Polynomials

MATHEMATICAL REASONING

1. $px^3 + qx^2 + rx + s = 0$ is said to be cubic polynomial, if _____.
(a) $s \neq 0$ (b) $r \neq 0$
(c) $q \neq 0$ (d) $p \neq 0$

2. If sum of all zeros of the polynomial $5x^2 - (3+k)x + 7$ is zero, then zeroes of the polynomial $2x^2 - 2(k+11)x + 30$ are
(a) 3, 5
(b) 7, 9
(c) 3, 6
(d) 2, 5

3. If the sum of the product of the zeroes taken two at a time of the polynomial $f(x) = 2x^3 - 3x^2 + 4tx - 5$ is -8, then the value of t is _____.
(a) 2 (b) 4
(c) -2 (d) -4

4. if a and b are the roots of the quadratic equation $x^2 + px + 12 = 0$ with the condition $a - b = 1$, then the value of 'p' is _____.
(a) 1
(b) 7
(c) -7
(d) 7 or -7

5. What will be the value of p(3), if 3 is one I of zeroes of polynomial $p(x) = x^3 + bx + D$?
(a) 3 (b) D
(c) 27 (d) 0

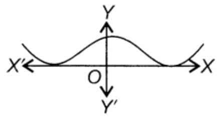
6. A cubic polynomial with sum of its zeroes, sum of the product of its zeroes taken two at a time and the product of its zeroes as $-3,\ 8,\ 4$ respectively, is _____.
(a) $x^3 - 3x^2 - 8x - 4$
(b) $x^3 + 3x^2 - 8x - 4$
(c) $x^3 + 3x^2 + 8x - 4$
(d) $x^3 - 3x^2 - 8x + 4$

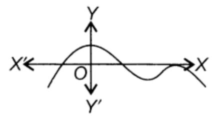
7. If p, q are the zeroes of the polynomial $f(x) = x^2 + k(x-1) - c$, then $(p-1)(q-1)$ is equal to _____.
(a) $c-1$ (b) $1-c$
(c) c (d) $1+c$

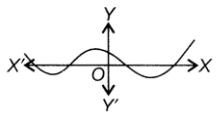
8. When $x^3 - 3x^2 + 3x + 5$ is divided by $x^2 - x + 1$, the quotient and remainder are
(a) $x+2,\ 7$ (b) $x-2, -7$
(c) $x-2,\ 7$ (d) $x+2, -7$

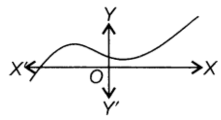
9. What should be subtracted from $f(x) = 6x^3 + 11x^2 - 39x - 65$ so that f(x) is exactly divisible by $x^2 + x - 1$?
(a) $38x + 60$ (b) $-38x - 60$
(c) $-19x - 30$ (d) $9x + 10$

10. Which of the following graph has more than three distinct real roots?

(a) 

(b) 

(c) 

(d) 

11. If one zero of the polynomial $f(x) = (k^2+4)x^2 + 13x + 4k$ is reciprocal of the other, then k is equal to ____.
(a) 2 (b) -2
(c) 1 (d) -1

12. A polynomial of the form $ax^5 + bx^3 + cx^2 + dx + e$ has at most ____ zeroes.
(a) 3 (b) 5
(c) 7 (d) 11

13. If α and β are the roots of the equation $2x^2-7x+8=0$, then the equation whose roots are $(3\alpha-4\beta)$ and $(3\beta-4\alpha)$ is____.

(a) $2x^2+7x+98=0$

(b) $x^2+7x+98=0$

(c) $2x^2-7x-98=0$

(d) $2x^2-7x+98=0$

14. For x^2+2x+5 to be a factor of $x^4+\alpha x^2+\beta$, the values of α and β should respectively be_____.

(a) 2, 5

(b) 5, 25

(c) 6, 25

(d) 5, 2

15. If α, β be two zeroes of the quadratic polynomial $ax^2+bx-c=0$, then find the value of $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}$.

(a) $\frac{b^2-2ac}{a^2}$

(b) $\frac{3abc-b^3}{c^3}$

(c) $\frac{3abc-b^3}{a^2c}$

(d) $\frac{b^3+3abc}{a^2c}$

EVERYDAY MATHEMATICS

16. Area of a triangular field is $(x^4-6x^3-26x^2+138x-35)m^2$ and base of the triangular field is $(x^2-4x+1)m$. Find the height of the triangular field.

(a) $2(x^2-2x-35)m$

(b) $\frac{1}{2}(x^2-2x-35)m$

(c) $2(3x^2-x-4)m$

(d) $\frac{1}{2}(3x^2-x-4)m$

17. A rectangular garden of length $(2x^3+5x^2-7)$ m has the perimeter $(4x^3-2x^2+4)m.$. Find the breadth of the garden.

(a) $(6x^2-9)m$

(b) $(-6x^2+9)m$

(c) $(2x^3-7x^2+11)m$

(d) $(6x^3+7x^2+9)m$

18. Raghav had $Rs.(6x^3+2x^2+3x)$ and he bought $(4x^2+3)$ shirts. The price of each shirt is $Rs.(x+5)$. How much money is left with Raghav?

(a) $Rs.(2x^3-18x^2-15)$

(b) $Rs.(4x^2+2x+3)$

(c) $Rs.(x^3-3x)$

(d) $Rs.(2x^3+2x^2-15)$

19. Two different container contains $(2x^3+2x^2+3x+3)L$ and $(4x^3-2x^2+6x-3)L$ water. What is biggest measure that can measure both quantities exactly?

(a) $(x^2+2x)L$

(b) $(2x^2+3)L$

(c) $(2x-1)L$

(d) $(x+1)L$

20. Length and breadth of a rectangular park are $(3x^2+2x)m$ and $(2x^3-3)m$ respectively. Find the area of the park, when $x=3$.

(a) $1924m^2$ (b) $1492m^2$

(c) $1881\ m^2$ (d) $1683m^2$

ACHIEVERS SECTION (HOTS)

21. Find the roots of ax^2+bx+6, if the polynomial $x^4+x^3+8x^2+ax+b$ is exactly divisible by x^2+1.

(a) $-1,3$ (b) $2,5$

(c) $-1,-6$ (d) $-3,2$

22. Which of the following options hold?

Statement - I: If p(x) and g(x) are two polynomials with $g(x) \neq 0$, then we can find polynomials q(x) and r(x) such that $p(x) = g(x) \times q(x) + r(x)$, where degree of r(x) is greater than degree of g(x).

Statement - II: When $4x^5 + 3x^3 + 2x^2 + 8$ is divided by $4x^2 + 2x + 1$, then degree of remainder is 1.

(a) Both Statement - I and Statement - II are true.
(b) Statement -1 is true but Statement - II is false.
(c) Statement -1 is false but Statement - II is true.
(d) Both Statement - I and Statement -Hi are false.

23. Obtain all the zeroes of the polynomial $f(x) = 3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeros are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

(a) $1, -1$ (b) $1, 1$
(c) $-1, -1$ (d) $1, 0$

24. Match the following.

Column -I	Column - II
(P) If one of the zero of the polynomial $f(x) = (k^2 + 4)x^2 + 13x + 4$ is reciprocal of the other, then k is equal to	(i) 1
(Q) Sum of the zeroes of the polynomial $f(x) = 2x^3 + kx^2 + 4x + 5$ is 3, then A-is equal to	(ii) 0
(R) If the polynomial $f(x) = ax^3 + bx + c$ is exactly divisible by $g(x) = x^2 + bx + c$, then ab is equal to	(iii) –6

(a) (P) $\rightarrow$ (iii); (Q) $\rightarrow$ (i); (R) $\rightarrow$ (ii)
(b) (P) $\rightarrow$ (ii); (Q) $\rightarrow$ (iii); (R) $\rightarrow$ (i)
(c) (P) $\rightarrow$ (i); (Q) $\rightarrow$ (iii); (R) $\rightarrow$ (ii)
(d) (P) $\rightarrow$ (ii); (Q) $\rightarrow$ (i); (R) $\rightarrow$ (iii)

25. If 1 and -1 are zeroes of polynomial $Lx^4 + Mx^3 + Nx^2 + Rx + P$, then

Find:
(i) $L + N + P$
(ii) $M + R$
(iii) $M^3 + R^3$

	(i)	(ii)	(iii)
(a)	1	1	-1
(b)	0	-1	0
(c)	0	0	0
(d)	-1	1	1

ANSWER KEY

1.	D	**2.**	A	**3.**	D	**4.**	D	**5.**	D
6.	C	**7.**	B	**8.**	C	**9.**	B	**10.**	C
11.	A	**12.**	B	**13.**	A	**14.**	C	**15.**	D
16.	A	**17.**	B	**18.**	A	**19.**	B	**20.**	D
21.	C	**22.**	C	**23.**	C	**24.**	B	**25.**	C

HINTS AND SOLUTION

1. (d): For a cubic polynomial to exist, coefficient of term x^3 must not be equal to zero.

2. (a) : Sum of zeroes of polynomial $5x^2 - (3+k)x + 7$ is $\frac{-[-(3+k)]}{5}$ i.e., $\frac{3+k}{5}$

According to question, $\frac{3+k}{5} = 0 \Rightarrow k = -3$

Now, $2x^2 - 2(k+11)x + 30$ becomes $2x^2 - 16x + 30$.

i.e., $2x^2 - 16x + 30 = 0$ or $x^2 - 8x + 15 = 0$

$\Rightarrow \quad x = 3, 5$

Hence, zeroes of polynomial $2x^2 - 16x + 30$ are 3, 5.

3. (d) : Given polynomial is $2x^3 - 3x^2 + 4tx - 5$

Sum of product of roots taken two at a time is $\frac{4t}{2}$.

$\therefore \quad \frac{4t}{2} = -8 \Rightarrow t = -4$

4. (d) : Given equation is $x^2 + px + 12 = 0$

Now, if a and b are its roots, then
Sum of roots, $a + b = -p$ and
Product of roots, $a \times b = 12$
Also, $a - b = 1$ (Given)
We know that, $(a-b)^2 = (a+b)^2 - 4ab$

$\Rightarrow \quad 1 = p^2 - 4 \times 12 \quad \Rightarrow 1 = p^2 - 48$

$\Rightarrow \quad p^2 = 49 \Rightarrow \ = \pm 7$

5. (d) : Since, 3 is one of the zeroes of polynomial $p(x)$. So, $p(3) = 0$

6. (c): For a cubic polynomial, $ax^3 + bx^2 + cx + d$

Sum of zeroes $= -\dfrac{b}{a}$

Sum of the product of zeroes taken two at a time $= \dfrac{c}{a}$

Product of zeroes $= -\dfrac{d}{a}$

We have, $-\dfrac{b}{a} = -3, \dfrac{c}{a} = 8$ and $\dfrac{-d}{a} = 4$

$\therefore \quad x^3 + 3x^2 + 8x - 4$ is the required polynomial.

7. (b) : Given equation is $x^2 + k(x-1) - c$

$= x^2 + kx - (k+c)$

Since, p and q are the zeroes,
$\therefore \quad p + q = -k$ and $pq = -(k+c)$
Now, $(p-1)(q-1) = pq - q - p + 1$
$= pq - (p+q) + 1 = -(k+c) - (-k) + 1$
$= -k - c + k + 1 = 1 - c$

8. (c) :

9. (b) : By long division method, we have

$$
\begin{array}{r|l}
 & 6x + 5 \\
\hline
x^2 + x - 1 & 6x^3 + 11x^2 - 39x - 65 \\
 & \underset{-}{6x^3} \underset{-}{+ 6x^2} \underset{+}{- 6x} \\
\cline{2-2}
 & 5x^2 - 33x - 65 \\
 & \underset{-}{5x^2} \underset{-}{+ 5x} \underset{+}{- 5} \\
\cline{2-2}
 & -38x - 60
\end{array}
$$

We must subtract the remainder so that f(x) is exactly divisible by $x^2 + x - 1$

Hence, $-38x - 60$ is to be subtracted.

10. (c) : For more than three distinct real roots the graph must cut x-axis at least four times. So, graph in option (C) has more than three distinct real roots.

11. (a) : $f(x) = (k^2+4)x^2 + 13x + 4k$

Now, let α and β be the roots, then according to the question, $\alpha = \dfrac{1}{\beta} \Rightarrow \alpha\beta = 1$

Now, we know that $\alpha\beta = \dfrac{4k}{k^2+4}$

$\Rightarrow \quad 1 = \dfrac{4k}{k^2+4} \Rightarrow k^2 - 4k + 4 = 0$

$\Rightarrow \quad (k-2)^2 = 0 \Rightarrow k - 2 = 0 \Rightarrow k = 2$

12. (b) : Since, degree of given polynomial is 5, so $ax^5 + bx^3 + cx^2 + dx + e$ has atmost 5 zeroes.

13. (a) :

14. (c) : For $x^2 + 2x + 5$ to be a factor of $x^4 + \alpha x^2 + \beta$, remainder should be zero.

$$
\begin{array}{r|l}
 & x^2 - 2x + 5 \\
\hline
x^2 + 2x + 5 & x^4 + \alpha x^2 + \beta \\
 & \underset{-}{x^4} \underset{-}{+ 2x^3} \underset{-}{+ 5x^2} \\
\cline{2-2}
 & -2x^3 + (\alpha - 5)x^2 + \beta \\
 & \underset{+}{-2x^3} \underset{+}{- 4x^2} \underset{+}{- 10x} \\
\cline{2-2}
 & (\alpha - 1)x^2 + 10x + \beta \\
 & \underset{-}{5x^2} \underset{-}{+ 10x} \underset{-}{+ 25} \\
\cline{2-2}
 & (\alpha - 6)x^2 + \beta - 25
\end{array}
$$

Now, remainder should be equal to zero.

$\therefore \quad \alpha - 6 = 0$ and $\beta - 25 = 0$

$\Rightarrow \quad \alpha = 6$ and $\beta = 25$

15. (d) : Since, α and β are the zeroes of quadratic equation $ax^2 + bx - c = 0$

$\therefore \quad \alpha + \beta = \dfrac{-b}{a}$ and $\alpha\beta = \dfrac{-c}{a}$

Now, $\dfrac{\alpha^2}{\beta} + \dfrac{\beta^2}{\alpha} = \dfrac{\alpha^2 + \beta^3}{\alpha\beta}$

$=\dfrac{(\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta)}{\alpha\beta}$

$=\dfrac{-b}{a}\times\dfrac{-a}{c}[(\alpha+\beta)^2-3\alpha\beta]=\dfrac{b}{c}\left[\dfrac{b^2}{a^2}+\dfrac{3c}{a}\right]$

$=\dfrac{b}{c}\left[\dfrac{b^2+3ca}{a^2}\right]=\dfrac{b^3+3abc}{a^2c}$

16. (a) : Base of the triangular field

$= (x^2-4x+1)m$

Area of the triangular field $=\dfrac{1}{2}\times Base\times Height$

Now, $x^4-6x^3-26x^2+138x-35$

$\dfrac{1}{2}\times\left(x^2-4x+1\right)\times Height$

$\Rightarrow$Height $=\dfrac{2(x^4-6x^3-26x^2+138x-35)}{x^2-4x+1}$

$$\begin{array}{r|l} & x^2-2x-35 \\ \hline x^2-4x+1 & x^4-6x^3-26x^2+138x-35 \\ & x^4-4x^3+x^2 \\ & -\quad+\quad- \\ \hline & -2x^3-27x^2+138x-35 \\ & -2x^3+8x^2-2x \\ & +\quad-\quad+ \\ \hline & -35x^2+140x-35 \\ & -35x^2+140x-35 \\ & +\quad-\quad+ \\ \hline & 0 \end{array}$$

17. (b) : Length of the garden

$=(2x^3+5x^2-7)m$

Perimeter of the garden $=2\times$(length + breadth)

$\therefore\ 4x^3-2x^2+4=2(2x^3+5x^2-7+breadth)$

$\Rightarrow\ 2x^3-x^2+2=(2x^3+5x^2-7)+$ breadth

So, breadth of the rectangle

$=2x^3-x^2+2-2x^3-5x^2+7=(-6x^2+9)m$

18. (a) : Total amount Raghav had $=Rs.(6x^3+2x^2+3x)$

Cost of one shirt $=Rs.(x+5)$

Number of shirts he bought $=4x^2+3$

$\therefore$ Amount spent by him

$=Rs.(x+5)(4x^2+3)$

$=Rs.(4x^3+20x^2+3x+15)$

Hence, money left with Raghav

$=Rs.(6x^3+2x^2+3x-4x^3-20x^2-3x-15)$

$=Rs.(2x^3-18x^2-15)$

19. (b) : Capacity of both the containers is $(2x^3+2x^2+3x+3)\,L$ and $(4x^3-2x^2+6x-3)\,L$

i.e., $(2x^2+3)(x+1)\,L$ and $(2x^2+3)(2x-1)\,L$

Required measure is the H.C.F. of capacity of both the containers i.e., $(2x^2+3)\,L$

20. (d) : Length of rectangular park $=(3x^2+2x)\,m$

Breadth of rectangular park $=(2x^3-3)\,m$

Area of park = length × breadth

$=(3x^2+2x)(2x^3-3)=(6x^5+4x^4-9x^2-6x)\,m$

For $x=3,\ 6x^5+4x^4-9x^2-6x$

$=\ 6\times243+4\times81-9\times9-6\times3=1683$

Hence, area of park $=1683\ m^2$

21. (c) : $x^4+x^3+8x^2+ax+b$ is exactly divisible by x^2+1

$\Rightarrow$ Remainder must be zero.

$$\begin{array}{r|l} & x^2+x+7 \\ \hline x^2+1 & x^4+x^3+8x^2+ax+b \\ & x^4\quad\ +x^2 \\ & -\qquad - \\ \hline & x^3+7x^2+ax+b \\ & x^3\qquad +x \\ & -\qquad - \\ \hline & 7x^2+(a-1)x+b \\ & 7x^2\qquad +7 \\ & -\qquad - \\ \hline & (a-1)x+(b-7) \end{array}$$

$(a-1)\,x+(b-7)=0$

$\Rightarrow\ a-1=0$ and $b-7=0\ \Rightarrow\ a=1$ and $b=7$

Now, ax^2+bx+6 becomes x^2+7x+6 .

$x^2+7x+6=x^2+6x+x+6=0$

$\Rightarrow\quad x(x+6)+1(x+6)=0$

$\Rightarrow\ (x+1)(x+6)=0\ \Rightarrow\ x=-1,-6$

22. (c) : Statement - I is false because if p(x) and g(x) are two polynomials with $g(x)\neq 0$, . then we can find polynomials q(x) and r(x) such that

$$p(x) = g(x) \times q(x) + r(x)$$

where $r(x) = 0$ or degree of $r(x)$ < degree of $g(x)$.

Statement - II is false as when $4x^5 + 3x^3 + 2x^2 + 8$ is divided by $4x^2 + 2x + 1$, the remainder is $-\frac{5x}{4} + \frac{31}{4}$ which is a polynomial of degree 1.

23. (c) : $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$ are the zeroes of polynomial $f(x)$

$\therefore \left(x - \sqrt{\frac{5}{3}}\right), \left(x + \sqrt{\frac{5}{3}}\right)$ are factors of

.i.e., $\left(x^2 - \frac{5}{3}\right)$ exactly divides f(x).

Now, $3x^4 + 6x^3 - 2x^2 - 10x - 5$

$$= \left(x^2 - \frac{5}{3}\right)(3x^2 + 6x + 3) = 3\left(x^2 - \frac{5}{3}\right)(x+1)^2$$

For zeroes of polynomial f(x), $f(x) = 0$

$$\Rightarrow \quad 3\left(x^2 - \frac{5}{3}\right)(x+1)^2 = 0 \quad \Rightarrow \quad x = \sqrt{\frac{5}{3}}, -\sqrt{\frac{5}{3}}, -1, -1$$

24. (b): (P) $f(x) = (k^2 + 4)x^2 + 13x + 4$

Let one root be a, then other root must be $\frac{1}{\alpha}$

$\therefore$ Product of roots $= \frac{1}{\alpha} \times \alpha = 1$

$$\therefore \quad 1 = \frac{4}{k^2 + 4} \Rightarrow k^2 + 4 = 4 \Rightarrow k^2 = 0 \Rightarrow k = 0$$

(Q) $f(x) = 2x^3 + kx^2 + 4x + 5$

Sum of zeroes of $f(x) = -\frac{k}{2}$

According to question, $-\frac{k}{2} = 3 \Rightarrow k = -6$

(R) $f(x)$ is exactly divisible by $g(x)$, i.e., when $f(x)$ is divided by g(x) remainder must be zero.

$$\begin{array}{r|l} & ax - ab \\ \hline x^2 + bx + c & ax^3 + bx - c \\ & ax^3 + abx^2 + acx \\ & - \quad - \quad - \\ \hline & -(ab)x^2 + (b - ac)x - c \\ & -(ab)x^2 - ab^2x - abc \\ & + \quad + \quad + \\ \hline & (b - ac + ab^2)x + (abc - c) \end{array}$$

$\therefore \quad (b - ac + ab^2)x + (abc - c) = 0$

$\Rightarrow \quad b - ac + ab^2 = 0$ and $abc - c = 0$

$\therefore \quad abc - c = 0 \quad \Rightarrow \quad ab = 1$

25. (c) : Since, 1 and -1 are zeroes of $Lx^4 + Mx^3 + Nx^2 + Rx + P$.

$\therefore \qquad L + M + N + R + P = 0 \qquad$ ...(1)

and $L - M + N - R + P = 0 \qquad$ ...(2)

Adding (1) and (2), we get

$2L + 2N + 2P = 0 \Rightarrow L + N + P = 0$

Subtracting (1) from (2), we get

$-2M - 2R = 0 \quad \Rightarrow \quad M + R = 0$

Now, $(M + R)^3 = 0 \qquad (\because M + R = 0)$

$\Rightarrow \qquad M^3 + R^3 + 3MR(M + R) = 0$

$\Rightarrow M^3 + R^3 + 3MR \times 0 = 0 \quad [\because M + R = 0]$

$\Rightarrow \qquad M^3 + R^3 = 0$