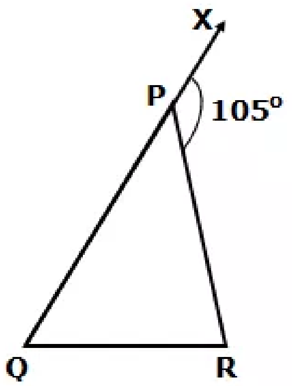
Chapter 11. Triangles and Their Congruency

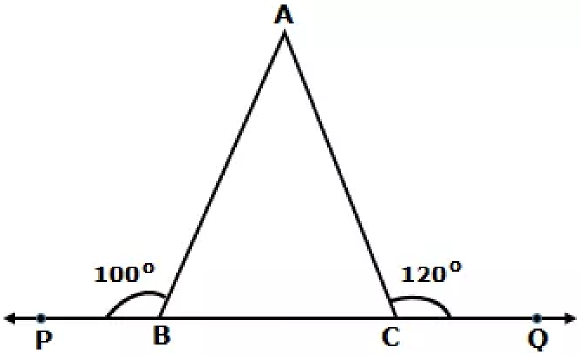
Ex 11.1

Answer 1.



$\angle Q : \angle R = 1 : 2$
Let $\angle Q = x^\circ$
$\Rightarrow \angle R = 2x^\circ$
Now, $\angle RPX = \angle Q + \angle R$ ....[Exterior angle property]
$\Rightarrow 105^\circ = x^\circ + 2x^\circ$
$\Rightarrow 105^\circ = 3x^\circ$
$\Rightarrow x^\circ = 35^\circ$
$\Rightarrow \angle Q = x^\circ = 35^\circ$ and $\angle R = 2x^\circ = 70^\circ$

Answer 2.



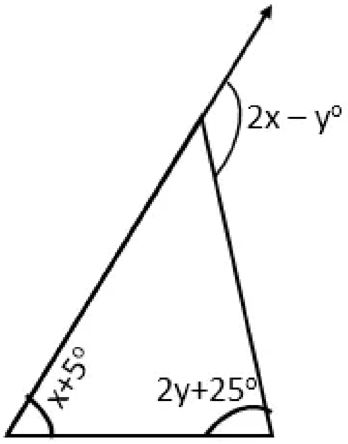
$\angle ABP + \angle ABC = 180^\circ$ ....(Linear pair)
$\Rightarrow 100^\circ + \angle ABC = 180^\circ$
$\Rightarrow \angle ABC = 180^\circ - 100^\circ = 80^\circ$

$\angle ACQ + \angle ACB = 180^\circ$ ....(Linear pair)
$\Rightarrow 120^\circ + \angle ACB = 180^\circ$
$\Rightarrow \angle ACB = 180^\circ - 120^\circ = 60^\circ$

Now, in ΔABC,
$\angle A + \angle B + \angle C = 180^\circ$ ....(Angle sum property of a triangle)
$\Rightarrow \angle A + 80^\circ + 60^\circ = 180^\circ$
$\Rightarrow \angle A = 180^\circ - 80^\circ - 60^\circ = 40^\circ$

Hence, the angles of a triangle are 40°, 60° and 80°.

Answer 3.



$(2x - y)^{\circ} = (x + 5)^{\circ} + (2y + 25)^{\circ}$ ....(Exterior angle property)

$\Rightarrow 2x^{\circ} - y^{\circ} = x^{\circ} + 5^{\circ} + 2y^{\circ} + 25^{\circ}$

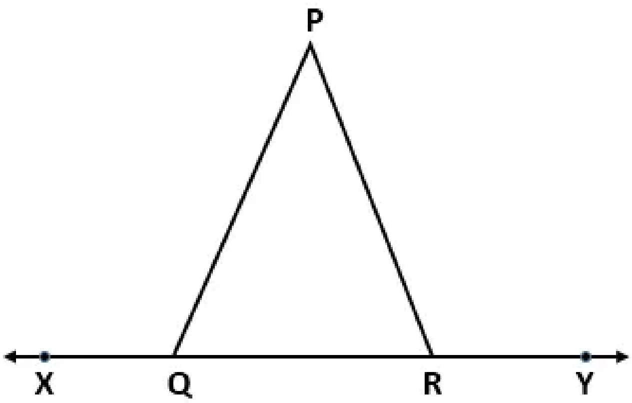
$\Rightarrow 2x^{\circ} - x^{\circ} = 2y^{\circ} + y^{\circ} + 30^{\circ}$

$\Rightarrow x^{\circ} = 3y^{\circ} + 30^{\circ}$

When $y = 15^{\circ}$, we have

$x^{\circ} = 3 \times 15^{\circ} + 30^{\circ} = 45^{\circ} + 30^{\circ} = 75^{\circ}$

Answer 4.



In ΔPQR,

$\angle P + \angle Q = 130^{\circ}$ ....(given)

Now, $\angle P + \angle Q = \angle PRY$ ....(Exterior angle property)

$\Rightarrow \angle PRY = 130^{\circ}$

$\angle PRY + \angle R = 180^{\circ}$ ....(Linear pair)

$\Rightarrow 130^{\circ} + \angle R = 180^{\circ}$

$\Rightarrow \angle R = 180^{\circ} - 130^{\circ} = 50^{\circ}$

Also, $\angle P + \angle R = 120^\circ$ ....(given)

Now, $\angle P + \angle R = \angle PQX$ ....(Exterior angle property)

$\Rightarrow \angle PQX = 120^\circ$

$\angle PQX + \angle Q = 180^\circ$ ....(Linear pair)

$\Rightarrow 120^\circ + \angle Q = 180^\circ$

$\Rightarrow \angle Q = 180^\circ - 120^\circ = 60^\circ$

In ΔPQR,

$\angle P + \angle Q + \angle R = 180^\circ$ ....(Angle sum property of a triangle)

$\Rightarrow \angle P + 60^\circ + 50^\circ = 180^\circ$

$\Rightarrow \angle P = 180^\circ - 110^\circ = 70^\circ$

Thus, the angles of ΔPQR are as follws:

$\angle P = 70^\circ$, $\angle Q = 60^\circ$ and $\angle R = 50^\circ$

Answer 5.

For any triangle, sum of measures of all three angles = 180°

Thus, we have

$(x + 10)^\circ + (x + 30)^\circ + (x - 10)^\circ = 180^\circ$

$\Rightarrow x^\circ + 10^\circ + x^\circ + 30^\circ + x^\circ - 10^\circ = 180^\circ$

$\Rightarrow 3x^\circ + 30^\circ = 180^\circ$

$\Rightarrow 3x^\circ = 150^\circ$

$\Rightarrow x = 50$

Now,

$(x + 10)^\circ = (50 + 10)^\circ = 60^\circ$

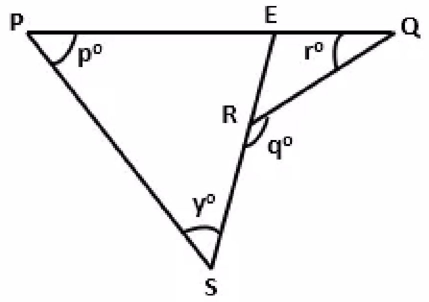
$(x + 30)^\circ = (50 + 30)^\circ = 80^\circ$

$(x - 10)^\circ = (50 - 10)^\circ = 40^\circ$

Thus, the angles of a triangle are 60°, 80° and 40°.

Answer 6.

SR is produced to meet PQ at E.



In ΔPSE,

$\angle P + \angle S + \angle PES = 180^\circ$ ....(Angle sum property of a triangle)

$\Rightarrow p^\circ + y^\circ + \angle PES = 180^\circ$

$\Rightarrow \angle PES = 180^\circ - p^\circ - y^\circ$ ....(i)

In ΔRQE,

$\angle R + \angle Q + \angle REQ = 180^\circ$ ....(Angle sum property of a triangle)

$\Rightarrow (180^\circ - q^\circ) + r^\circ + \angle REQ = 180^\circ$

$\Rightarrow \angle REQ = 180^\circ - (180^\circ - q^\circ) - r^\circ$

$\Rightarrow \angle REQ = q^\circ - r^\circ$ ....(ii)

Now, $\angle PES + \angle REQ = 180^\circ$ ....(Linear pair)

$\Rightarrow (180^\circ - p^\circ - y^\circ) + (q^\circ - r^\circ) = 180^\circ$ ....[From (i) and (ii)]

$\Rightarrow -p^\circ - y^\circ + q^\circ - r^\circ = 0$

$\Rightarrow -y^\circ = -q^\circ + p^\circ + r^\circ$

$\Rightarrow y^\circ = q^\circ - p^\circ - r^\circ$

Answer 7.

In ΔPQR,

$\angle P + \angle Q + \angle R = 180^\circ$ ....(angle sum property)

$\Rightarrow 4x^\circ + 5x^\circ + 9x^\circ = 180^\circ$

$\Rightarrow 18x^\circ = 180^\circ$

$\Rightarrow x = 10$

$\Rightarrow \angle P = 4x^\circ = 4 \times 10^\circ = 40^\circ$

$\angle Q = 5x^\circ = 5 \times 10^\circ = 50^\circ$

$\angle QPR = \angle PRS$ ....(Alternate angles)

And, $\angle QPR = 40^\circ$

$\Rightarrow \angle PRS = 40^\circ$

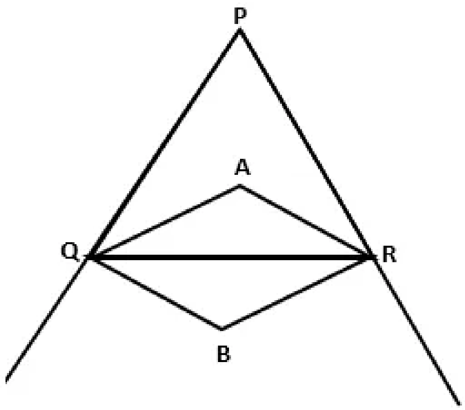
By exterior angle property,

$\angle PQR + \angle QPR = \angle PRS + y^\circ$

$\Rightarrow 40^\circ + 50^\circ = 40^\circ + y^\circ$

$\Rightarrow y = 50^\circ$

Answer 8.



By exterior angle property,

$\angle RQS = \angle P + \angle R$ and $\angle QRT = \angle P + \angle Q$

Since QB bisects $\angle RQS$,

$\angle BQR = \frac{1}{2}\angle RQS = \frac{1}{2}(\angle P + \angle R)$

Also RB bisects $\angle QRT$,

$\angle BRQ = \frac{1}{2}\angle QRT = \frac{1}{2}(\angle P + \angle Q)$

In ΔQBR,

$\angle QBR + \angle BRQ + \angle BQR = 180°$

$\Rightarrow \angle QBR + \frac{1}{2}(\angle P + \angle Q) + \frac{1}{2}(\angle P + \angle R) = 180°$

$\Rightarrow \angle QBR + \frac{1}{2}(\angle P + \angle Q + \angle P + \angle R) = 180°$

$\Rightarrow \angle QBR + \frac{1}{2}(\angle P + 180°) = 180°$ ....[$\angle P + \angle Q + \angle R = 180°$]

$\Rightarrow 2\angle QBR + \angle P + 180° = 360°$

$\Rightarrow 2\angle QBR = 180° - \angle P$ ....(i)

Since QB bisects $\angle PQR$,

$\angle AQR = \frac{1}{2}\angle PQR$

Also RA bisects $\angle PRQ$,

$\angle QRA = \frac{1}{2}\angle PRQ$

In ΔAQR,

$\angle AQR + \angle QRA + \angle QAR = 180°$

$\Rightarrow \frac{1}{2}\angle PQR + \frac{1}{2}\angle PRQ + \angle QAR = 180°$

$\Rightarrow \frac{1}{2}(\angle PQR + \angle PRQ) + \angle QAR = 180°$

$\Rightarrow \angle PQR + \angle PRQ + 2\angle QAR = 360°$

$\Rightarrow 2\angle QAR = 360° - \angle PQR - \angle PRQ$

$\Rightarrow 2\angle QAR = 180° + (180° - \angle PQR - \angle PRQ)$

$\Rightarrow 2\angle QAR = 180° + \angle P$ ....(ii)

Adding (i) and (ii),

$\Rightarrow 2\angle QAR + 2\angle QBR = 180° + \angle P + 180° - \angle P$

$\Rightarrow 2\angle QAR + 2\angle QBR = 360°$

$\Rightarrow \angle QAR + \angle QBR = 180°$

Answer 9.

By exterior angle property,

$\angle p = \angle PQR + \angle PRQ$

$\angle q = \angle QPR + \angle PRQ$

$\angle r = \angle PQR + \angle QPR$

Now, $\angle p + \angle q + \angle r = \angle PQR + \angle PRQ + \angle QPR + \angle PRQ + \angle PQR + \angle QPR$

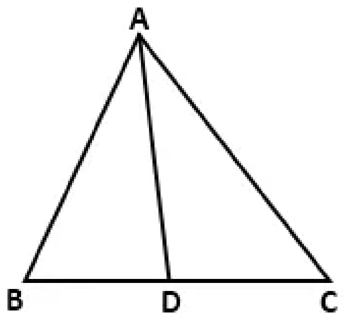
$\Rightarrow \angle p + \angle q + \angle r = 2\angle PQR + \angle 2PRQ + 2\angle QPR$

$\Rightarrow \angle p + \angle q + \angle r = 2(\angle PQR + \angle PRQ + \angle QPR)$

$\Rightarrow \angle p + \angle q + \angle r = 2 \times 180°$ [Angle sum property: $\angle PQR + \angle PRQ + \angle QPR = 180°$]

$\Rightarrow \angle p + \angle q + \angle r = 360°$

Answer 10.



Given, $\angle CAD = \angle B$ ....(i)

By exterior angle property,

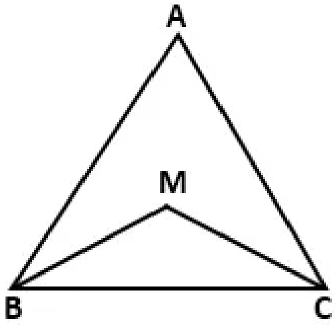
$\angle ADB = \angle CAD + \angle C$

Also, $\angle ADC = \angle BAD + \angle B$

$\Rightarrow \angle ADC = \angle BAD + \angle CAD$ ....[From (i)]

$\Rightarrow \angle ADC = \angle BAC$

Answer 11.



Sinec BM and CM are bisectors of $\angle ABC$ and $\angle ACB$,

$\angle B = 2\angle OBC$ and $\angle C = 2\angle OCB$ ....(i)

In ΔABC,

$\angle A + \angle B + \angle C = 180^\circ$

$\Rightarrow \angle A + 2\angle OBC + 2\angle OCB = 180^\circ$ ....[From (i)]

$\Rightarrow \frac{\angle A}{2} + \angle OBC + \angle OCB = 90^\circ$ ....[Dividing both sides by 2]

$\Rightarrow \angle OBC + \angle OCB = 90^\circ - \frac{\angle A}{2}$ ....(ii)

Now, in ΔBMC,

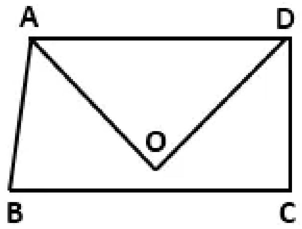
$\angle OBC + \angle OCB + \angle BMC = 180^\circ$

$\Rightarrow 90^\circ - \frac{\angle A}{2} + \angle BMC = 180^\circ$ ....[From (ii)]

$\Rightarrow \angle BMC = 180^\circ - 90^\circ + \frac{\angle A}{2}$

$\Rightarrow \angle BMC = 90^\circ + \frac{\angle A}{2}$

Answer 12.



Since AO and DO are bisectors of $\angle A$ and $\angle D$ of quadrilateral ABCD,

$\angle A = 2\angle OAD$ and $\angle D = 2\angle ODA$ ....(i)

In ΔAOD,

$\angle OAD + \angle ODA + \angle AOD = 180^\circ$

$\Rightarrow 2\angle OAD + 2\angle ODA + 2\angle AOD = 360^\circ$ ....[Multiplying both sides by 2]

$\Rightarrow 2\angle OAD + 2\angle ODA = 360^\circ - 2\angle AOD$ ....(ii)

In quadrlateral ABCD,

$\angle A + \angle B + \angle C + \angle D = 360^\circ$

$\Rightarrow 2\angle OAD + \angle B + \angle C + 2\angle ODA = 360^\circ$ ....[From (i)]

$\Rightarrow \angle B + \angle C = 360^\circ - 2\angle OAD - 2\angle ODA$

$\Rightarrow \angle B + \angle C = 360^\circ - (2\angle OAD + 2\angle ODA)$

$\Rightarrow \angle B + \angle C = 360^\circ - (360^\circ - 2\angle AOD)$ ....[From (ii)]

$\Rightarrow \angle B + \angle C = 360^\circ - 360^\circ + 2\angle AOD$

$\Rightarrow \angle B + \angle C = 2\angle AOD$

Answer 13.

Consider ΔABC.

Now, $\angle A < \angle B + \angle C$

$\Rightarrow \angle A + \angle A < \angle A + \angle B + \angle C$

$\Rightarrow 2\angle A < 180^\circ$

$\Rightarrow \angle A < \dfrac{180^\circ}{2}$

$\Rightarrow \angle A < 90^\circ$

Similarly, we have

$\angle B < 90^\circ$ and $\angle C < 90^\circ$.

Hence, the triangle is acute-angled.

Answer 14.

Let the angles of a triangle be 2x, 4x and 6x.

Then, we have

$2x + 4x + 6x = 180°$

$\Rightarrow 12x = 180°$

$\Rightarrow x = 15°$

$\Rightarrow 2x = 2 \times 15° = 30°$

$4x = 4 \times 15° = 60°$

$6x = 6 \times 15° = 90°$

Since one angle is 90°, the triangle is a right-angled triangle.

Answer 15.

Let ABC be a triangle such that

$\angle A + \angle B = 139°$ ....(i)

and, $\angle A - \angle B = 5°$ ....(ii)

Adding (i) and (ii), we get

$2\angle A = 144°$

$\Rightarrow \angle A = 72°$

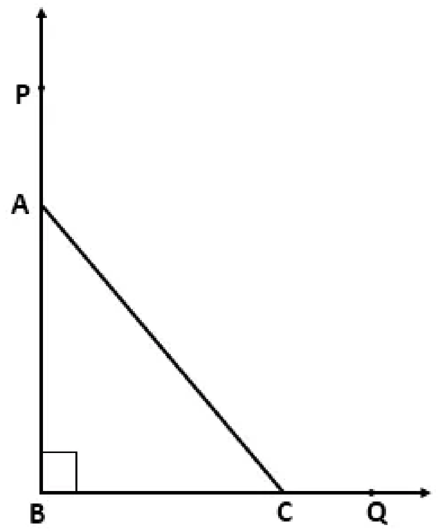
From (i), we have

$\angle B = 139° - 72° = 67°$

Now, 3^{rd} angle $= 180° - (\angle A + \angle B) = 180° - 139° = 41°$

Thus, the angles of a triangle are 72°, 67° and 41°.

Answer 16.



In $\Delta ABC, \angle B = 90°$

And, $\angle ABC + \angle BAC + \angle ACB = 180°$

$\Rightarrow \angle BAC + \angle ACB = 180° - \angle ABC$

$\Rightarrow \angle BAC + \angle ACB = 180° - 90°$

$\Rightarrow \angle BAC + \angle ACB = 90°$ ....(i)

By exterior angle property,

$\angle PAC = \angle ABC + \angle ACB$ ....(ii)

$\angle QCA = \angle ABC + \angle BAC$ ....(iii)

Adding (ii) and (iii), we get

$\angle PAC + \angle QCA = \angle ABC + \angle ACB + \angle ABC + \angle BAC$

$\Rightarrow \angle PAC + \angle QCA = (\angle ACB + \angle BAC) + 2\angle ABC$

$\Rightarrow \angle PAC + \angle QCA = 90° + 2 \times 90°$ ....[From (i)]

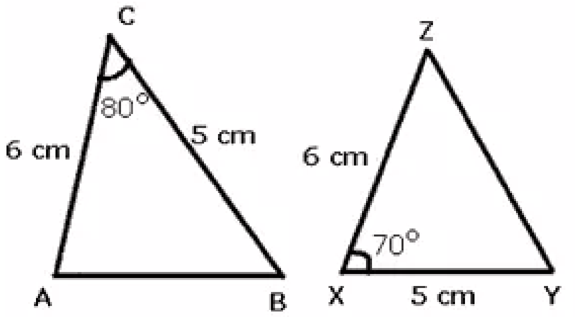
$\Rightarrow \angle PAC + \angle QCA = 90° + 180°$

$\Rightarrow \angle PAC + \angle QCA = 270°$

Ex 11.2

Answer 1.

(i)



In ΔABC and ΔXYZ

AC = XZ

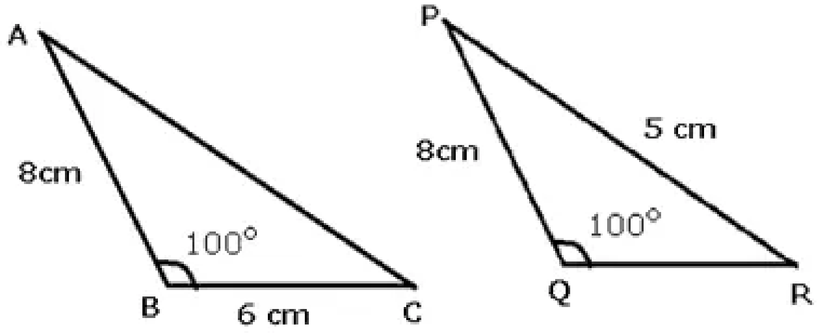
BC = XY

The included angle ∠C=80° is not equal to∠X i.e. 70° .

Now, for ΔABC to be congruent to ΔXYZ, AB should be equal to XY and YZ should be equal to BC. Then, ∠A = ∠C and ∠X = ∠Z. So, the measure of ∠B will not be equal to ∠Y.

Therefore, ΔABC cannot be congruent to ΔXYZ.

(ii)



In ΔABC and ΔPQR

AB = PQ

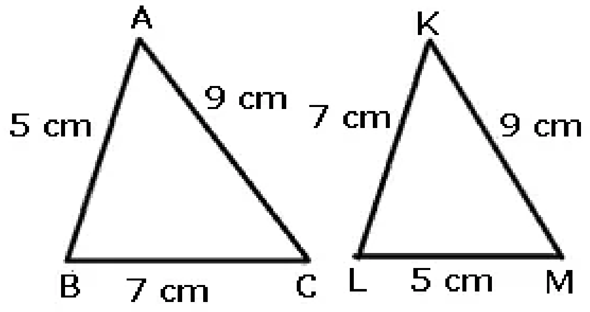
∠B=∠Q =

BC can be equal to QR or AC can be equal to RP

Therefore,

ΔABC can be congruent to ΔPQR .

(iii)



In ΔABC and ΔKLM

AB = LM

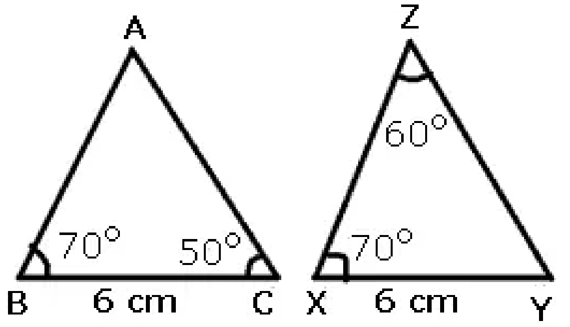
BC = KL

AC = KM

Therefore,

$\Delta ABC \cong \Delta KLM$ (SSS criteria)

(iv)



In ΔABC and ΔXYZ

$\angle B = \angle X$

BC = XY

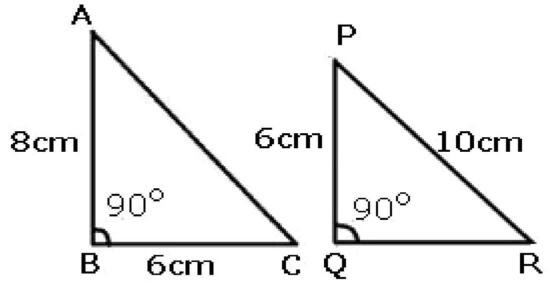
$\angle Y = 180° - (70° + 60°) = 50°$

$\angle C = \angle Y$

Therefore,

$\Delta ABC \cong \Delta XYZ$ (ASA criteria)

(v)



In ΔABC and ΔPQR

$\angle B = \angle Q$

$BC = PQ$

By Pythagoras theorem,

$PR^2 = PQ^2 + QR^2$

$10^2 = 6^2 + QR^2$

$100 = 36 + QR^2$

$QR = \sqrt{100-36}$

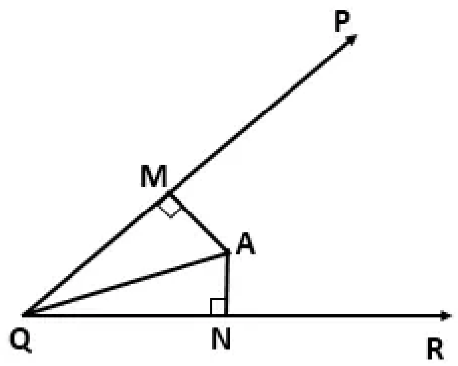
$QR = \sqrt{64} = 8cm$

$AB = QR$

Therefore,

$\Delta ABC \cong \Delta PQR$ (SAS and RHS criteria)

Answer 2.



Given,

$AM \perp PQ$ and $AN \perp QR$

$AM = AN$

In ΔAQM and AQN,

$AM = AN$ ....(given)

$AQ = AQ$ ....(common)

$\angle AMQ = \angle ANQ$ ....(Each = 90°)

So, by RHS congruence, we have

$\Delta AQM \cong \Delta AQN$

$\Rightarrow \angle AQM = \angle AQN$ ....(cpct)

$\Rightarrow \angle AQP = \angle AQR$

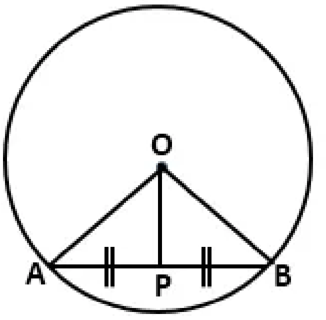
Answer 3.

Given:

In the figure, O is centre of the circle and AB is chord.

P is the mid-point of AB $\Rightarrow AP = PB$

To prove: $OP \perp AB$



Construction: Join OA and OB

Proof:

In ΔOAP and ΔOBP

$OA = OB$ ....[radii of the same circle]

$OP = OP$ ....[common]

$AP = PB$ ....[given]

$\therefore$ By Side-Side-Side criterion of congruency,

$\Delta OAP \cong \Delta OBP$

The corresponding parts of the congruent triangles are congruent.

$\therefore \angle OPA = \angle OPB$

But $\angle OPA + \angle OPB = 180°$ [linear pair]

$\therefore \angle OPA = \angle OPB = 90°$

Hence $OP \perp AB$.

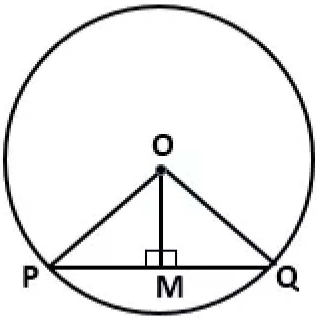
Answer 4.

Given:

In the figure, O is centre of the circle and PQ is a chord.

OM ⊥ PQ

To prove: PM = QM



Construction: Join OP and OQ

Proof:

In right triangles ΔOPM and ΔOQM,

OP = OQ ....[radii of the same circle]

OM = OM ....[common]

∴ By Right angle-Hypotenuse-Side criterion of congruency,

ΔOPM ≅ ΔOQM

The corresponding parts of the congruent triangles are congruent.

∴ PM = QM

Answer 5.

In Δ ABC and Δ PQR and

AB = PQ

BC = QR

$\angle ABX + \angle ABC = \angle PQY + \angle PQR = 180^\circ$

$\angle ABX = \angle PQY$

$\Rightarrow \angle ABC = \angle PQR$

Therefore,

Δ ABC ≅ Δ PQR (SAS criteria)

Answer 6.

Given:

D is mid-point of BC $\Rightarrow$ BD = DC

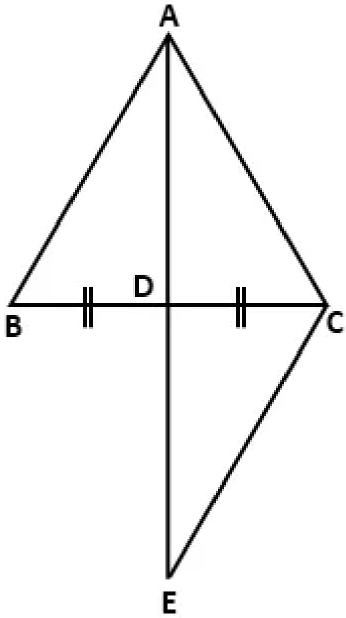
DE = AD

To prove:

a. $\Delta ABD \cong \Delta ECD$

b. AB = EC

c. AB $\parallel$ EC



a. In ΔABD and ΔECD,

BD = DC ....(given)

$\angle ADB = \angle CDE$ ....(vertically opposite angles)

AD = DE ....(given)

$\therefore$ By Side-Angle-Side criterion of congruence,

$\Delta ABD \cong \Delta ECD$

b. The corresponding parts of the congruent triangles are congruent.

$\therefore$ AB = EC

c. Also, $\angle DAB = \angle DEC$ ....(cp.c.t)

$\therefore$ AB $\parallel$ EC ....($\angle DAB$ and $\angle DEC$ are alternate angles)

Answer 7.

In ΔBAP and ΔCAP

$\angle BAP = \angle CAP$ (AD is the bisector of $\angle BAC$)

$AP = AP$

$\angle BPD + \angle BPA = \angle CPD + \angle CPA = 180°$

$\angle BPD = \angle CPD$

$\Rightarrow \angle BPA = \angle CPA$

Therefore,

$\Delta CAP \cong \Delta BAP$ (ASA criteria)

Hence, CP = BP.

Answer 8.

In ΔGCB and ΔDCE and

$\angle 1 + \angle GBC = \angle 2 + \angle DEC = 180°$

$\angle 1 = \angle 2 =$

$\Rightarrow \angle GBC = \angle DEC$

$BC = CE$

$\angle GCB = \angle DCE =$ (vertically opposite angles)

Therefore,

$\Delta GCB \cong \Delta DCE$ (ASA criteria)

Answer 9.

In ΔABC,
Since $AB = AC$
$\angle C = \angle B$ (angles opposite to the equal sides are equal)

BO and CO are angle bisectors of $\angle B$ and $\angle C$ respectively

Hence, $\angle ABO = \angle OBC = \angle BCO = \angle ACO$

Join AO to meet BC at D

In ΔABO and ΔACO and

$AO = AO$

$AB = AC$

$\angle C = \angle B =$

Therefore, $\Delta ABO \cong \Delta ACO$ (SAS criteria)

Hence, $\angle BAO = \angle CAO$
$\Rightarrow$ AO bisects angle BAC
In ΔABO and ΔACO
and $AB = AC$
$AO = AO$
$\angle BAD = \angle CAD =$ (proved)
$\Delta ABO \cong \Delta ACO$ (SAS criteria)
Therefore,
$BO = CO$

Answer 10.

In ΔABD and ΔFEC
$AB = FE$
$BD = CE$ ($BC = DE$; CD is common)
$\angle B = \angle E$
$\Delta ABD \cong \Delta FEC$ (SAS criteria)

Answer 11.

In ΔBMR and ΔDNR

$BM = DN$

$\angle BMR = \angle DNR = 90^\circ$

$\angle BRM = \angle DRN =$ (vertically opposite angles)

Hence, $\angle MBR = \angle NDR$ (sum of angles of a triangle = 180°)

$\Delta BMR \cong \Delta DNR$ (ASA criteria)

Therefore, $BR = DR$

So, AC bisects BD.

Answer 12.

In ΔQLM and ΔRNM

$QM = MR$

$LM = MN$

$\angle QLM = \angle RNM = 90^\circ$

Therefore, $\Delta QLM \cong \Delta RNM$ (RHS criteria)

Hence, $QL = RN$ .... (i)

Join PM

In ΔPLM and ΔPNM and

$PM = PM$ (common)

$LM = MN$

$\angle PLM = \angle PNM = 90^\circ$

Therefore, $\Delta PLM \cong \Delta PNM$ (RHS criteria)

Hence, $PL = PN$ .... (ii)

From (i) and (ii)

$PQ = PR$

Answer 13.

$\angle 1 = 2\angle 2$ and $\angle 4 = 2\angle 3$

1 = 22 and 4 = 2 3 $\angle 1 = \angle 4$ (vertically opposite angles)

$\Rightarrow 2\angle 2 = 2\angle 3$ or $\angle 2 = \angle 3$ ....(i)

$\angle R = \angle S$ = (since RT = TS and angle opposite to equal sides are equal)

$\Rightarrow \angle TRB = \angle TSA$ = ...(ii)

In ΔRBT and ΔSAT.

RT = TS

$\angle TRB = \angle TSA$

$\angle RTB = \angle STA$ = (common)

Therefore, $\Delta RBT \cong \Delta SAT$. (ASA criteria)

Answer 14.

In ΔCAD and ΔCBE

CA = CB (Isosceles triangle)

$\angle CDA = \angle CEB = 90°$

$\angle ACD = \angle BCE$ = (common)

Therefore, $\Delta CAD \cong \Delta CBE$ (AAS criteria)

Hence, CE = CD

But, CA = CB

$\Rightarrow AE + CE = BD + CD$

$\Rightarrow AE = BD$

Answer 15.

In ΔABC

$AB = AC$

$AX = AY$

$\Rightarrow BX = CY$

In ΔBXC and ΔCYB

$BX = CY$

$BC = BC$

$\angle B = \angle C$ = C (AB = AC and angles opposite to equal sides are equal)

Therefore, $\Delta BXC \cong \Delta CYB$ (SAS criteria)

Hence, $CX = BY$

Answer 16.

Given:

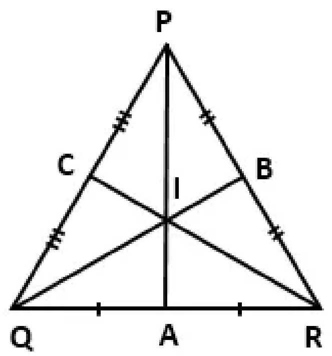
In ΔPQR,

PA is the perpendicular bisector of QR $\Rightarrow QA = RA$

RC is the perpendicular bisector of PQ $\Rightarrow PC = QC$

QB is the perpendicular bisector of PR $\Rightarrow PB = RB$

PA, RC and QB meet at I.



To prove: $IP = IQ = IR$

Proof:

In ΔQIA and ΔRIA

$QA = RA$ ....[Given]

$\angle QAI = \angle RAI$ ....[Each = 90°]

$IA = IA$ ....[Common]

$\therefore$ By Side-Angle-Side criterion of congruence,

$\Delta QIA \cong \Delta RIA$

The corresponding parts of the congruent triangles are congruent.
∴ IQ = IR ....(i)

Similarly, in ΔRIB and ΔPIB

RB = PB ....[Given]

$\angle RBI = \angle PBI$ ....[Each = 90°]

IB = IB ....[Common]

∴ By Side-Angle-Side criterion of congruence,
$\Delta RIB \cong \Delta PIB$
The corresponding parts of the congruent triangles are congruent.
∴ IR = IP ....(ii)

From (i) and (ii), we have
IP = IQ = IR

Answer 17.

In ΔADE and ΔBAC
AE = AC

AB = AD

$\angle BAD = \angle EAC$

$\angle DAC = \angle DAC = DAC$ (common)

$\Rightarrow \angle BAC = \angle EAD = EAD$

Therefore, $\Delta ADE \cong \Delta BAC$ (SAS criteria)

Hence, BC = DE

Answer 18.

Given:

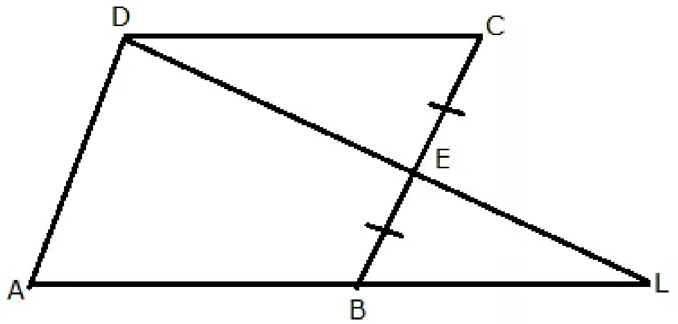
ABCD is a parallelogram, where BE = CE

To prove:

a. $\Delta DCE \cong \Delta LBE$

b. AB = BL

c. $DC = \frac{AL}{2}$



a. In ΔDCE and ΔLBE

$\angle DCE = \angle EBL$ ....[DC || AB, alternate angles]

$CE = BE$ ....[given]

$\angle DEC = \angle LEB$ ....[vertically opposite angles]

$\therefore$ By Angle-Side-Angle criterion of congruence,

$\Delta DCE \cong \Delta LBE$

The corresponding parts of the congruent triangles are congruent.

$\therefore DC = LB$ ....(1)

b. DC = AB ....(2)[opposite sides of a parallelogram]

From (1) and (2),

AB = BL ....(3)

c. AL = AB + BL

$\Rightarrow AL = AB + AB$ ....[From (3)]

$\Rightarrow AL = 2AB$

$\Rightarrow AL = 2DC$ ....[From (2)]

Answer 19.

$\angle BCD = \angle ADC$

$\angle ACB = \angle BDA$

$\angle BCD + \angle ACB = \angle ADC + \angle BDA$

$\Rightarrow \angle ACD = \angle BDCACD = BDC$

In ΔACD and ΔBCD

$\angle ACD = \angle BDCACD = BDC$

$\angle ADC = \angle BCD$

ADC = BCD CD = CD

Therefore, $\Delta ACD \cong \Delta BCD$ (ASA criteria)

Hence, AD = BC and $\angle A = \angle B$.

Answer 20.

Since AP and BQ are perpendiculars to the line segment AB, therefore AP and BQ are parallel to each other.

In ΔAOP and ΔBOQ

$\angle PAO = \angle QBO = 90°$

$\angle APO = \angle BQO$ (alternate angles)

AP = BQ

Therefore, $\Delta AOP \cong \Delta BOQ$ AOP BOQ (ASA criteria)

Hence, AO = OB and PO = OQ

Thus, O is the mid-point of line segments AB and PQ.

Answer 21.

CE is median to AB

$\Rightarrow AE = BE$ ....(i)

BD is median to AC

$\Rightarrow AD = DC$ ....(ii)

But $AB = AC$......(iii)

Therefore from (i), (ii) and (iii)

$BE = CD$

In $\triangle BEC$ and $\triangle BDC$

$BE = CD$

$\angle EBC = \angle DCB$ (angles opposite to equal sides are equal)

$BC = BC$ (common)

Therefore, $\triangle BEC \cong \triangle BDC$ (SAS criteria)

Hence, $BD = CE$

Answer 23.

In $\triangle ABC$ and $\triangle PQR$

$BC = QR$

AD and PM are medians of BC and QR respectively

$\Rightarrow BD = DC = QM = MR$

In $\triangle ABD$ and $\triangle PQM$

$AB = PQ$

$AD = PM$

$BD = QM$

Therefore, $\triangle ABD \cong \triangle PQM$ABD PQM (SSS criteria)

Hence, $\angle B = \angle Q$

Now in $\triangle ABC$ and $\triangle PQR$

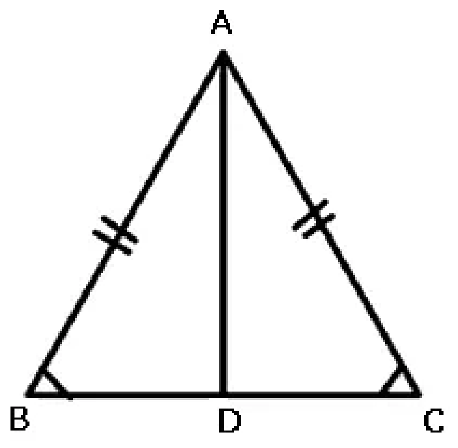
$AB = PQ$

$BC = QR$

$\angle B = \angle Q$

Therefore, $\triangle ABC \cong \triangle PQR$ABC PQR (SAS criteria)

Answer 24.



Now in ΔABD and ΔADC

$AB = AC$

$AD = AD$

$\angle B = \angle C$

Therefore, $\Delta ABD \cong \Delta ADC$ (SSA criteria)

Hence, $BD = DC$

Thus, AD bisects BC

Answer 25.

Since $AB = AC$

$\angle ABC = \angle ACB$

But $\angle DBC = \angle DCB$

$\Rightarrow \angle ABD = \angle ACD$

Now in ΔABD and ΔADC

$AB = AC$

$AD = AD$

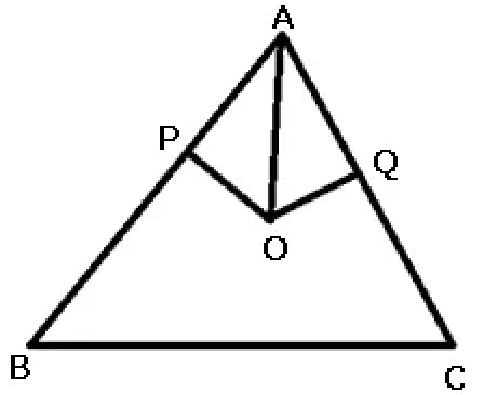
$\angle ABD = \angle ACD$

Therefore, $\Delta ABD \cong \Delta ADC$ (SSA criteria)

Hence, $\angle BAD = \angle CAD$

Thus, AD bisects $\angle BAC$

Answer 26.



In $\triangle POA$ and $\triangle QOA$

$\angle OPA = \angle OQA = 90°$

OP = OQ (given)

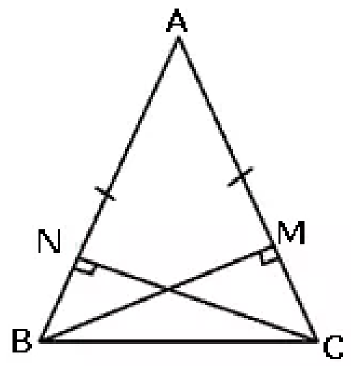
AO = AO

Therefore, $\triangle POA \cong \triangle QOA$ (SSA criteria)

Hence, $\angle PAO = \angle QAO$

Thus, OA bisects $\angle BAC$

Answer 27.



In $\triangle BNC$ and $\triangle CMB$

$\angle BNC = \angle CMB = 90°$

$\angle NBC = \angle MCB$ (AB = AC)

BC = BC

Therefore, $\triangle BNC \cong \triangle CMB$ (AAS criteria)

Hence, BM = CN

Answer 28.

In $\triangle ABC$

$AB = AC$

$\angle ABC = \angle ACB$ (equal sides have equal angles opposite to them)...(i)

$\angle GBC = \angle HCB = 90^\circ$ ....(ii)

Subtracting (i) from (ii)

$\angle GBA = \angle HCA$ ....(iii)

In $\triangle GBA$ and $\triangle HCA$

$\angle GBA = \angle HCA$ (from iii)

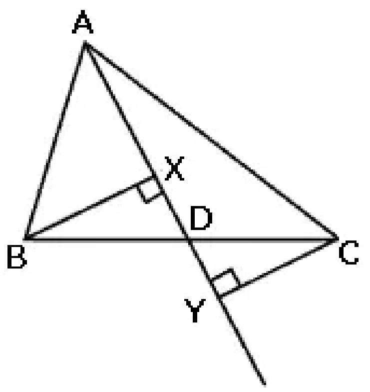
$\angle BAG = \angle CAH$ (vertically opposite angles)

$BC = BC$

Therefore, $\triangle GBA \cong \triangle HCA$ (ASA criteria)

Hence, $BG = CH$ and $AG = AH$

Answer 29.



In $\triangle BXD$ and $\triangle CYD$

$\angle BXD = \angle CYD$ (90°)

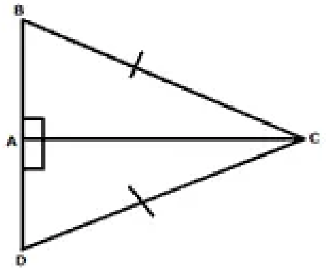
$\angle XDB = \angle YDC$ (vertically opposite angles)

$BD = DC$ (AD is median on BC)

Therefore, $\triangle BXD \cong \triangle CYD$ (AAS criteria)

Hence, $BX = CY$

Answer 30.



In ΔABC and ΔADC

$\angle BAC = \angle DAC$ (90°)

$BC = DC$

$AC = AC$ (common)

Therefore, $\Delta ABC \cong \Delta ADC$ (SSA criteria)

Hence, $\angle BCA = \angle DCA$

Thus, AC bisects $\angle BCD$

Answer 31.

$\angle PQT = \angle RQU$ .....(i)

$\angle TQS = \angle UQS$ .....(ii)

Adding (i) and (ii)

$\angle PQS = \angle RQS$

In ΔPQS and ΔRQS

$\angle PQS = \angle RQS$

$PQ = RQ$ (given)

$QS = QS$ (common)

Therefore, $\Delta PQS \cong \Delta RQS$ (SAS criteria)

Hence, $\angle QPS = \angle QRS$

Now in ΔPQT and ΔRQU

$\angle QPS = \angle QRS$

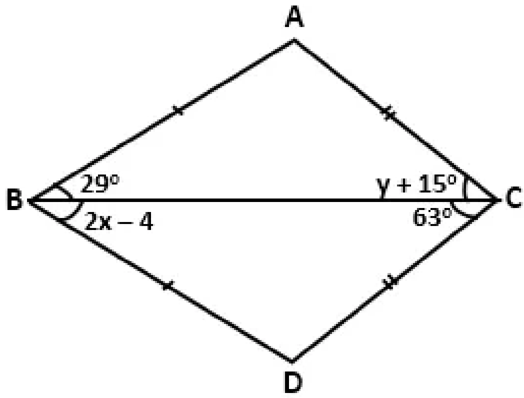
$PQ = RQ$ (given)

$\angle PQT = \angle RQU$ (given)

Therefore, $\Delta PQT \cong \Delta RQU$ (ASA criteria)

Hence, $QT = QU$.

Answer 32.



In ΔABC and ΔDBC

$AB = DB$ ....[given]

$AC = DC$ ....[given]

$BC = BC$ ....[common]

$\therefore$ By Side-Side-Side criterion of congruence,

$\Delta ABC \cong \Delta DBC$

$\therefore \angle ACB = \angle DCB$ ....[c.p.c.t.]

$\Rightarrow y + 15° = 63°$

$\Rightarrow y = 63° - 15°$

$\Rightarrow y = 48°$

Now, $\angle ABC = \angle DBC$ ....[c.p.c.t.]

$\Rightarrow 29° = 2x - 4°$

$\Rightarrow 2x = 29° + 4°$

$\Rightarrow 2x = 33°$

$\Rightarrow x = \dfrac{33°}{2}$

$\Rightarrow x = 16.5°$

Hence, $x = 16.5°$ and $y = 48°$