# ☆ Stability.

⇒ For an LTI System, the LTI System is said to be stable if ~~the~~ it satisfies the following Conditions.

① If the input is bounded, the output must be bounded.

② If the input signal to the system is zero, the output must be zero irrespective of all the initial conditions

⇒ This Stabilities are Classified into the two way based on operating Conditions.

① <u>Conditional Stable System:</u>

⇒ Here, the system is Stable for certain range of System. Components.

② <u>Absolutely Stable System.</u>

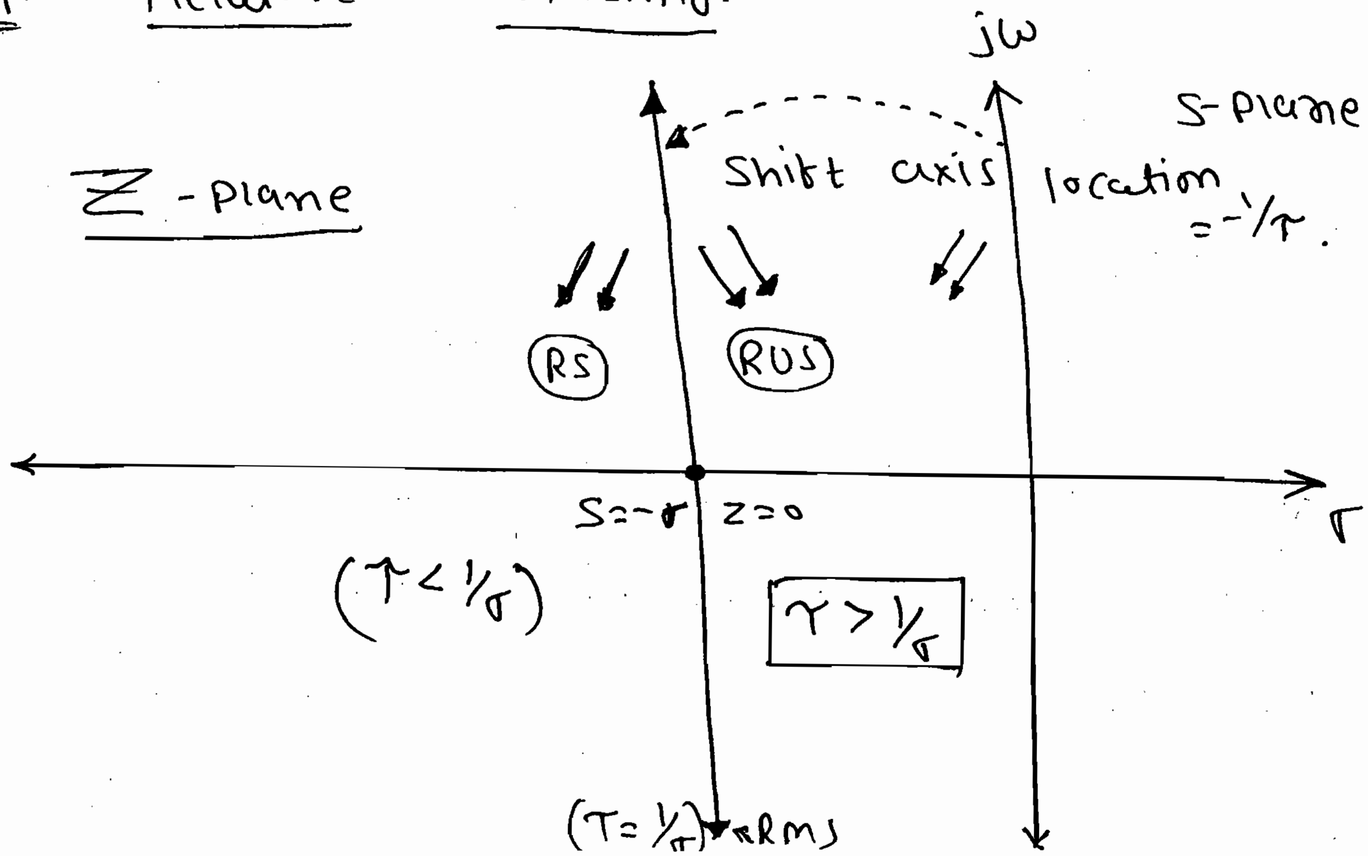
⇒ Here, the system is stable for all the values of system components.

(3) **Marginal (or) Critical (or) limitedly Stable System:**

⇒ A linear time Invariant system is said to be marginal stable if for the bounded input, the output maintains the constant amplitude and freq. of oscillation.

→ The non-repeated Pole on imaginary axis gives the constant amplitude and freq. of oscillation & the system is marginal Stable.

## * Relative Stability:



⇒ The relative Stability Concept is applicable only for Stable system.

⇒ By using relatively Stable concept we can find system time constant, settling time and time required to reach steady state.

* Techniques used for calculate stability are:

1) Routh - Hurwitz Criterion.
2) Root - Locus.
3) Bode plot.
4) Nyquist Plot.
5) Nicholas Chart.

* Routh - Hurwitz Criterion:

(RH Criterian):

* Purpose:

① To find the Closed Loop System Stability.

② To find the no. of Closed Loop Poles lies in the right, left, an imaginary axis of the s-plane.

③ ⇒ The main purpose of the RH- criteria is to find the no. of poles in right of s-plane only.

⇒ ④ To find the range of K-value for CL system stability.

⑤ To find the k value to become the system marginal stable. (or) Undamped system.

⑥ To find the natural freq. of oscillation (or) Undamped oscillation.

⑦ To find the relative stability.

→ By using relative stability concept we can find system time constant, setteling time $t_s$.

⑧ ⇒ To find a CL stability by using RH - criteria required char. eqn.

i.e. $\boxed{1+GH=0.}$

⇒ whereas in remain all the stability techniques required OLTF of a unity (or) Non-UFB system.

⇒ CL Stability

RH

$\xrightarrow{CE}$ $1 + G(s) \cdot H(s) = 0$

RL

BP

NP

⇓

$G(s) \cdot H(s)$.

→ OLTF of unity (or) Non UFB system.

⇒ The $n^{th}$ order general form of Char. eq$^n$ is,

$$\xrightarrow{CE} \boxed{a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots + a_{n-1} s^1 + a_n s^0 = 0.}$$

$a_0, a_1, a_2 \ldots$ are coefficient.

$$
\begin{array}{c|cccc|c|c}
s^{n} & a_0 & a_2 & a_4 & \cdots & +/- & + \\
s^{n-1} & a_1 & a_3 & a_5 & \cdots & +/- & + \\
s^{n-2} & \dfrac{a_1 \cdot a_2 - a_0 a_3}{a_1} & \dfrac{a_1 a_4 - a_0 a_5}{a_1} & & \cdots & +/- & - \\
\vdots & \vdots & \vdots & & & \vdots & + \\
 & & & & & & - \\
s^{0} & a_n & --- & & & +/- & \vdots \; + \\
 & & & & & S & US
\end{array}
$$

⇒ The condition for the system stability stability, are

① All coefficient in the first column should have same sign and no-coefficient should be '0' in the first column.

② The no. of sign changes in the first column equal to no. of poles in the right plane and the system become unstable.

$\boxed{Q}$ Find the System Stability to the following Char. eqn's

① $s+10=0$.

② $s^2+25=0$.

③ $s^2+10s+10=0$.

④ $s^3+25s^2+8s+10=0$.

⑤ $s^3+7s^2+6s+100=0$.

⑥ $s^3+8s^2+4s+32=0$.

Soln:

① $s+a=0$.

$\therefore \xrightarrow{CE} as+b=0$.

$$\left.\begin{array}{c|c} s^1 & a \\ s^0 & b \end{array}\right\} \quad \begin{array}{l} b, a>0 \quad \text{(or)} \\ b, a<0 \Rightarrow (S) \end{array}$$

$\Rightarrow s+10=0$ is Stable.

② $s^2+25=0$

$\xrightarrow{CE} as^2+bs+c=0$.

$$\left.\begin{array}{c|cc} s^2 & a & c \\ s^1 & b & \\ s^0 & c & \end{array}\right\} \Rightarrow \text{if } \begin{array}{l} a,b,c>0 \text{ (or)} \\ a,b,c<0 \end{array}$$

$\Downarrow$ (S)

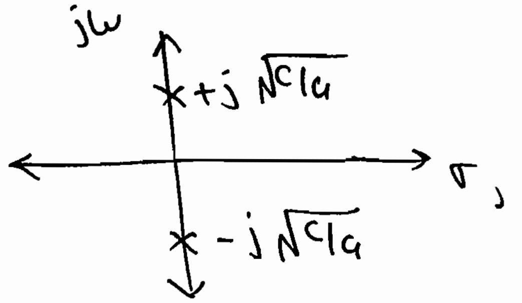
$\Rightarrow$ if $b=0$, $a,c>0$

$\Rightarrow$ (MS)

$\Rightarrow$

$\xrightarrow{CE}$ $as^2 + c = 0$

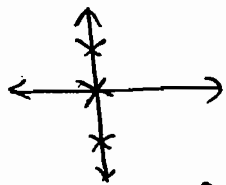
$\Rightarrow \quad s = \pm j\sqrt{c/a}$

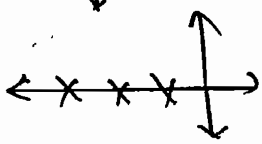


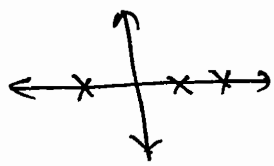

MS .

$\xrightarrow{CE}$ $as^3 + bs^2 + cs + d = 0.$

| | | |
|---|---|---|
| $s^3$ | $a$ | $c$ |
| $s^2$ | $b$ | $d$ |
| $s^1$ | $\frac{bc - ad}{b}$ | |
| $s^0$ | $d$ | |

$bc = ad \longrightarrow$  MS

$bc > ad \longrightarrow$  S

$bc < ad \rightarrow$  $\rightarrow$ US

$\Rightarrow$ even powers of s- terms gives f.o.o.

$bs^2 + d = 0.$

$\therefore \quad s = \pm j\sqrt{\frac{d}{b}}$

$j\omega_n = \pm j\sqrt{d/b}$

$$\boxed{\omega_n = \sqrt{d/b} \text{ rad/sec.}}$$

$\Rightarrow \quad s^2 + 25 = 0 \Rightarrow$ MS .

10

④ $s^3 + 25s^2 + 8s + 10 = 0.$

200

$\rightarrow (bc = 200) > (ad = 10) \Rightarrow$ (S).

100

⑤ $s^3 + 7s^2 + 6s + 100 = 0.$

42

$\rightarrow (bc = 7 \times 6 = 42) < (ad = 100) \Rightarrow$ (US).

32

⑥ $s^3 + 8s^2 + 4s + 32 = 0.$

32

$\rightarrow (bc = 32) = (ad = 32).$

$\Rightarrow$ (MS).

F.O.O. $8s^2 + 32 = 0$

$s^2 = 4$

$s = \pm j2$

$\therefore \boxed{\omega_n = 2 \text{ rad/sec}}$

$\boxed{Q}$ Find the no. of poles in the right of S-plane to the given char. eqn.

$s^4 + 2s^3 + 3s^2 + 4s + 5 = 0.$

Soln.

| | | | |
|---|---|---|---|
| $s^4$ | 1 | 3 | 5 |
| $s^3$ | 2 | 4 | 2R, 1C |
| ① ( $s^2$ | 1 | 5 | |
| $s^1$ | $-6$ | 2R, 1C. | |
| ② ( $s^0$ | 5 | | |

$\Rightarrow$ 2 sign changes.

hence 2 poles lies on RHS plane

System Unstable.

$\Rightarrow$ Total 4 pole and 2 left & 2 Right.

$\boxed{Q}$ Find the no. of Poles on Right to the given char. eqn:

① $s^4 + 2s^3 + 3s^2 + 2s + 1$.

Soln:

| | | | |
|---|---|---|---|
| $s^4$ | 1 | 3 | 1 |
| $s^3$ | 2 | 2 | |
| $s^2$ | 2 | 1 | |
| $s^1$ | 1 | | |
| $s^0$ | 1 | | |

No sign change.

$\rightarrow$ Satable

$\rightarrow$ No Poles on RH s-plane.

$\rightarrow$ 4 Poles on LH s-plane.

② $s^4 + 2s^3 + 3s^2 + s + 2 = 0$.

Soln:

| | | | |
|---|---|---|---|
| $s^4$ | 1 | 3 | 2 |
| $s^3$ | 2 | 1 | |
| $s^2$ | $5/2$ | 2 | |
| $s^1$ | $-3/5$ | | |
| $s^0$ | 2 | | |

① ②

$\rightarrow$ 2 sign changes

$\rightarrow$ System $\Rightarrow$ US

$\rightarrow$ 2 poles on RH s-plane.

$\rightarrow$ 2 Poles on LH s-plane.

③ $s^4 + 2s^3 + 2s^2 + 4s + 8 = 0.$

Sol^n:

| | | | |
|---|---|---|---|
| $s^4$ | 1 | 2 | 8 |
| $s^3$ | 2 | 4 | |
| $s^2$ | $\cancel{0}\,\epsilon$ | 8 | |
| $s^1$ | $\frac{4\epsilon-16}{\epsilon}$ | | |
| $s^0$ | 8 | | |

① $s^2 \to s^1$

② $s^1 \to s^0$

$-\infty$

$$\lim_{\epsilon \to 0} \frac{4\epsilon - 16}{\epsilon} = -\infty.$$

→ 2 - sign changes, 2 - Poles on RH-splane.

2 - poles on LH - splane.

Difficulty - 1

⇒ Whenever any one element is '0', Replace '0' by smallest positive Constant '$\epsilon$' and continue the Routh Stability. Finally for $\epsilon = 0$ check the no. of sign Changes.

④ $s^5 + s^4 + 2s^3 + 2s^2 + 3s + 15 = 0.$

| | | | |
|---|---|---|---|
| $s^5$ | 1 | 2 | 3 |
| $s^4$ | 1 | 2 | 15 |
| $s^3$ | $\cancel{0}\,\epsilon$ | -12 | 0 |
| $s^2$ | $\frac{\overset{A}{2\epsilon+12}}{\epsilon}$ | 15 | |
| $s^1$ | $\frac{-12A-15\epsilon}{A}$ | | |
| $s^0$ | 15 | | |

① $s^3 \to s^2$

② $s^1 \to s^0$

$-\infty$

$-12$

$$\lim_{\epsilon \to 0} \frac{2\epsilon + 12}{\epsilon} = \infty.$$

$$\lim_{\epsilon \to 0} -12 - \frac{15\epsilon}{A} = -12.$$

2 - sign changes

2 - Poles on Rh-splane

(5) $s^5 + s^4 + 3s^3 + 3s^2 + 2s + 2 = 0.$

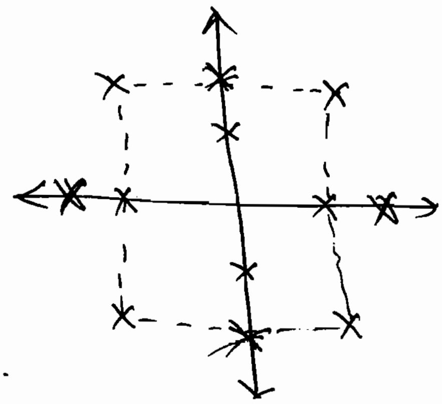
Sol^n:

| | | | |
|---|---|---|---|
| $s^5$ | 1 | 3 | 2 |
| $s^4$ | 1 | 3 | 2 |
| $s^3$ | 0 (4) | $\cancel{0}$ 6 | 0 |
| $s^2$ | $3/2$ | 2 | |
| $s^1$ | $2/3$ | | |
| $s^0$ | 2 | | |

AE: $1.s^4 + 3s^2 + 2 = 0.$

$\therefore \frac{dAE}{ds} = 4s^3 + 6s.$

AE : $1.s^4 + 3s^2 + 2 = 0. \rightarrow$



system $\rightarrow$ Marginaly Stable.

Difficulty - 2:

$\Rightarrow$ whenever in the Routh tabular form Row of zero occurs then we required to form the Auxiliary eq^n by using the above row of zero coefficients and differentiate the Auxiliary equitation and replace zeros by the coefficients of differential auxiliary eq^n and continue the Routh Stability.

$\rightarrow$ In the Routh tabular form row of zero occurs means the pole must lies

Symmetrical about the origin.

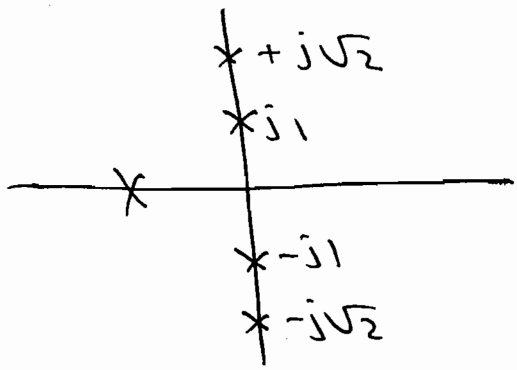
⇒ The Auxillary eqn must consist only even power of s- terms because the Roots of Auxillary eqns must be symmetrical about origin.

→ The roots of auxillary eqn are CL poles which are symmetrical,

$$s^4 + 3s^2 + 2 = 0.$$

$$\therefore (s^2+1)(s^2+2) = 0$$

$$s = \pm j1 \quad , \quad s = \pm\sqrt{2}j$$




⇒ Non-repeated poles on jω-axis hence MS.

⇒ Whenever in the Routh tabular form only once the Row of zero occurs and all the co-efficients in the 1st Column are +ve then the system is Marginal Stable because the poles must lies on the imaginary axis which are non-repeated.

→ 1 time Row of zero occurs means

the poles are symmetrical about the origin but not repeated.

⇒ The no. of time rows of zero indicates no. of poles are repeated.

⇒

| | | | |
|---|---|---|---|
| $s^5$ | − | | |
| $s^4$ | + | | |
| $s^3$ | + $\cancel{0}$ | 0 | 0 |
| $s^2$ | + | | |
| $s^1$ | + | | |
| $s^0$ | − | | |

① No sym.

AE ($4^{th}$ order)
↓
4 poles symmetrical @ origin.

① Sym.

**Note:**

⇒ The sign change occurs below the AE there must be a symmetrical pole in the left to the pole placed in Right.

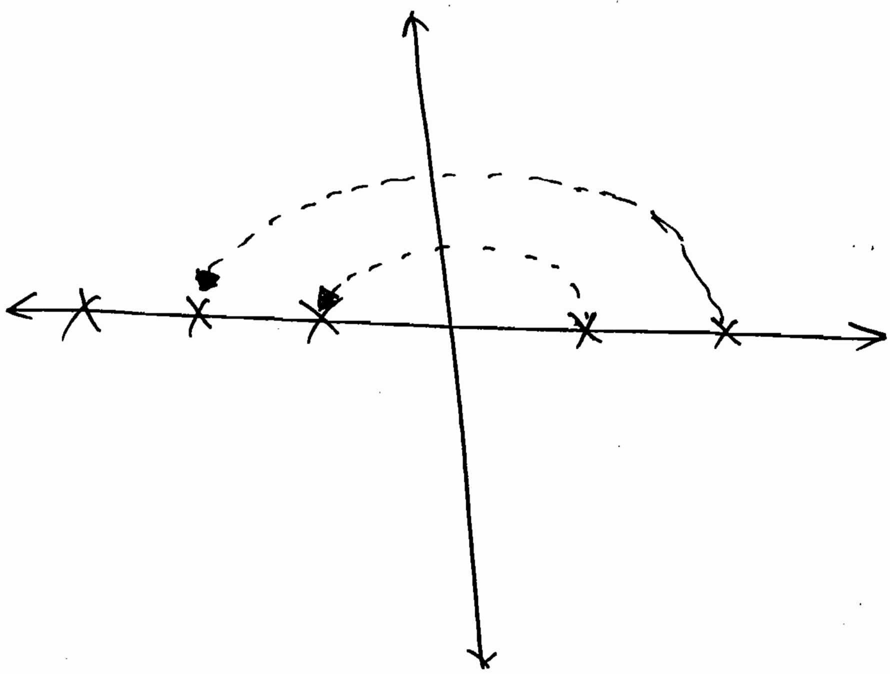
⇒ The sign change occur above the Auxillary eqn there is no symmetrical pole in the left ~~to~~ the pole placed in the Right side.

⟹

| | | |
|---|---|---|
| $s^5$ | + | No symm |
| $s^4$ | + | ⟷ AE → 4th order (4 pole sym. about origin) |
| $s^3$ | $\cancel{0}$ 0 0 | 1 Row |
| $s^2$ | + | |
| $s^1$ | − | ① |
| $s^0$ | + | ② [2 poles in Real axis] |

Symm

⟹



**(e)** $s^6 + 3s^5 + 4s^4 + 6s^3 + 5s^2 + 3s + 2 = 0.$

Soln:

| | | | | |
|---|---|---|---|---|
| $s^6$ | 1 | 4 | 5 | 2 |
| $s^5$ | 3 | 6 | 3 | |
| $s^4$ | 2 | 4 | 2 | |
| $s^3$ | $\cancel{0}$ 8 | $\cancel{0}$ 8 | 0 | |
| $s^2$ | 2 | 2 | | |
| $s^1$ | 0 | 0 | | |
| $s^0$ | 2 | | | |

$\rightarrow AE_1 \Rightarrow 2s^4 + 4s^2 + 2 = 0.$

$\rightarrow$ 1st row.

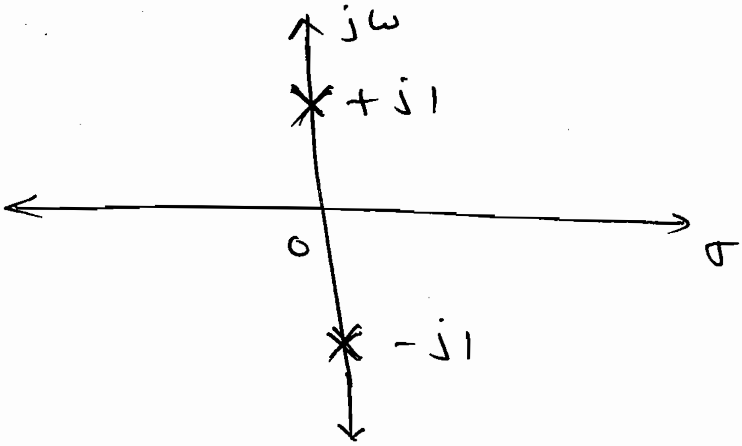
$\rightarrow AE_2 \Rightarrow 2s^2 + 2 = 0$

$\rightarrow$ 2nd row. → Repeated nature of pole.

$AE_2: \quad 2s^2 + 2s^0 = 0$

$$\boxed{s = \pm j1}$$

US




⇒ Whenever many times Row of zeros occurs and all the Co-efficients in the 1st Column are positive then the system is unstable the poles must lies on the imaginary axis which are repeated.

**Q** Find the no. of Poles in the Right the right of s-plane to the given char. eqn $s^4 + s^3 - s - 1 = 0$.

Soln:

$$\begin{array}{c|ccc} s^4 & 1 & 0 & -1 \\ s^3 & 1 & -1 & \\ \hline s^2 & 1 & -1 & \\ s^1 & \cancel{0} & \cancel{0} & \\ s^0 & -1 & & \end{array}$$

AE: $s^2 - 1 = 0$

1 Row

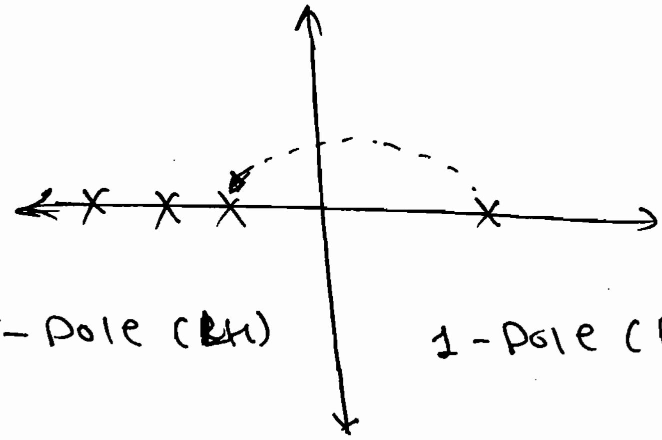
1

$s = \pm 1$

$$\boxed{s = \pm 1}$$

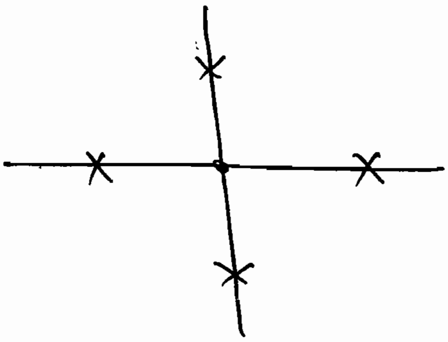
① - sign Change. below AE.

Hence, Symmetrical pole.




**a** Identify the Routh tabular form to the given poles locations in the s-plane.

①



Soln: ⇒ 4 pole are symmetrical → 1 Row of zero.

⇒ 1 pole ~~below~~ ~~AE~~ RH → 1 sign change.

below AE because symm. Pole in the left side.

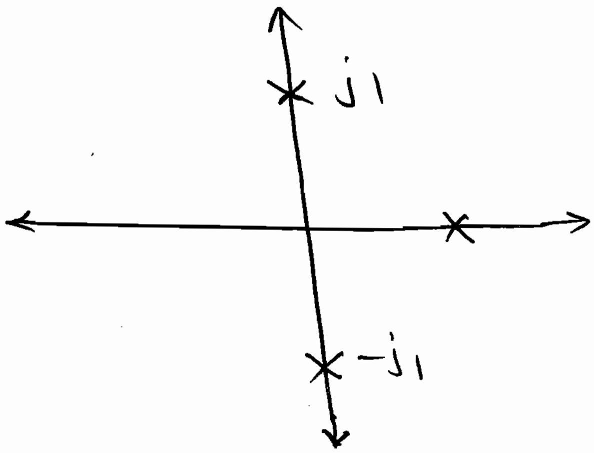
$\therefore \quad (s^2-1)(s^2+1)=0.$

$s^4-1=0$

| | | | | |
|---|---|---|---|---|
| $s^4$ | 1 | 0 | −1 | AE: $s^4-1=0$ (4th order). |
| $s^3$ | $\cancel{0}$ 4 | 0 | 0 | 1 Row of zero. |
| $s^2$ | $\cancel{0}$ $\epsilon$ | −1 | | |
| $s^1$ | $-4/\epsilon$ | | | $\lim_{\epsilon \to 0} -\frac{4}{\epsilon} = -\infty$ |
| $s^0$ | −1 | | | |

① sign change ($s^2 \to s^1$)

② 




$\underline{\underline{\text{Sol}^n}}$:

$$(s^2+1)(s-1)=0.$$

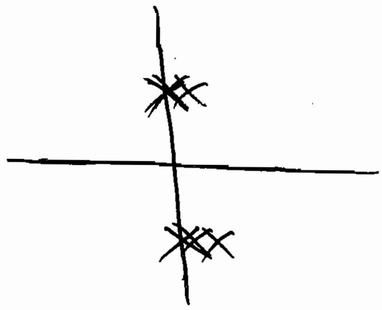
$\underline{CE} \rightarrow s^3 - s^2 + s - 1 = 0.$

| | | |
|---|---|---|
| $s^3$ | 1 | ① 1 |
| $s^2$ | -1 | -1 |
| $s^1$ | ~~0~~ -2 | ~~0~~ → ROZ. |
| $s^0$ | -1 | |

AE : $s^2+1=0$

$\boxed{s=\pm j}$

③



$\underline{\underline{\text{Sol}^n}}$:

$$(s^2+1)^3=0.$$

$s=\pm j1,\ \pm j1,\ \pm j1$

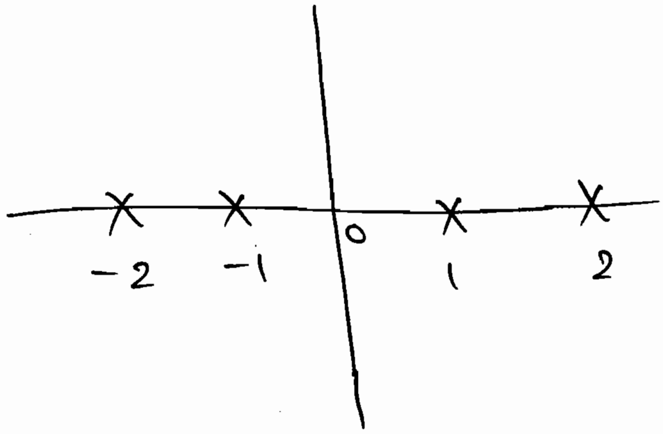
$= s^6 + 1 + 3s^4 + 3s^2$

$= s^6 + 3s^4 + 3s^2 + 1$

| | | | | |
|---|---|---|---|---|
| $s^6$ | 1 | 3 | 3 | 1 |
| $s^5$ | ~~0~~ 6 | ~~0~~ 12 | ~~0~~ 6 | ~~0~~ → 1st row |
| $s^4$ | 1 | 2 | 1 | |
| $s^3$ | ~~0~~ 4 | ~~0~~ 4 | ~~0~~ → 2nd row | |
| $s^2$ | 1 | 1 | 0 | |
| $s^1$ | ~~0~~ | ~~0~~ → 3rd row. | | |
| $s^0$ | 1 | | | |

**(a)**

**HW**




**Soln:** 4 Poles are symmetrical and 1 Row of zero.

Two poles are RH splane and sym.

Hence 2 sign changes below the AE.

$\Rightarrow$ CE $\rightarrow$ $(s^2-1)(s^2-4)=0$

$\therefore\ s^4 - 5s^2 + 4 = 0.$

| | | | |
|---|---|---|---|
| $s^4$ | 1 | -5 | 4 ← AE |
| $s^3$ | ~~0~~ 4 | ~~0~~ -10 | ~~0~~ 0 → 1 Ro2 |
| $s^2$ | -5/2 | 4 | |
| $s^1$ | 18/5 | | |
| $s^0$ | 4 | | |

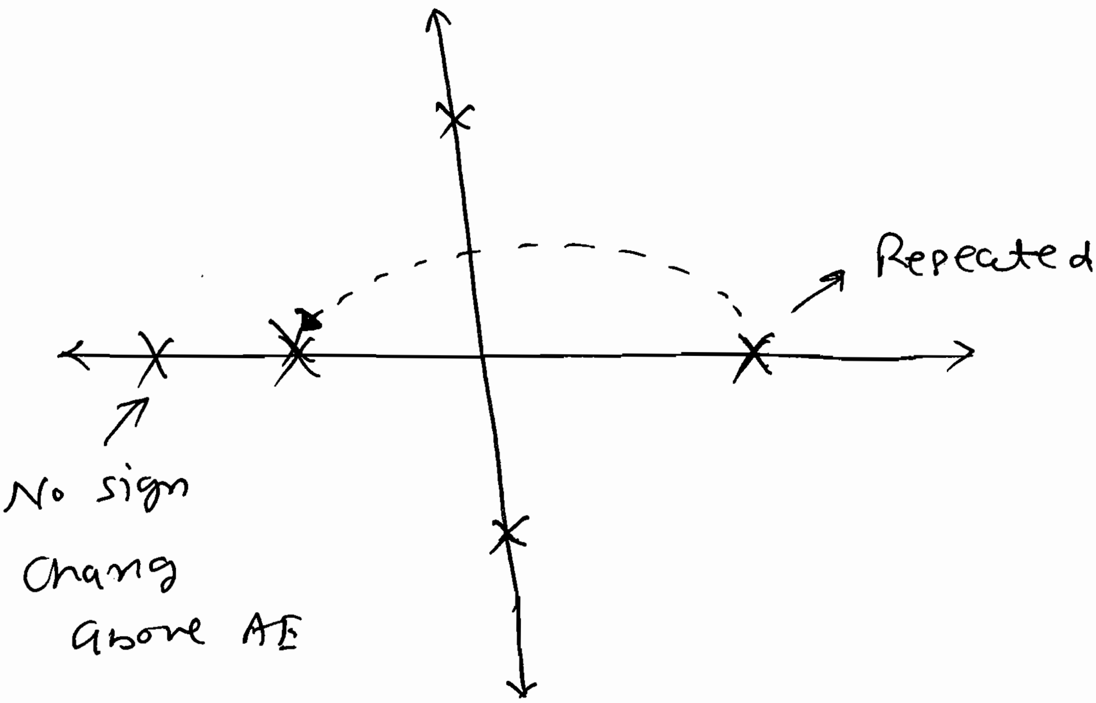
① ($s^2 \to s^1$)

② ($s^1 \to s^0$)

Q Identify the no. of poles on the imaginary axis, in the left and Right of s plane to the given sample Routh tabular form:

| | | | | |
|---|---|---|---|---|
| $s^7$ | + | | | |
| $s^6$ | + | | | $AE_1$   6th order AE $\Rightarrow$ 6 pole sym. @ origin |
| $s^5$ | 0 | 0 | 0 | $\rightarrow$ 1 RoZ |
| $s^4$ | + | | | 2 times RoZ $\Rightarrow$ 2 poles are repeated. |
| $s^3$ | 0 | 0 | 0 | $\rightarrow$ 2 RoZ |
| $s^2$ | + | | | |
| $s^1$ | − | | | 1 |
| $s^0$ | + | | | 2 |

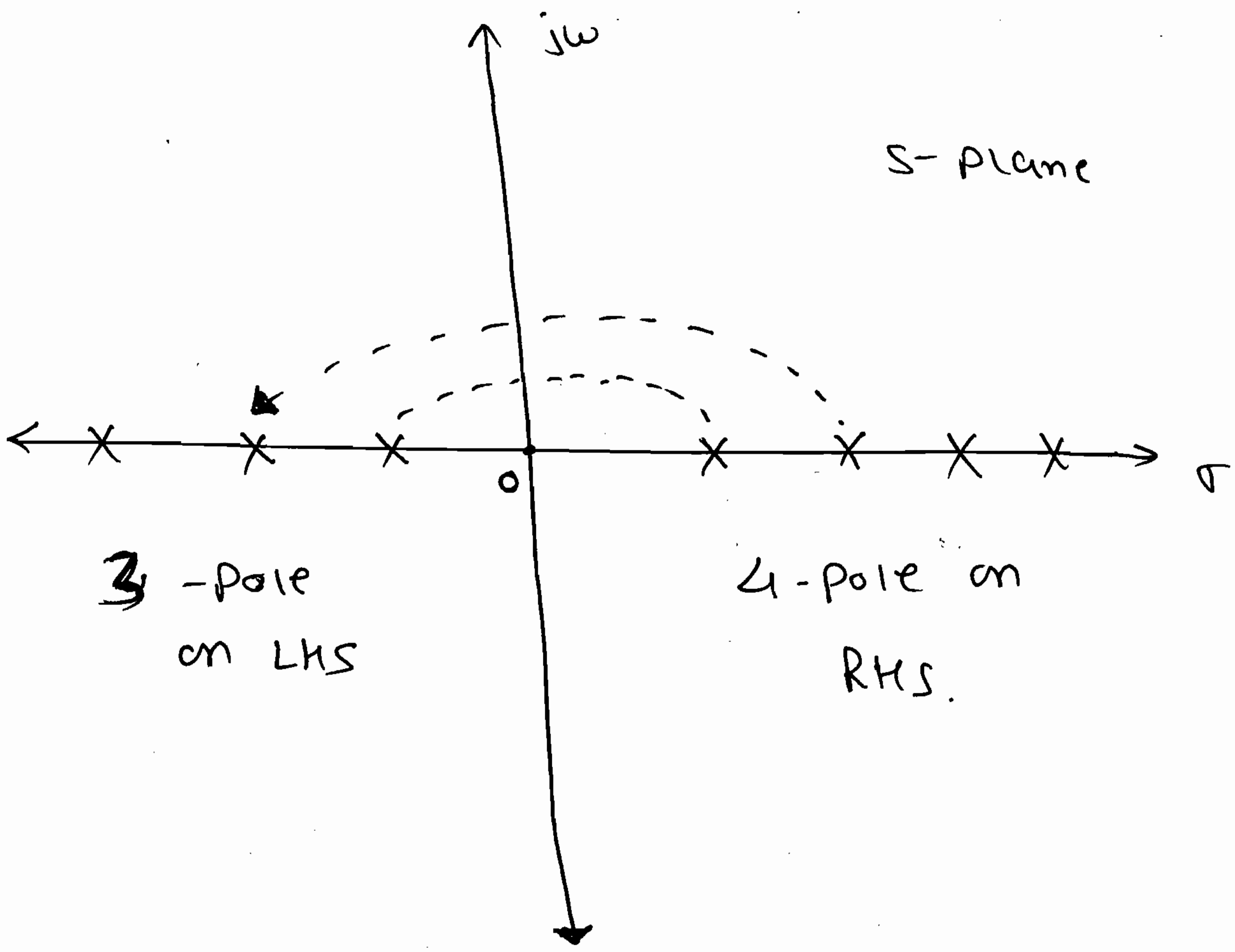
Sol:
=



②

$s^7$ | + ① Two sign change above AE hence two pole on RH-s-plane.
$s^6$ | − ②
$s^5$ | +
$s^4$ | + ← AE: 4th order 4 pole
$s^3$ | + → IRO2 sym. © origin.
$s^2$ | + ②
$s^1$ | − → 2 sign change below AE hence 2 sym. pole on RH-splane.
$s^0$ | + ②

Soln:



# * Conditional Stable System:-

a) Find the range of k value for system stability.

b) Find the k value to become the system marginal stable (or) Undamped system.

c) Find the natural freq$^n$ of osc$^n$ when the system is marginal stable to the char. eq$^n$.

$$s^3 + 8s^2 + 4s + k = 0.$$

Sol$^n$:

| | | |
|---|---|---|
| $s^3$ | 1 | 4 |
| $s^2$ | 8 | k |
| (MS) → $s^1$ | $\frac{32-k}{8} > \cancel{0}$ (S) | |
| × $s^0$ (MS) | $k > 0$ (S) | |

→ For stable,

$$\frac{32-k}{8} > 0 \quad \& \quad \boxed{k > 0}$$

$$\Rightarrow 32 - k > 0$$

$$\boxed{k < 32}$$

So, $\boxed{0 < k < 32}$ ⇓ (S).

For, (ms),

$$\frac{32-K}{8} = 0$$

$\Rightarrow \boxed{K = 32} \Rightarrow \boxed{K_{mar.} = 32}$

**Note:**

$\Rightarrow$ For K marginal value consider only odd power of s power rows.

$$AE \rightarrow \quad 8s^2 + K = 0$$

$$8s^2 = -K_{marg.}$$

$$s^2 = -32/8$$

$$s = \pm j2.$$

$$j\omega_n = \pm j2$$

$$\Rightarrow \boxed{\omega_n = 2 \text{ rad/sec}}$$

$\boxed{a}$ $2s^3 + 5s^2 + 10s + (K+5) = 0.$

**Soln:**

| | | |
|---|---|---|
| $s^3$ | 2 | 10 |
| $s^2$ | 5 | $K+5$ |
| $s^1$ | $\frac{50-2(K+5)}{5}$ | |
| $s^0$ | $K+5$ | |

$\rightarrow$ For Stable,

$$\frac{40-2K}{5} > 0$$

$40 > 2K \Rightarrow \boxed{K < 20}$

$$K+5 > 0$$

$$\boxed{K > -5}$$

$\Rightarrow \boxed{-5 < K < 20} \Rightarrow$ (S)

⇒ For MS,

$$\frac{40-2K}{5} = 0$$

$$\boxed{K_{max} = 20}$$

AE → $5s^2 + K + 5 = 0$

$5s^2 + 25 = 0$

$s^2 = -5$

$s = \pm j\sqrt{5}$

$\therefore \boxed{\omega_n = \sqrt{5} \text{ rad/sec}}$

Another method:

$$\underbrace{2s^3 + 5s^2 + 10s + (K+5)}_{50} = 0 \qquad 2K+10$$

$\therefore \quad 2K + 10 = 50$

$K = 40/2$

$\Rightarrow \boxed{K_{max} = 20}$

(c) Repeat the above problem for given

$$GH(s) = \frac{K}{s(s+2)(s+4)(s+6)}.$$

Soln:

CE → $1 + GH(s) = 0.$

$s(s+2)(s+4)(s+6) + K = 0.$

$\therefore (s^2+2s)(s^2+10s+24) + K = 0.$

$\Rightarrow \quad s^4 + 10s^3 + 24s^2 + 2s^3 + 20s^2 + 48s + K = 0.$

$\therefore \quad s^4 + 12s^3 + 44s^2 + 48s + K = 0.$

$$
\begin{array}{c|ccc}
s^4 & 1 & 44 & K \\
s^3 & \cancel{12}\ 1 & \cancel{48}\ 4 & \\
s^2 & 40 & K & \\
s^1 & \frac{160-K}{40} & & \\
s^0 & K & &
\end{array}
$$

$\Rightarrow$ For, Stable,

$$\boxed{K > 0} \quad \& \quad \frac{160-K}{40} > 0 \Rightarrow \boxed{K < 160}$$

$$\therefore \quad \boxed{0 < K < 160} \Rightarrow S.$$

$\Rightarrow$ For MS,

$$\therefore \quad \frac{160 - K_{mar}}{40} = 0$$

$$\therefore \quad \boxed{K_{mar} = 160.}$$

$$AE \longrightarrow \quad 40s^2 + K_{mar} = 0$$

$$\therefore \quad 40s^2 + 160 = 0.$$

$$s^2 = -4.$$

$$\Rightarrow \quad \boxed{s = \pm 2j.}$$

$\Rightarrow \quad \boxed{\omega_n = 2 \text{ rad/sec}}$

$\boxed{Q}$ Find k and b values so that the $G(s) = \dfrac{k(s+1)}{s^3+bs^2+3s+1}$, $H(s)=1$

oscillates with a freq. of 2 rad/sec.

Sol^n:

$\omega_n = 2$ rad/sec

$\Rightarrow$ MS.

CE $\rightarrow$ $1 + GH(s) = 0$.

$$\therefore\ 1 + \frac{K(s+1)}{s^3+bs^2+3s+1} = 0.$$

$$\therefore\ s^3 + bs^2 + 3s + 1 + Ks + K = 0.$$

$$s^3 + bs^2 + (K+3)s + (K+1) = 0.$$

$$\begin{array}{c|cc}
s^3 & 1 & K+3 \\
s^2 & b & (K+1) \\
s^1 & \frac{b(K+3)-(K+1)}{b} & \\
s^0 & K+1 &
\end{array}$$

$\Rightarrow$ For MS, $\dfrac{b(K+3)-(K+1)}{b} = 0.$

$$\therefore\ b(K+3) - (K+1) = 0$$

$$b = \frac{K+1}{K+3} \quad -(1).$$

$\therefore$ AE $\rightarrow$ $bs^2 + (K+1) = 0.$

$$\therefore \quad s^2 = -\left(\frac{K+1}{b}\right).$$

$$\therefore \quad s = \pm j\sqrt{\frac{K+1}{b}}.$$

$$\Rightarrow \quad \omega_n = \sqrt{\frac{K+1}{b}} = 2 \text{ rad/sec.}$$

$$\therefore \quad 4 = \frac{K+1}{b}.$$

from - eq$^n$ ①

$$4 = \frac{\cancel{K+1}}{\cancel{K+1}} \times K+3$$

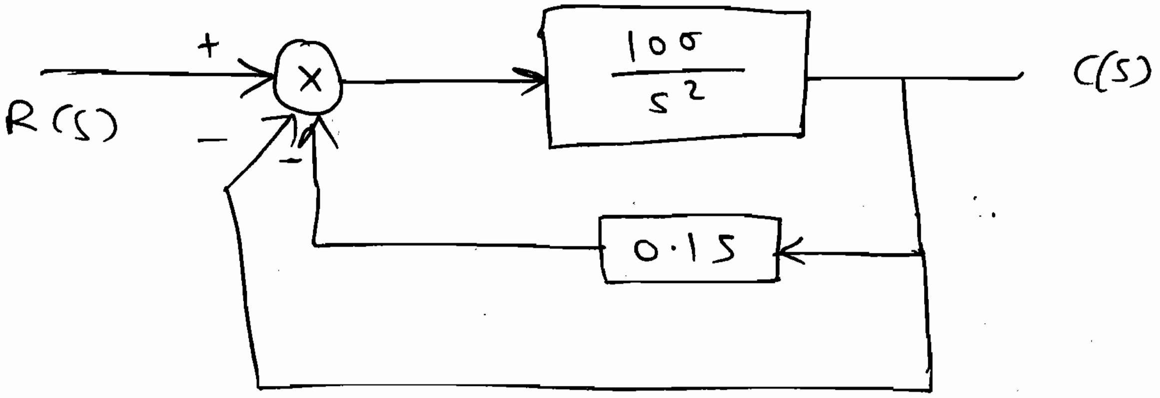
$$\boxed{K = 1}$$

$$\therefore \quad b = \frac{K+1}{4}$$

$$\Rightarrow \quad b = 1+1/4$$

$$\boxed{b = 0.5}$$

$\boxed{Q}$ Check the stability to the given BD.




Soln:

Char. eq$^n$ $1 + GH(s) = 0.$

$$\therefore \quad 1 + \frac{100}{s^2}\left[1 + 0.1s\right] = 0.$$

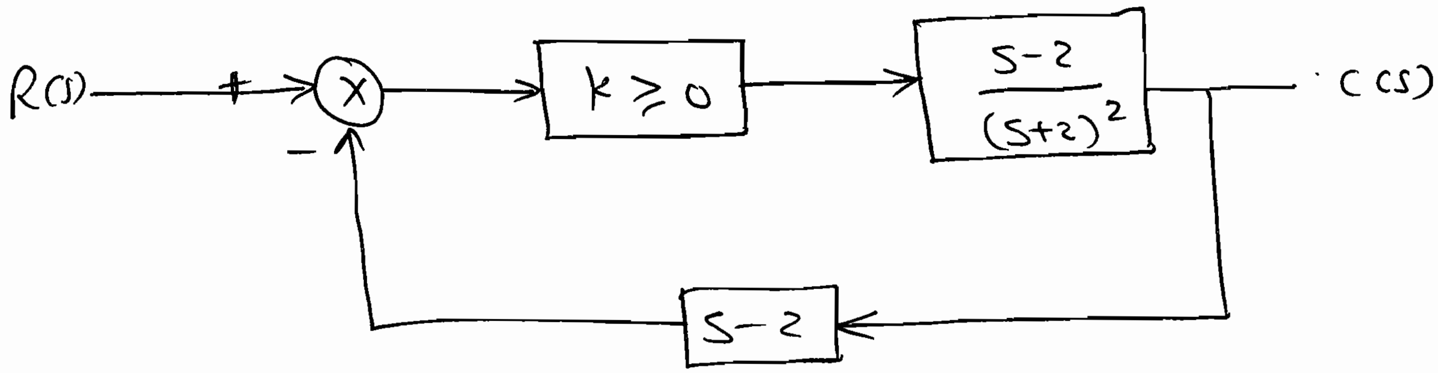
$$\therefore \quad 1 + \frac{\overset{10}{\cancel{100}}}{s^2}\left[\frac{s+10}{\cancel{10}}\right] = 0$$

$$\Rightarrow s^2 + 10s + 100 = 0.$$

$$\begin{array}{c|cc} s^2 & 1 & 100 \\ s^1 & 10 & \\ s^0 & 100 & \end{array}$$

No sign Change. $\Rightarrow$ (S).

$\boxed{Q}$ Find the Range of k value for system to be stable.




Soln: CE $\longrightarrow$ $1 + GH(s) = 0.$

$$\therefore \quad 1 + \frac{K(s-2)^2}{(s+2)^2} = 0.$$

$$\therefore \quad s^2 + 4s + 4 + Ks^2 - 4Ks + 4K = 0$$

$$\therefore \quad s^2(1+K) + s(4-4K) + 4K+4 = 0$$

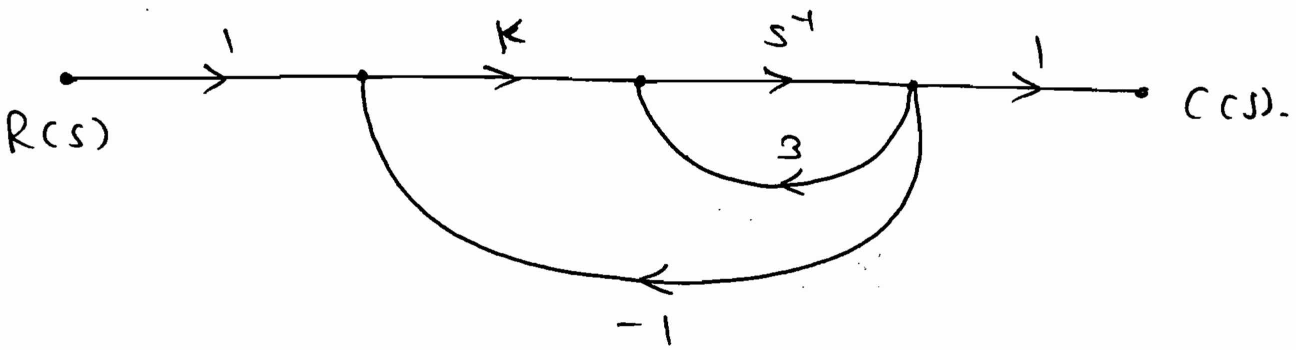
$$\begin{array}{c|ll} s^2 & 1+K>0 & 4+4K \\ s^1 & 4-4K>0 & \\ s^0 & 4+4K>0 & \end{array}$$

$\Rightarrow$

$$\begin{array}{l|l|l} K+1>0 & 4-4K>0 & 4+4K>0 \\ K>-1 & 4>4K & 4>-4K \\ & K<1 & K>-1 \end{array}$$

So, $\boxed{0 \le K < 1}$ ( $\because$ $K \geq 0$ given).

**Q** Find the range of K value.




Soln:

CE $\rightarrow$ $1 + GH(s) = 0.$

$\therefore \quad 1 - 3s^{-1} + Ks^{-1} = 0$

$1 - 3/s + \frac{K}{s} = 0.$

$\therefore \quad s + (K-3) = 0.$

for (S), $K - 3 > 0$

$\boxed{K > 3}$

$\boxed{a}$ The Loop Gain of the system

$$GH = \frac{k}{s(s+1)(s+2)}$$, the value of $k$

for which the system just becomes

the unstable is, —?

Soln:

CE → $1 + GH(s) = 0.$

$$\therefore \quad 1 + \frac{k}{s(s+1)(s+2)} = 0.$$

$$\therefore \quad s(s^2 + 3s + 2) + k = 0.$$

$$\therefore \quad s^3 + 3s^2 + 2s + k = 0.$$

⇒ $\underbrace{\text{just becomes the Unstable}}$

⇓

Marginal Stable.

$$\therefore \quad \begin{array}{c|cc} s^3 & 1 & 2 \\ s^2 & 3 & k \\ s^1 & \frac{6-k}{3} & \\ s^0 & k & \end{array}$$

For, MS $\quad \frac{6-k}{3} = 0$

$$\Rightarrow \boxed{K_{mar} = 6}$$

**Q** A system has $G(s) = \frac{K}{s^3 + 8s^2 + 4s}$, $H(s) = 1$. For what value of K the system will produce continuous oscⁿ.

**Solⁿ:**

$$1 + GH(s) = 0.$$

$$\therefore 1 + \frac{K}{s^3 + 8s^2 + 4s} = 0.$$

$$\therefore s^3 + 8s^2 + 4s + K = 0.$$

Contⁿ oscillation → MS.

$$\begin{array}{c|cc} s^3 & 1 & 4 \\ s^2 & 8 & K \\ s^1 & \frac{32-K}{8} & \\ s^0 & K & \end{array}$$

for MS, $\frac{32 - K_{mar}}{8} = 0.$

$$\Rightarrow \boxed{K_{mar} = 32}$$

**Another method:**

$$s^3 + 8s^2 + 4s + K = 0$$

$$\therefore \boxed{K = 32}$$

# ☆ Relative Stability

⇒ The Relative Stability Concept applicable for only stable system.

**Q** A system has $G(s) = \dfrac{2}{s(s+1)(s+2)}$, $H(s) = 1$. With RH criteria determine its Relative Stability about the line $s = -1$. [or]

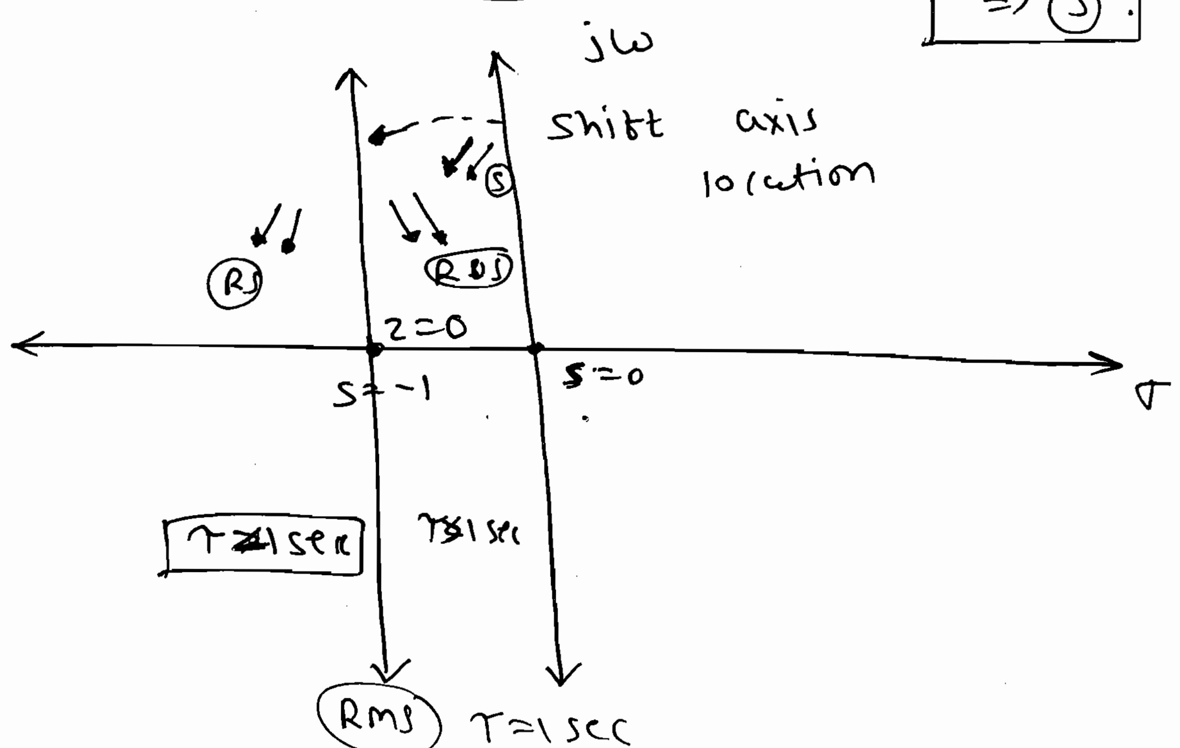
Check whether the time const. greater (or) lesser (or) equal to 1 sec to the given system.

Sol$^n$:

CE → $1 + G(s)H(s) = 0$

⇒ $s^3 + 3s^2 + 2s + 2 = 0.$

$\boxed{\begin{array}{c} 6 > 2 \\ \Rightarrow S \end{array}}$ ✱✱



$\Rightarrow \quad s+1 = 0 = z$

$\therefore \boxed{s = z-1}$

$$\therefore G(z) = \frac{2}{(z-1)(z)(z+1)}.$$

$$\therefore 1 + G(z)\cdot H(z) = 1 + \frac{2}{z(z^2-1)}.$$

$$= z^3 - z + 2.$$

$$\begin{array}{c|cc} z^3 & 1 & -1 \\ z^2 & \cancel{0}\,\epsilon & 2 \\ z^1 & \frac{-\epsilon - 2}{\epsilon} = -\infty & \\ z^0 & 2 & \end{array}$$

① ②

Two sign change betn $s=0$ & $s=-1$.

$\therefore$ 2 poles betn $s=0$ & $s=-1$.

$\Rightarrow \quad S = z +$ Axis shift location

$S = (z - 1/\tau)$

RS $\Rightarrow (\tau) < (\ ).$

RMS $\Rightarrow \tau = (\ )!$

RUS $\Rightarrow \tau > (\ ).$

## * Limitation of RH criteria:

① The exact location of pole can not be determine.

② The RH criteria is not applicable for exponential sine (or) cosine terms because it gives the infinite series.

③ RH criteria is applicable to finite no. of terms.

Note: By using RH criteria we can get approximation soln to exponential term.

Q Find the value of K for stability.

$$G(s) \cdot H(s) = \frac{K \cdot e^{-sT}}{s(s+1)}.$$

Soln: CE $\Rightarrow$ $1 + GH(s) = 0 \Rightarrow 1 + \frac{K \cdot e^{-sT}}{s(s+1)} = 0$

$$\Rightarrow 1 + \frac{K(1 - sT)}{s(s+1)} = 0$$

$$\Rightarrow s^2 + s + K - KsT = 0$$

$$\Rightarrow s^2 + (1 - KT)s + K = 0.$$

$$\begin{array}{c|cc} s^2 & 1 & K \\ s^1 & 1-K\tau & \\ s^0 & K. & \end{array}$$

$1 - K\tau > 0$ , $K > 0$

$\boxed{K < 1/\tau.}$

$\therefore$ $\Rightarrow$ $\boxed{0 < K < 1/\tau}$ for (S).