Control Systems Test I

Number of Questions: 35 **Time: 60 min.**

Directions for questions 1 to 35: Select the correct alternative from the given choices.

1. A system has a damping ratio of 1.25, a natural frequency of 400rad/sec and *DC* gain of 1. The response of the system to a unit step input is

(A) $1+\frac{800}{3}\left(e^{-800t}-e^{-200t}\right)$

(B) $1-\frac{800}{3}\left(e^{-200t}+e^{-800t}\right)$

(C) $1-\frac{800}{3}\left(e^{-200t}-e^{-800t}\right)$

(D) $1+\frac{800}{3}\left(e^{-200t}-e^{-800t}\right)$

2. Gain margin is the amount of gain to make the system

(A) Oscillatory (B) Stable
(C) Exponential (D) Unstable

3. A system with gain margin close to unity or a phase margin close to zero is

(A) relatively stable (B) highly stable
(C) oscillatory (D) none

4. The unit step response of a particular control system is $c(t) = 1 - 5e^{-t}$, then the transfer function is

(A) $\frac{1+4s}{(s+1)s}$ (B) $\frac{-(1+4s)}{s+1}$

(C) $\frac{1-4s}{(s+1)}$ (D) $\frac{-1+4s}{(s+1)s}$

5. A phase – lead compensator will

(A) Improve the speed of the response
(B) Increases Bandwidth
(C) Reduces the amount of overshoot
(D) All the above

6. There are three poles and two zeros of $G(s)\,H(s)$. there will be

(A) One root locus (B) Two root loci
(C) Three root loci (D) Five root loci

7. A control system has $G(s)$

$H(s) = \frac{K}{s(s+3)(s^2+4s+20)}$, for $(0 < K < \infty)$.

Then the number of break away points occurs in the root locus are

(A) One (B) Two
(C) Three (D) Four

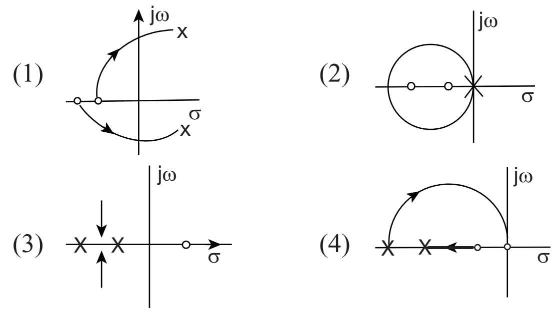
8. The output of a given system settles within ± 5% for a unit step input. Then the settling time is $G(s) = \frac{s+4}{s^2+6s+36}$

(A) 0.5 (B) 0.75
(C) 0.9 (D) 1

9. Which one of the following is correct when a pole is added to a forward path transfer function.

(A) System becomes less stable
(B) Bandwidth of system Decreases
(C) Stability improves
(D) Both A and B

10. Consider the sketch shown below



The root locus can be

(A) 1 and 3 (B) 2 and 4
(C) 1, 2 and 4 (D) None of the above

11. Consider a feedback control system with loop transfer function $G(S)\,H(S) = \frac{K}{s(1+s)(1+2s)}$ then order of the given system is

(A) 1 (B) 3
(C) 2 (D) 4

12. A minimum phase unity feedback system has a Bode plot with a constant slope of 0dB/decade for all frequencies. What is the value of the maximum phase margin for the system?

(A) 180° (B) 90°
(C) –90° (D) 0°

13. The inter section of Asymptotes of root loci of a system with open loop transfer function

$G(s)H(s) = \frac{K(s+2)}{s(s+1)(s+3)}$ is

(A) –2 (B) –1
(C) –0.67 (D) 1

14. Which one of the following characteristic equations of results in the stable operation of feedback system

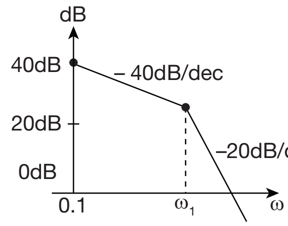
(A) $s^3 + 7s^2 + 5s - 6 = 0$
(B) $s^3 + 2s^2 + 3s + 7 = 0$
(C) $s^3 + 4s^2 + 10s + 11 = 0$
(D) $s^4 + s^3 - 3s^2 + 5s + 6 = 0$

15. Transfer function of a lag compensator is given by $G(s)$ $H(s) = \frac{s+0.2}{s+0.026}$ then the frequency corresponding to maximum phase lead angle is

(A) 7.69 rad/sec (B) 4.9 rad/sec

(C) 2.77 rad/sec (D) 0.072 rad/sec

16. Find the Transfer function for the given bode plot of a unit feedback system



(A) $\dfrac{0.33}{s(s+0.316)^2}$ (B) $\dfrac{0.331}{s^2(s+0.316)}$

(C) $\dfrac{1}{s^2(1+0.3165)}$ (D) $\dfrac{1}{s(1+0.3165)^2}$

17. What should be the value of λ in order to the given system is controllable

$\dot{x}_1 = x_2 + \mu;\ \dot{x}_2 = \lambda x_2 - 6x_1 - 2\mu$

(A) $\lambda = -5$ (B) $\lambda \neq -5$

(C) $\lambda = -6$ (D) $\lambda \neq -6$

18. For a given transfer function choose the state variable matrix $\dfrac{C(s)}{R(s)} = \dfrac{30}{s^3+8s^2+7s+30}$

(A) $\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -30 & -7 & -8 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ 30 \end{bmatrix} r$

(B) $\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 30 & 7 & 8 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ 30 \end{bmatrix} r$

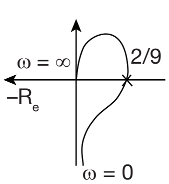
(C) $\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -8 & -7 & -30 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ 30 \end{bmatrix} r$

(D) $\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -8 & -7 & -30 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ 30 \end{bmatrix} r$

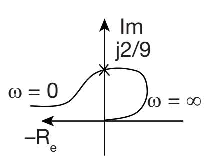
19. A unity feed back has open – loop transfer function

$G(s) = \dfrac{1}{s(4s+1)(s+2)}$

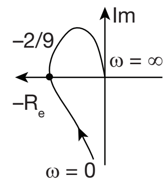
(A)



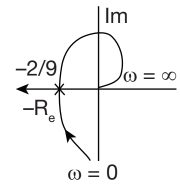
(B)



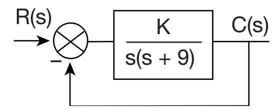
(C)



(D)



20. The unity feedback system $R(s)$ has



(A) Steady state velocity error of k/9units

(B) Steady state position error of K/9units

(C) Zero steady state velocity error

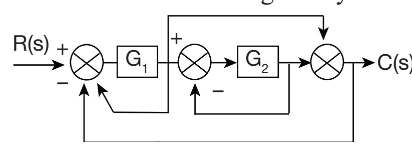
(D) Zero steady state position error

21. If maximum peak overshoot is 16.3% then the resonant peak is

(A) 0.5 (B) 0.15

(C) 1.15 (D) 00.86

22. Determine Transfer function of given system



(A) $\dfrac{G_1G_2 + G_1}{1 + G_1 + G_2 + G_1G_2}$

(B) $\dfrac{2G_1G_2 + G_1}{1 + 2G_1 + G_2 + G_1G_2}$

(C) $\dfrac{G_1G_2 + G_1}{1 + 2G_1 + G_2 + 3G_1G_2}$

(D) $\dfrac{2G_1G_2 + G_1}{1 + 2G_1 + G_2 + 3G_1G_2}$

23. The radius of M and N circles are $\sqrt{2}$, 1 respectively then the values of M and N respectively are

(A) $\sqrt{2}$, 1 (B) $\sqrt{2}, 1/\sqrt{3}$

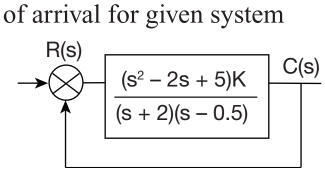
(C) $1/\sqrt{3}, \sqrt{2}$ (D) ∞, 0

24. Maximum peak overshoot in the unit step response is 0.4 and peak time is 2sec. Then settling time for 5% error is

(A) 0.65 (B) 0.57

(C) 0.75 (D) 0.4564

25. Find Angle of arrival for given system



(A) 160.355° (B) 109.65°

(C) 199.645° (D) 189.7°

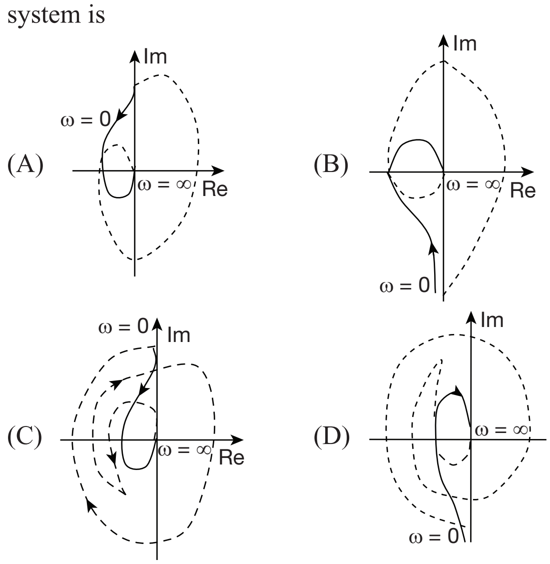
26. Find the phase margin of a system with open loop transfer function $G(S) = \dfrac{1}{s(1+s)}$

(A) –141.78° (B) 38.21°
(C) –38° (D) 180°

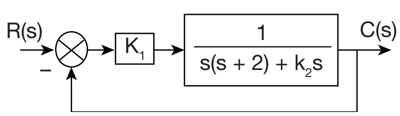
27. The open loop transfer function of a feedback control system is given by $GH(s) = \dfrac{K\,(s+2)}{s(1+as)(1+5s)}$, then find error due to unit parabolic input if $K = 9$ and $a = 3$

(A) ∞ (B) 0
(C) 9 (D) 3

28. The open loop transfer function of a system is $GH(s) = \dfrac{A(1+s)^2}{s^3}$, then Nyquist plot for given system is



29. The system has damping ratio 0.8 and a frequency of damped oscillations of 10rad/sec. Then the value of K_2



(A) 16.7 (B) 26.7
(C) 24.7 (D) 5.16

30. Forward path transfer function of a unity, feedback system is given by

$$G(s)H(s) = \frac{125}{(s+1)(s+2)(s+3)(0.25s+1)},$$

then the system is

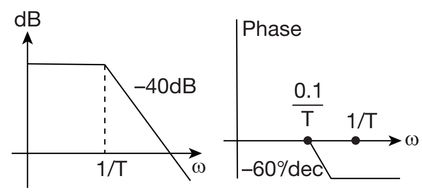
(A) Stable
(B) Unstable
(C) Marginally stable
(D) More information is required

31. For a second order system overshoot is 20% and settling time is 0.6seconds then characteristic equation roots are.

(A) $-6.6576 \pm j13$
(B) $-6.6576 \pm j14.6$
(C) $13 \pm j14.6$
(D) $14.6 \pm j13$

Common Data for Questions 32 and 33:

The Bode plot of the transfer function $\dfrac{b_0}{(1+s\tau)}$ is given in below figures.



32. The error in phase angle at $\omega = \dfrac{0.5}{T}$ is

(A) 3.44° (B) 26.56°
(C) 60° (D) 30°

33. The error in gain at $\omega = \dfrac{0.5}{T}$ is

(A) –40 dB (B) –30 dB
(C) 0 dB (D) 0.97dB

Linked Answer Questions 34 and 35:

The open loop transfer function of a unity feedback control system is given by

$$G(S) = \frac{K}{(s+3)(s+1)(s^2+6s+25)}$$

34. Find the values of K which will cause sustained oscillations in the closed – looped system.

(A) 3993.6 (B) –75
(C) –118 (D) 399.36

35. Find the oscillating frequency.

(A) 11.8 rad/sec
(B) 3.43 rad/sec
(C) 0.547 rad/sec
(D) 1.8 rad/sec

Answer Keys

1. D	**2.** D	**3.** C	**4.** C	**5.** D	**6.** C	**7.** C	**8.** D	**9.** D	**10.** D
11. B	**12.** A	**13.** B	**14.** C	**15.** D	**16.** B	**17.** B	**18.** A	**19.** C	**20.** D
21. C	**22.** D	**23.** B	**24.** D	**25.** C	**26.** B	**27.** A	**28.** C	**29.** C	**30.** B
31. A	**32.** A	**33.** D	**34.** D	**35.** B					

Hints and Explanations

1. $T(S) = \dfrac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$ ₹

$= \dfrac{160000}{s^2 + 2 \times 1.25 \times 400s + (400)^2}$

$= \dfrac{160000}{s^2 + 1000s + 160000}$

$C(S) = \dfrac{160000}{s\left(s^2 + 1000s + 160000\right)}$

$= \dfrac{1}{s} + \dfrac{1600}{6(s+200)} - \dfrac{1600}{6(s+800)}$

$C(t) = 1 + \dfrac{800}{3}e^{-200t} - \dfrac{800}{3}e^{-800t}$ Choice (D)

2. Choice (D)

3. Choice (C)

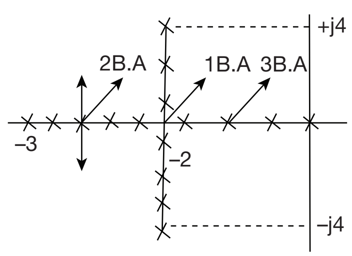
4. $C(S) = \dfrac{1}{s} - \dfrac{5}{s+1} = \dfrac{s+1-5s}{s(s+1)} = \dfrac{1-4s}{s(s+1)}$

$T(S) = \dfrac{1-4s}{\dfrac{s(s+1)1}{s}} = \dfrac{1-4s}{s+1}$ Choice (C)

5. Choice (D)

6. Choice (C)

7.



Choice (C)

8. On 5% basis the setting time for a system is given by

$t_s = \dfrac{3}{\xi\omega_n} = \dfrac{3}{\frac{6}{2}} = 1$ Choice (D)

9. Choice (D)

10. Root locus path should starts from pole to zero so none of the given diagram root locus starts from pole. Choice (D)

11. $GH(S) = \dfrac{K}{s(1+s)(1+2s)} = \dfrac{K}{2s^3 + 3s^2 + s}$

So order is 3. Choice (B)

12. Choice (A)

13. Intersection of asymptotes i.e., centroid

$= \dfrac{\Sigma \text{ real parts of poles} - \Sigma \text{real part of zeros}}{\text{number of poles} - \text{number of zeros}}$

$= \dfrac{-1-3-(-2)}{3-1} = \dfrac{-4+2}{2} = -1$

Choice (B)

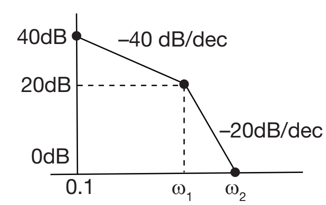
14. For stable operation, all co-efficient of the characteristic equation should be real & have the same sign and none of the co-efficient should be zero. Choice (C)

15. Transfer function of a lag compensator $G(S) = \dfrac{s + \frac{1}{T}}{s + \frac{1}{\beta T}}$

$\dfrac{1}{T} = 0.2;\ \dfrac{1}{\beta T} = 0.026 \Rightarrow \beta = 7.69$

$\omega_m = \dfrac{1}{T\sqrt{\beta}} = \dfrac{0.2}{\sqrt{7.69}} = 0.072$ rad/sec Choice (D)

16.



$40 - 20 = -40(\log(0.1) - \log\omega_1)$

$\Rightarrow \omega_1 = 0.316$rad/sec

$20 = -20(1\log 0.316 - \log\omega_2)$

$\omega_2 = 3.16$

$40 = 20\left(\log \dfrac{K}{(0.1)^2(0.31)}\right)$

$K = 0.331$

Transfer function of system

$= \dfrac{0.331}{s^2(s+0.316)}$ Choice (B)

17. $\dot{x} = \begin{bmatrix} 0 & 1 \\ -6 & \lambda \end{bmatrix} X + \begin{bmatrix} 1 \\ -2 \end{bmatrix} U$

From above $A = \begin{bmatrix} 0 & 1 \\ -6 & \lambda \end{bmatrix}$ and $B = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

Controllability $Q_c = [B : AB]$

$AB = \begin{bmatrix} 0 & 1 \\ -6 & \lambda \end{bmatrix}\begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} -2 \\ -6-2\lambda \end{bmatrix}$

$Q_C = \begin{bmatrix} 1 & -2 \\ -2 & -6-2\lambda \end{bmatrix}$

If $|Q_C| \neq 0$ then system is said to be controllable

$-6 - 2\lambda - 4 = 0$

$\Rightarrow \quad 2\lambda = -10$

$\lambda = -5$

$\lambda \neq -5$

For the system to be controllable Choice (B)

18. $\dfrac{C(s)}{R(s)} = \dfrac{b_0}{s^3 + a_2 s^2 a_1 s + a_0}$

$\Rightarrow \quad (s^3 + a_2 s^2 + a_1 s + a_0)C(s) = R(s)\, b_0$

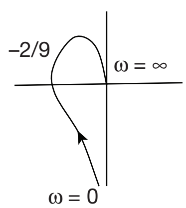
$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_0 & -a_1 & -a_2 \end{bmatrix} X + \begin{bmatrix} 0 \\ 0 \\ b_0 \end{bmatrix} r$$

$$\Rightarrow \quad \dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -30 & -7 & -8 \end{bmatrix} X + \begin{bmatrix} 0 \\ 0 \\ 30 \end{bmatrix} r$$

Choice (A)

19. Given system is $\dfrac{1}{s(4s+1)(s+2)}$

$\phi = -90° - \tan^{-1}(4\omega) - \tan^{-1}(\omega/2)$



Transfer function cuts the real axis when imaginary part is equal to zero.

$-180 = -90 - \tan^{-1}(4\omega) - \tan^{-1}\omega/2$

$90 = \tan^{-1}4\omega + \tan^{-1}\omega/2$

$1 - 4\omega \times \omega/2 = 0$

$2\omega^2 = 1$

$\omega = \dfrac{1}{\sqrt{2}}$

Magnitude Nyquist plot can be real axis is

$= \dfrac{1}{\omega\sqrt{16\omega^2+1}\sqrt{\omega^2+4}} = \dfrac{1}{1/\sqrt{2}\sqrt{16\times 1/2+1}\sqrt{1/2+4}}$

$= 2/9$ Choice (C)

20. For unit step i/p the steady state error is $\dfrac{A}{1+K_p}$

$K_P = \underset{S\to 0}{Lt}\, GH(s)$

$= \underset{S\to 0}{Lt} \dfrac{K}{s(s+9)} = \infty$

$e_{ss} = \dfrac{A}{1+\infty} = \dfrac{A}{\infty} = 0$

$\Rightarrow$ for unit ramp input $e_{ss} = \dfrac{A}{K_V}$

$K_V = Lt_{S\to 0}\, s \dfrac{K}{s(s+9)} = K/9$

$e_{ss} = 9/k$

Hence steady state error is $9/K$ for ramp input.

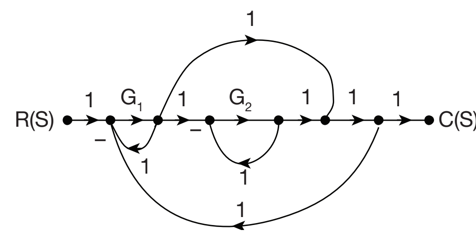
Choice (D)

21. $M_P = e^{-\pi\xi/\sqrt{1-\xi^2}} = 0.163$

$\Rightarrow \quad \xi = 0.5$

$M_r = \dfrac{1}{2\xi\sqrt{1-\xi^2}} = 1.15$ Choice (C)

22. Draw signal flow graph



$T/F = \dfrac{G_1G_2 + G_1(1+G_2)}{1+G_2+G_1+G_1G_2+G_1+G_1G_2+G_1G_2}$

$= \dfrac{2G_1G_2 + G_1}{1+2G_1+G_2+3G_1G_2}$ Choice (D)

23. radius of M circle $= \sqrt{\dfrac{M^2}{(M^2-1)^2}} = \sqrt{2}$

$\dfrac{M}{M^2-1} = \sqrt{2}$

$\Rightarrow \quad M = \sqrt{2}$

Radius of N circle $= \sqrt{\dfrac{1}{4} + \dfrac{1}{(2N)^2}}$

$1 = \dfrac{1}{4} + \dfrac{1}{(2N)^2} \Rightarrow \dfrac{3}{4} = \dfrac{1}{(2N)^2}$

$\Rightarrow \quad \dfrac{1}{2N} = \dfrac{\sqrt{3}}{2} \quad \Rightarrow N = \dfrac{1}{\sqrt{3}}$ Choice (B)

24. $M_P = e^{-\xi\pi/\sqrt{1-\xi^2}} = 0.4$

$\Rightarrow \quad \xi = 0.28$

$t_p = 2\text{sec} = \dfrac{\pi}{\omega_n\sqrt{1-\xi^2}}$

$\Rightarrow \quad \omega_n = \dfrac{3.14}{2\sqrt{1-(0.28)^2}} = 1.63$ rad/sec

$T_s = \dfrac{3}{\xi\omega_n} = \dfrac{3}{0.28\times 1.63} = 0.4564$

Choice (D)

25. $\frac{C(s)}{R(s)} = \frac{(s^2-2s+5)K}{(s+2)(s-0.5)}$

$= \frac{K(s-1-2j)(s-1+2j)}{(s+2)(s-0.5)}$

$\angle GH(S) = \frac{K(1+2j-1+2j)}{(1+2j+2)(1+2j-0.5)}$ at $s = 1 + 2j$

$= \frac{K(4j)}{(3+2j)(0.5+2j)}$

$\angle GH(S) = 90° - \tan^{-1}2/3 - \tan^{-1}2/0.5$

$= -19.645°$

$\phi_A = 180 - \phi = 199.645°$ Choice (C)

26. Given that $G(S) = \frac{1}{s(1+s)}$. At gain cross over frequency ω_{gc} the magnitude of $G(j\omega)$ is unity.

At $\omega = \omega_{gc}$

$\Rightarrow |G(j\omega)| = \frac{1}{\omega^2_{gc}\sqrt{1+\omega^2_{gc}}} = 1$

$\Rightarrow \omega_{gc}\sqrt{1+\omega^2_{gc}} = 1$

$\Rightarrow \omega^2_{gc}(1+\omega^2_{gc}) = 1$

$\omega^4_{gc} - \omega^2_{gc} - 1 = 0$

$\omega^2_{gc} = 1.27$

$\phi_{jc} = -90° - \tan^{-1}\omega_{gc}$

$= -141.78°$

Phase margin = $180° + \phi jc = 38.21°$ Choice (B)

27. Given system

$G(S)\,H(S) = \frac{9(s+2)}{s(1+3s)(1+5s)}$

Error due to Ramp input is = $1/K_a$

Where $K_a = \lim_{s\to 0} s^2 G(S) = 0$

Unit parabolic $i/p = 1/K_a$

$= \frac{1}{0} = \infty$ Choice (A)

28. $GH(j\omega) = \frac{K(1+j\omega)^2}{(j\omega)^3}$

$\Rightarrow |GH| = \frac{K(1+\omega^2)}{\omega^3}$

$\angle GH(j\omega) = -270° + 2\tan^{-1}\omega$

As ω increases from 0 to ∞, phase $-270°$ to $-90°$. Due to S^3 term there will be 3 infinite semicircles.

Choice (C)

29. $\frac{C(s)}{R(s)} = \frac{K_1}{s^2+(2+K_2)s+K_1}$

$\omega_d = \omega_n\sqrt{1-\xi^2}$

$10 = \omega_n\sqrt{1-(0.8)^2}$

$\omega_n = \frac{10}{0.6} = 16.66$

$\Rightarrow 2 + K_2 = 2\xi\omega_n$

$2 + k_2 = 2 \times 0.8 \times 16.7$

$\Rightarrow K_2 = 24.66$ Choice (C)

30. Closed loop transfer function

$= \frac{125\times 4}{(s^2+3s+2)(s+3)(s+4)+125\times 4}$

$= \frac{500}{(s^2+3s+2)(s^2+7s+12)+500}$

Characteristic equation

$= (s^2 + 3s + 2)(s^2 + 7s + 12) + 500$

$\Rightarrow s^4 + 10s^3 + 35s^2 + 50s + 524 = 0$

S^4	1	35	524
S^3	10	50	0
S^2	30	524	
S^1	–124.7	0	
S^0	524		

There are two sign changes so it unstable.

Choice (B)

31. $M_p = e^{-\xi\pi/\sqrt{1-\xi^2}} = 0.2$

$\Rightarrow \xi = 0.456$

$T_s = \frac{4}{\xi\omega_n}$

$\Rightarrow \omega_n = \frac{4}{\xi T_s} = \frac{4}{0.456\times 0.6} = 14.6$

Then poles are $= -\xi\omega_n \pm j\omega_n\sqrt{1-\xi_2}$

$= -14.6\times 0.456 \pm j14.6\sqrt{1-(0.456)^2}$

$= -6.6576 \pm j13$ Choice (A)

32. $\omega = \frac{0.5}{T}$

$\angle G\left(\omega = \frac{0.5}{T}\right) = -\tan^{-1}\omega T$

$= -\tan^{-1}0.5 = -26.56°$

From the phase plot at $\omega = \frac{0.5}{T}$

$\angle GH(S) = -30°$

Error $= -26.56° + 30° = 3.44°$ Choice (A)

33. At $\omega = \frac{0.5}{T}$

$$|GH| = \left|\frac{K}{1 + jT\frac{0.5}{T}}\right| = \frac{K}{\sqrt{1.25}}$$

$20\log|GH| = 20\log K - 10\log 1.25$

$= 20\log K - 0.97$

From the plot at $\omega = \frac{0.5}{T} = 20\log K$

So error in gain = 0.97 dB Choice (D)

34. Given system is open loop transfer function

$$G(S) = \frac{K}{(s+3)(s+1)(s^2+6s+25)+K}$$

⇒ Characteristic equation is

$(s^2 + 4s + 3)(s^2 + 6s + 25) + K = 0$

$s^4 + 10s^3 + 52s^2 + 118s + 75 = 0$

S^4	1	52	75+K
S^3	10	118	0
S^2	40.2	75 +1C	
S^1	$\frac{3993.6 - 10K}{40.2}$		
S^0	75 + K		

⇒ $75 + K > 0$

$K > -75$

⇒ $3993.6 - 10K > 0$

⇒ $K < 399.36$

⇒ The range of K for the system to be stable

$0 < K < 399.36$

[∴ K range is starts from zero]

⇒ For $K = 399.36$ system will oscillate

Choice (D)

35. If $K = 399.36$ then S^1 row become zero. The co-efficient of auxiliary equation are given by the s^2 row.

$40.2s^2 + 75 + 399.36 = 0$

$s^2 = -11.8$

⇒ $s = \pm j3.43$

When $K = 399.36$, the system has roots on imaginary axis and so it oscillates. The frequency of oscillation is given by the value of root on imaginary axis.

Frequency of oscillation ω= 3.43 rad/sec

Choice (B)