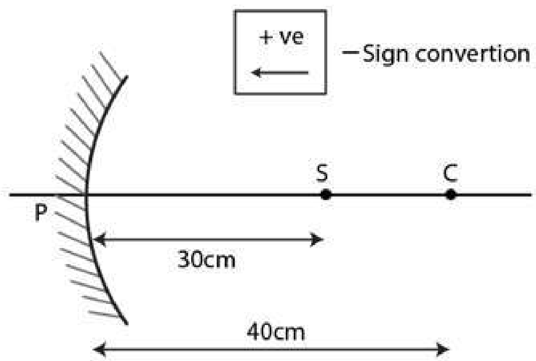
Geometrical Optics

Exercise Solutions

Solution 1:
Here u = -30 cm and R = -40 cm



From the mirror equation,

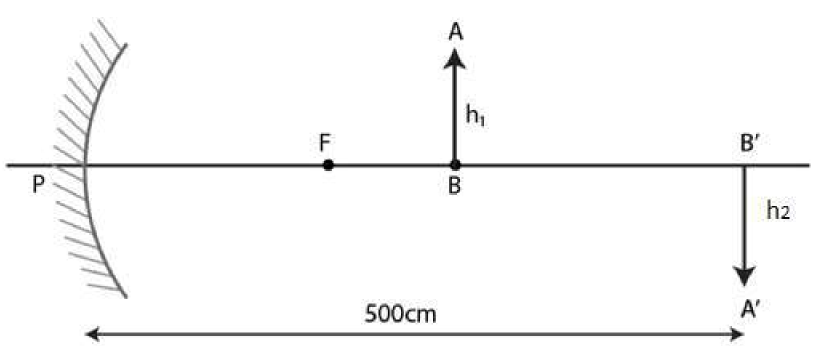
$1/v + 1/u = 2/R$

$\Rightarrow 1/v = 2/-40 - 1/-30 = 1/60$

Therefore, the image will be formed at the distance of 60 cm in front of the mirror.

Solution 2:
Height of the image = h_1= 20cm
Height of the image = h_2 = 50cm

The distance of screen from the mirror = v = 5.0 m



Since, $-v/u = h_2/h_1$

=> u = [-500x2]/5

= -200 cm = 2 m

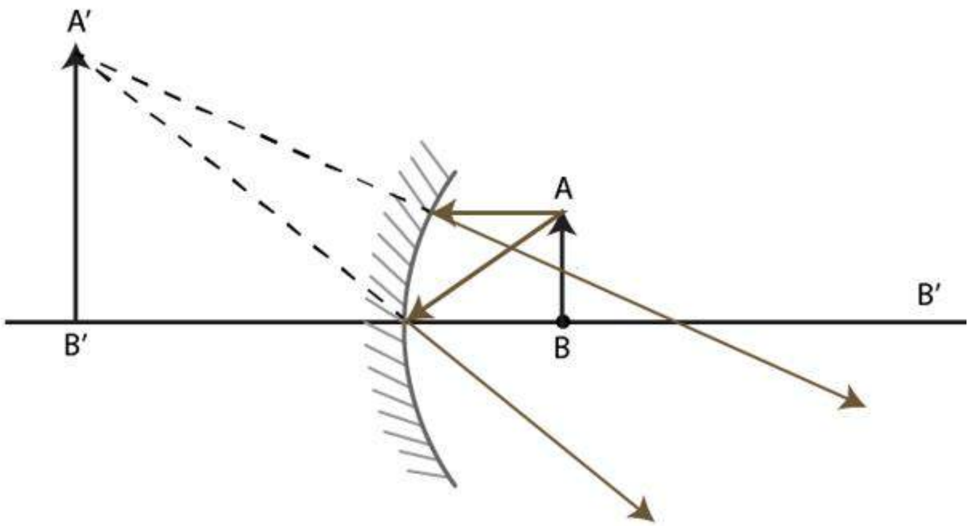
From the mirror formula,

1/v + 1/u = 1/f

=> 1/-5 + 1/-2 = 1/f

=> f = -1.44 m -> [focal length]

Solution 3:
Focal length of the concave mirror = f = -20cm
Magnification of the mirror = M = v/u = 2
=> v = 2u



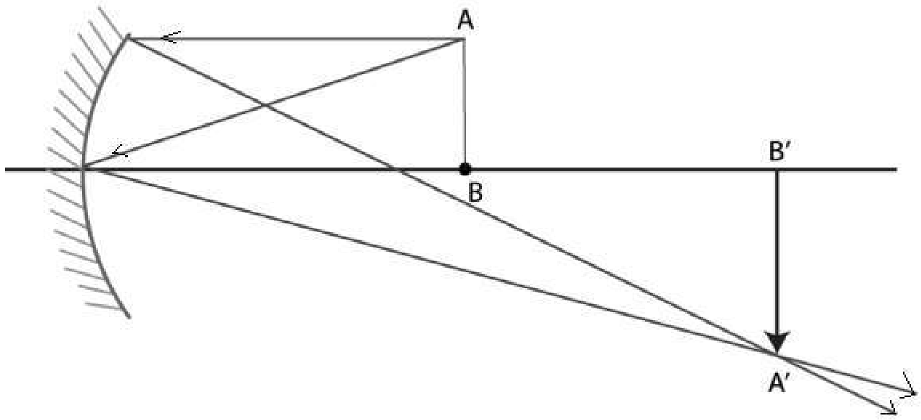
Let us consider the image formed will be virtual and beyond the mirror.
Hence, v = -20u

From the mirror formula,

1/v + 1/u = 1/f

=> 1/-2u + 1/u = 1/-20
u = 10 cm

When image formed will be real and inverted. Hence, v = 20u



=> 1/2u + 1/u = 1/20
u = 30 cm
Therefore, the positions are 10 cm and 30 cm from the concave mirror.

Solution 4:

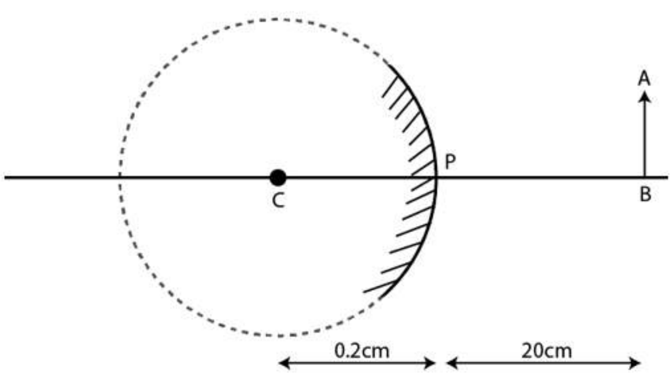
Here f = 7.5 cm and M(Magnification) = 0.6 cm = v/u
=> v = 0.6u

From the mirror formula,

1/v + 1/u = 1/f

=> 1/0.6u - 1/u = 1/7.5
u = 5 cm

Solution 5:
The distance of object from the ball =u = 20cm
Diameter of ball bearing= d= 0.4cm
Height of the object = h= 1.6 cm



Consider distance of the object from the ball = u=-20cm

From the mirror formula,

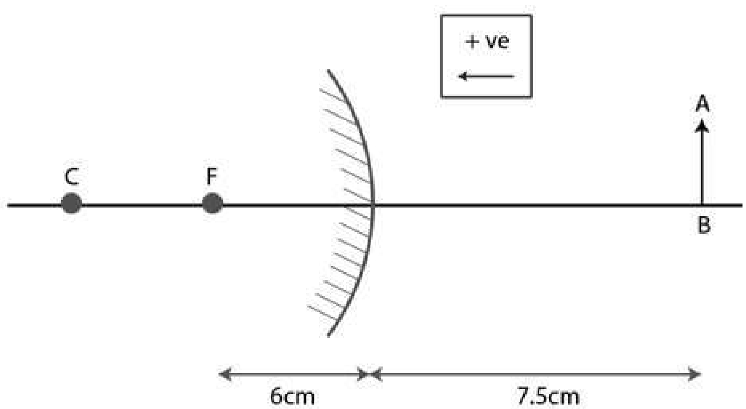
1/v + 1/-20 = 2/0.2

=> v = 0.1 cm

Now, Magnification= M = -v/u = 0.1/-20 = 1/200

So, height of the image = 16/200 = 0.8 m = 0.8 mm

Solution 6:
AB = 3 cm, u = -7.5 cm and f = 6 cm



From the mirror formula,
1/v + 1/u = 1/f

1/v + 1/-7.5 = 1/6

=> v = 10/3 cm

Now, Magnification= M = -v/u = 10/[7.5x3]

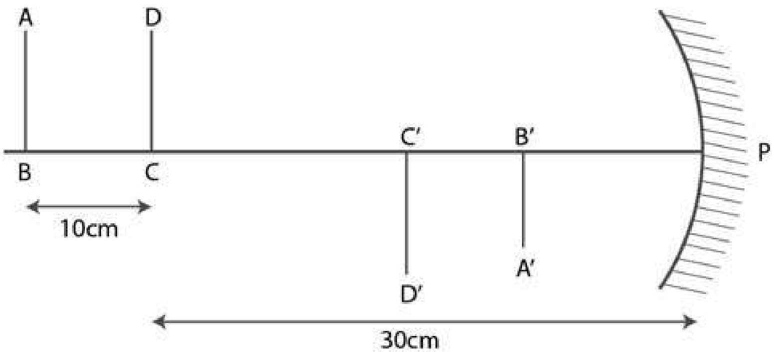
=> A'B'/AB = 10/[7.5x3]

=> A'B' = 100/72 = 1.33 cm
Image will form at a distance of 10/3 cm. From the pole and image is 1.33 cm.

Solution 7:

R = 20 cm, f = R/2 = -10 cm
For part AB, PB = 30+10 =40 cm

Here u = - 40 cm



From the mirror equation,
1/v + 1/u = 1/f

1/v + 1/-40 = 1/-10

=> v = -13.3 cm

So, PB' = 13.3 cm

Now, Magnification= M = -v/u = -13.3/-40 = -1/3

=> A'B'= -10/3 cm

For part CD, PC = 30 cm

Here u = - 30 cm

From the mirror equation,
1/v + 1/u = 1/f

1/v + 1/-30 = 1/-10

=> v = -15 cm

So, PC' = -15 cm

Now, Magnification= M = -v/u = -15/-30 = -1/2

=> C'D' = 5 cm

Now, B'C' = PC'- PB' = 15 - 13.3 = 17 cm

Therefore, total length = A'B' + B'C' + C'D' = 3.3 + 1.7 + 5 = 10 cm

Solution 8:
Here u = 25 cm
Magnification in the image = 1.4

=> A'B'/AB = -v/u

=> 1.4 = -v/-25

=> v = 35 cm

Using mirror formula,

1/v + 1/u = 1/f

=> 1/f = 1/35 + 1/25 = -2/175

f = -87.5 cm
Which is focal length of the concave mirror.

Solution 9:
Here f = 7.6 m and diameter of the moon is 3450 km

Using mirror formula,

1/v + 1/u = 1/f
using u = - 3.8 x 10^5 km and f = - 7.6 m

=> v = -7.6 cm
Since distance of moon from the earth is very large as compared to focal length it can be taken as infinity.

Now, Magnification, m = -v/u = [distance of the image]/[distance of the object]

=> -(-76)/[- 3.8 x 10^5] = [distance of the image]/[3450x10^3]

=> Distance of the image = 0.069 m = 6.9 cm

Solution 10:
Here u = -30 cm and f = -20 cm

Using mirror formula,

1/v + 1/u = 1/f

=> v = -60 cm

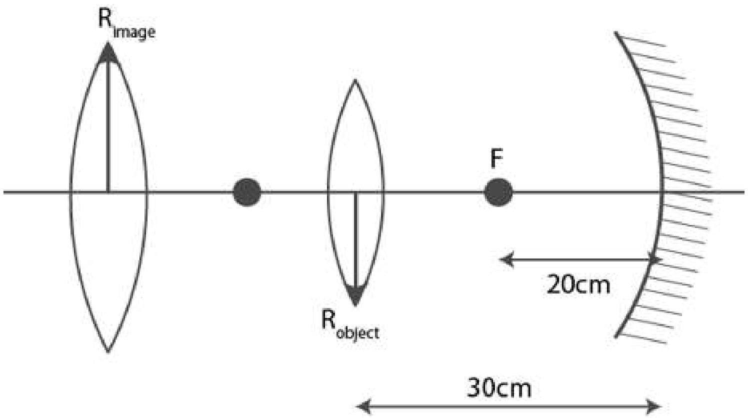


Image of the circle formed at distance 60 cm in front of the mirror.

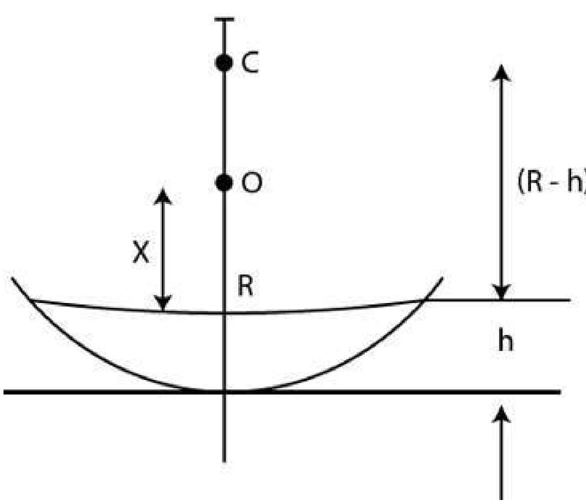
=> m = -v/u = [Radius of the image]/[Radius of the object]

=> -(-60)/(-30) = [Radius of the image]/2

=> Radius of the image = 4 cm

Hence, radius of the image formed = 4cm

Solution 11:
Let the object placed at a height x above the surface of water.



If the image has to be formed at the same position, then the apparent position of the object with respect to mirror should be at the center of curvature.

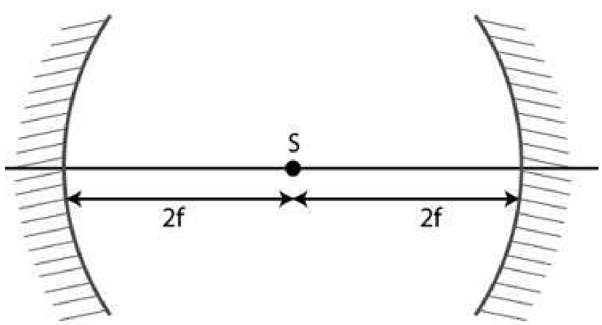
=> [Real depth]/[apparent depth] = $1/\mu$

Now, $x/(R-h) = 1/\mu$

=> $x = [R-h]/\mu$

Solution 12:
Both the mirrors have equal focal length f. (Given)

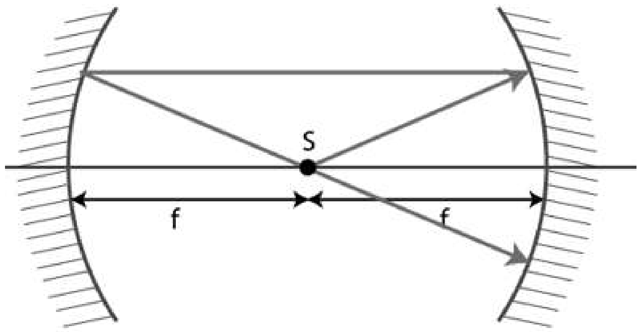
Case 1:
The source is at distance '2f' from each mirror, that is the object is at center of curvature of the mirrors. Then the image will be produced at the same point S.



=> $d = 2f + 2f = 4f$

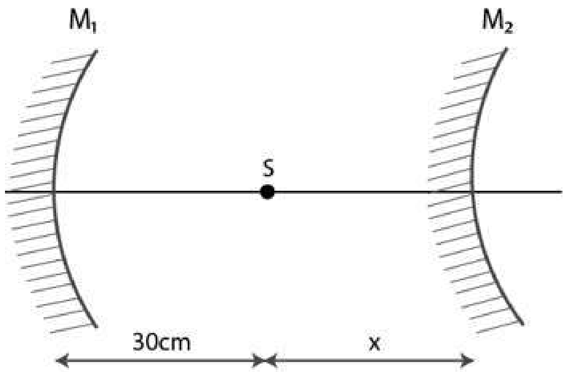
Case 2:

When the source s is at a distance f from each mirror, the rays from the source after reflecting from one mirror will become parallel and so these parallel rays after the reflection from the other mirror the object itself. In this case only one image is formed.



=> $d = f + f = 2f$

Solution 13:
The focal length of both the mirrors is same, i.e. 20 cm, and the image is also formed at the focus after the reflection from both the mirror surfaces.



For mirror M_1:

The distance of object from the mirror, u= -30cm (converging mirror)

Focal length, f= -20cm

Using mirror formula,

1/v + 1/u = 1/f ...(1)
Here u = 30 and f = -20

=> v = -60 cm

For mirror M_2:
Object distance from the mirror will be the difference of the image formed by first mirror and the distance between the two mirrors.
=> u = 60 - (30 + x) cm = 30-x cm

Distance of image from the mirror, v= -x cm

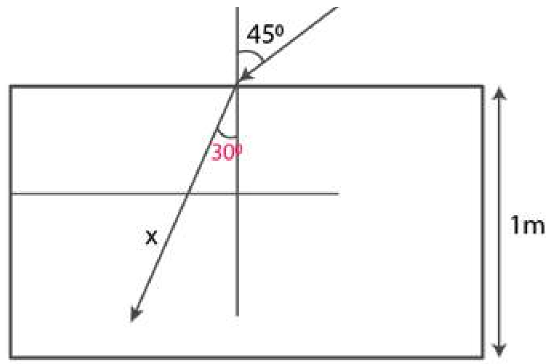
Focal length of the mirror, f= 20cm

mirror formula => -1/x + 1/(30-x) = 1/20

=> x = 20 cm or -30 cm

Therefore, total distance between the two lines is 20+30 = 50 cm.

Solution 14:
Angle of incidence i= 45°
Angle of refraction r= 30°



Using the Snell's law:

[sin i]/[sin r] = c/v

Where c = velocity of light in are and v = velocity of light in medium

=> [sin i]/[sin r] = [3×10^8]/v

=> sin 45°/sin30° = [3×10^8]/v

=> v = [3×10^8]/√2 m/s

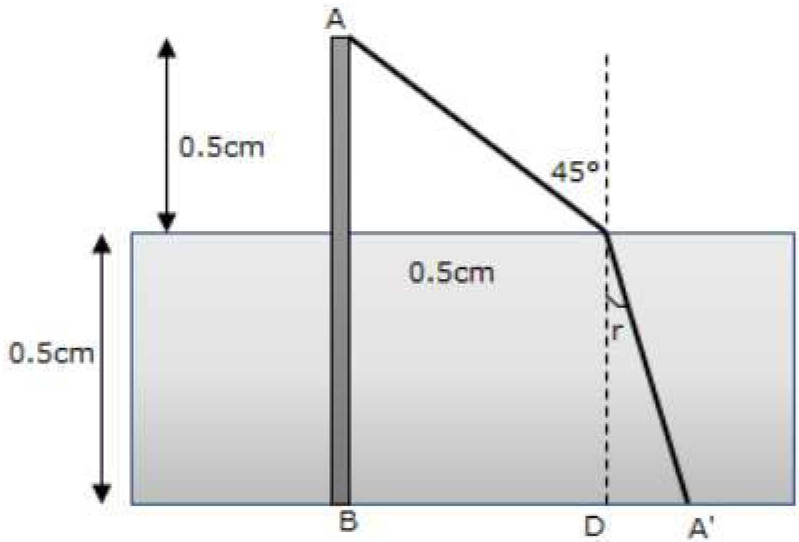
Now, Distance travelled by light in the slab is x

x = 1/cos30° = 2/√3

Hence, time taken by light = 2√2/[√3 x 3 x 10^8]

= 5.4×10^{-9} sec

Solution 15:
Shadow length = BA' = BD + A'D = 0.5 + 0.5 tan r



Now, 1.33 = sin45°/sin r

=> sin r = 0.53

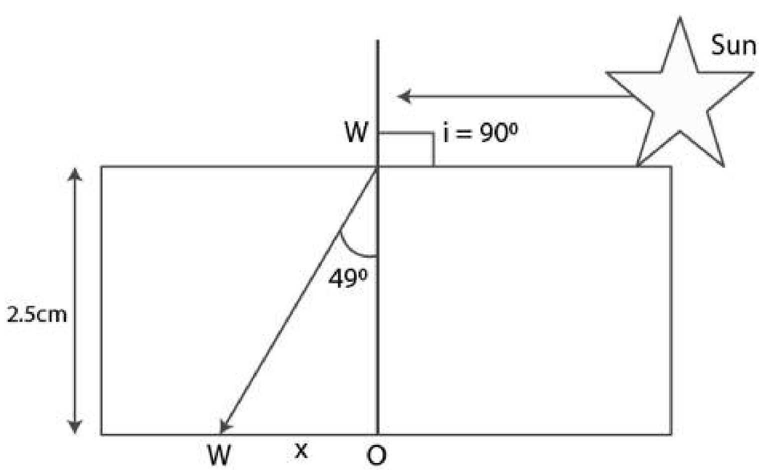
=> cos r = √(1-sin²r = √[1-(0.53)²] = 0.85

=> tan r = 0.6235

Therefore, shadow length = 0.5(1+0.6235) = 81.2 cm

Solution 16:

Height of the lake= 2.5m



Using the Snell's law:

[sin i]/[sin r] = μ_2/μ_1 = (4/3)/1

=> sin r = 3/4

=> r = 49°

From diagram,
tan r = x/2.5

=> tan 49° = x/2.5

=> x = 2.8 m

Solution 17:
The thickness of the glass slab, d = 2 cm
and refractive index of the slab μ = 1.5
shift due to the glass slab: $\Delta T = (1-1/\mu)\, d = (1-1/1.5)2.1 = 0.7$ cm

Microscope should be shifted 0.70cm to focus the object again.

Solution 18:

Let Δt_o is shift due to oil and Δt_w = shift due to water

$$\Delta t_w = \left(1-\frac{1}{\mu}\right)d = \left(1-\frac{1}{1.33}\right)20 = 5cm$$

and,

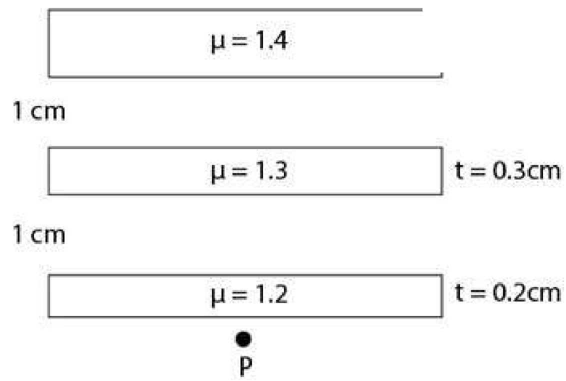
$$\Delta t_o = \left(1-\frac{1}{\mu}\right)d = \left(1-\frac{1}{1.3}\right)20 = 4.6cm$$

Total shift Δt = 5 + 4.6 = 9.6 cm

Apparent depth = 40 - (9.6) = 30.4 cm below the surface.

Solution 19:

The presence of air medium in between the sheets does not affect the shift.



Hence, the shift will be due to 3 sheets of different reactive index other than air

= (1 - 1/1.2)0.2 + (1- 1/13)(0.3) + (1- 1/14)(0.4)

= 0.2 cm above point P

Solution 20:

Total number of transparent slabs= k

Refractive index= $\mu_1, \mu_2, \mu_3, \ldots \mu_k$

Thickness of the slabs = $t_1, t_2, t_3, \ldots t_k$

Let refractive index, μ, of combination of slabs and image is formed at same place.

Then the shift:

$$\triangle t = \left(1-\frac{1}{\mu_1}\right)t_1 + \left(1-\frac{1}{\mu_2}\right)t_2 + \left(1-\frac{1}{\mu_3}\right)t_3 + \cdots .. + \left(1-\frac{1}{\mu_k}\right)t_k \quad \ldots(1)$$

and,

$$\triangle t = \left(1-\frac{1}{\mu_1}\right)(t_1 + t_2 + t_3 + \ldots\ldots\ldots\ldots\ldots + t_k) \quad \ldots(2)$$

From (1) and (2)

$$\left(1-\frac{1}{\mu_1}\right)t_1 + \left(1-\frac{1}{\mu_2}\right)t_2 + \left(1-\frac{1}{\mu_3}\right)t_3 + \ldots .. + \left(1-\frac{1}{\mu_k}\right)t_k =$$

$$\left(1-\frac{1}{\mu_1}\right)(t_1 + t_2 + t_3 + \ldots\ldots t_k)$$

$$= (t_1 + t_2 + t_3 + \ldots\ldots + t_k) - \left(\frac{t_1}{\mu_1}\right) + \left(\frac{t_2}{\mu_2}\right) + \ldots\ldots\ldots + \left(\frac{t_k}{\mu_k}\right)$$

$$\Rightarrow -\frac{1}{\mu}\sum_{i=1}^{k} t_i = -\sum_{i=1}^{k}\left(\frac{t_i}{\mu_i}\right)$$

$$\Rightarrow \mu = \frac{\sum_{i=1}^{k} t_i}{\sum_{i=1}^{k}\left(\frac{t_i}{\mu_i}\right)}$$

Solution 21: Volume of water in cylindrical vessel = 800 π cm^3

Volume of cylindrical water column after putting the glass piece inside it:

$\pi r^2 h = 800\pi + \pi r_g^2 h_1$

r_g = Radius of glass piece = 4 cm
h_1 = Height of glass piece = 8cm
and r = 6 cm

=> h = 25.7 cm

There will two shifts due to the presence of glass and water:

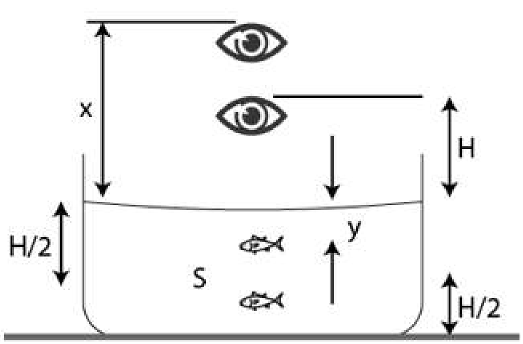
1st shift for glass and 2nd shift for water are as

$$\Delta t_1 = \left(1 - \frac{1}{\mu_g}\right) t_g = \left(1 - \frac{1}{3/2}\right) \times 8 = 2.26 \text{ cm}$$

$$\Delta t_2 = \left(1 - \frac{1}{\mu_w}\right) th_w = \left(1 - \frac{1}{4/3}\right) \times (25.7 - 8) = 4.44 \text{cm}$$

Total shift = Δt_1 + Δt_2 = (2.26+ 4.44) cm= 7.1 cm above the bottom.

Solution 22:



(a)Let x be the distance of the image of the eye formed above the surface as seen by the fish

=> H/x = [Real depth]/[Apprent depth] = $1/\mu$

=> $x = \mu H$

and [$H/2 + \mu H$] is the distance of the direct image.

Similarly, [$H/2 + (H+x)$] is the distance of image through mirror
which implies [$H/2 + (H+x)$] = $H(\mu + 3/2)$

(b) Here $\mu = (H/2)/y$ => $y = H/2\mu$
Where y is the distance of the image of fish below the surface as seen by eye.

Direct image = $H + y = H + H/2\mu = H(1 + 1/2\mu)$

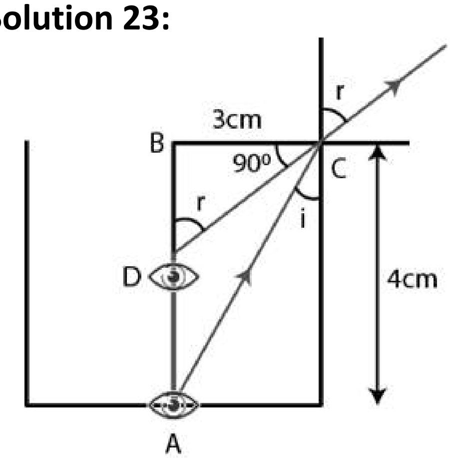
Another image of fish will be formed H/2 below the mirror. So, the real depth for that

image of fish becomes 3H/2.

Apparent depth for the surface of water = $3H/2\mu$

Therefore, the distance of image from the eye is $H + 3H/2\mu = H(1 + 3/2\mu)$

Solution 23:



By using Snell's law;

$\sin i/\sin r = 1/\mu_w = 1/1.33 = 3/4$

Where μ_w = Refractive Index of water = 1.33

=> $\sin r = (4/3) \sin i$ ...(A)

From the diagram, $\cos r = x/3$ ....(1)
and $\sin i = 3/5$

(A) => $\sin r = 4/5$

Therefore, $\cot r = 3/4$ ...(2)

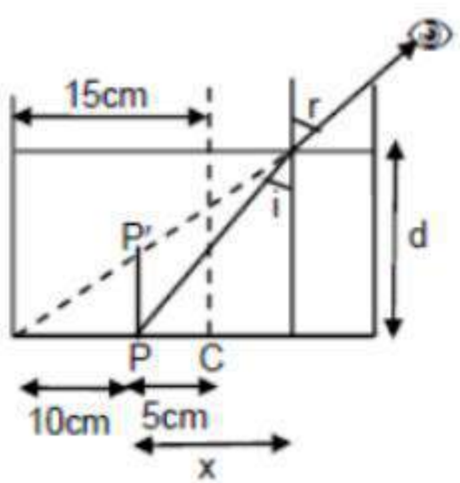
Equating (1) and (2)

$x/3 = 3/4$

=> $x = 9/4 = 2.25$ cm; apparent depth

Hence, The ratio of real depth to apparent depth = 4 : (2.25) = 1.78.

Solution 24:



For the given cylindrical vessel;
Diameter (d) = 30cm, Radius (r) = 15 cm, Height (h) = 30 cm and d = height of water

By using Snell's law;

$\sin i/\sin r = 1/\mu_w = 1/1.33 = 3/4$

Where μ_w = Refractive Index of water = 1.33

here Angle of refraction = 45^o

=> $\sin i = 3/(4\sqrt{2})$

The Point "P" will be visible when the refracted ray makes angle of 45 degree at the point of refraction.

Let x cm distance from P to X.

$\tan 45^o = (x+10)/d$

=> $d = x + 10$ ...(1)

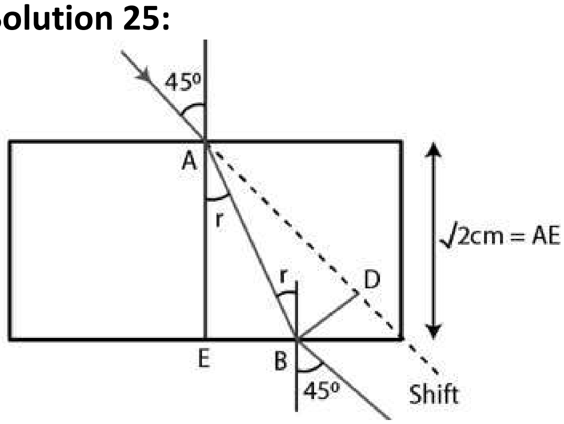
Now, $\tan i = x/d$

Using value of sin i, we have the value of $\tan i = 3/\sqrt{23}$

=> $3/\sqrt{23} = (d-10)/d$

=> d = 26.7 cm

Solution 25:



From the diagram,

$(\sin 45^o)/\sin r = 2/1$

$\Rightarrow \sin r = 1/(2\sqrt{2})$

or $r = 21^o$

Here $\theta = (45^o - 21^o) = 24^o$

and BD = shift in path = AB $\sin 24^o$

= 0.406 x AB

= $AE/\cos 21^o \times 0.406$

= 0.62 cm

Solution 26:
By Snell's law;

$\sin i/\sin r = \sin\theta_c/\sin 90^o = \mu_g/\mu_o$

Where μ_o = Refractive index of optical fibre = 1.72
and μ_g = Refractive index of glass coating = 1.50

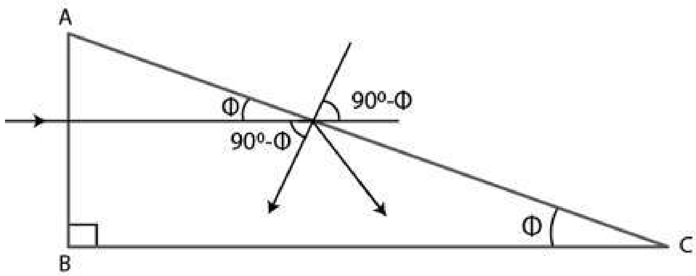
Also, θ_c = critical angle = $\sin^{-1}(1/\mu)$

$\Rightarrow \sin\theta_c/\sin 90^o = 1.50/1.72 = 75/86$

$\Rightarrow \theta_c = 75/86$

Therefore, critical angle is $\sin^{-1}(75/86)$

Solution 27:



By Snell's law;

sin i/sin r = 1/μ

Also, θ_c = critical angle = $\sin^{-1}(1/\mu)$

=> $\sin \theta_c / \sin 90^\circ = 1/\mu = 1/1.50 = 2/3$

=> $\theta_c = \sin^{-1}(2/3)$

Now, for total internal reflection, $90^\circ - \phi > \theta_c$ (from diagram)

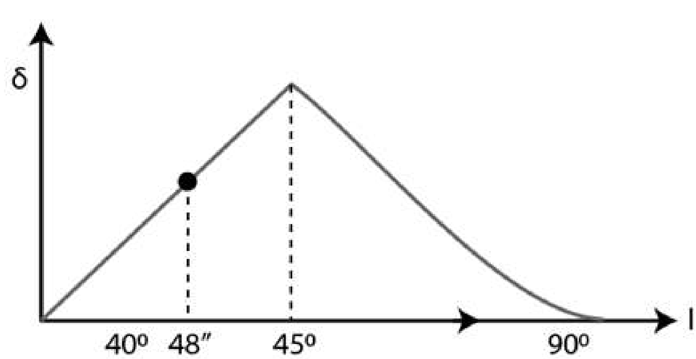
=> $\phi < 90^\circ - \theta_c = \cos^{-1}(2/3)$

Thus, largest angle ϕ for which the light ray is totally reflected at the surface,
AC = $\cos^{-1}(2/3)$

Solution 28:

From the definition of critical angle, if a refracted ray makes an angle more than 90 degrees, then total internal reflection will occur. So, the maximum angle of refraction is 90 degrees.

Solution 29:



Θ_c = critical angle = $\sin^{-1}(1/\mu)$

and $\sin \theta_c/\sin r = \mu_a/\mu_g$

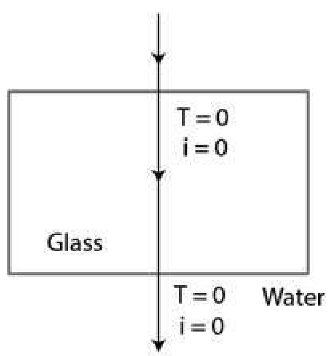
Here $\mu_a = 1.0$ and $\mu_g = 1.5$ and $r = 90^o$

=> $\sin \theta_c/\sin 90^o = 2/3$

=> $\theta_c = 40^o48''$

The angle of deviation due to refraction from glass to air increases as the angle of incidence increases from 0° to 40°48''. The angle of deviation due to total internal reflection further increases for 40°48'' to 45° and then it decreases.

Solution 30:



For two angles of incidence,

1) When light passes through the normal:
Angle of incidence = 0^o, Angle of refraction= 0^o and Angle of deviation = 0^o

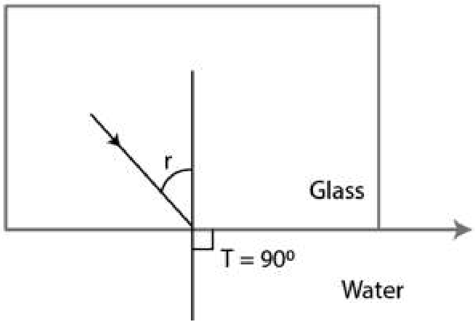
(2)When light incident at critical angle :

$\sin \theta_c/\sin r = \mu_w/\mu_g$

here $\mu_W = 1.33 = 4/3$ and $\mu_g = 1.5 = 3/2$

Therefore, $\sin \theta_c = 8/9$

and $\theta_c = \sin^{-1}(8/9)$



Also, the Angle of deviation = $90^\circ - \theta_c$

$= 90^\circ - \sin^{-1}(8/9)$

$= \cos^{-1}(8/9)$

So, If angle of incidence increases beyond critical angle, total internal reflection will occur.
Thus, the range of angle of deviation is from 0° to $\cos^{-1}(8/9)$.

Solution 31:
$\sin\theta_c = 8/9$

We know, critical angle = $\theta_c = \sin^{-1}(1/\mu) = 41.8^\circ$
Here $\mu = 1.5$

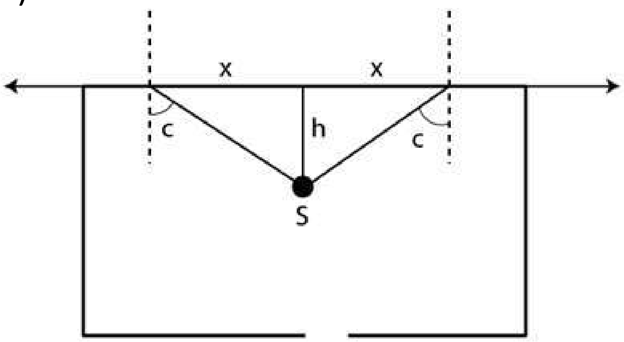
Maximum angle of deviation in refraction = $90^\circ - 41.8^\circ = 47.2^\circ$

In this case, total internal reflection must have taken place. In reflectio, deviation = $180^\circ - 2i = 90^\circ$

$\Rightarrow i = 45^\circ$

Solution 32:

(a)



Here S be the source point and x be the radius of circular area.

We know, critical angle = θ_c

So, tan θ_c = x/h

=>

$$\frac{x}{h} = \frac{\sin\theta_c}{\sqrt{1-\sin^2\theta_c}} = \frac{\frac{1}{\mu}}{\sqrt{1-\frac{1}{\mu^2}}}$$

[we know, sin θ_c = (1/μ)]

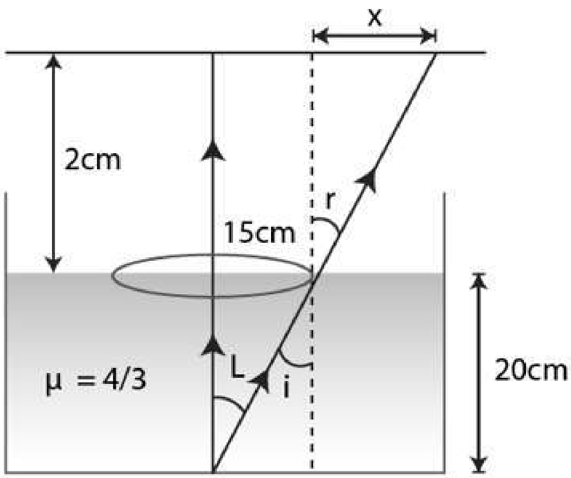
=> x = h/√(μ^2-1)

Light escapes from the circular area at a fixed desistance r on the water surface, directly above the point source S.

(b) As, sin θ_c = (1/μ)

=> θ_c = $\sin^{-1}$(1/μ)

Solution 33:



(a)Height of water container= h= 20 cm ..(given)
Ceiling of the room is 2.0 m above the water surface. ..(given)

From diagram, sin i = 15/25

By Snell's law,

sin i/sin r = 1/μ = 3/4
[Refractive index of water = 4/3 (given]

=> sin i = 4/5

Again tan r = x/2 (from diagram)

Therefore,

$$\sin r = \frac{\tan r}{\sqrt{1+\tan^2 r}} = \frac{\frac{x}{2}}{\sqrt{1+\frac{x^2}{4}}}$$

$$\Rightarrow \frac{x}{\sqrt{4+x^2}} = \frac{4}{5}$$

=> $x = 8/3$ cm

Total radius of the shadow = 8/3 + 0.15 = 2.81 m

(b) For Maximum angle of refraction: $i = \theta_c$
Let R is the maximum radius.

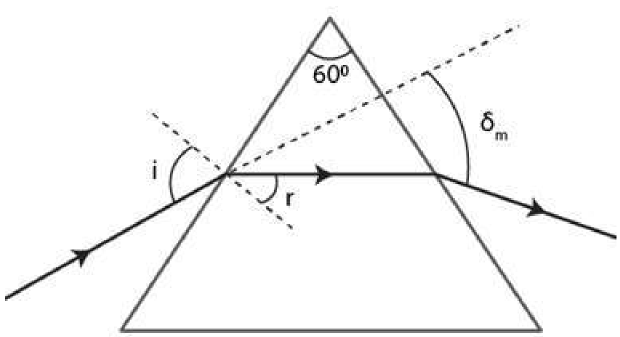
$\sin \theta_c = \sin i / \sin r$

=> $\sin \theta_c = \sin \theta_c / \sin r$

=> $R / \sqrt{[R^2+20]} = 3/4$

=> $16R^2 = 9R^2 + 9x400$
=> R = 22.67 cm

Solution 34:



Let Angle of prism = A and δ_m is the minimum deviation.

=> 1.732 x sin 30^o = sin[(Angle of minimum deviation +60^o)/2]

=> 1.732 x (1/2) = sin[δ_m +60^o)/2]

[Here 1.732 x sin 30^o = √3/2 which is equal to sin 60^o]

So, sin 60^o = sin[(δ_m+60^o)/2]

or δ_m + 60^o = 2 x 60^o

$\Rightarrow \delta_m = 60^o$

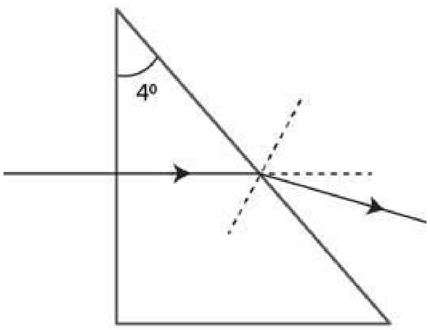
Also, $\delta_m = 2i - A$

$\Rightarrow 2i = 120^o$

or $i = 60^o$

Therefore, the minimum angle of deviation, $\delta_m = 60^o$.
Required angle of incidence = $i = 60^o$

Solution 35:



We know, Angle of minimum deviation,

$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

Find angle of deviation, δ_m

For small angle, $\sin\theta = \theta$ (approx.)]

$\Rightarrow \mu = [(A+\delta_m)/2]/[\sin(A/2)]$

Here $A = 4^0$

$\Rightarrow 1.5 = [4^0 + \delta_m]/2^o$

Given $\mu = 1.5$

$\Rightarrow \delta_m = 2^o$