PHYSICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 92

Total Marks : 9

Max. Time : 9 min.

Topic : Stokes' Law

Type of Questions		**M.M., Min.**
Comprehension ('–1' negative marking) Q.1 to Q.3	**(3 marks, 3 min.)**	**[9, 9]**

COMPREHENSION

STOKES' LAW

Stokes proved that the viscous drag (F) on a **<u>spherical body</u>** of radius r moving with relative velocity v in a fluid of viscosity η is given by $F = 6\pi\eta r v$. This force is opposite to relative velocity. This is called Stokes' law. The work done by the force is negative and it dissipates in the form of heat.

TERMINAL VELOCITY

When a body is dropped in a viscous fluid, it first accelerates and then its acceleration becomes zero and it attains a constant velocity called terminal velocity.

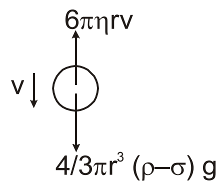
Calculation of Terminal Velocity

Let us consider a small ball, whose radius is r and density is ρ, falling freely in a liquid (or gas) whose density is σ and coefficient of viscosity η . When it attains a terminal velocity v. It is subjected to two forces :

(i) effective force acting downward

$$= V(\rho - \sigma)\, g = \frac{4}{3}\pi r^3 (\rho - \sigma) g,$$

(ii) viscous force acting upward $= 6\pi\eta rv$.



Since the ball is moving with a constant velocity v i.e., there is no acceleration in it, the net force acting on it must be zero. That is

$$6\pi\eta rv = \frac{4}{3} p r^3 (\rho - \sigma) g \quad \text{or} \quad v = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{\eta}$$

Thus, terminal velocity of the ball is directly proportional to the square of its radius

Important point

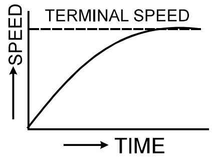
Air bubble in water always goes up. It is because density of air (ρ) is less than the density of water (σ). So the terminal velocity for air bubble is Negative, which implies that the air bubble will go up. Positive terminal velocity means the body will fall down.

Applications of Stokes' Formula

(i) In determining the Electronic Charge by Millikan's Experiment : Stokes' formula is used in Millikan's method for determining the electronic charge. In this method the formula is applied for finding out the radii of small oil-drops by measuring their terminal velocity in air.

(ii) Velocity of Rain Drops : Rain drops are formed by the condensation of water vapour on dust particles. When they fall under gravity, their motion is opposed by the viscous drag in air. As the velocity of their fall increases, the viscous drag also increases and finally becomes equal to the effective force of gravity. The drops then attain a (constant) terminal velocity which is directly proportional to the square of the radius of the drops. In the beginning the raindrops are very small in size and so they fall with such a small velocity that they appear floating in the sky as cloud. As they grow in size by further condensation, then they reach the earth with appreciable velocity,

(iii) **Parachute** : When a soldier with a parachute jumps from a flying aeroplane, he descends very slowly in air.



In the beginning the soldier falls with gravity acceleration g, but soon the acceleration goes on decreasing rapidly until in parachute is fully opened. Therefore, in the beginning the speed of the falling soldier increases somewhat rapidly but then very slowly. Due to the viscosity of air the acceleration of the soldier becomes ultimately zero and the soldier then falls with a constant terminal speed. In Fig graph is shown between the speed of the falling soldier and time.

Illus. 1. A spherical ball is moving with terminal velocity inside a liquid. Determine the relationship of rate of heat loss with the radius of ball.

Sol. Rate of heat loss = power = F × v = $6\pi\eta r v \times v = 6\pi\eta r v^2 = 6p\eta r\left[\frac{2}{9}\frac{gr^2(\rho_0-\rho_\ell)}{\eta}\right]^2$

Rate of heat loss $\alpha\ r^5$

Illus. 2. A drop of water of radius 0.0015 mm is falling in air. If the coefficient of viscosity of air is 1.8×10^{-5} kg /(m-s), what will be the terminal velocity of the drop? (density of water = 1.0×10^3 kg/m^2 and g = 9.8 N/kg.) Density of air can be neglected.

Sol. By Stokes' law , the terminal velocity of a water drop of radius r is given by

$$\nu = \frac{2}{9}\ \frac{r^2(\rho-\sigma)g}{\eta}$$

where ρ is the density of water, σ is the density of air and η the coefficient of viscosity of air. Here σ is negligible and r = 0.0015 mm = 1.5×10^{-3} mm = 1.5×10^{-6} m. Substituting the values :

$$\nu = \frac{2}{9} \times \frac{(1.5\times10^{-6})^2\times(1.0\times10^3)\times9.8}{1.8\times10^{-5}} = \mathbf{2.72 \times 10^{-4}\ m/s}$$

Now answer the follwing :

1. A ball bearing of radius of 3 mm made of iron of density 7.85 g cm^{-3} is allowed to fall through a long column of glycerine of density 1.25 g cm^{-3}. It is found to attain a terminal velocity of 2.20 cm s^{-1}. Determine the viscosity of glycerine in centipoise. (Take g = 10 m/s^2)

2. An air bubble of 1 cm radius is rising at a steady rate of 0.5 cm s^{-1} through a liquid of density 0.81 gcm^{-3}. Calculate the coefficient of viscosity of the liquid. Neglect the density of air.(Take g = 10 m/s^2)

3. A metallic sphere of radius 1.0×10^{-3} m and density 1.0×10^4 kg/m^3 enters a tank of water, after a free fall through a distance of h in the earth's gravitational field. If its velocity remains unchanged after entering water, determine the value of h. Given : coefficient of viscosity of water = 1.0×10^{-3} N-s/m^2, g = 10 m/s^2 and density of water = 1.0×10^3 kg/m^3.

Answers Key

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1. 6000 **2.** 360 poise **3.** 20 m

Hint & Solutions

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1. **Ans.** 6000

2. **Ans.** 360 poise

3. The velocity attained by the sphere in falling freely from a height h is

$\nu = \sqrt{2gh}$ ....(i)

This is the terminal velocity of the sphere in water. Hence by Stokes''s law, we have

$$\nu = \frac{2}{9}\,\frac{r^2(\rho-\sigma)g}{\eta}$$

where r is the radius of the sphere, ρ is the density of the material of the sphere

σ (= 1.0 × 10^3 kg/m^3) is the density of water and η is coefficient of viscosity of water.

$\therefore \quad \nu$ =

$$\frac{2\times(1.0\times10^{-3})^2(1.0\times10^4-1.0\times10^3)\times10}{9\times1.0\times10^{-3}}$$

= 20 m/s

from equation (i), we have

$$h = \frac{\nu^2}{2g} = \frac{20\times20}{2\times10} = 20 \text{ m}$$