Engineering Mathematics Test 4

Number of Questions: 25 **Time: 60 min.**

(Probability and Statistics)

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. If A, B and C are mutually exclusive such that $5P(B) = 8\ P(A)$, $4P(C) = 3P(B)$ and $19P(A \cup B) = 13$ then $P(A \cup B \cup C)$ is ________.
   (A) 0.75
   (B) 1
   (C) 0.625
   (D) Cannot be determined

2. 240 passengers travelling in a plane from Hyderabad to Sharjah like one or more of the three meals among sandwich, burger and pizza as given below. 140 passengers like sandwich,110 passengers like burger and 90 passengers like pizza. 40 of them like both sandwich and burger, 50 of them like both Sandwich and Pizza, 40 of them like both burger and pizza where as 30 of them like all the three meals. What is the probability that a randomly selected passenger likes pizza only?
   (A) 0.125 (B) 0.250
   (C) 0.375 (D) 0.500

3. Let S be the set of all 4 digit numbers that can be formed using the digits 2, 3, 5, 7, 8 and 9. Probability that a randomly selected number of S has all digits distinct is _______.
   (A) $\frac{5}{12}$ (B) $\frac{5}{18}$
   (C) $\frac{5}{24}$ (D) $\frac{5}{36}$

4. An unbiased coin is tossed until it shows up the same face in two consecutive throws. What is the probability that the number of tosses is not more than 4?
   (A) $\frac{3}{4}$ (B) $\frac{1}{8}$
   (C) $\frac{7}{8}$ (D) $\frac{1}{4}$

5. What is the probability that a quadratic equation $ax^2 + bx + c = 0$ has equal roots if a, b and c are distinct and are taken from {1, 2, 3, 4, 6, 8, 9}?
   (A) $\frac{1}{35}$ (B) $\frac{2}{35}$
   (C) $\frac{1}{105}$ (D) $\frac{2}{105}$

6. A bag contains 4 five rupee coins, 3 two rupee coins and 3 one rupee coins. If 6 coins are drawn from the bag at random, what are the odds in favour of the draw yielding maximum amount?
   (A) 1 : 70 (B) 1 : 69
   (C) 69 : 70 (D) 70 : 1

7. Kids and Toys factory is transporting balls of 5 different colours – yellow, blue, red, green and white. Mr. Bholeram, a worker in the factory has to separate these balls as per their colours into different boxes and label them with the corresponding coloured labels.

   Mr. Bholeram, after separating the balls, sealed the boxes and then labelled the boxes at random. What is the probability that all the boxes are incorrectly labelled?
   (A) 1 (B) 0
   (C) $\frac{11}{120}$ (D) $\frac{11}{30}$

8. While shuffling a pack of cards, 4 cards are accidentally dropped. The probability that all of them are numbered cards (2 to 10) of the same suit is
   (A) $\frac{4 \times {}^{9}C_4}{{}^{52}C_4}$ (B) $\frac{\left({}^{9}C_4\right)^4}{{}^{52}C_4}$
   (C) $\frac{4 \times {}^{9}C_1}{{}^{52}C_4}$ (D) $\frac{\left({}^{9}C_1\right)^4}{{}^{52}C_4}$

9. Arpit and Bipin pick up a ball at random from a bag containing 5 violet, 2 red and 3 orange balls one after the other, replacing it every time till one of them gets an orange ball and the one who first gets an orange ball is declared a winner. If Arpit begins the game, then the probability of Bipin winning the game is
   (A) $\frac{10}{17}$ (B) $\frac{7}{17}$
   (C) $\frac{7}{10}$ (D) $\frac{3}{10}$

10. An urn A contains 6 white balls and 7 black balls. And urn B contains 8 white balls and 6 black balls. A person draws a ball at random from one of the two urns. It turns out to be black. What is the probability that the ball was drawn from urn A?
    (A) $\frac{7}{14}$ (B) $\frac{49}{88}$
    (C) $\frac{39}{88}$ (D) None of the above

11. The bivariate probability distribution of X and Y is as follows.

X \ Y	0	1	2
0	$\frac{1}{40}$	$\frac{2}{40}$	$\frac{3}{40}$
1	$\frac{2}{40}$	$\frac{3}{40}$	$\frac{1}{40}$
2	$\frac{3}{40}$	$\frac{1}{40}$	$\frac{7}{40}$
3	$\frac{4}{40}$	$\frac{5}{40}$	$\frac{8}{40}$

Find $P(X \le 1, Y = 2)$.

(A) $\frac{2}{5}$ (B) $\frac{1}{5}$
(C) $\frac{3}{10}$ (D) $\frac{1}{10}$

12. In a book of 500 pages, there are 50 typing errors. Assuming that the number of errors per page follows poission distribution, find the probability that randomly chosen 5 pages will contain no error.

(A) 0.6065 (B) 0.6078
(C) 0.6538 (D) 0.3935

13. The continuous random variable X is uniformly distributed with mean 2 and variance 12. Find $P(X > 0)$.

(A) $\frac{1}{3}$ (B) $\frac{4}{5}$
(C) $\frac{2}{3}$ (D) $\frac{1}{5}$

14. X and Y are two independent normal variates with means 3, 6 and variances, 1, 9 respectively. Find the value of k such that $P(X + Y \le k) = P(9X - Y \ge 2k)$.

(A) 9.3 (B) 9.6
(C) 8.6 (D) 10.3

15. Bag A contains 9 white balls and 5 green balls. Bag B contains 6 white balls and 7 green balls. One ball is drawn from bag A and is placed in bag B. Now one ball is drawn at random from bag B. It is found that the ball is green. Find the probability that white ball is transferred from bag A.

(A) $\frac{20}{103}$ (B) $\frac{63}{103}$
(C) $\frac{80}{103}$ (D) $\frac{75}{103}$

16. A dice is rolled twice the sum of the numbers appearing is 7, what is the probability that at least one dice shows 3?

(A) $\frac{3}{7}$ (B) $\frac{2}{3}$
(C) $\frac{1}{3}$ (D) $\frac{4}{7}$

17. A random variable X has the following probability distribution.

X = xi	0	1	2	3	4
P(x = xi)	K	2K	3K	5K	4K

Then find $P(X \ge 2)$.

(A) $\frac{4}{5}$ (B) $\frac{1}{5}$
(C) $\frac{2}{5}$ (D) $\frac{1}{15}$

18. The standard error is ______.

(A) accepting the null hypothesis when it is false.
(B) rejecting the null hypothesis when it is true.
(C) the standard deviation of the sampling distribution of a statistic.
(D) the probability that the test statistic does not lie in the critical region.

19. In large sampling, the sampling distribution of means follows ______

(A) Normal distribution
(B) t – distribution
(C) F – distribution
(D) x^2 – distribution

20. Which of the following distributions is used to test the equality of variances of two populations from which two small random samples are drawn?

(A) Normal distribution
(B) t – distribution
(C) F – distribution
(D) x^2 – distribution

21. If a statistic s follows t – distribution with $\nu = 10$ degrees of freedom, then s^2 follows F – distribution with degrees of freedom (ν_1, ν_2) = ______.

(A) (1,9) (B) (1,10)
(C) (1,11) (D) (9,1)

22. In testing of hypothesis, if the test statistic is outside the critical region, then we will

P : Accept the null hypothesis
Q : Reject the null hypothesis
R : Accept the alternative hypothesis
S : Reject the alternative hypothesis

Which of the following is true?

(A) P only
(B) R only
(C) P and S only
(D) Q and R only

23. Three letters are placed into three addressed envelopes randomly. A random variable X denotes the number of letters placed into corresponding envelopes. Find the variance of X.

(A) $\frac{5}{6}$ (B) 2
(C) 1 (D) 3

24. The variance of the data $x, x + 3, x + 5, x + 7, x + 10$ is

(A) 11.2 (B) 11.6
(C) $11.6 + x$ (D) $11.2 + x$

25. The median of the following data can be 3, 8, 12, 28, 16, 15, x

(A) 13 (B) 14
(C) 15 (D) Any of the above

Answer Keys

1. B	**2.** A	**3.** B	**4.** C	**5.** C	**6.** B	**7.** D	**8.** A	**9.** B	**10.** B
11. D	**12.** A	**13.** C	**14.** B	**15.** B	**16.** C	**17.** A	**18.** C	**19.** A	**20.** C
21. B	**22.** C	**23.** C	**24.** B	**25.** D					

Hints and Explanations

1. Given $5P(B) = 8P(A)$ and $4P(C) = 3P(B)$

$\Rightarrow P(A) = \frac{5}{8}P(B)$ and $P(C) = \frac{3}{4}P(B)$ ----- (1)

Now $19\,P(A \cup B) = 13 \Rightarrow P(A \cup B) = \frac{13}{19}$

$P(A) + P(B) = \frac{13}{19}$ ($\because$ A and B mutually exclusive)

$\Rightarrow \frac{5}{8}P(B) + P(B) = \frac{13}{19}$

$\Rightarrow \frac{13}{8}P(B) = \frac{13}{19}$

$\Rightarrow P(B) = \frac{8}{19}$

Now $P(A \cup B \cup C) = P(A) + P(B) + P(C)$

$= \frac{5}{8}P(B) + P(B) + \frac{3}{4}P(B)$

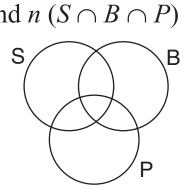
$= \left(\frac{5}{8} + 1 + \frac{3}{4}\right)P(B) = \frac{19}{8}P(B) = \frac{19}{8} \times \frac{8}{19} = 1.$

Choice (B)

2. The total number of passengers = 240

Let S, B and P denote the sets of passengers who like sandwich, burger and pizza respectively.

$\therefore$ $n(S) = 140$, $n(B) = 110$, $n(P) = 90$, $n(S \cap B) = 40$, $n(B \cap P) = 40$, $n(P \cap S)$ $= 50$ and $n(S \cap B \cap P) = 30$



$\therefore$ Probability that a randomly selected passenger likes only pizza

$= \frac{(\text{The number of passengers who like only pizza})}{(\text{The total number of passengers})}$

$= \frac{n(P) - n(P \cap S) - n(B \cap P) + n(S \cap B \cap P)}{240}$

$= \frac{90 - 40 - 50 + 30}{240} = \frac{1}{8} = 0.125.$ Choice (A)

3. The number of 4 digit numbers that can be formed using the digits 2, 3, 5, 7, 8 and 9

= The number of elements of $S = 6^4$

The number of 4 digit numbers of S that have all digits distinct = The number of 4 digit numbers that can be formed using the digits 2, 3, 5, 7, 8 and 9 = ${}^6P_4$

$\therefore$ Probability that a randomly selected number of S has all digits distinct $= \frac{{}^6P_4}{6^4} = \frac{5}{18}$ Choice (B)

4. The number of tosses may be 2 or 3 or 4.

The possible cases and their corresponding probabilities:

Case 1 : HH OR TT $\rightarrow 2\left(\frac{1}{2}\right)^2$

Case 2 : HTT OR THH $\rightarrow 2\left(\frac{1}{2}\right)^3$

Case 3 : $HTHH$ OR $THTT$ $\rightarrow 2\left(\frac{1}{2}\right)^4$

Hence, the required probability is

$2\left[\frac{1}{4} + \frac{1}{8} + \frac{1}{16}\right] = \frac{7}{8}$ Choice (C)

5. Considering different values of a, b and c from the set $\{1, 2, 3, 4, 6, 8, 9\}$, we get different quadratic equations. As a, b and c are distinct, ${}^7P_3 = 210$ different quadratic equations can be formed.

$\therefore$ Total ways are 210

For the quadratic equation $ax^2 + bx + c = 0$ to have equal roots, $b^2 = 4ac$.

The possible combinations of a, b and c respectively are 1, 6, 9 and 9, 6, 1.

Hence favourable cases are 2

$\therefore$ Required probability $= \frac{2}{210} = \frac{1}{105}$. Choice (C)

6. We have 4 five rupee coins, 3 two rupee coins and 3 one rupee coins.
For the draw to yield a maximum amount, of the 6 coins drawn 4 should be five rupee coins and 2 should be two rupee coins. The required probability is

$$\frac{{}^4C_4 \times {}^3C_2}{{}^{10}C_6} = \frac{3}{210} = \frac{1}{70}$$

Hence, odds in favour are favourable ways : unfavourable ways = 1 : 69. Choice (B)

7. There are 5 boxes and 5 labels. Hence the boxes can be labelled in 5! i.e. 120 different ways

P (all labelled incorrectly) $= \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!}$

$= \frac{44}{120} = \frac{11}{30}$ Choice (D)

8. There are 9 numbered cards in each suit.
P(all the 4 cards are numbered cards of same suit)

$$= \frac{{}^9C_4 + {}^9C_4 + {}^9C_4 + {}^9C_4}{{}^{52}C_4} = \frac{4 \times {}^9C_4}{{}^{52}C_4}$$ Choice (A)

9. The probability of picking up an orange ball is $\frac{3}{10}$ while not picking up an orange ball is $\frac{7}{10}$.

We compute the probability of Arpit (the beginner) winning the game.
Let A and B be the events of Arpit and Bipin picking up an orange ball respectively
The winning sequence of Arpit can be

$A, \bar{A}\,\bar{B}\,A, \quad \bar{A}\,\bar{B}\,\bar{A}\,\bar{B}\,A, \ldots\ldots$

As the above sequence indicates, Arpit may pick an orange ball right in the 1st trial with a probability of $\frac{3}{10}$ (or) in the third trial (as the 2nd trial is made by Bipin, and for Arpit to win, Bipin should not be getting an orange ball). The probability here being $\left(\frac{7}{10}\right)^2 \times \frac{3}{10}$ (or) in the fifth trial with a probability of $\left(\frac{7}{10}\right)^4 \times \frac{3}{10}$ and so on.

$$\therefore \quad P(A) = \frac{3}{10} + \left(\frac{7}{10}\right)^2 \times \frac{3}{10} + \left(\frac{7}{10}\right)^4 \times \frac{3}{10} + \ldots..$$

$$= \frac{\frac{3}{10}}{1 - \left(\frac{7}{10}\right)^2} = \frac{30}{51} = \frac{10}{17}$$

Probability of Bipin winning is the same as probability of Arpit losing i.e.,

$\therefore \quad P(B) = P(\bar{A}) = 1 - \frac{10}{17} = \frac{7}{17}$ Choice (B)

Note: If 'p' is the probability of success (in this case picking up an orange ball), the probability that the beginner wins the game $= \frac{1}{2-p}$

10. Probability of selecting urn A is $P(A) = \frac{1}{2}$.

and that of selecting urn B is $P(B) = \frac{1}{2}$

Probability of drawing a black ball (event E) when urn A is selected $P\left(\frac{E}{A}\right) = \frac{{}^7C_1}{{}^{13}C_1}$ and probability of E when urn B is selected $P\left(\frac{E}{B}\right) = \frac{{}^6C_1}{{}^{14}C_1}$

Probability of selecting black ball

$= P(A).P\left(\frac{E}{A}\right) + P(B).P\left(\frac{E}{B}\right)$

$\frac{1}{2}.\frac{{}^7C_1}{{}^{13}C_1} + \frac{1}{2}.\frac{{}^6C_1}{{}^{14}C_1}$

$$\text{Required Probability} = \frac{\frac{1}{2}.\frac{{}^7C_1}{{}^{13}C_1}}{\frac{1}{2}.\frac{{}^7C_1}{{}^{13}C_1} + \frac{1}{2}.\frac{{}^6C_1}{{}^{14}C_1}}$$

$$= \frac{\frac{7}{13}}{\frac{7}{13} + \frac{6}{14}} = \frac{\frac{7}{13}}{\frac{98+75}{13 \times 14}} = \frac{7 \times 14}{176} = \frac{49}{88}$$ Choice (B)

11. The marginal distributions are given below.

X \ Y	0	1	2	$P_x(x)$
0	$\frac{1}{40}$	$\frac{2}{40}$	$\frac{3}{40}$	$\frac{6}{40}$
1	$\frac{2}{40}$	$\frac{3}{40}$	$\frac{1}{40}$	$\frac{6}{40}$
2	$\frac{3}{40}$	$\frac{1}{40}$	$\frac{7}{40}$	$\frac{11}{40}$
3	$\frac{4}{40}$	$\frac{5}{40}$	$\frac{8}{40}$	$\frac{17}{40}$
$P_y(y)$	$\frac{10}{40}$	$\frac{11}{40}$	$\frac{19}{40}$	1

$P(X \le 1, Y = 2) = P(X = 0, Y = 2) + P(X = 1, Y = 2)$

$= \frac{3}{40} + \frac{1}{40} = \frac{4}{40} = \frac{1}{10}$ Choice (D)

12. Average number of errors per page $\lambda = \frac{50}{500} = \frac{1}{10}$

Average number of errors per 5 pages = $5 x \frac{1}{10} = \frac{1}{2}$

Probability of k errors per page is $P(x = k) = \frac{\lambda^k}{k!} e^{-\lambda}$.

$\therefore$ here $k = 0$

$\therefore$ $P(k = 0) = \frac{\lambda^k}{k!} e^{-\lambda} = e^{-\frac{1}{2}}$

$\therefore$ Probability that a random sample of 5 pages has no error = $e^{-0.5} = 0.6065$ Choice (A)

13. We know that X is uniform random variable in the interval $[a, b]$ then $p(x) = \frac{1}{b-a}$, $a < x < b$ and mean = $\frac{a+b}{2}$, variance = $\frac{(b-a)^2}{12}$

Given mean = 2 $\Rightarrow \frac{a+b}{2} = 2$

$\Rightarrow a + b = 4$ ---- (1)

Variance = 12 $\Rightarrow \frac{(b-a)^2}{12} = 12$

$\Rightarrow (b - a)^2 = 144 \Rightarrow b - a = 12$ ------ (2)

Solving (1) and (2) $a = -4$, $b = 8$

$\therefore$ $P(x) = \frac{1}{b-a} = \frac{1}{8-(-4)} = \frac{1}{12}$

$$P(x > 0) = \int_0^b p(x)dx = \int_0^8 \frac{1}{12} dx = \frac{1}{12} x \Big]_0^8 = \frac{8}{12} = \frac{2}{3}$$

Choice (C)

14. Given mean of $X = 3$
Variance of $X = 1$
Mean of $Y = 6$
Variance of $Y = 9$
$X = N(3, 1)$ $Y = N(6, 3)$ and X and Y are independent.
Let $u = x + y$; and $v = 9x - y$
Then u, v are also normal variates
$U = x + y = N(3 + 6, 1 + 9) = N(9, 10)$
$V = 9x - y = N(9(3) - 6, 81(1) + 9) = N(21, 90)$
By definition

$Z = \frac{u-9}{\sqrt{10}}$ and for $u = k \Rightarrow Z = \frac{k-9}{\sqrt{10}} = z$

Again $z = \frac{v-21}{\sqrt{90}}$ and for $v = 2k \Rightarrow Z = \frac{2k-21}{\sqrt{90}} = z_1$

Given $P(x + y \le k) = p(9x - y \ge 2k)$
$P(z \le z_1) = p(z \ge z_2)$
$P(z \le z_1) = p(z \le -2z)$

$= \frac{k-9}{\sqrt{10}} = \frac{-(2k-21)}{\sqrt{90}} = k - 9$

$= \frac{-2k+21}{3}$

$3k - 27 = -2k + 21$

$5k = 48 \Rightarrow k = \frac{48}{5} = 9.6$ Choice (B)

15. Let B_1: transfer of white ball to bag B.
B_2: transfer of green ball to bag B.

$P(B_1) = \frac{9}{14}; P(B_2) = \frac{5}{14}$

Let E be the event of drawing a green ball from bag B after transfer.

$P\left(\frac{E}{B_1}\right)$ = probability of drawing green ball if white ball is transferred to bag $B = \frac{7}{14}$

$P\left(\frac{E}{B_2}\right)$ = probability of drawing a green ball if green ball is transferred to bag $B = \frac{8}{14}$.

$\therefore$ $P(E) = P(B_1) . P\left(\frac{E}{B_1}\right) + P(B_2) . P\left(\frac{E}{B_2}\right)$

$= \frac{9}{14} . \frac{7}{14} + \frac{5}{14} . \frac{8}{14} = \frac{63}{196} + \frac{40}{196} = \frac{103}{196}$

$\therefore$ The required probability $P\left(\frac{B_1}{E}\right)$

$= \frac{P(B_1).P\left(\frac{E}{B_1}\right)}{P(E)} = \frac{\frac{9}{14}.\frac{7}{14}}{\frac{103}{196}} = \frac{63}{103}$ Choice (B)

16. Let A be the event that the number 3 appears atleast once.
B be the event that sum of the numbers appearing is 7.
$A \cap B$ be the event that the sum is 7 and 3 appear atleast once $P(B) = \frac{6}{36} \Rightarrow P(A \cap B) = \frac{2}{36}$

A/B denotes atleast one number show 3 while the sum of the numbers is 7.

$P\left(\frac{A}{B}\right) = \frac{P(B \cap A)}{P(B)} = \frac{\frac{2}{36}}{\frac{6}{36}} = \frac{1}{3}$ Choice (C)

17. We know $\sum P(X = x_1) = 1$

$\therefore k + 2k + 3k + 4k + 5k = 1$

$15k = 1 \Rightarrow k = \frac{1}{15}$

$P(X \geq 2) = P(X = 2) + P(X = 3) + P(X = 4)$

$= 3k + 5k + 4k = 12k = \frac{12}{15} = \frac{4}{5}$ Choice (A)

18. By definition. Choice (C)

19. Standard Result. Choice (A)

20. Standard Result. Choice (C)

21. We know that, if a statistic 's' follows t-distribution with degrees of freedom = ν, then 's^2' follows F–distribution with degrees of freedom $(1, \nu)$

Here ν = 10

∴ 's^2' follows F-distribution with degrees of freedom $= (1, \nu) = (1,10)$. Choice (B)

22. When the test statistic is outside the critical region, it lies in the acceptance region. So, we will accept the null hypothesis and reject the alternative hypothesis.

∴ Both P and S are true. Choice (C)

23. Three letters are placed into 3 addressed envelopes randomly in 3! = 6 ways.

X denotes the number of letters placed into corresponding addressed envelopes. The probability distribution table is as follows.

$X = x_i$	0	1	2	3
$P(x = x_i)$	$\frac{2}{6}$	$\frac{3}{6}$	0	$\frac{1}{6}$

∴ mean $(M) = \sum X_i P(x = x_i)$

$$= 0 \times \frac{2}{6} + 1 \times \frac{3}{6} + 2 \times \frac{0}{6} + 3 \times \frac{1}{6}$$

$$= \frac{0+3+0+3}{6} = \frac{6}{6} = 1$$

Variance $= \sum x_i^2 P(x = x_i) - \mu^2$

$$= 0 \times \frac{2}{6} + 1 \times \frac{3}{6} + 4 \times 0 + 9 \times \frac{1}{6} - 1$$

$$= \frac{3+9}{6} - 1 = 2 - 1 = 1.$$ Choice (C)

24. We know that variance $(x, x + 3, x + 5, x + 7, x + 10)$ = variance (0, 3, 5, 7, 10)

$$AM(0, 3, 5, 7, 10) = \frac{0+3+5+7+10}{5} = \frac{25}{5} = 5$$

$$\text{Variance } (0, 3, 5, 7, 10) = \frac{\sum (x_1 - A)^2}{n}$$

$$= \frac{(0-5)^2 + (3-5)^2 + (5-5)^2 + (7-5)^2 + (10-5)^2}{5}$$

$$= \frac{25+4+0+4+25}{5} = \frac{58}{5}$$

Variance $= \frac{58}{5} = 11.6$. Choice (B)

25. The ascending order of the given data except x is 3, 8, 12, 15, 16, 28

If $x < 12$, the fourth observation is 12 hence median is 12 if $x > 15$, the fourth observation is 15, hence median is 15. If $12 < x < 15$, the fourth observation is x, hence median is x.

Median is always lies between [12, 15]. Choice (D)