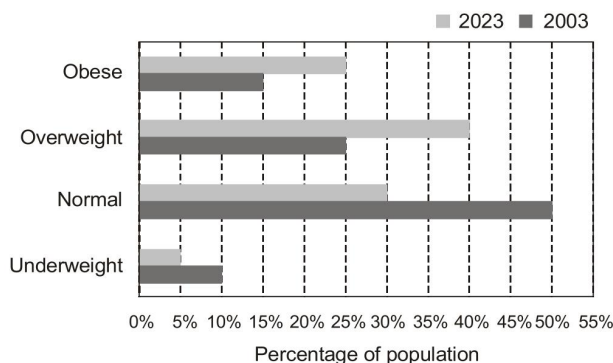


# Electronics Engineering (Forenoon Session) Exam Date- 11-02-2024

## SECTION - A

## GENERAL APTITUDE

- Q.1 The bar chart shows the data for the percentage of population falling into different categories based on Body Mass Index (BMI) in 2003 and 2023.



Based on the data provided, which one of the following options is INCORRECT?

- (a) The ratio of the percentage of population falling into overweight category to the percentage of population falling into normal category has increased in 20 years.
- (b) The ratio of the percentage of population falling into obese category to the percentage of population falling into normal category has decreased in 20 years.
- (c) The ratio of the percentage of population falling into underweight category to the percentage of population falling into normal category has decreased in 20 years.
- (d) The percentage of population falling into normal category has decreased in 20 years.

Ans. (b)

End of Solution

- Q.2 Sequence the following sentences (P, Q, R, S) in a coherent passage:

P: Shifu's student exclaimed, "Why do you run since the bull is an illusion?"

Q: Shifu said, "Surely my running away from the bull is also an illusion."

R: Shifu once proclaimed that all life is illusion.

S: One day, when a bull gave him chase. Shifu began running for his life.

- (a) RSPQ
- (b) SPRQ
- (c) SRPQ
- (d) RPQS

Ans. (a)

End of Solution

- Q.3 Five years ago, the ratio of Aman's age to his father's age was 1:4, and five years from now, the ratio will be 2:5. What was his father's age when Aman was born?

- (a) 32 years
- (b) 35 years
- (c) 28 years
- (d) 30 year

**Ans. (d)**

Five year ago

$$\text{Son's age} = x$$

$$\text{Father's age} = 4x$$

Five year after

$$\text{Son's age} = x + 10$$

$$\frac{x+10}{4x+10} = \frac{2}{5}$$

$$5x + 50 = 8x + 20$$

$$3x = 30$$

$$x = 10$$

Son's age five year ago = 10

Father's age five year ago = 40

At the time of son's birth father's age =  $40 - 10 = 30$  year

**End of Solution**

**Q.4** The greatest prime factor of  $(3^{199} - 3^{196})$  is

(a) 11

(b) 13

(c) 3

(d) 17

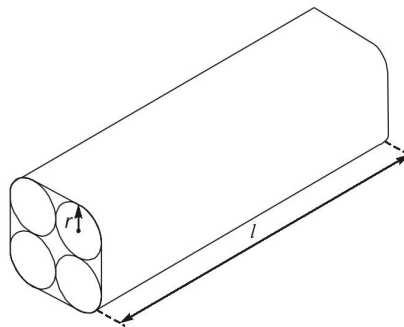
**Ans. (b)**

$$= 3^{196}(3^3 - 1) = 3^{196} \times 26 = 2 \times 3^{196} \times 13$$

Highest prime factors = 13

**End of Solution**

**Q.5** Four identical cylindrical chalk-sticks, each of radius  $r = 0.5$  cm and length  $l = 10$  cm. are bound tightly together using a duct tape as shown in the following figure



The width of the duct tape is equal to the length of the chalk-stick. The area (in  $\text{cm}^2$ ) of the duct tape required to wrap the bundle of chalk-sticks once, is

(a)  $10(8 + \pi)$

(b)  $20(4 + \pi)$

(c)  $10(4 + \pi)$

(d)  $20(8 + \pi)$

**Ans. (c)**

**End of Solution**

**Q.6** Two identical sheets A and B. of dimensions  $24 \text{ cm} \times 16 \text{ cm}$ , can be folded into half using two distinct operations. FO1 or FO2.

In FO1, the axis of folding remains parallel to the initial long edge, and in FO2, the axis of folding remains parallel to the initial short edge.

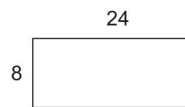
If sheet A is folded twice using FO1, and sheet B is folded twice using FO2, the ratio of the perimeters of the final shapes of A and B is

- (a)  $11 : 18$  (b)  $18 : 11$   
(c)  $11 : 14$  (d)  $14 : 11$

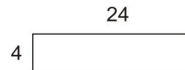
**Ans. (d)**

**Condition 1:**

Folding first time



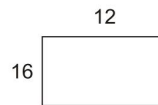
Folding first time



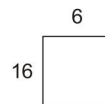
$$\text{Perimeter} = 2(24 + 4) = 56 \text{ cm}$$

**Condition 2:**

Folding first time



Folding second time



$$\text{Perimeter} = 2(16 + 6) = 44 \text{ cm}$$

$$\text{Ratio} = 56 : 44 = 14 : 11$$

**End of Solution**

**Q.7** If '→' denotes increasing order of intensity, then the meaning of the words [charm → enamor → bewitch] is analogous to [bored → \_\_\_\_\_ weary].

Which one of the given options is appropriate to fill the blank?

- (a) worsted (b) dead  
(c) jaded (d) baffled

**Ans. (c)**

**End of Solution**

**Q.8** For a real number  $x > 1$ ,

$$\frac{1}{\log_2 x} + \frac{1}{\log_3 x} + \frac{1}{\log_4 x} = 1$$

The value of  $x$  is

- (a) 24 (b) 12  
(c) 4 (d) 36

**Ans. (a)**

$$\log_x^2 + \log_x^3 + \log_x^4 = 1$$

$$\log_x^{24} = 1$$

$$x = 24$$

**End of Solution**

**Q.9** P, Q, R, S, and T have launched a new startup. Two of them are siblings. The office of the startup has just three rooms. All of them agree that the siblings should not share the same room.

If S and Q are single children, and the room allocations shown below are acceptable to all,

P	R	T	S	Q	P	Q	R	T	S
---	---	---	---	---	---	---	---	---	---

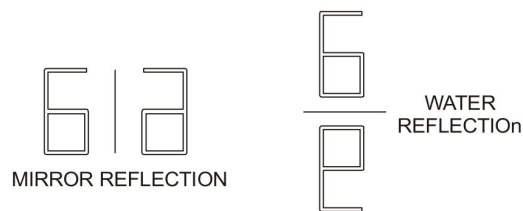
then, which one of the given options is the siblings?

- (a) T and R (b) T and Q  
(c) P and S (d) P and T

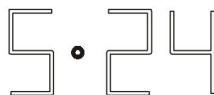
**Ans. (d)**

**End of Solution**

**Q.10** Examples of mirror and water reflections are shown in the figures below:

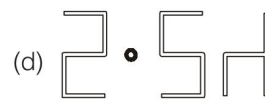
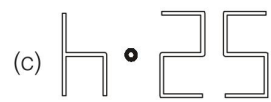
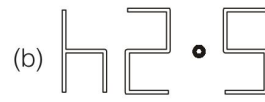
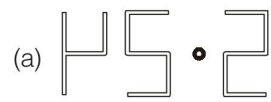


An object appears as the following image after first reflecting in a mirror and then reflecting on water.





The original object is



Ans. (b)

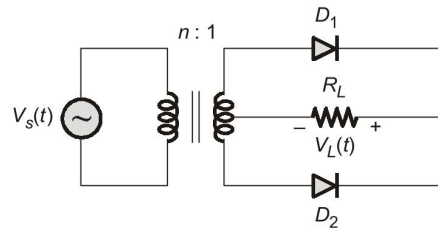
End of Solution



## SECTION - B

## TECHNICAL

- Q.1** In the circuit shown, the  $n : 1$  step-down transformer and the diodes are ideal. The diodes have no voltage drop in forward biased condition. If the input voltage (in Volts) is  $V_s(t) = 10\sin\omega t$  and the average value of load voltage  $V_L(t)$  (in Volts) is  $2.5/\pi$ , the value of  $n$  is \_\_\_\_\_.



- (a) 8  
(b) 12  
(c) 4  
(d) 16

**Ans. (c)**

Given,

$$V_{DC} = \frac{2V_M}{\pi} = \frac{2.5}{\pi}$$

$$V_M = \frac{2.5}{2}$$

$\therefore$

$$2V_M = V_s = 2.5 \text{ V}$$

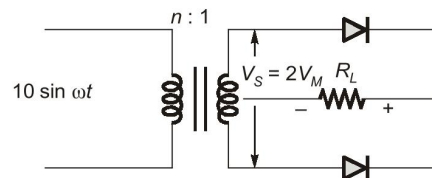
$$n : 1 = \frac{V_p}{V_s} : 1$$

$$= \frac{10}{2.5} : 1$$

$$= 4 : 1$$

$\therefore$

$$n = 4$$



**End of Solution**

- Q.2** Let  $p(x, y, z, t)$  and  $u(x, y, z, t)$  represent density and velocity, respectively, at a point  $(x, y, z)$  and time  $t$ . Assume  $\frac{\partial \rho}{\partial t}$  is continuous. Let  $V$  be an arbitrary volume in space enclosed by the closed surface  $S$  and  $\hat{n}$  be the outward unit normal of  $S$ .

Which of the following equations is/are equivalent  $\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho u) = 0$ ?

(a)  $\int_V \frac{\partial \rho}{\partial t} dv = \int_V \nabla \cdot (\rho u) dv$

(b)  $\int_V \frac{\partial \rho}{\partial t} dv = - \oint_S \rho u \cdot \hat{n} ds$

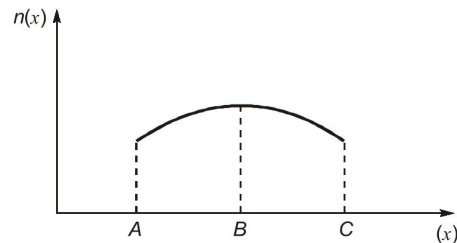
(c)  $\int_V \frac{\partial \rho}{\partial t} dv = \oint_S \rho u \cdot \hat{n} ds$

(d)  $\int_V \frac{\partial \rho}{\partial t} dv = - \int_V \nabla \cdot (\rho u) dv$

**Ans. (b, d)**

**End of Solution**

- Q.3** The free electron concentration profile  $n(x)$  in a doped semiconductor at equilibrium is shown in the figure, where the points A, B, and C mark three different positions. Which of the following statements is/are true?



- (a) For  $x$  between B and C, the electron diffusion current is directed from C to B.
- (b) For  $x$  between B and A, the electric field is directed from A to B.
- (c) For  $x$  between B and A, the electron drift current is directed from B to A.
- (d) For  $x$  between B and C, the electric field is directed from B to C.

**Ans.** (a, c, d)

End of Solution

- Q.4** A machine has a 32-bit architecture with 1-word long instructions. It has 24 registers and supports an instruction set of size 40. Each instruction has five distinct fields, namely opcode, two source register identifiers, one destination register identifier, and an immediate value. Assuming that the immediate operand is an unsigned integer, its maximum value is \_\_\_\_\_.

**Ans.** (2047)

Given 32-bit architecture {1 word long instructions}

$$1 \text{ word} = 32\text{-bits}$$

$$\text{Number of registers} = 24, \text{ bits required} \Rightarrow 2^n = 24$$

$$n = \log_2(24)$$

$$\therefore n = 5\text{-bits}$$

$$\text{Number instructions} = 40$$

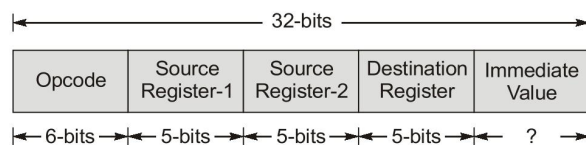
$$\therefore \text{Opcode bits would be } 2^n = 40$$

$$\therefore n = \log_2(40)$$

$$n = 6\text{-bits}$$

Each register field requires 5-bits.

Instruction format would be



$$\therefore \text{Immediate value bits} = 32 - (6 + 5 + 5 + 5) = 32 - 21 = 11\text{-bits}$$

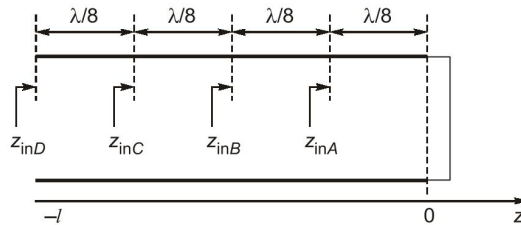
The range of unsigned values = 0 to  $2^n - 1$

$$= 0 \text{ to } 2^{11} - 1 = 0 \text{ to } (2048 - 1) = 0 \text{ to } 2047$$

$$\therefore \text{Maximum unsigned integer} = 2047$$

End of Solution

- Q.5** Consider a lossless transmission line terminated with a short circuit as shown in the figure below. As one moves towards the generator from the load, the normalized impedances  $z_{inA}$ ,  $z_{inB}$ ,  $z_{inC}$  and  $z_{inD}$  (indicated in the figure) are \_



- (a)  $z_{inA} = +0.4j \Omega$ ,  $z_{inB} = \infty$ ,  $z_{inC} = -0.4j \Omega$ ,  $z_{inD} = 0$   
 (b)  $z_{inA} = -1j \Omega$ ,  $z_{inB} = 0$ ,  $z_{inC} = +1j \Omega$ ,  $z_{inD} = \infty$   
 (c)  $z_{inA} = \infty$ ,  $z_{inB} = +0.4j \Omega$ ,  $z_{inC} = 0$ ,  $z_{inD} = +0.4j \Omega$   
 (d)  $z_{inA} = +1j \Omega$ ,  $z_{inB} = \infty$ ,  $z_{inC} = -1j \Omega$ ,  $z_{inD} = 0$

**Ans. (d)**

$$Z_{S/C} = jZ_0 \tan \beta l.$$

⇒ Normalized impedance,

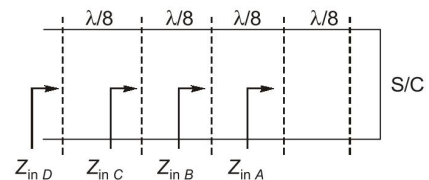
$$\bar{Z}_{S/C} = \frac{Z_{S/C}}{Z_0} = j \tan \beta l$$

$$\bar{Z}_{inA} = j \tan \left( \frac{2\pi}{\lambda} \cdot \frac{\lambda}{8} \right) = j \tan (\pi/4) = j1 \Omega$$

$$\bar{Z}_{inB} = j \tan \left( \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} \right) = \infty$$

$$\bar{Z}_{inC} = j \tan \left( \frac{2\pi}{\lambda} \cdot \frac{3\lambda}{8} \right) = -j1 \Omega$$

$$\bar{Z}_{inD} = j \tan \left( \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} \right) = 0$$



**End of Solution**

- Q.6** A causal and stable LTI system with impulse response  $h(t)$  produces an output  $y(t)$  for an input signal  $x(t)$ . A signal  $x(0.5t)$  is applied to another causal and stable LTI system with impulse response  $h(0.5t)$ . The resulting output is \_\_\_\_\_.

- (a)  $2y(0.5t)$  (b)  $4y(0.5t)$   
 (c)  $0.25y(2t)$  (d)  $0.25y(0.25t)$

**Ans. (a)**

$$y(t) = x(t) * h(t)$$

$$x(t) \leftrightarrow X(\omega)$$

$$x(0.5t) \leftrightarrow 2X(2\omega)$$

$$h(0.5t) \leftrightarrow 2H(2\omega)$$

$$y(t) \leftrightarrow Y(\omega)$$

$$Y(\omega) \leftrightarrow X(\omega) \times H(\omega)$$

After scaling,  $Y_1(\omega) = 2X(2\omega) \times 2H(2\omega)$

$$= 4 X(2\omega) \times H(2\omega)$$

After taking inverser Fourier transform

$$y_1(t) = 2y(0.5t)$$

End of Solution

Q.7 For the Boolean function

$$F(A, B, C, D) = \sum m(0, 2, 5, 7, 8, 10, 12, 13, 14, 15)$$

the essential prime implicants are \_\_\_\_\_.

(a)  $BD, \bar{B}\bar{D}, AB$

(b)  $BD, \bar{B}\bar{D}$

(c)  $AB, \bar{B}\bar{D}$

(d)  $BD, AB$

Ans. (b)

$$f(A, B, C, D) = \sum m(0, 2, 5, 7, 8, 10, 12, 13, 14, 15)$$

		CD			
		$\bar{C}\bar{D}$	$\bar{C}D$	$CD$	$C\bar{D}$
AB	$\bar{A}\bar{B}$	1 0	1 1	3 3	2 2
	$\bar{A}B$	4 4	5 1	7 1	6 6
AB	$A\bar{B}$	12 1	13 1	15 1	14 1
	$AB$	8 1	9 9	11 11	10 1

P.I

$BD \quad \bar{B}\bar{D}$   
E.P.I    E.P.I

End of Solution

Q.8 The general form of the complementary function of a differential equation is given by  $y(t) = (At + B)e^{-2t}$ , where  $A$  and  $B$  are real constants determined by the initial condition. The corresponding differential equation is \_\_\_\_\_.

(a)  $\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y = f(t)$

(b)  $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = f(t)$

(c)  $\frac{d^2y}{dt^2} + 4y = f(t)$

(d)  $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = f(t)$

Ans. (b)

∴ Given

$$\text{Complementary function} = (At + B)e^{-2t} \approx (C_1 + C_2t)e^{-2t}$$

i.e. roots of auxiliary equation are  $m = -2, -2$

So, AE is  $(m + 2)(m + 2) = 0$

$$m^2 + 4m + 4 = 0$$

Replace  $m \rightarrow D$  we get,

$$(D^2 + 4D + 4)y = 0$$

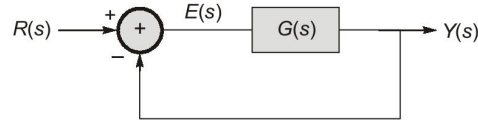
i.e. the required different

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = f(t)$$

End of Solution

**Q.9**

In the feedback control system shown in the figure below  $G(s) = \frac{6}{s(s+1)(s+2)}$ .

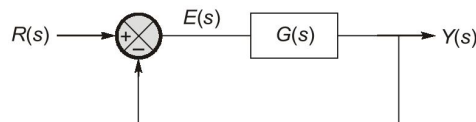


$R(s)$ ,  $Y(s)$ , and  $E(s)$  are the Laplace transforms of  $r(t)$ ,  $y(t)$ , and  $e(t)$ , respectively. If the input  $r(t)$  is a unit step function, then \_\_\_\_\_.

- (a)  $\lim_{t \rightarrow \infty} e(t) = 0$  (b)  $\lim_{t \rightarrow \infty} e(t)$  does not exist,  $e(t)$  is oscillatory  
(c)  $\lim_{t \rightarrow \infty} e(t) = \frac{1}{3}$  (d)  $\lim_{t \rightarrow \infty} e(t) = \frac{1}{4}$

**Ans. (b)**

Given system is



$$G(s) = \frac{6}{s(s+1)(s+2)}$$

The characteristic equation,

$$1 + G(s) = 0$$

$$1 + \frac{6}{s(s+1)(s+2)} = 0$$

$$s(s+1)(s+2) + 6 = 0$$

$$s^3 + 3s^2 + 2s + 6 = 0$$

Clearly, here the internal coefficient product is equal to external coefficient product i.e.,

$$3 \times 2 = 6 \times 1$$

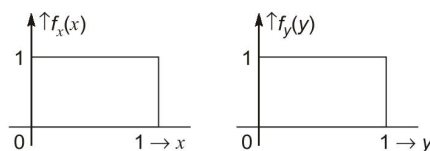
Hence, the given system is marginally stable system.

$\therefore e(t)$  not exist because system is oscillatory system.

**End of Solution**

**Q.10** Suppose  $X$  and  $Y$  are independent and identically distributed random variables that are distributed uniformly in the interval  $[0, 1]$ . The probability that  $X \geq Y$  is \_\_\_\_.

**Ans. (0.5)**



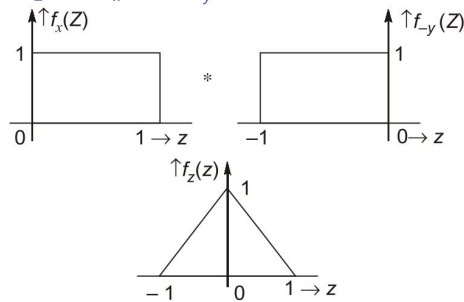
$$P(X \geq Y) = P(X - Y \geq 0)$$

$$Z = X + (-Y)$$

Let

$$P(Z \geq 0) = \int_0^{\infty} f_z(Z) d_z$$

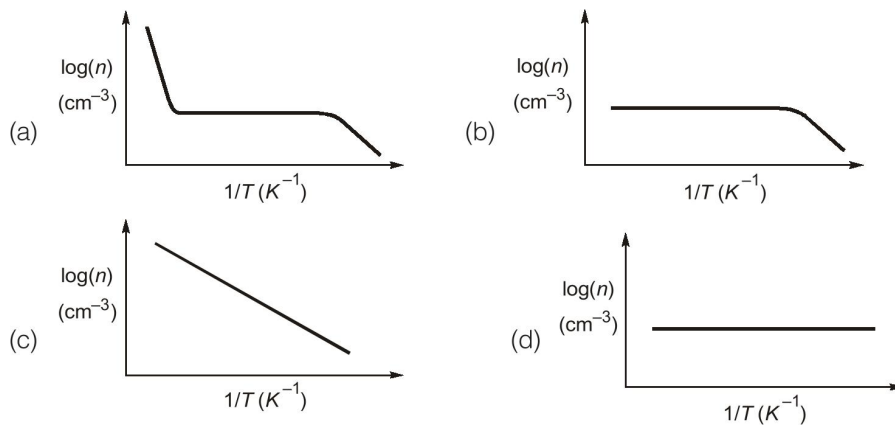
$$f_z(z) = f_x(z) * f_y(z)$$



$$P(Z \geq 0) = \frac{1}{2}$$

End of Solution

**Q.11** For non-degenerately doped n-type silicon, which one of the following plots represents the temperature ( $T$ ) dependence of free electron concentration ( $n$ )?



**Ans. (a)**

End of Solution

**Q.12** In a number system of base  $r$ , the equation  $x^2 - 12x + 37 = 0$  has  $x = 8$  as one of its solutions. The value of  $r$  is \_\_\_\_\_.

**Ans. (11)**

End of Solution

**Q.13** An amplitude modulator has output (in Volts)

$$s(t) = A \cos(400\pi t) + B \cos(360\pi t) + B \cos(440\pi t)$$

The carrier power normalized to  $1 \Omega$  resistance is 50 Watts. The ratio of the total sideband power to the total power is  $1/9$ . The value of  $B$  (in Volts, rounded off to two decimal places) is \_\_\_\_\_.

Ans. (2.5)

$$P_C = \frac{A^2}{2} = 50$$

$$A = 10$$

$$\frac{P_{SB}}{P_t} = \frac{\mu^2}{2 + \mu^2} = \frac{1}{9}$$

$$\mu = \frac{1}{2}$$

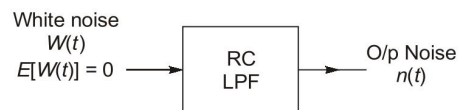
$$B = \frac{A_c \mu}{2} = \frac{10}{2} \times \frac{1}{2} = 2.5$$

End of Solution

**Q.14** A White Gaussian noise  $w(t)$  with zero mean and power spectral density  $\frac{N_0}{2}$ , when applied to a first-order  $RC$  low pass filter produces an output  $n(t)$ . At a particular time  $t = t_k$ , the variance of the random variable  $n(t_k)$  is \_\_\_\_\_.

- (a)  $\frac{N_0}{4RC}$  (b)  $\frac{2N_0}{RC}$   
 (c)  $\frac{N_0}{RC}$  (d)  $\frac{N_0}{2RC}$

Ans. (a)



$$E[n(t)] = E[W(t)] \cdot H(0)$$

$$E[n(t)] = 0$$

$$E[n^2(t)] = \{E(n(t))^2 + \text{var}[n(t)]\}$$

$$\text{var}[n(t)] = E[n^2(t)]$$

at  $t = t_k$

$$\text{var}[n(t)]|_{t=t_k} = E[n^2(t_k)] = E[n(t_k) \cdot n(t_k)] = R_n(0)$$

$$S_n(f) = S_w(f) \cdot |H(f)|^2$$

$$= \frac{N_0}{2} \cdot \frac{1}{1 + (\omega RC)^2}$$

$$R_n(\tau) = \text{IFT}[S_n(f)] = \frac{N_0}{4RC} \cdot e^{\frac{-|\tau|}{RC}}$$

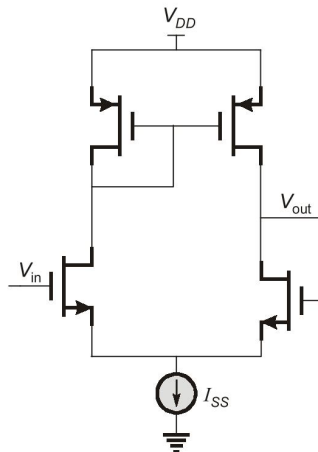
$$R_n(0) = \frac{N_0}{4RC}$$

End of Solution



**Q.15** For the closed loop amplifier circuit shown below, the magnitude of open loop low frequency small signal voltage gain is 40. All the transistors are biased in saturation. The current source  $I_{SS}$  is ideal. Neglect body effect, channel length modulation and intrinsic device capacitances. The closed loop low frequency small signal voltage gain

$\frac{V_{out}}{V_{in}}$  (rounded off to three decimal places) is \_\_\_\_\_.



- (a) 0.976                      (b) 1.000  
(c) 0.488                      (d) 1.025

**Ans. (a)**

$$A_{OL} = 40$$

Given circuit is differential amplifier with current mirror active load.

$$V_f = V_{out}$$

$$\frac{V_f}{V_{out}} = \beta = 1$$

$$A_{CL} = \frac{A_{OL}}{1 + \beta A_{OL}} = \frac{40}{1 + 1 \times 40}$$

$$A_{CL} = 0.976$$

**End of Solution**

**Q.16** A digital communication system transmits through a noiseless bandlimited channel  $[-W \ W]$ . The received signal  $z(t)$  at the output of the receiving filter is given by

$$z(t) = \sum_n b[n] x(t - nT) \text{ where } b[n] \text{ are the symbols and } x(t) \text{ is the overall system response}$$

to a single symbol. The received signal is sampled at  $t = mT$ . The Fourier transform of  $x(t)$  is  $X(f)$ . The Nyquist condition that  $X(f)$  must satisfy for zero intersymbol interference at the receiver is \_\_\_\_\_.

$$(a) \sum_{m=-\infty}^{\infty} X\left(f + \frac{m}{T}\right) = T$$

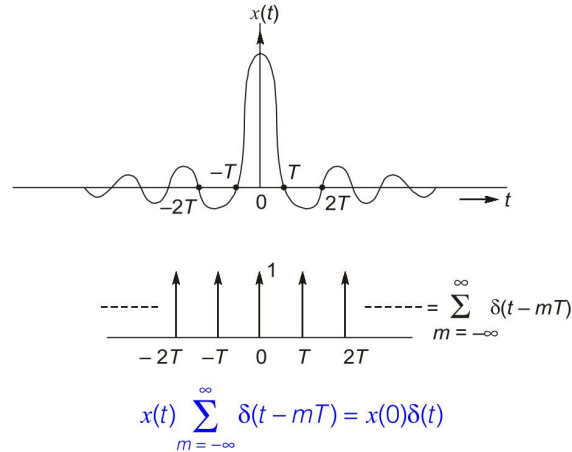
$$(b) \sum_{m=-\infty}^{\infty} X\left(f + \frac{m}{T}\right) = \frac{1}{T}$$

$$(c) \sum_{m=-\infty}^{\infty} X(f + mT) = T$$

$$(d) \sum_{m=-\infty}^{\infty} X(f + mT) = \frac{1}{T}$$

Ans. (a)

For zero ISI,  $x(t)$  should have zero crossings at  $\pm T, \pm 2T, \pm 3T, \dots$



$$\frac{1}{T} \sum_{m=-\infty}^{\infty} X\left(f - \frac{m}{T}\right) = x(0)$$

$$\sum_{m=-\infty}^{\infty} X\left(f - \frac{m}{T}\right) = x(0)T$$

$$(or) \sum_{m=-\infty}^{\infty} X\left(f + \frac{m}{T}\right) = x(0)T$$

End of Solution

**Q.17** For a causal discrete-time LTI system with transfer function

$$H(z) = \frac{2z^2 + 3}{\left(z + \frac{1}{3}\right)\left(z - \frac{1}{3}\right)}$$

which of the following statements is/are true?

- (a) The system is stable.
- (b) The final value of the impulse response is 0.
- (c) The system is a minimum phase system.
- (d) The initial value of the impulse response is 2.

Ans. (a, b, d)

$$\text{Given: } H(z) = \frac{2z^2 + 3}{\left(z + \frac{1}{3}\right)\left(z - \frac{1}{3}\right)}$$

System is non-minimum phase system. Since zero are lying outside the unit circle.

**Final value:** 
$$h(\infty) = \lim_{z \rightarrow 1} (z-1) \frac{2z^2+3}{\left(z+\frac{1}{3}\right)\left(z-\frac{1}{3}\right)} = 0$$

**Initial value:** 
$$\begin{aligned} h(0) &= \lim_{z \rightarrow \infty} \frac{(2z^2+3)}{\left(z+\frac{1}{3}\right)\left(z-\frac{1}{3}\right)} = \lim_{z \rightarrow \infty} \frac{z^2 \left[2+\frac{3}{z^2}\right]}{z^2 \left[1+\frac{1}{3z}\right] \left[1-\frac{1}{3z}\right]} \\ &= \lim_{z \rightarrow \infty} \frac{\left[2+\frac{3}{z^2}\right]}{\left[1+\frac{1}{3z}\right] \left[1-\frac{1}{3z}\right]} = \frac{2}{1 \times 1} = 2 \end{aligned}$$

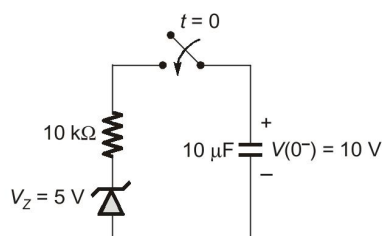
$$z = -\frac{1}{3}, z = \frac{1}{3}$$

$$|z| = \frac{1}{3} = 0.33$$

System is stable. Since  $|\text{pole}| < 1$  i.e. pole are lying inside the unit circle.

**End of Solution**

- Q.18** As shown in the circuit, the initial voltage across the capacitor is 10 V, with the switch being open. The switch is then closed at  $t = 0$ . The total energy dissipated in the ideal Zener diode ( $V_z = 5 \text{ V}$ ) after the switch is closed (in mJ, rounded off to three decimal places) is \_\_\_\_\_.



**Ans. (0.125)**

For  $t > 0$ : Capacitor starts discharging through 10 K resistor and zener diode.

$$V_c(t) = 10e^{-t/RC}, R \times C = 0.1 \text{ sec}$$

Zener diode remains on till  $V_c$  becomes 5 V.

$$5 = 10e^{-t_1/RC} \Rightarrow t_1 = RC \ln 2 = 0.0693 \text{ sec}$$

Current in the circuit is  $i(t) = I_0 e^{-t/RC}$

where 
$$I_0 = \frac{10-5}{10\text{k}} = 0.5 \text{ mA}$$

Total energy dissipated in zener diode is,

$$W = \int_0^{t_1} V_z \times i(t) dt$$

$$\begin{aligned}
 W &= \int_0^{0.0693} 5 \times 0.5 e^{-t/RC} \text{ mJ} \\
 &= 2.5[-0.1] \left[ e^{-t/0.1} \right]_0^{0.0693} \\
 W &= 0.125 \text{ mJ}
 \end{aligned}$$

End of Solution

**Q.19** Let  $\hat{i}$  and  $\hat{j}$  be the unit vectors along  $x$  and  $y$  axes, respectively and let  $A$  be a positive constant. Which one of the following statements is true for the vector fields  $\vec{F}_1 = A(\hat{i}y + \hat{j}x)$  and  $\vec{F}_2 = A(\hat{i}y - \hat{j}x)$ ?

- (a) Neither  $\vec{F}_1$  nor  $\vec{F}_2$  is an electrostatic field.
- (b) Only  $\vec{F}_2$  is an electrostatic field
- (c) Only  $\vec{F}_1$  is an electrostatic field.
- (d) Both  $\vec{F}_1$  and  $\vec{F}_2$  are electrostatic fields.

**Ans. (c)**

For an electrostatic field,  $\nabla \times \vec{F} = 0$

$$\vec{F}_1 = A[y\hat{i} + x\hat{j}]$$

$$\begin{aligned}
 \Rightarrow \quad \Delta \times \vec{F}_1 &= A \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y & x & 0 \end{vmatrix} \\
 &= A[0\hat{i} - 0\hat{j} + (1-1)\hat{k}] \\
 &= 0
 \end{aligned}$$

$$\vec{F}_2 = A[y\hat{i} - x\hat{j}]$$

$$\begin{aligned}
 \Rightarrow \quad \Delta \times \vec{F}_2 &= A \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y & -x & 0 \end{vmatrix} \\
 &= A[0\hat{i} - 0\hat{j} + (-1-1)\hat{k}] \\
 &= -2A\hat{k}
 \end{aligned}$$

Hence,  $\vec{F}_1$  is electrostatic,  $\vec{F}_2$  is not electrostatic.

End of Solution

**Q.20** In the context of Bode magnitude plots, 40 dB/decade is the same as

- (a) 12 dB/octave (b) 10 dB/octave  
(c) 6 dB/octave (d) 20 dB/octave

**Ans. (a)**

$$20 \times n \text{ dB/decade} = 6 \times n \text{ dB/octave}$$

$$40 \text{ dB/decade} = 12 \text{ dB/octave}$$

**End of Solution**

**Q.21** Let  $R$  and  $R^3$  denote the set of real numbers and the three dimensional vector space over it, respectively. The value of  $\alpha$  for which the set of vectors

$$\{[2 \ -3 \ \alpha], [3 \ -1 \ 3], [1 \ -5 \ 7]\}$$

does not form a basis of  $R^3$  is \_\_\_\_\_.

**Ans. (5)**

$\therefore$  Given vectors form basis so there must be L.I and its condition is,

$$|A| \neq 0$$

or

$$\begin{vmatrix} 2 & 3 & 1 \\ -3 & -1 & -5 \\ a & 3 & 7 \end{vmatrix} \neq 0$$

$$2[-7 + 15] - 3[-21 + 5a] + 1[-9 + a] \neq 0$$

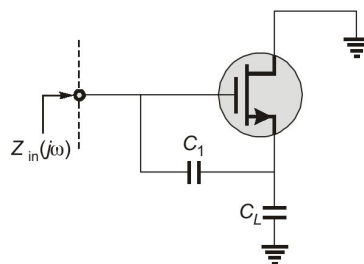
$$16 + 63 - 15a - 9 + a \neq 0$$

$$-14a \neq -70$$

$$\Rightarrow a \neq 5$$

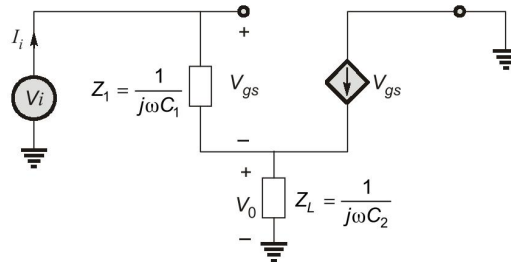
**End of Solution**

**Q.22** In the circuit below, assume that the long channel NMOS transistor is biased in saturation. The small signal trans-conductance of the transistor is  $g_m$ . Neglect body effect, channel length modulation and intrinsic device capacitances. The small signal input impedance  $Z_{in}(j\omega)$  is \_\_\_\_\_.



- (a)  $\frac{-g_m}{C_1 C_L \omega^2} + \frac{1}{j\omega C_1} + \frac{1}{j\omega C_L}$  (b)  $\frac{-g_m}{C_1 C_L \omega^2} + \frac{1}{j\omega C_1 + j\omega C_L}$   
(c)  $\frac{1}{j\omega C_1} + \frac{1}{j\omega C_L}$  (d)  $\frac{g_m}{C_1 C_L \omega^2} + \frac{1}{j\omega C_1} + \frac{1}{j\omega C_L}$

Ans. (a)



$$I_i = \frac{V_i - V_0}{Z_i} = \frac{V_i}{Z_i} - \frac{V_0}{Z_i} \quad \dots(1)$$

$$V_0 = (I_i + g_m V_{gs}) Z_L$$

$$V_0 = I_i Z_L + g_m V_{gs} Z_L$$

$$V_0 = I_i (Z_L + g_m Z_1 Z_L)$$

$$I_i = \frac{V_i}{Z_1} - \frac{I_i (Z_L + g_m Z_1 Z_L)}{Z_1}$$

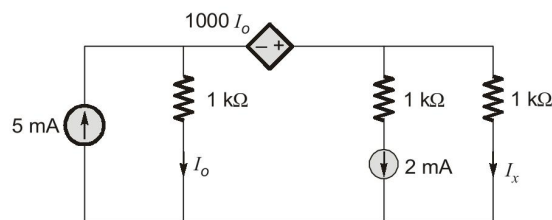
$$I_i \frac{[Z_1 + Z_L + g_m Z_1 Z_L]}{Z_1} = \frac{V_i}{Z_1}$$

$$\frac{V_i}{I_i} = Z_1 + Z_L + g_m Z_1 Z_L$$

$$Z_{in} = \frac{1}{j\omega C_1} + \frac{1}{j\omega C_L} - \frac{g_m}{\omega^2 C_1 C_L}$$

End of Solution

Q.23 In the given circuit, the current  $I_x$  (in mA) is \_\_\_\_.



Ans. (2)

$V_1$  and  $V_2$  are super node. KCL at  $V_1$  and  $V_2$ .

$$\frac{V_1}{10^3} + \frac{V_2}{1 \times 10^3} + 2 \times 10^{-3} = 5 \times 10^{-3} \quad \dots(1)$$

$$I_0 = \frac{V_1}{10^3}$$

$$V_2 - V_1 = 10^3 I_0 = 10^3 \times \frac{V_1}{10^3} = V_1$$

$$V_2 = 2V_1$$

$$V_1 = \frac{V_2}{2}$$

...(ii)

Put in equation (i)

$$\frac{1}{2}V_2 + V_2 = 3$$

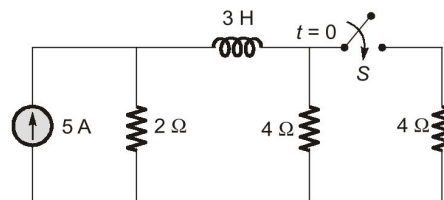
$$V_2 = 2 \text{ V}$$

$$I_x = \frac{V_2}{10^3} = 2 \text{ mA}$$

$$I_x = 2 \text{ mA}$$

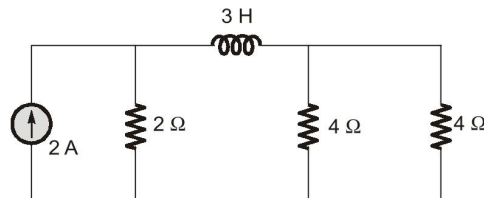
End of Solution

**Q.24** In the circuit given below, the switch S was kept open for a sufficiently long time and is closed at time  $t = 0$ . The time constant (in seconds) of the circuit for  $t > 0$  is \_\_\_\_\_.

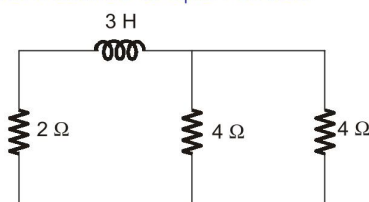


**Ans. (0.75)**

After the switch closed



For time constant, current source is open circuit.



$\therefore$

$$\tau = \frac{L_{eq}}{R_{eq}} = \frac{3}{2+2} = \frac{3}{4} = 0.75 \text{ sec}$$

End of Solution

**Q.25** A source transmits symbols from an alphabet of size 16. The value of maximum achievable entropy (in bits) is \_\_\_\_\_.

**Ans. (4)**

$$H_{\max} = \log_2 16 = 4$$

End of Solution

**Q.26** A uniform plane wave with electric field  $\vec{E}(x) = A_y \hat{a}_y e^{-j\frac{2\pi x}{3}}$  V/m is travelling in the air (relative permittivity,  $\epsilon_r = 1$  and relative permeability,  $\mu_r = 1$ ) in the  $+x$  direction ( $A_y$  is a positive constant,  $\hat{a}_y$  is the unit vector along the  $y$  axis). It is incident normally on an ideal electric conductor (conductivity,  $\sigma = \infty$ ) at  $x = 0$ . The position of the first null of the total magnetic field in the air (measured from  $x = 0$ , in metres) is \_\_\_\_\_.

- (a)  $-6$  (b)  $-\frac{3}{2}$   
(c)  $-3$  (d)  $-\frac{3}{4}$

**Ans. (d)**

At perfect conductor,

$$\Gamma = -1$$

$$\Rightarrow \Gamma = 1\pi$$

As we know that  $H_{\min}$  occurs at  $E_{\max}$ .

$$\text{Hence, } E_{\max} = E_0[1 + |\Gamma|]$$

$$H_{\min} = \frac{E_0}{\eta}[1 - |\Gamma|] \text{ at } 2\beta x_{\max} = 2n\pi + \theta_\Gamma$$

$$\text{So, } 2\beta x_{\max} = 2n\pi + \theta_\Gamma$$

$$\Rightarrow \frac{4\pi}{\lambda} x_{\max} = 2n\pi + \pi$$

$$\Rightarrow x_{\max} = (2n+1)\frac{\lambda}{4}; \quad n = 0, 1, 2, 3\ldots$$

So, for 1<sup>st</sup> value of  $x$  for H-field to be zero is

$$x_{\max} = \frac{\lambda}{4} \quad [\text{at } n = 0]$$

Now, from the equation given,

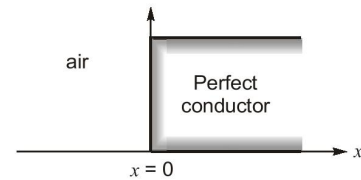
$$\beta = \frac{2\pi}{3}$$

$$\Rightarrow \frac{2\pi}{\lambda} = \frac{2\pi}{3}$$

$$\Rightarrow \lambda = 3$$

$$\text{Hence, } x_{\max} = \frac{3}{4}$$

If the reference is not at interference then  $x = -\frac{3}{4}$ .



End of Solution



**Q.27** The information bit sequence (1 1 1 0 1 0 1 0 1) is to be transmitted by encoding with Cyclic Redundancy Check 4 (CRC-4) code, for which the generator polynomial is  $C(x) = x^4 + x + 1$ . The encoded sequence of bits is \_\_\_\_\_.

- (a) {1 1 1 0 1 0 1 0 1 1 1 0 1}      (b) {1 1 1 0 1 0 1 0 1 1 1 1 0}  
 (c) {1 1 1 0 1 0 1 0 1 0 1 0 0}      (d) {1 1 1 0 1 0 1 0 1 1 1 0 0}

**Ans. (c)**

Given information bit sequence  $\rightarrow d = 111010101$

Generator polynomial  $\rightarrow C(x) = x^4 + x + 1$

$$C = 10011$$

Generator polynomial having 5 bits, so append  $5 - 1 = 4$  bits to information sequence.

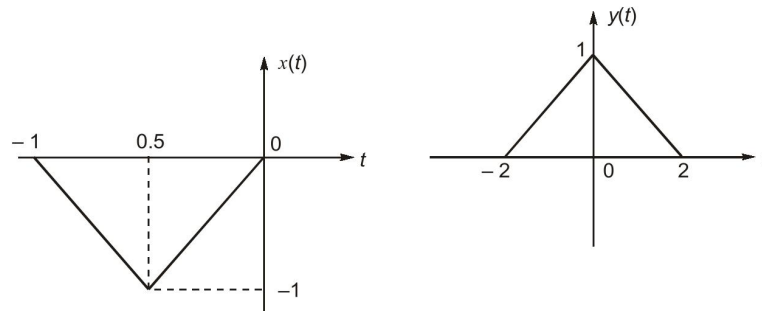
$$d = 1110101010000$$

$$\begin{array}{r}
 10011 \overline{) 111010101(11111} \\
 \underline{10011} \phantom{000000000} \\
 011100101 \phantom{000000000} \\
 \underline{10011} \phantom{000000000} \\
 01111101 \phantom{000000000} \\
 \underline{10011} \phantom{000000000} \\
 0110001 \phantom{000000000} \\
 \underline{10011} \phantom{000000000} \\
 010111 \phantom{000000000} \\
 \underline{10011} \phantom{000000000} \\
 00100 \phantom{000000000} \\
 \hline
 \text{CRC} = 0100
 \end{array}$$

$$\text{Encoded sequence} = 1110101010100$$

**End of Solution**

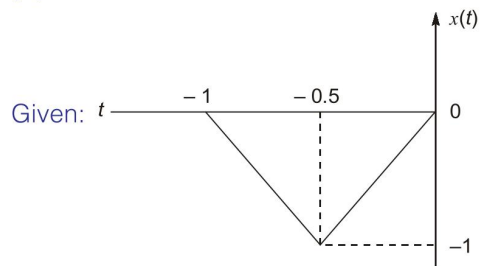
**Q.28** Consider two continuous time signals  $x(t)$  and  $y(t)$  as shown below



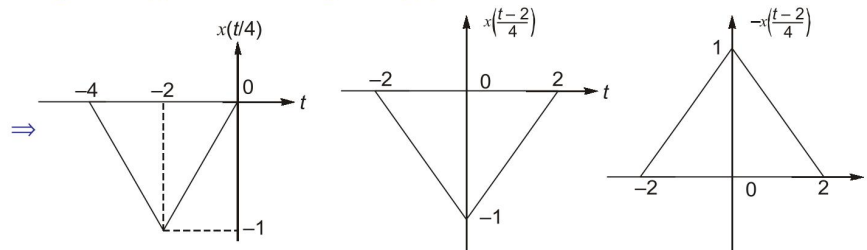
If  $X(f)$  denotes the Fourier transform of  $x(t)$ , then the Fourier transform of  $y(t)$  is \_\_\_\_\_.

- (a)  $-\frac{1}{4}X(f/4)e^{-j\pi f}$       (b)  $-4X(4f)e^{-j\pi f}$   
 (c)  $-4X(4f)e^{-j4\pi f}$       (d)  $-\frac{1}{4}X(f/4)e^{-j4\pi f}$

Ans. (c)



Length of  $x(t)$  is 1 and length of  $y(t)$  is 4.



$$\therefore y(t) = -x\left(\frac{t-2}{4}\right)$$

$$y(t) = -x\left(\frac{t}{4} - \frac{2}{4}\right) = -x\left(\frac{t}{4} - \frac{1}{2}\right)$$

On taking Fourier transform, we get

$$x(t) \leftrightarrow X(f)$$

$$x\left(\frac{t}{4} - \frac{1}{2}\right) \leftrightarrow 4X(4f)e^{-j4\pi f}$$

$$-x\left(\frac{t}{4} - \frac{1}{2}\right) \leftrightarrow -4X(4f)e^{-j4\pi f}$$

End of Solution

**Q.29** Consider a system  $S$  represented in state space as

$$\frac{dx}{dt} = \begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} r, y = \begin{bmatrix} 2 & -5 \end{bmatrix} x$$

Which of the state space representations given below has/have the same transfer function as that of  $S$ ?

(a)  $\frac{dx}{dt} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} r, y = \begin{bmatrix} 0 & 2 \end{bmatrix} x$  (b)  $\frac{dx}{dt} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} r, y = \begin{bmatrix} 1 & 2 \end{bmatrix} x$

(c)  $\frac{dx}{dt} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} x + \begin{bmatrix} -1 \\ 3 \end{bmatrix} r, y = \begin{bmatrix} 1 & 1 \end{bmatrix} x$  (d)  $\frac{dx}{dt} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} r, y = \begin{bmatrix} 1 & 2 \end{bmatrix} x$

Ans. (b, c)

From given state model, (observable form)

$$TF = C[(sI - A)^{-1}B] + D$$

...(B)

$$= \frac{(2s+1)}{s^2+3s+2}$$

From this controllable form can be written as

$$X^0 = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} X + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$Y = [1 \ 2] X + [0]u$$

Another possibility,  $TF = \frac{2s+1}{s^2+3s+2} = \frac{-1}{s+1} + \frac{3}{s+2}$

$$X^0 = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} X + \begin{bmatrix} -1 \\ 3 \end{bmatrix} v$$

$$Y = [1 \ 1]X + [0]u$$

Diagonal form of state mode.

**End of Solution**

**Q.30** A full scale sinusoidal signal is applied to a 10-bit ADC. The fundamental signal component in the ADC output has a normalized power of 1 W, and the total noise and distortion normalized power is 10  $\mu$ W. The effective number of bits (rounded off to the nearest integer) of the ADC is \_\_\_\_\_.

- (a) 10 (b) 9  
(c) 8 (d) 7

**Ans. (c)**

$$\frac{S}{N_q} = \frac{1}{10 \times 10^{-6}}$$

$$\frac{S}{N_q} = 10^5$$

$$\left( \frac{S}{N_q} \right)_{dB} = 10 \log_{10} 10^5$$

$$= 50 \text{ dB}$$

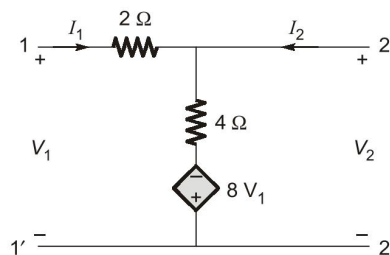
$$1.76 + 6.02n = 50$$

$$n = 8.01$$

rounded off to nearest integer  $\rightarrow n \simeq 8$

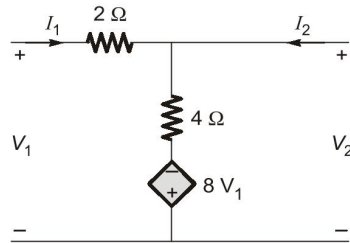
**End of Solution**

**Q.31** For the two port network shown below, the value of the  $Y_{21}$  parameter (in Siemens) is \_\_\_\_\_.

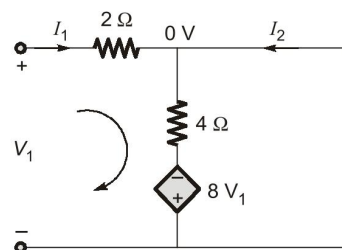


Ans. (1.5)

Given two-port network,



$$Y_{21} = \left. \frac{I_2}{V_1} \right|_{V_2=0}$$



by applying KVL in the loop,

$$V_1 - 2I_1 - 4(I_1 + I_2) + 8V_1 = 0$$

$$9V_1 - 6I_1 - 4I_2 = 0$$

but

$$I_1 = \frac{V_1 - 0}{2} \Rightarrow \frac{V_1}{2} = I_1$$

$$\therefore 9V_1 - 6\left(\frac{V_1}{2}\right) - 4I_2 = 0$$

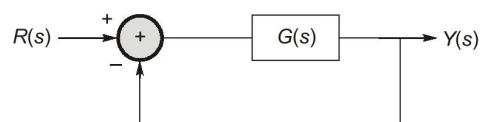
$$6V_1 = 4I_2$$

$$\therefore \frac{I_2}{V_1} = \frac{6}{4} = \frac{3}{2} = 1.5 \text{ S}$$

End of Solution

Q.32 Consider a unity negative feedback control system with forward path gain

$$G(s) = \frac{K}{(s+1)(s+2)(s+3)} \text{ as shown.}$$



The impulse response of the closed-loop system decays faster than  $e^{-t}$  if \_\_\_\_\_.

(a)  $-4 \leq K \leq -1$

(b)  $1 \leq K \leq 5$

(c)  $7 \leq K \leq 21$

(d)  $-24 \leq K \leq -6$

Ans. (b)

Given,  $G(s) = \frac{K}{(s+1)(s+2)(s+3)}$

Impulse response of system is

$$Y(s) = \frac{K}{(s+1)(s+2)(s+3)+K}$$

Given: At impulse response of closed loop system decay faster than  $e^{-t}$

Hence, put  $s = s - 1$ ,

$\therefore$  The new characteristic equation is,

$$(s - 1 + 1)(s - 1 + 2)(s - 1 + 3) + K = 0$$

$$s(s + 1)(s + 2) + K = 0$$

$$s^3 + 3s^2 + 2s + K = 0$$

By RH criterion,

$$\begin{array}{c|cc} s^3 & 1 & 2 \\ s^2 & 3 & K \\ s^1 & \frac{6-K}{3} & \\ s^0 & K & \end{array}$$

$$\therefore K > 0 ; \quad \frac{6-K}{3} > 0 \Rightarrow K < 6$$

$$\therefore 0 < K < 6$$

Hence, option (b) satisfies above range of  $K$ .

End of Solution

**Q.33** Consider the Earth to be a perfect sphere of radius  $R$ . Then the surface area of the region, enclosed by the  $60^\circ\text{N}$  latitude circle, that contains the north pole in its interior is \_\_\_\_\_.

- (a)  $(2 - \sqrt{3})\pi R^2$  (b)  $\frac{(\sqrt{2} - 1)\pi R^2}{2}$   
(c)  $\frac{(2 + \sqrt{3})\pi R^2}{8\sqrt{2}}$  (d)  $\frac{2\pi R^2}{3}$

Ans. (a)

From the figure

Earth's North pole =  $90^\circ$

Earth's equator =  $0^\circ$

In spherical co-ordinate system

Earth's North pole,  $\theta = 0^\circ$

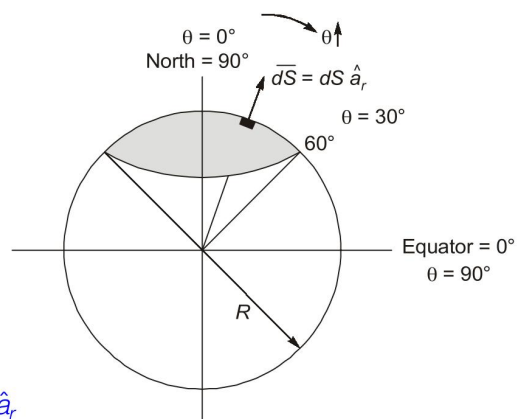
Earth's equator,  $\theta = 90^\circ$

The area of the shaded portion,

$$dS = dS \hat{a}_r$$

Hence, in spherical coordinate system

$$\Rightarrow dS = r^2 \sin\theta d\theta d\phi \hat{a}_r$$



$$\begin{aligned}
\Rightarrow S &= \int dS = \int r^2 \sin\theta d\theta d\phi \\
&= r^2 \int_{\theta=0^\circ}^{30^\circ} \sin\theta d\theta \int_{\phi=0}^{2\pi} d\phi \quad \text{at } r = R \\
&= R^2 - \left[ -\cos\theta \right]_0^{30^\circ} \cdot \phi \Big|_0^{2\pi} \\
&= R^2 \{ -[\cos 30^\circ - \cos 0^\circ] \} \cdot 2\pi \\
&= R^2 \left\{ -\left[ \frac{\sqrt{3}}{2} - 1 \right] \right\} \cdot 2\pi = R^2 \cdot \frac{2-\sqrt{3}}{2} \cdot 2\pi \\
&= (2-\sqrt{3})\pi R^2
\end{aligned}$$

End of Solution

**Q.34** The photocurrent of a PN junction diode solar cell is 1 mA. The voltage corresponding to its maximum power point is 0.3 V. If the thermal voltage is 30 mV, the reverse saturation current of the diode (in nA, rounded off to two decimal places) is \_\_\_\_\_.

**Ans. (4.13)**

**Note:** In this question voltage corresponds to max. power,  $V_m$  is given

i.e.,  $V_m = 0.3$  Volt

( $V_{OC}$  is not given be careful)

$$\begin{aligned}
I_0 &= \frac{I_L}{\left(1 + \frac{V_m}{V_T}\right) e^{V_m/V_T} - 1} = \frac{1 \text{ mA}}{\left(1 + \frac{0.3}{0.03}\right) e^{0.3/0.03} - 1} \\
&= 4.127 \text{ nA} \simeq 4.13 \text{ nA}
\end{aligned}$$

End of Solution

**Q.35** A continuous time signal  $x(t) = 2 \cos(8\pi t + \pi/3)$  is sampled at a rate of 15 Hz. The sampled signal  $x_s(t)$  when passed through an LTI system with impulse response

$$h(t) = \left( \frac{\sin 2\pi t}{\pi t} \right) \cos(38\pi t - \pi/2)$$

produces an output  $x_o(t)$ . The expression for  $x_o(t)$  is \_\_\_\_\_.

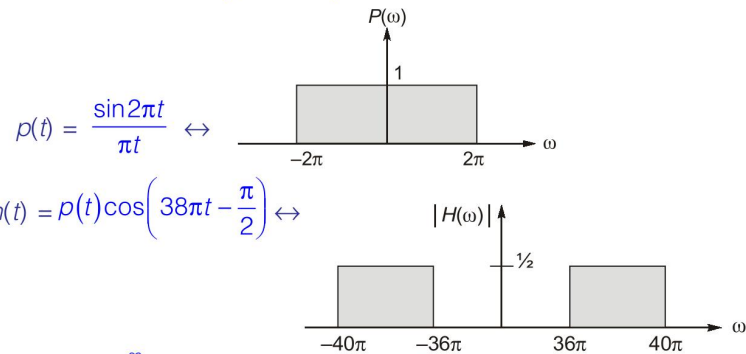
- (a)  $15 \sin(38\pi t + \pi/3)$  (b)  $15 \cos(38\pi t - \pi/6)$   
(c)  $15 \sin(38\pi t + \pi/6)$  (d)  $15 \sin(38\pi t - \pi/3)$

**Ans. (b)**

$$\begin{aligned}
x(t) &= 2 \cos\left(8\pi t + \frac{\pi}{3}\right), \quad \omega_o = 8\pi \\
f_s &= 15 \text{ Hz} \\
x(t) &\xrightarrow{\text{sample}} x_s(t) \xrightarrow{h(t)} x_o(t) \\
\Rightarrow h(t) &= \frac{\sin 2\pi t}{\pi t} \cos\left(38\pi t - \frac{\pi}{2}\right)
\end{aligned}$$

Let  $p(t) = \frac{\sin 2\pi t}{\pi t}$

$$h(t) = p(t) \cos\left(38\pi t - \frac{\pi}{2}\right)$$



After sampling:  $X_s(\omega) = f_s \sum_{n=-\infty}^{\infty} X(\omega - n\omega_s)$

Frequency components present in sampler output  $n\omega_s \pm \omega_o$

$$\omega_o, \omega_s \pm \omega_o, 2\omega_s \pm \omega_o, \dots$$

$$8\pi, 30\pi \pm 8\pi, 60\pi \pm 8\pi, \dots$$

$$8\pi, 22\pi, 38\pi, 52\pi, 68\pi, \dots \text{ (rad/sec)}$$

System will pass '38π' component of input.

$$x_o(t) = 2 \times \frac{f_s}{2} \cos\left[38\pi t + \frac{\pi}{3} - \frac{\pi}{2}\right] = 15 \cos\left(38\pi t - \frac{\pi}{6}\right)$$

End of Solution

**Q.36** A 4-bit priority encoder has inputs  $D_3, D_2, D_1$  and  $D_0$  in descending order of priority. The two-bit output  $AB$  is generated as 00, 01, 10, and 11 corresponding to inputs  $D_3, D_2, D_1$  and  $D_0$ , respectively. The Boolean expression of the output bit  $B$  is \_\_\_\_\_.

(a)  $\overline{D_3} D_2 + \overline{D_3} \overline{D_1}$

(b)  $\overline{D_3} \overline{D_2}$

(c)  $\overline{D_3} \overline{D_1}$

(d)  $D_3 \overline{D_2} + \overline{D_3} D_1$

**Ans. (\*)**

Given 4-bit priority encoder has inputs  $D_3, D_2, D_1$  and  $D_0$  in descending order of priority.

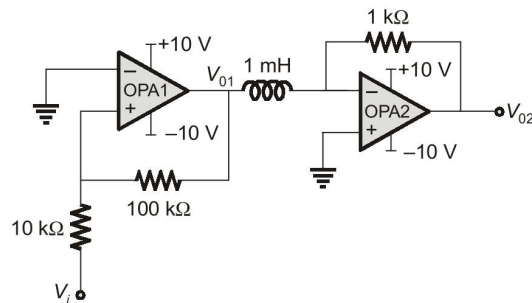
$D_3$	$D_2$	$D_1$	$D_0$	$A$	$B$
0	0	0	1	0	0
0	0	1	X	0	1
0	1	X	X	1	0
1	X	X	X	1	1

The Boolean expression of the output bit  $B = \overline{D_3} \overline{D_2} D_1 + D_3$

Therefore,  $B = D_3 + \overline{D_2} D_1$

End of Solution

**Q.37** The opamps in the circuit shown are ideal, but have saturation voltages of  $\pm 10$  V.



Assume that the initial inductor current is 0 A. The input voltage ( $V_i$ ) is a triangular signal with peak voltages of  $\pm 2$  V and time period of  $8 \mu\text{s}$ . Which one of the following statements is true?

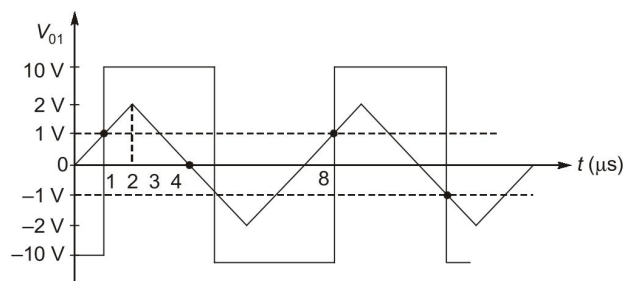
- (a)  $V_{01}$  is delayed by  $1 \mu\text{s}$  relative to  $V_i$ , and  $V_{02}$  is a trapezoidal waveform.
- (b)  $V_{01}$  is delayed by  $2 \mu\text{s}$  relative to  $V_i$ , and  $V_{02}$  is a triangular waveform.
- (c)  $V_{01}$  is not delayed relative to  $V_i$ , and  $V_{02}$  is a trapezoidal waveform.
- (d)  $V_{01}$  is not delayed relative to  $V_i$ , and  $V_{02}$  is a triangular waveform.

**Ans. (b)**

Opamp  $A_1$  is non-inverting type Schmitt Trigger.

$$V_{UT} = \frac{R_1}{R_2} \times V_{Sat} = \frac{10}{100} \times 10 \text{ V} = 1 \text{ V}$$

$$V_{LT} = -\frac{R_1}{R_2} \times V_{sat} = \frac{-10}{100} \times 10 = -1 \text{ V}$$



$V_{01}$  is square wave. It is delayed by  $1 \mu\text{s}$  wrt input  $V_i$ .

Opamp  $A_2$  is integrator. It converts square wave into triangular wave. Hence  $V_{02}$  is triangular wave.

\*Answer is option (b).

**End of Solution**

**Q.38** Let  $z$  be a complex variable. If  $f(z) = \frac{\sin(\pi z)}{z^2(z-2)}$  and  $C$  is the circle in the complex plane

with  $|z| = 3$  then  $\oint_C f(z) dz$  is \_\_\_\_\_.



$$(a) -\pi^2 j$$

$$(b) j\pi\left(\frac{1}{2} - \pi\right)$$

$$(c) j\pi\left(\frac{1}{2} + \pi\right)$$

$$(d) \pi^2 j$$

Ans. (a)

Poles of  $f(z)$  are  $z = \underset{\substack{\downarrow \\ \text{simple pole}}}{2}$  and  $\underset{\substack{\downarrow \\ \text{Double pole}}}{0}$

$R_1 = \text{Residue of } f(z) = \text{at } (z = 2)$

$$= \lim_{z \rightarrow 2} (z-2)f(z) = \lim_{z \rightarrow 2} \left( \frac{\sin \pi z}{z^2} \right) = 0$$

$R_2 = \text{Residue of } f(z) \text{ (at } (z = 0, m = 2))$

$$R_2 = \frac{1}{2!} \left[ \frac{d^{2-1}}{dz^{2-1}} (z-0)^2 f(z) \right]_{z=0}$$

$$\begin{aligned} \left[ \frac{d}{dz} \left[ \frac{\sin \pi z}{z-2} \right] \right]_{z=0} &= \left[ \frac{(z-2) \cos \pi z (\pi) - \sin \pi z}{(z-2)^2} \right]_{z=0} \\ &= \frac{(0-2) \cos(2\pi) \cdot \pi - \sin 0}{(0-2)^2} = \frac{-2\pi}{4} = \frac{-\pi}{2} \end{aligned}$$

By C-R.T,

$$\begin{aligned} I &= \oint_c f(z) dz = 2\pi j (R_1 + R_2) \\ &= 2\pi j \left( 0 - \frac{\pi}{2} \right) = -\pi^2 j \end{aligned}$$

End of Solution

**Q.39** Consider a MOS capacitor made with p-type silicon. It has an oxide thickness of 100 nm, a fixed positive oxide charge of  $10^{-8} \text{ C/cm}^2$  at the oxide-silicon interface, and a metal work function of 4.6 eV. Assume that the relative permittivity of the oxide is 4 and the absolute permittivity of free space is  $8.85 \times 10^{-14} \text{ F/cm}$ . If the flatband voltage is 0 V, the work function of the p-type silicon (in eV, rounded off to two decimal places) is \_\_\_\_.

Ans. (4.32)

$$\begin{aligned} C_{ox} &= \frac{\epsilon_{ox}}{t_{ox}} = \frac{4 \times 8.85 \times 10^{-14}}{100 \times 10^{-9} \times 100} \\ &= 4 \times 8.85 \times 10^{-14} \times 10^5 = 35.4 \times 10^{-9} \text{ F/cm}^2 \end{aligned}$$

$$V_{FB} = \frac{Q_{ox}}{C_{ox}} + \phi_{ms}$$

$$0 = \frac{Q_{ox}}{C_{ox}} + \phi_{ms}$$

$$\frac{Q_{ox}}{C_{ox}} = \frac{10^{-8}}{35.4 \times 10^{-9}} = \frac{10}{35.4} \text{ volt}$$

$Q_{ox} \rightarrow$  Positive

$\therefore \frac{Q_{ox}}{C_{ox}} \rightarrow$  Negative

$$0 = \frac{-10}{35.4} + \phi_{ms}$$

$$\phi_{ms} = \frac{10}{35.4}$$

$$\phi_m - \phi_s = \frac{10}{35.4}$$

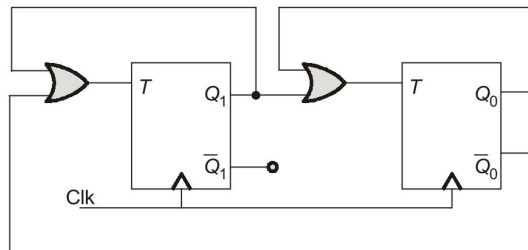
$$\phi_s = \phi_m - \frac{10}{35.4} = 4.6 - \frac{10}{35.4} = 4.317 \text{ volt}$$

$\therefore \phi_s = 4.317 \simeq 4.32 \text{ eV}$

Work function energy of semiconductor.

End of Solution

**Q.40** The sequence of states ( $Q_1 Q_0$ ) of the given synchronous sequential circuit is \_\_\_\_\_.



- (a) 11  $\rightarrow$  00  $\rightarrow$  10  $\rightarrow$  01  $\rightarrow$  00      (b) 00  $\rightarrow$  01  $\rightarrow$  10  $\rightarrow$  00  
(c) 01  $\rightarrow$  10  $\rightarrow$  11  $\rightarrow$  00  $\rightarrow$  01      (d) 00  $\rightarrow$  10  $\rightarrow$  11  $\rightarrow$  00

**Ans. (a)**

clk	$Q_1 + \bar{Q}_0$		$Q_1 + Q_0$	
	$T_1$	$T_0$	$Q_1$	$Q_0$
			0	0
	1	0	1	0
	1	1	0	1
	0	1	0	0

00  $\rightarrow$  10  $\rightarrow$  01  $\rightarrow$  00

End of Solution

**Q.41** The radian frequency value(s) for which the discrete time sinusoidal signal  $x[n] = A \cos(\Omega n + \pi/3)$  has a period of 40 is/are \_\_\_\_\_.

- (a)  $0.15\pi$       (b)  $0.3\pi$   
(c)  $0.45\pi$       (d)  $0.225\pi$

Ans. (a, b, c)

$$x[n] = A \cos \left[ \Omega n + \frac{\pi}{3} \right]$$

$$N = 40$$

$$\Omega = ?$$

$$\frac{\Omega}{2\pi} = \frac{m}{N}$$

$$\frac{\Omega}{2\pi} = \frac{m}{40}$$

$$\Omega = m \cdot \frac{2\pi}{40} = \pi \cdot m(0.05)$$

$$\Omega = 0.05\pi$$

$$m = 1$$

$$\Omega = 0.05\pi$$

$$m = 2$$

$$\Omega = 0.10\pi$$

$$m = 3$$

$$\Omega = 0.15\pi$$

$$m = 4$$

$$\Omega = 0.20\pi$$

$$m = 5$$

$$\Omega = 0.25\pi$$

$$m = 6$$

$$\Omega = 0.30\pi$$

$$m = 7$$

$$\Omega = 0.35\pi$$

$$m = 8$$

$$\Omega = 0.40\pi$$

$$m = 9$$

$$\Omega = 0.45\pi$$

End of Solution

**Q.42** Which of the following statements is/are true for a BJT with respect to its DC current gain  $\beta$ ?

- (a) Under high-level injection condition in forward active mode,  $\beta$  will decrease with increase in the magnitude of collector current.
- (b)  $\beta$  will be lower when the BJT is in saturation region compared to when it is in active region.
- (c) Under low-level injection condition in forward active mode, where the current at the emitter-base junction is dominated by recombination-generation process,  $\beta$  will decrease with increase in the magnitude of collector current.
- (d) A higher value of  $\beta$  will lead to a lower value of the collector-to-emitter breakdown voltage.

Ans. (a, b, d)

End of Solution

**Q.43** An NMOS transistor operating in the linear region has  $I_{DS}$  of 5  $\mu$ A at  $V_{DS}$  of 0.1 V. Keeping  $V_{GS}$  constant, the  $V_{DS}$  is increased to 1.5 V.

Given that  $\mu_n C_{ox} \frac{W}{L} = 50 \mu\text{A/V}^2$ , the transconductance at the new operating point (in  $\mu\text{A/V}$ , rounded off to two decimal places) is \_\_\_\_\_.

Ans. (52.5)

For linear region of MOS,

$$I_D = \mu_n C_{ox} \left( \frac{W}{L} \right) \left[ (V_{GS} - V_T) V_{DS} - \frac{1}{2} V_{DS}^2 \right]$$

$$5 \times 10^{-6} = 50 \times 10^{-6} \left[ (V_{GS} - V_T) 0.1 - \frac{1}{2} (0.1)^2 \right]$$

$$\frac{1}{10} = (V_{GS} - V_T) 0.1 - \frac{1}{2} \times 0.1^2$$

$$0.1 + 0.5(0.1)^2 = (V_{GS} - V_T) 0.1$$

$$1 + 0.5(0.1) = (V_{GS} - V_T)$$

$$1.05 \text{ volt} = V_{GS} - V_T$$

Now:  $V_{GS} \rightarrow \text{fix}$

$$V_{DS} = 1.5$$

$$V_{DS} > V_{GS} - V_T \Rightarrow \text{MOS is in saturation}$$

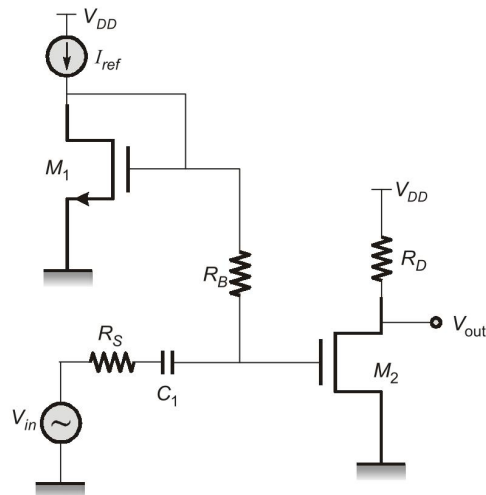
$$g_{m(\text{sat})} = \mu_n C_{ox} \left( \frac{W}{L} \right) (V_{GS} - V_T)$$

$$= 50 \times 10^{-6} \times 1.05$$

$$= 52.5 \times 10^{-6} \text{ A/V} = 52.5 \mu\text{A/V}$$

End of Solution

**Q.44** In the circuit shown below, the transistors  $M_1$  and  $M_2$  are biased in saturation. Their small signal transconductances are  $g_{m1}$  and  $g_{m2}$  respectively. Neglect body effect, channel length modulation and intrinsic device capacitances.



Assuming that capacitor  $C_1$  is a short circuit for AC analysis, the exact magnitude of

small signal voltage gain  $\left| \frac{V_{out}}{V_{in}} \right|$  is \_\_\_\_\_.

$$(a) \frac{g_{m2}R_D \left( \frac{1}{g_{m1}} \right)}{\frac{1}{g_{m1}} + R_s}$$

$$(b) g_{m2}R_D$$

$$(c) \frac{g_{m2}R_D \left( R_B + \frac{1}{g_{m1}} \right)}{R_B + \frac{1}{g_{m1}} + R_s}$$

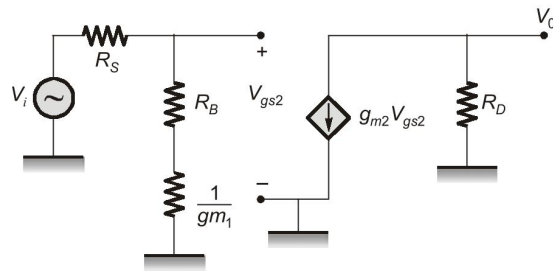
$$(d) \frac{g_{m2}R_D \left( R_B + \frac{1}{g_{m1}} + R_s \right)}{R_B + \frac{1}{g_{m1}}}$$

Ans. (c)

$$V_0 = -g_{m2}V_{gs}R_D$$

$$V_{gs} = V_i \times \frac{R_B + \frac{1}{g_{m1}}}{R_s + R_B + \frac{1}{g_{m1}}}$$

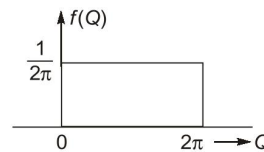
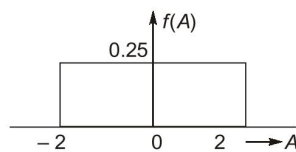
$$\left| \frac{V_0}{V_{in}} \right| = \frac{g_{m2}R_D \left( R_B + \frac{1}{g_{m1}} \right)}{R_s + R_B + \frac{1}{g_{m1}}}$$



End of Solution

**Q.45** Let  $X(t) = A \cos(2\pi f_0 t + \theta)$  be a random process, where amplitude  $A$  and phase  $\theta$  are independent of each other, and are uniformly distributed in the intervals  $[-2, 2]$  and  $[0, 2\pi]$ , respectively.  $X(t)$  is fed to an 8-bit uniform mid-rise type quantizer. Given that the autocorrelation of  $X(t)$  is  $R_x(\tau) = \frac{2}{3} \cos(2\pi f_0 \tau)$ , the signal to quantization noise ratio (in dB, rounded off to two decimal places) at the output of the quantizer is \_\_\_\_\_.

Ans. (45.15)



$$S = R_x(0) = \frac{2}{3}$$

$$N_q = \frac{\Delta^2}{12}$$

$$\Delta = \frac{2 - (-2)}{2^8} = \frac{4}{2^8}$$

$$N_q = \frac{4^2}{\frac{2^{16}}{12}} = \frac{2^4}{12 \times 2^{16}} = \frac{1}{12 \times 2^{12}}$$

$$\frac{S}{N_q} = \frac{2}{3} \times \frac{12 \times 2^{12}}{1} = 32,768$$

$$\left( \frac{S}{N_q} \right)_{dB} = 10 \log 32768 = 45.15 \text{ dB}$$

End of Solution

**Q.46** The relationship between any N-length sequence  $x[n]$  and its corresponding N-point discrete Fourier transform  $X[k]$  is defined as

$$X[k] = F\{x[n]\}.$$

Another sequence  $y[n]$  is formed as below

$$y[n] = F\{F\{F\{F\{x[n]\}\}\}\}.$$

For the sequence  $x[n] = \{1, 2, 1, 3\}$ , the value of  $Y[0]$  is \_\_\_\_\_.

**Ans. (112)**

$$y[n] = N^2 x[n]$$

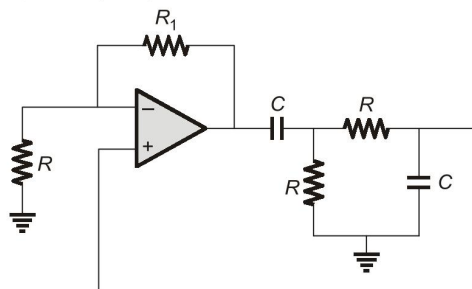
$$y[n] = N^2 x[n]$$

$$\therefore N = 4$$

$$\begin{aligned} Y[k] \big|_{k=0} &= \sum_{n=0}^3 y[n] = \sum_{n=0}^3 N^2 x[n] \\ &= N^2 [x[0] + x[1] + x[2] + x[3]] \\ &= (4)^2 [1 + 2 + 1 + 3] = 16 \times 7 = 112 \end{aligned}$$

End of Solution

**Q.47** In the circuit below, the opamp is ideal.



If the circuit is to show sustained oscillations, the respective values of  $R_1$  and the corresponding frequency of oscillation are \_\_\_\_\_.

(a)  $2R$  and  $1/(2\pi\sqrt{6}RC)$

(b)  $29R$  and  $1/(2\pi RC)$

(c)  $2R$  and  $1/(2\pi RC)$

(d)  $29R$  and  $1/(2\pi\sqrt{6}RC)$

Ans. (c)

For given circuit, frequency of oscillations is  $f_o = \frac{1}{2\pi RC}$ .

For sustained oscillations, minimum gain should be 3.

$$1 + \frac{R_1}{R} = 3$$

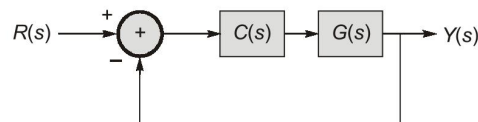
$$\Rightarrow R_1 = 2R$$

Answer is option (c)

End of Solution

**Q.48** A satellite attitude control system, as shown below, has a plant with transfer function

$G(s) = \frac{1}{s^2}$  cascaded with a compensator  $C(s) = \frac{K(s + \alpha)}{s + 4}$ , where  $K$  and  $\alpha$  are positive real constants.



In order for the closed-loop system to have poles at  $-1 \pm j\sqrt{3}$ , the value of  $\alpha$  must be \_\_\_\_\_.

- (a) 0 (b) 3  
(c) 1 (d) 2

Ans. (c)

$$G(s)C(s) = \frac{1}{s^2} \frac{k(s + \alpha)}{(s + 4)}$$

Characteristic equation is

$$s^3 + 4s^2 + ks + \alpha = 0 \quad \dots(I)$$

Poles of the system present at  $-1 \pm j\sqrt{3}$

Characteristic equation is

$$(s + a)[(s + 1)^2 + 3] = 0$$

$$(s + a)(s^2 + 2s + 4) = 0$$

$$s^3 + (2 + a)s^2 + (4 + 2a)s + 4a = 0 \quad \dots(II)$$

Compare equation (I) and (II)

$$2 + a = 4$$

$$a = 2$$

$$4 + 2a = k$$

$$k = 8$$

$$k\alpha = 4a$$

$$\alpha = \frac{8}{8} = 1$$

$$\alpha = 1$$

End of Solution

**Q.49** A source transmits a symbol  $s$ , taken from  $(-4, 0, 4)$  with equal probability, over an additive white Gaussian noise channel. The received noisy symbol  $r$  is given by  $r = s + w$ , where the noise  $w$  is zero mean with variance 4 and is independent of  $s$ . Using

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{t^2}{2}} dt, \text{ the optimum symbol error probability is } \underline{\hspace{2cm}}.$$

- (a)  $\frac{4}{3}Q(1)$  (b)  $\frac{2}{3}Q(1)$   
(c)  $\frac{4}{3}Q(2)$  (d)  $\frac{2}{3}Q(2)$

**Ans. (a)**

$$r = s + w$$

$$f_w(w) = \frac{1}{\sqrt{2\pi}} e^{-\frac{w^2}{2 \times 4}} = N(0, 4)$$

transmission of  $-4 \rightarrow r = -4 + w$

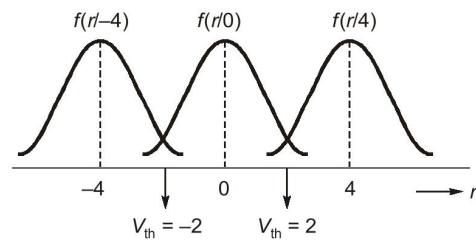
$$f(r|-4) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(r+4)^2}{2 \times 4}}$$

transmission of  $0 \rightarrow r = w$

$$f(r|0) = \frac{1}{\sqrt{2\pi}} e^{-\frac{r^2}{2 \times 4}}$$

transmission of  $4 \rightarrow r = 4 + n$

$$f(r|4) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(r-4)^2}{2 \times 4}}$$



$$P_e = P(-4) P_{e-4} + P(0) P_{e0} + P(4) P_{e4}$$

Given  $P(-4) = P(0) = P(4) = \frac{1}{3}$

$$P_{e-4} = P(r > -2) \quad \text{where, } r = N(-4, 4)$$

$$= 1 - (1 - F_r(2))$$

$$= 1 - \left( 1 - Q\left(\frac{-2+4}{2}\right) \right)$$

$$= Q(1)$$

$$P_{e0} = P(r < -2) + P(r > 2) \quad \text{where, } r = N(0, 4)$$

$$= F_r(-2) + \{1 - F_r(2)\}$$

$$= \left( 1 - Q\left(\frac{-2-0}{2}\right) \right) + Q\left(\frac{2-0}{2}\right)$$

$$= [1 - Q(-1)] + Q(1) = 2Q(1)$$



$$\begin{aligned}
P_{\text{ed}} &= P(r < 2) \\
&= F_r(2) \\
&= 1 - Q\left(\frac{2-4}{2}\right) \\
&= 1 - Q(-1) \\
&= Q(1) \\
P_e &= \frac{1}{3}Q(1) + \frac{2}{3}Q(1) + \frac{1}{3}Q(1) \\
&= \frac{4}{3}Q(1)
\end{aligned}$$

where,  $r = N(4, 4)$

**End of Solution**

**Q.50** Let  $F_1$ ,  $F_2$ , and  $F_3$  be functions of  $(x, y, z)$ . Suppose that for every given pair of points  $A$  and  $B$  in space, the line integral  $\int_C (F_1 dx + F_2 dy + F_3 dz)$  evaluates to the same value along any path  $C$  that starts at  $A$  and ends at  $B$ . Then which of the following is/are true?  
 (a) There exists a differentiable scalar function  $f(x, y, z)$  such that

$$F_1 = \frac{\partial f}{\partial x}, F_2 = \frac{\partial f}{\partial y}, F_3 = \frac{\partial f}{\partial z}.$$

$$(b) \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z} = 0.$$

$$(c) \frac{\partial F_3}{\partial y} = \frac{\partial F_2}{\partial z}, \frac{\partial F_1}{\partial z} = \frac{\partial F_3}{\partial x}, \frac{\partial F_2}{\partial x} = \frac{\partial F_1}{\partial y}.$$

$$(d) \text{ For every closed path } \Gamma, \text{ we have } \oint_{\Gamma} (F_1 dx + F_2 dy + F_3 dz) = 0.$$

**Ans. (a, c, d)**

$\therefore$  Given integral is independent of contour 'C'

$$\int_C (F_1 dx + F_2 dy + F_3 dz) = \int_C \vec{F} \cdot \vec{dr} \text{ is independent of 'C'}$$

Then  $\vec{F}$  is conservative field

$$\therefore \vec{F} = \nabla \phi$$

$$\Rightarrow F_1 = \frac{\partial \phi}{\partial x}, F_2 = \frac{\partial \phi}{\partial y}, F_3 = \frac{\partial \phi}{\partial z}$$

$\therefore$  Option (a) is true.

$\therefore \vec{F}$  is conservative

$$\text{Then, } \nabla \times \vec{F} = 0$$

$$\Rightarrow \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix} = \vec{0}$$

$$= i \left( \frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z} \right) - j \left( \frac{\partial F_3}{\partial x} - \frac{\partial F_1}{\partial z} \right) + k \left( \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right) = 0\hat{i} + 0\hat{j} + 0\hat{k}$$

Equate on both sides,

We get,  $\frac{\partial F_3}{\partial y} = \frac{\partial F_2}{\partial z}, \quad \frac{\partial F_3}{\partial x} = \frac{\partial F_1}{\partial z}, \quad \frac{\partial F_2}{\partial x} = \frac{\partial F_1}{\partial y}$

∴ Option (c) is true.

If  $\vec{F}$  is irrotational then,

$$\oint_C F_1 dx + F_2 dy + F_3 dz = 0$$

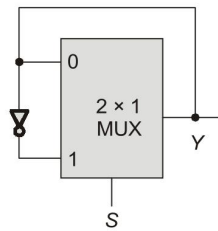
∴ Option (d) is also true.

But,  $\vec{\nabla} \cdot \vec{F} = \vec{\nabla} \cdot (\vec{\nabla} \phi) \neq 0$

∴ Option (b) is false.

End of Solution

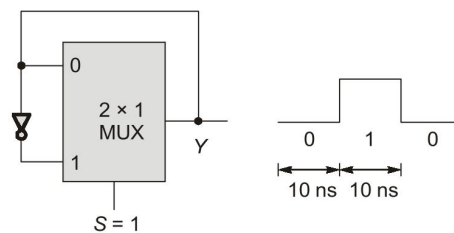
**Q.51** The propagation delay of the  $2 \times 1$  MUX shown in the circuit is 10 ns. Consider the propagation delay of the inverter as 0 ns.



If  $S$  is set to 1 then the output  $Y$  is \_\_\_\_\_.

- (a) constant at 1
- (b) a square wave of frequency 100 MHz
- (c) a square wave of frequency 50 MHz
- (d) constant at 0

**Ans. (c)**



∴  $T_c = 10 + 10 = 20 \text{ nsec}$

∴  $f_c = \frac{1}{T_c} = \frac{10^9}{20} = 50 \text{ MHz}$

End of Solution

**Q.52** A non-degenerate n-type semiconductor has 5% neutral dopant atoms. Its Fermi level is located at 0.25 eV below the conduction band ( $E_C$ ) and the donor energy level ( $E_D$ ) has a degeneracy of 2. Assuming the thermal voltage to be 20 mV. The difference between  $E_C$  and  $E_D$  (in eV, rounded off to two decimal places) is \_\_\_\_\_.

**Ans. (0.18)**

5% neutral donor atoms/dopant atoms means 95% donor atoms are ionized.  
Concentration of electrons occupying the donor level is given as

$$n_d = \frac{N_d}{1 + \frac{1}{g} \exp\left(\frac{E_D - E_F}{kT}\right)}$$

and  $n_d = N_d - N_d^*$  where  $N_d^*$  is ionized donor atoms concentration.

according to question

$$n_d = 5\% \text{ of } N_d$$

$$g = 2 \text{ degeneracy factor}$$

$$\frac{5}{100} N_d = \frac{N_d}{1 + \frac{1}{g} \exp\left(\frac{E_D - E_F}{kT}\right)}$$

$$\frac{1}{20} = \frac{1}{1 + \frac{1}{g} \exp\left(\frac{E_D - E_F}{kT}\right)}$$

$$1 + \frac{1}{g} \exp\left(\frac{E_D - E_F}{kT}\right) = 20$$

$$\exp\left(\frac{E_D - E_F}{kT}\right) = 38$$

$$\frac{E_D - E_F}{kT} = \ln(38) \quad \left\{ \text{given } V_T = \frac{kT}{q} = 20 \text{ mV} \right\}$$

$$E_D - E_F = \ln(38) \times 0.020$$

$$= 0.07275 \text{ eV}$$

given:

$$E_C - E_F = 0.25 \text{ eV}$$

$$\therefore E_C - E_D + E_D - E_F = 0.25 \text{ eV}$$

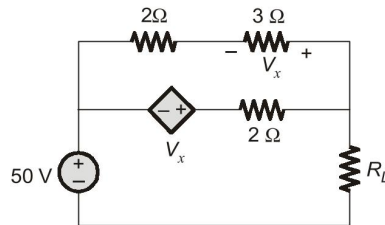
$$E_C - E_D = 0.25 \text{ eV} - (E_D - E_F)$$

$$= 0.25 \text{ eV} - 0.07275 \text{ eV}$$

$$= 0.17725 \text{ eV} \simeq 0.18 \text{ eV}$$

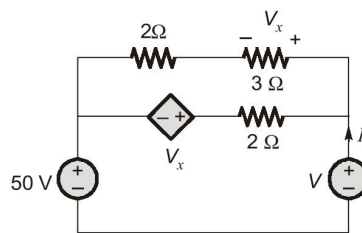
End of Solution

**Q.53** In the network shown below, maximum power is to be transferred to the load  $R_L$ .



The value of  $R_L$  (in  $\Omega$ ) is \_\_\_\_\_.

**Ans. (2.5)**



$$I = \frac{V - V_x}{2} + \frac{V}{5} = \frac{V}{2} - \frac{V_x}{2} + \frac{V}{5}$$

But,  $V_x = \frac{V}{5} \times 3$

$$\therefore I = \frac{V}{2} - \frac{1}{2} \times \frac{3V}{5} + \frac{V}{5} = \frac{V}{2} - \frac{3V}{10} + \frac{V}{5}$$

$$I = \frac{5V - 3V + 2V}{10} = \frac{4V}{10}$$

$$\therefore \frac{V}{I} = \frac{10}{4} = 2.5 \Omega$$

**End of Solution**

**Q.54** A lossless transmission line with characteristic impedance  $Z_0 = 50 \Omega$  is terminated with an unknown load. The magnitude of the reflection co-efficient is  $|\Gamma| = 0.6$ . As one moves towards the generator from the load, the maximum value of the input impedance magnitude looking towards the load (in  $\Omega$ ) is \_\_\_\_\_.

**Ans. (200)**

As we know that

$$Z_{\max} = SZ_0$$

Now,

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.6}{1 - 0.6} = \frac{1.6}{0.4} = 4$$

$$\therefore Z_{\max} = 4 \times 50 = 200 \Omega.$$

**End of Solution**

**Q.55** Consider the matrix  $\begin{bmatrix} 1 & k \\ 2 & 1 \end{bmatrix}$ , where  $k$  is a positive real number. Which of the following vectors is/are eigenvector(s) of this matrix?

(a)  $\begin{bmatrix} 1 \\ \sqrt{2/k} \end{bmatrix}$

(b)  $\begin{bmatrix} 1 \\ -\sqrt{2/k} \end{bmatrix}$

(c)  $\begin{bmatrix} \sqrt{2k} \\ -1 \end{bmatrix}$

(d)  $\begin{bmatrix} \sqrt{2k} \\ 1 \end{bmatrix}$

**Ans.** (a, b)

C Eq is  $|A - \lambda I| = 0$

or  $\lambda^2 - 2\lambda + (1 - 2K) = 0$

$$\lambda = 1 \pm \sqrt{2K}$$

E vector for  $\lambda = 1 \pm \sqrt{2K}$  :

$$AX = \lambda X$$

$$(A - \lambda I)X = 0$$

$$\begin{bmatrix} -\sqrt{2K} & K \\ 2 & -\sqrt{2K} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow -\sqrt{2K}x_1 + Kx_2 = 0$$

or  $x_1 = \sqrt{\frac{K}{2}}x_2$

$$X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \sqrt{K/2} \\ 1 \end{bmatrix} \approx \begin{bmatrix} 1 \\ \sqrt{\frac{2}{K}} \end{bmatrix}$$

Similarly other E vector is

$$X = \begin{bmatrix} -\sqrt{\frac{K}{2}} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -\sqrt{\frac{2}{K}} \end{bmatrix}$$

End of Solution

