(0, 2, 0)$ or $(0, -6, 0)$

Q No 5; A point R with x-coordinate 4 lies on the line segment joining the points $P(2, -3, 4)$; $Q(8, 0, 10)$. Find the coordinates of R.

Sol: Let R divides the line segment PQ in the ratio $k:1$



$\therefore R$ is $\left(\dfrac{8k+2}{k+1}, \dfrac{-3}{k+1}, \dfrac{10k+4}{k+1}\right)$

From the given condition $\frac{8K+2}{K+1} = 4$. $\therefore 8K+2 = 4K+4$

$\Rightarrow 4K = 2 \Rightarrow K = \frac{1}{2}$

$\therefore$ R is $\left(4, \frac{-3}{\frac{1}{2}+1}, \frac{10\times\frac{1}{2}+4}{\frac{1}{2}+1}\right)$ i.e. $(4, -2, 6)$.

Q No 6. If A and B be the points $(3,4,5)$ and $(-1,3,-7)$ resp. find equation of the set of points P such that $PA^2 + PB^2 = K^2$; where K is any constant.

Sol: Given points are $A(3,4,5)$ and $B(-1,3,-7)$ and let $P(x,y,z)$

From given condition

$$PA^2 + PB^2 = K^2$$

$$\therefore (x-3)^2 + (y-4)^2 + (z-5)^2 + (x+1)^2 + (y-3)^2 + (z+7)^2 = K^2$$

$$\therefore x^2 + 9 - 6x + y^2 + 16 - 8y + z^2 + 25 - 10z + x^2 + 1 + 2x + y^2 + 9 - 6y + z^2 + 49 + 14z = K^2$$

$$\therefore 2x^2 + 2y^2 + 2z^2 - 4x - 14y + 4z = K^2 - 100$$

$$\therefore x^2 + y^2 + z^2 - 2x - 7y + 2z = \frac{K^2 - 100}{2}.$$

Which is required equation

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