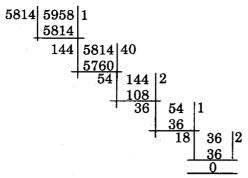
Real Number

IIT Foundation Material

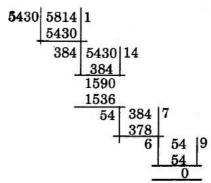
SECTION -

Straight Objective Type

1.



18 is G.CD of 5814, 5958



G.CD of 5814, 598 = 6

12 is the largest possible value of n Hence (b) is the correct option.

2.
$$\frac{x_1}{x_2} = \frac{x_2}{x_3} = \frac{x_3}{x_4} = \dots = \frac{x_{15}}{x_{16}}$$
If $x_1 + x_2 + x_3 + x_4 = 20$,
$$x_5 + x_6 + x_7 + x_8 = 320$$

 $x_{13} + x_{14} + x_{15} + x_{16} = 19680$

Hence (b) is the correct option.

3. Unit digit of $5^{2003} = 5$

Unit digit of
$$\frac{1}{5^{2003}} = 8$$

Since

$$\frac{1}{5} = 0.2$$
, $\frac{1}{5^2} = 0.04$,

$$\frac{1}{5^3} = 0.008$$
, $\frac{1}{5^4} = 0.0016$

$$\frac{1}{5^5} = 0.00032, \ \frac{1}{5^6} = 0.000064...$$

$$2003 = 5 \times 400 + 3$$

Hence (b) is the correct option.

4. $S_n = 1 - 2 + 3 - 4 + 5 - 6 + 7 - 8 + \dots + n$ terms

$$=\frac{n}{2}$$
 if n is even

$$=\frac{n+1}{2}$$
 if n is odd

$$S_{2002} \frac{2002}{2} = -1001$$

$$S_{2003} \frac{2003+1}{2} = -1002$$

$$S_{2004} \frac{2004}{2} = -1002$$

$$\therefore \ \ S_{2002} - S_{2003} + S_{2004}$$

$$=-1001-1002-1002$$

$$=-3005$$

- **5.** Let $x = 121^2 25^n + 1900^n (-4)^n$ by mathematical induction method $\forall n \in Nx$ is divisible by 2000 \Rightarrow Remainder is zero Hence (c) is the correct option.
- **6.** Highest power of n that divides P!

$$=\sum_{k=1}^{\alpha} \left[\frac{P}{n^h} \right]$$

Highest power of 2003 that divides

$$2003! = \sum_{k=1}^{\alpha} \left[\frac{2003}{2003^k} = 101 \right]$$

Hence (c) is the correct option.

- 7. The smallest positive integer n such $\sqrt{n} \sqrt{n-1} < 0.01$ is 2501 Hence (c) is the correct option.
- 8. $n^{200} < 5^{300}$ $\Rightarrow (n^2)^{100} < (5^3)^{100}$ $\Rightarrow (n^2)^{100} < (125)^{100}$ $\Rightarrow n^2 < 125$ $\Rightarrow 125 > n^2$

 $121=11^2$ is nearest possible volume for n. Hence **(d)** is the correct option.

9. Unit digit of $(3^{1001} \times 7^{1002} \times 13^{1003})$ Unit digit of $3^{1001} = 9$ Unit digit of $7^{1002} = 3$ Unit digit of $13^{1003} = 1$ Unit digit of $13^{1001} \times 7^{1002} \times 13^{1003}$ = UD of $(9 \times 3 \times 1)$ = UD of (27)= 7 Hence (d) is the correct option.

- 10. $1260x = N^{3}$ $\Rightarrow 1260 = 22 \times 32 \times 7 \times 5 \times x = N^{3}$ $\Rightarrow x = 2 \times 3 \times 7^{2} \times 5^{2} = 7350$ Hence (d) is the correct option.
- 11. $2^{13-1} \equiv 1 \pmod{13}$ $2^{12} \equiv 1 \pmod{13}$ $(2^{12})^{83} \equiv 1^4 \pmod{13}$ $2^{996} \equiv 1 \pmod{13}$ $2^4 \equiv 3 \pmod{13}$ $\Rightarrow 2^{1000} \equiv 3 \pmod{13}$ Hence (c) is the correct option.
- 12. 51+61+71+....+341 $=\frac{29}{2}[51+34]$ $\left[s_n = \frac{n}{2}[a+l]\right]$ $=\frac{29}{2}[51+341]$ $=\frac{29}{2} \times 392$ =5684Hence (a) is the correct option.
- **13.** The number of positive integral solution of

$$2x + 3y = 763$$
is 128

- **14.** 895 Hence (b) is the correct option.
- **16.** $(a-1)^2 + (b-2)^2 + (c-3)^2 + (d-4)^2 = 0$ $\Rightarrow a-1 = 0, b-2 = 0, c-3 = 0, d-4 = 0$ $\Rightarrow a = 1, b = 2.c = 3, d = 4$ $\Rightarrow a \times b \times c \times d + 1$ $= 1 \times 2 \times 3 \times 4 + 1 = 25 = 5^2$ Hence (c) is the correct option.
- **17.** (29, 37) Hence (d) is the correct option.
- 18. $422 \times 9 = 3798$ \Rightarrow The sum of the digits of the number $10^{422} - 1 = 3798$ $\Rightarrow n = 422$ Hence (c) is the correct option.
- 19. $29030 = 2 \times 3 \times 5 \times 7 \times 11 \times 13$ Hence (c) is the correct option.

- 20. Greater than 350 Hence (d) is the correct option.
- $1260n = 2^2 = 32 \times 7 \times 5 \times n$ is a perfect cube 21.
- $n = 7^2 \times 5^2 \times 3 \times 2$ \Rightarrow
- 1000 < n < 1000 \Rightarrow Hence (d) is the correct option.
- $\sqrt{9-(n+2)^2} > 0$ **22**.
- $-(n+2)^2 > -9$ \Rightarrow
 - $(n+2)^2 < 9$
 - $n^2 + 4n + 4 9 < 0$
 - $n^2 + 4n 5 < 0$
 - (n+5)(n-1)<0
- -5 < n < 1 and $n \in \mathbb{Z}$
- $n = \{-4, -3, -2, -1, 0\}$
- possible values for n is 5 \Rightarrow Hence (b) is the correct option.
- 23. 99|7*ab*73
 - \Rightarrow 11|7ab73 and 9|7ab73

$$(7+6+3)-(a+7)=0$$
 or multiple of 11

- b-a+3=0 b-a=-3 $\begin{vmatrix} 17+a+b \\ 17+a+b=18 \end{vmatrix}$ = multiple of 9
- a + b = 1
- -a + b = -3
- a + b = 1
 - b = -1

not possible

Let
$$17+a+b=21$$

$$a+b=4$$

$$-a+b=8$$

$$2b=12$$
and
$$b-a+3$$

$$b-a+8$$

Hence number of solution for (a, b) is zero.

Hence (c) is the correct option.

b=6 and a=-2

- **24.** $x^n y^n$ is divisible by x y $\forall n \in \mathbb{N}$
- \Rightarrow 31|107⁹⁰ 76⁹⁰
- $\Rightarrow 62 | 107^{90} 76^{90}$ Hence (b) is the correct option.
- **25.** When $1^{2003} + 3^{2003} + \dots + 2003^{2003}$ is divided by 2004 then the remainder is 1 Hence (b) is the correct option.
- **26.** 18 Hence (c) is the correct option.
- **27.** When 9999 is divided by 4, it leaves the remainder 3
- \Rightarrow the unit digit of 7^{9999}

= unit digit of
$$7^3$$

similarly
$$= 3$$

Ten's digit of
$$7^{9999}$$
 = 2

Hundred's digit of $7^{9999} = 5$

- **28.** The number of items the digit l acears in 123456 979899100 is 20 Hence (c) is the correct option.
- **29.** $n!=1\times 2\times 3\times \dots \times n$ has four zero at the end $\Rightarrow n!=20! \text{ or } 21! \text{ or } 22! \text{ or } 24!$ (n+1)!=25! has six zeros at the end
- \Rightarrow n = 24 Hence (g) is the correct option.
- **30.** Number of ordered pairs satisfying the equation Hence (b) is the correct option.

SECTION - II

Assertion - Reason Questions

- 31. $17|5|17^8 5^8$ $\Rightarrow 12|17^8 - 5^8$ $\Rightarrow 3|17^8 - 5^8$ $13+2|13^7 + 2^7$ $\Rightarrow 15|13^7 - 2^7 \Rightarrow 3|13^7 + 2^7$ $\Rightarrow 3|17^8 - 5^8 + 13^7 + 2^7$ since $x - y|x^n - y^n \quad \forall n \in \mathbb{N}$ and $x + y|x^n - y^n$ if n is odd Hence (b) is the correct option.
- **32.** Let X = A3640548981270644B $99 \mid X \Rightarrow 9 \mid X$ and $11 \mid X$ $9 \mid X \Rightarrow 71 + A + B = \text{multiple of } 9$ $11 \mid X \Rightarrow (37 + A) - (34 + B)$ = either 0 or multiple of 11

$$71 + A + B = 81$$

 $A + B = 10$
 $|(37 + A) - (34 + B) = 11|$
 $A - B + 3 = 11|$
 $A - B = 8$
 $\Rightarrow A + B = 10$
 $A - B = 8$

$$\oplus 2 A = 18 \implies A = 9, B = 1$$

Statement 1 is correct

7 or 13 divideds a number which is n

The form ABC ABC

Hence Statement 2 is correct but

Statement 2 is not a correct e5cplination for Statement 1.

Hence (b) is the correct option.

33. Let

Hence 7 divides X

Statement 1 is correct

 $x^n + y^n$ is divisible by x + y It n is odd is also true

Statement 2 is a correct explanation for Statement 1.

Hence (a) is the correct option.

34.
$$5^{11-1} \equiv 1 \pmod{11}$$

$$3^{11-1} \equiv 1 \pmod{11}$$

$$\Rightarrow 5^{10} - 3^{10} \equiv 0 \pmod{11}$$

$$\Rightarrow$$
 5¹⁰ -3¹⁰ is divisible by 11

If P is a prime number, G. CD of (a, P) = 1

Them $a^{P-1} \equiv 1 \pmod{P}$ is well known as formet theorem.

Statement 2 is a correct explanation for Statement one.

Hence (a) is the correct option

35.
$$3600 = 2^4 \times 3^2 \times 5^2$$

The number of +ve integer ≤ 3600 that co-prime to 3600

$$= 3600 \times \left[1 - \frac{1}{2}\right] \left[1 - \frac{1}{3}\right] \left[1 - \frac{1}{5}\right]$$

$$=960$$

Hence (a) is the correct option.

- **36.** The product of three consecutive integers is always divisible by 6 $n(n^2-1) = (n-1)n(n+1)$
- \Rightarrow 6| $n(n^2-1)$

Hence (a) is the correct option.

37.
$$9504 = 2^5 \times 3^3 \times 11^1$$

Number of factors of

$$9504 = (5+1)(3+1)(1+1)$$

$$=6\times4\times2$$

$$= 48$$

38.
$$2160 = 2^4 \times 3^3 \times 5^1$$

Sum of the factors of 2160

$$= \left[\frac{2^{4-1}-1}{2-1}\right] \left[\frac{3^{3+1}-1}{3-1}\right] \left[\frac{5^{1+1}-1}{5-1}\right]$$

$$=31\times40\times6$$

$$=7440$$

Hence (a) is the correct option.

39. The highest power of 7 contains in 1000!

$$= \sum_{k=1}^{\alpha} \left[\frac{1000}{7^k} \right]$$

$$= \left[\frac{1000}{7} \right] + \left[\frac{1000}{7^2} \right] + \left[\frac{1000}{7^3} \right] + \left[\frac{1000}{7^4} \right] + = 164$$

The highest power of 3 contains in 1000!

$$= \sum_{k=1}^{\alpha} \left[\frac{1000}{3^k} \right]$$

$$= \left[\frac{1000}{3} \right] + \left[\frac{1000}{3^2} \right] + \left[\frac{1000}{3^3} \right] + \left[\frac{1000}{3^4} \right]$$

$$+ \left[\frac{1000}{3^5} \right] + \left[\frac{1000}{3^6} \right] + = 498$$

Hence (b) is the correct option.

40. The number of zero at the end

$$= \sum_{k=1}^{\alpha} \left[\frac{4000}{5^k} \right]$$

$$= \left[\frac{400}{5} \right] + \left[\frac{400}{5^2} \right] + \left[\frac{400}{5^3} \right] + \left[\frac{400}{5^7} \right] + \dots$$

$$= 80 + 16 + 3 + 0 + \dots$$

$$= 99$$

SECTION - III

Linked Comprehension Type

41.
$$2^{7-1} \equiv 1 \pmod{7}$$

 $\Rightarrow 2^6 \equiv 1 \pmod{7}$
 $2^{18} \equiv 4 \pmod{7}$
 $2^2 \equiv 4 \pmod{7}$
 $\Rightarrow 220 \equiv 4 \pmod{7}$
Hence (a) is the correct option.

42.
$$3^{23-1} \equiv 1 \pmod{23}$$

 $3^{22} \equiv 1 \pmod{23}$
 $3^3 \equiv 4 \pmod{23}$
 $(3^3)^3 \equiv 4^3 \pmod{23}$
 $\equiv -5 \pmod{23}$
 $3^9 \equiv -5 \pmod{23}$
 $3^{18} \equiv 2 \pmod{23}$
 $\Rightarrow 3^{40} \equiv 2 \pmod{23}$

43.
$$2^3 \equiv 0 \pmod{8}$$

 $(2^3)^{19} \equiv 0 \pmod{8}$
 $2^{57} \equiv 0 \pmod{18}$
Hence (c) is the correct option.

- **44.** The last three digit of $25^{63} \times 63^{25}$ are 3 and 7 Hence (b) is the correct option.
- **45.** The last three digit of $25^{63} \times 63^{25}$ are 4, 7 and 5 Hence (c) is the correct option.

46.
$$56789$$
 $\equiv -11 \pmod{100}$
 $(56789)^2$ $\equiv 21 \pmod{100}$
 $(56789)^4$ $\equiv 41 \pmod{100}$
 $(56789)^6$ $\equiv 61 \pmod{100}$
 $(56789)^{10}$ $\equiv 1 \pmod{100}$
 $(56789)^{40}$ $\equiv 1 \pmod{100}$
 $(56789)^{41}$ $\equiv 89 \pmod{100}$
Hence (c) is the correct option.

47. When the number

$$10^{10} + 10^{10^2} + \dots + 10^{\left(10^{10}\right)}$$
 is divisible by 7 it leaves the remainder 4 Hence (a) is the correct option.

- **48.** $13^{73} + 14^3$ is divided by 11 it leaves the remainder 5 Hence (c) is the correct option.
- **49.** $(2000)^{13-1}$ $\equiv 1 \pmod{13}$ $(2000)^{12}$ $\equiv 1 \pmod{13}$ $(2000^{12})^{83}$ $\equiv 1 \pmod{13}$ 2000^{996} $\equiv 1 \pmod{13}$ 2000 $\equiv 1 \pmod{13}$ 2000^2 $\equiv 11^2 \pmod{13}$

 2000^2 = 4 (mod 13) 2000^4 = 4 (mod 13)

 $= 4 \pmod{13}$ $2000^{1000} = 3 \pmod{13}$

50.
$$4^{4n} - 5^{3n} = (256)^n - (125)^n$$
$$256 - 125 | 4^{4n} - 5^{3n}$$
$$= 131 | 4^{4n} - 5^{3n}$$

Hence (a) is the correct option.

51.
$$2^{4n}1-| = (2^4)^n - 1^n$$

= $16^n - 1^n$
 $16-1|16^n - 1^n \Rightarrow 15|16^n - 1^n$
Hence (b) is the correct option.

52. $3^{2n} + 7$ is divisible by 16 Hence (d) is the correct option.

53.
$$\left[\frac{1}{2}\right] + \left[\frac{1}{2} + \frac{1}{100}\right] + \dots \left[\frac{1}{2} + \frac{99}{100}\right]$$

= 45
Hence (c) is the correct option.

54. The number of zeros at the end of

$$400! = \sum_{k=1}^{\alpha} \left[\frac{400}{5^2} \right] + \dots$$

$$= 80 + 16 + 3 + 0$$

$$= 99$$

Hence (b) is the correct option.

55.
$$\left[\frac{n+1}{2} \right] + \left[\frac{n+2}{4} \right] + \left[\frac{n+4}{8} \right] + \dots$$

$$= \left[\frac{n}{4} \right]$$

SECTION - IV

Matrix - Match Type

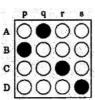
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